Rapidity Renormalization Group
... and its Applications

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Outline

1. Motivation - Event Shapes
2. Soft Collinear Effective Field Theory (SCET)
3. η-Regulator and ν-Renormalization Group
4. Numerics and Data
5. Other Applications
6. Conclusion

Angularities

... as a subclass of event shape observable

- Berger, Kucs, Sterman, 03

$$\tau_a = \frac{1}{\sqrt{s}} \sum_{i \in X} E_i \,(\sin\theta_i)^a \,(1 - |\cos\theta_i|)^{1-a} = \frac{1}{\sqrt{s}} \sum_{i \in X} |p_{i,\perp}| \, e^{-|\eta_i|(1-a)}$$

  where rapidity $\eta = \frac{1}{2}\log\left(\frac{E+p_{\|}}{E-p_{\|}}\right)$
- Infrared safety: $-\infty < a < 2$
- Factorizability: $-\infty < a < 1$? (Hornig, Lee, Ovanesyan)
- For $a \ll 1$, $|p_{soft,\perp}|/|p_{coll.,\perp}| \sim e^{-\eta_{coll.}} \ll 1$
  E.g., a=0, thrust distribution, $\tau = 1 - T$.
- For $a \sim 1$, $|p_{soft,\perp}| \sim |p_{coll.,\perp}|$.
  E.g., a=1, jet broadening observable, B.

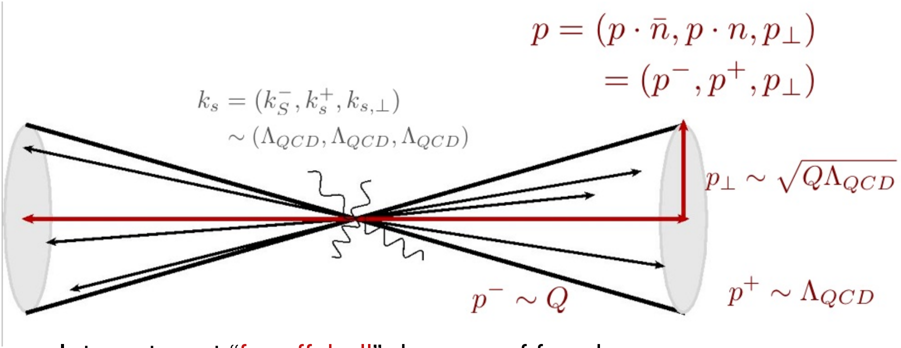
Factorization of Angularities

- Factorization Theorem in QCD
  - Collins, Soper, Sterman,...
  - Berger, Kucs, Sterman (03)
    Angularities for $a < 1$ been calculated to NLL/LO
- Factorization Theorem in Traditional SCET (SCET I)
  - Bauer, Manohar,Wise, Lee, Sterman, Becher, Schwartz, Fleming, Hoang, Mantry, Stewart, ...
  - Hornig, Lee, Ovanesyan Calculated angularities in SCET to NLL/LO for $a < 1$.
- Fail at a=1?!
  - Spurious divergences in each sector ($a \geq 1$), yet disappear after summing over sectors as long as $a \leq 2$.
  - Rapidity divergence
- Other cases with divergence in similar nature?

Soft Collinear Effective Theory (SCET I)

(Luke, Bauer, Fleming, Pirjol, Stewart)

- Describe interactions between energetic particles $E \sim Q$.
- Fluctuations, (Λ_{QCD} or other low energy scales, about light cone coordinate $n = (1, 0, 0, 1)$

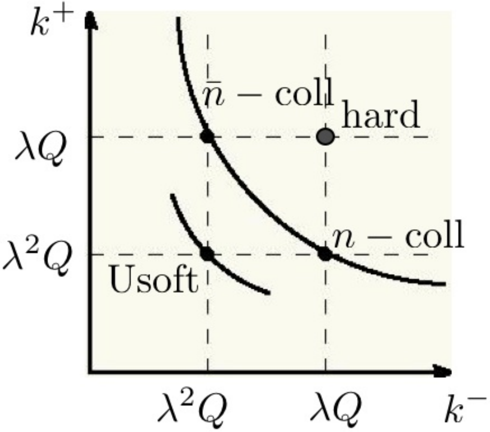


- Integrate out "far offshell" degrees of freedom.
  - soft-collinear decoupling

SCET degrees of freedom (modes)

$$p^\mu = (p^-, p^+, p^\perp); \qquad p^2 = p^+ p^- + p_\perp^2$$

- Light Cone Coordinates:
  $n = (1, \vec{n}) \sim (1, 0, 0, 1)$
- power counting parameter
  $\lambda \equiv \frac{\Lambda_{QCD}}{Q}$
- hard modes: $p^2 \sim Q^2$
  integrated out
- n-collinear
  $p^\mu \sim Q(1, \lambda^2, \lambda)$
- $\bar{n}$-collinear
  $p^\mu \sim Q(\lambda^2, 1, \lambda)$
- usoft ($p^2 \sim Q^2\lambda^4$)
  $p^\mu = Q(\lambda^2, \lambda^2, \lambda^2)$



Factorization Theorem for Angularities $\tau_{a<1}$

Bauer, Fleming, Lee, Sterman, 08

- Matching from full QCD onto SCET

$$j_i^\mu(x) = \sum_n \sum_{\tilde{p}_n,\tilde{p}_{\bar{n}}} C_{n\bar{n}}(\tilde{p}_n, \tilde{p}_{\bar{n}}; \mu) \mathcal{O}_{n,\bar{n}}(x; \tilde{p}_n, \tilde{p}_{\bar{n}}; \mu),$$

in which,

$$\mathcal{O}_{n,\bar{n}}(x; \tilde{p}_n, \tilde{p}_{\bar{n}}; \mu) = e^{i(\tilde{p}_n - \tilde{p}_{\bar{n}})} \bar{\chi}_{n,p_n}(x) Y_n(x) \Gamma_i^\mu \bar{Y}_{\bar{n}}(x) \chi_{\bar{n},p_{\bar{n}}}(x)$$

- Write the event shape distribution in SCET in a factorized form

$$\frac{d\sigma}{de} = \frac{1}{6Q^2} \sum_n |C_{n\bar{n}}(\tilde{p}_n, \tilde{p}_{\bar{n}}; \mu)|^2 \int dx \int de_n de_{\bar{n}} de_s \delta(e - e_n - e_{\bar{n}} - e_s) \frac{1}{N_C^2}$$

$$\times \mathrm{Tr}\langle 0|\chi_{n,Q}(x)_\beta \delta(e_n - \hat{e}_n) \bar{\chi}_{n,Q}(0)_\gamma |0\rangle \mathrm{Tr}\langle 0|\bar{\chi}_{n,-Q}(x)_\alpha \delta(e_{\bar{n}} - \hat{e}_{\bar{n}}) \bar{\chi}_{n,Q}(0)_\delta |0\rangle$$

$$\times \mathrm{Tr}\langle 0|\bar{Y}_{\bar{n}}^\dagger(x) Y_n^\dagger(x) \delta(e_s - \hat{e}_s) Y_n \bar{Y}_{\bar{n}}(0)|0\rangle \sum_{i=V,A} L^i \left((\bar{\Gamma}_i^\mu)\right)_{\alpha\beta} \left(\Gamma_{i\mu}\right)_{\gamma\delta}$$

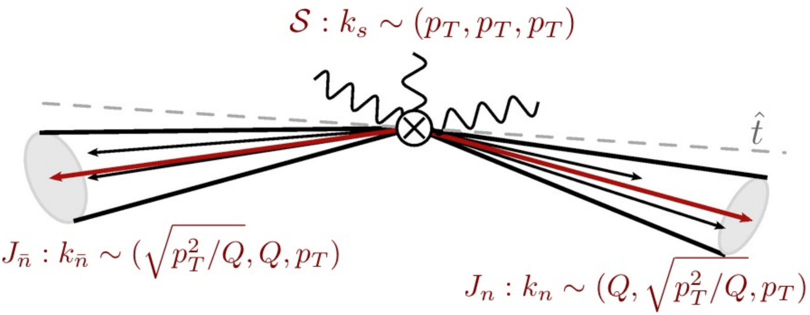
- Final result for differential event shape distribution

$$\frac{1}{\sigma_0}\frac{d\sigma}{de} = H(s;\mu) \int de_n de_{\bar{n}} de_s \delta(e - e_n - e_{\bar{n}} - e_s) J_n(e_n;\mu) J_{\bar{n}}(e_{\bar{n}};\mu) \mathcal{S}(e_s;\mu)$$

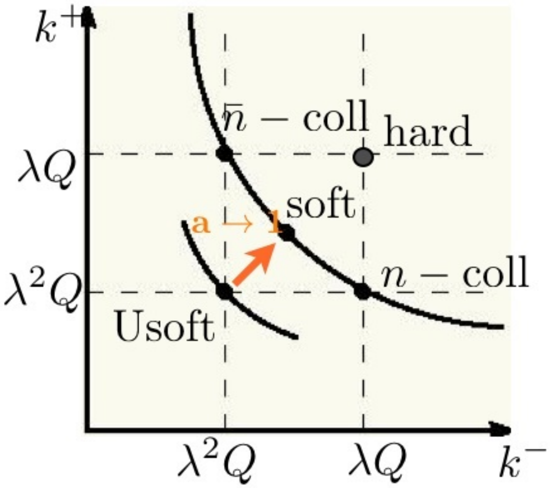
When $a \to 1$

$$\tau_a = \frac{1}{\sqrt{s}} \sum_{i \in X} |p_{i,\perp}| \, e^{-|\eta_i|(1-a)}$$

- For $a \ll 1$, $e_s \sim e_n \sim e_{\bar{n}}$, but $|p^s_\perp| \ll |p^n_\perp| \sim |p^{\bar{n}}_\perp|$ and $|\eta^s| \ll |\eta^n| \sim |\eta^{\bar{n}}|$.
  Soft radiation decoupled from the collinear radiation.
- For $a \to 1$, $\tau_{a=1} = \frac{1}{\sqrt{s}} \sum |p^i_\perp|$ is independent of the rapidity of the rapidity of each sate. For the $e \sim e_s \sim e_n$, we need $|p^s_\perp| \sim |p^n_\perp|$.



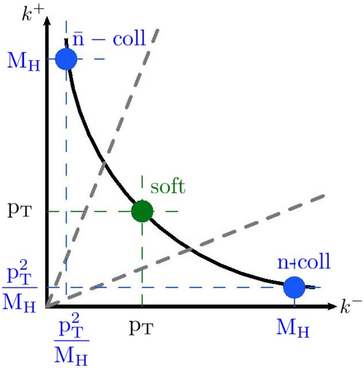
SCET Modes



k^+
$\bar{n}$ − coll
hard
λQ
soft
a → 1
$\lambda^2 Q$
n − coll
Usoft
$\lambda^2 Q$
λQ
k^-

[Not Quite] Back-to-Back Jets in SCET

- Jet Broadening Event Shape
  $B_T = \frac{1}{2}\tau_{a=1} = \frac{1}{2}\sum \frac{|\vec{k}_{i,\perp}|}{Q}$
- Demanding $B_T \ll 1$ for dijet events
- Relevant on-shell modes with $|\vec{p}_t| \sim \lambda Q$
- soft modes: $k_s \sim Q(\lambda, \lambda, \lambda)$
  collinear modes: $k_n \sim Q(1, \lambda^2, \lambda)$
  $k_{\bar{n}} \sim Q(\lambda^2, 1, \lambda)$



- Invariant massess of soft and collinear modes are on the same order $\mathcal{O}(Q^2\lambda^2)$
- Rapidity divergence arises as jets go soft or soft radiations go collinear since they are all on the same parabola.

Factorization Theorem for Jet Broadening $B_T = \frac{1}{2}\tau_{a=1}$

Define $e \equiv \tau_{a=1} = 2B_T$ from now on to simplify the notification

$$\frac{d\sigma}{de} = \sigma_0 H(s) \int de_n de_{\bar{n}} de_s \delta(e - e_n - e_{\bar{n}} - e_s) \times \int d\vec{p}_{1t} d\vec{p}_{2t} J_n(Q_-, e_n, \vec{p}_{1t}) J_{\bar{n}}(Q_+, e_{\bar{n}}, \vec{p}_{2t}) \mathcal{S}(e_s, \vec{p}_{1t}, \vec{p}_{2t}),$$

where in covariant guages $(\bar{d} \equiv 2 - 2\epsilon)$

$$J_n = \frac{2\pi\Omega_{\bar{d}}}{N_c} \langle 0|\, \bar{\chi}_n \delta(\hat{P}^- - Q^-) \delta(\hat{e} - e_n) \delta(\hat{P}_\perp + \vec{p}_{1\perp}) \frac{\bar{\not{n}}}{2} \chi_n |\, 0\rangle,$$

$$J_{\bar{n}} = \frac{2\pi\Omega_{\bar{d}}}{N_c} \langle 0|\, \frac{\not{n}}{2} \chi_{\bar{n}} \delta(\hat{P}^+ - Q^+) \delta(\hat{e} - e_{\bar{n}}) \delta(\hat{P}_\perp + \vec{p}_{2\perp}) \bar{\chi}_{\bar{n}} |\, 0\rangle,$$

$$\mathcal{S} = p_{1t}^{1-2\epsilon} p_{2t}^{1-2\epsilon} \Omega_{\bar{d}} \int \frac{d\Omega_{12}}{N_c} \times \langle 0 \mid S_n^\dagger S_{\bar{n}} \delta(\hat{e} - e_s) \delta^{\bar{d}}(\hat{P}_{n\perp} - \vec{p}_{1\perp}) \delta^{\bar{d}}(\hat{P}_{\bar{n}\perp} - \vec{p}_{2\perp}) S_{\bar{n}}^\dagger S_n |\, 0\rangle.$$

Naive Calculation with Pure Dim-Reg

- Bare jet function:

$$J_n(e_n, p_{i,t} = 0) = \frac{\alpha_s C_F}{\pi} \left(\frac{\mu^2}{Q^2 e_n^2} \right)^{\epsilon} \frac{1}{e_n} \int_0^1 \frac{1 + (1-z)^2}{z},$$

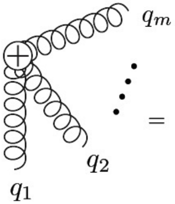
  where $z \equiv l^-/Q$, and l is the momentum of the gluon going across the cut.
- Integral ill-defined as $z \to 1$, the soft region.
- Leftover $\frac{1}{\epsilon}$ divergence multiplies non-zero e_n terms that virtual diagrams, which are always proportional to $\delta(e_n)$, cannot cancel.
- Traditional dim-reg regulating the $\vec{k}_\perp$ part of the real radiation dose not regulate the phase space integral while p_T is fixed.

New Regulator and ν-Renomalization Group

- Goal:
  - Multiplicatively Renormalizable
  - In the spirit of dimensional regularization
  - Does not introduce new dimensionful scales in the integrants, and maintains manifest power counting in the effective theory.
- η-regulator

$$
\begin{aligned}
W_n &= \left[\sum_{\text{perm}} \exp\left(\frac{-g}{\bar{n}\cdot\hat{P}} \left[\frac{|\bar{n}\cdot\hat{P}|^{-\eta}}{\nu^{-\eta}} \, \bar{n}\cdot A_{n,q}(0) \right] \right) \right] \\
S_n &= \left[\sum_{\text{perm}} \exp\left(\frac{-g}{n\cdot\hat{P}} \left[\frac{|2\hat{P}^3|^{-\eta/2}}{\nu^{-\eta/2}} \, n\cdot A_{s,q}(0) \right] \right) \right]
\end{aligned}
$$

η-Regulator



$$= \nu^{m\eta}\frac{g^m}{m!}\prod_{i=1}^{m}|\overline{n}.q_i|^{-\eta}\sum_{\text{perms}}\frac{(\overline{n}^{\mu_m}T^{A_m})\ldots(\overline{n}^{\mu_1}T^{A_1})}{[\overline{n}.q_1][\overline{n}.(q_1+q_2)]\ldots[\overline{n}.\sum_{j=1}^{m}q_j]}$$

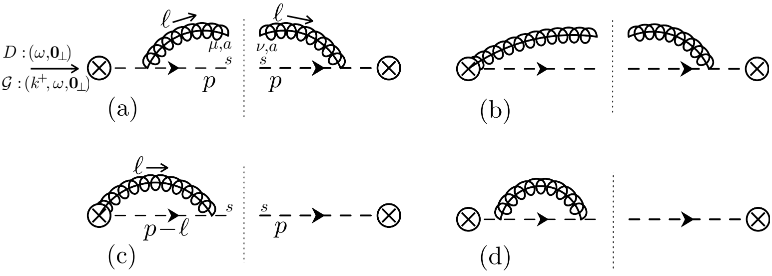
- Regulates the energy of each gluon coming off Wilson Line.
- Preserves Exponentiation Theorems.
- Preserves modes and their power counting.
- Dim-reg. style evolution equations.
- Does not hide soft functions, as analytic regulators do.
- η contribution spontaneously goes to zero when rapidity divergence is not present

Regulator Comparison

- Analytic regulator

  (QCD: Smirnov, Rakhmetov, 99; Beneke, Feldmann, 04; SCET: JC, Golf, Kelley, Manohar,07; Becher, Neubert, 10;)
  - Hides soft contributions (although still get fix-order matrix element correct after summing different sectors).
  - Each collinear sector has complicated regulator dependent and is meaningless before summing all sectors together...
    $\Rightarrow$breaks factorization
  - Does not exponentiation.
- Δ-regulator JC, Fuhrer, Hoang, Kelley, Manohar, 09
  - Introduces additional scales into integrals
  - Exponentiates after proper zero-bin subtraction
  - No known evolutions equation.
- Off the light cone Collins, Soper, 81
  - Introduces more scales into integrals
  - Proof of exponentiation straightforward
  - Introduces gauge modes that are not appropiate by strict power counting.
  - Understood evolution equation.

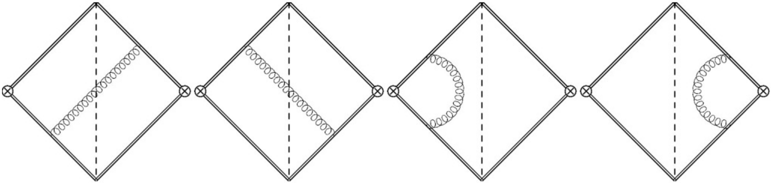
Jet Function Calculation with η-regulator



Total jet function in n-direction including real and virtual yields

$$J_n = \frac{\alpha_s C_F}{\pi}\left\{\frac{1}{2}\frac{1-\epsilon}{\Gamma(1-\epsilon)}\left(\frac{\mu^2}{Q^2}e^{\gamma_E}\right)^{\epsilon}\frac{1}{2}\left[\left(\frac{e_n}{2}\right)^{-1-2\epsilon}\right]_+^{(+\infty)} + 2\frac{\Gamma(-\eta)}{\Gamma(1-\epsilon)\Gamma(2-\eta)}\left(\frac{\mu^2}{Q^2}e^{\gamma_E}\right)^{\epsilon}\left(\frac{\nu}{Q}\right)^{\eta}\frac{1}{2}\left[\left(\frac{e_n}{2}\right)^{-1-2\epsilon}\right]_+^{(+\infty)}\right\}$$

Soft Function Calculation with η-regulator



$$\mathcal{S} = \frac{\alpha_s(\mu) C_F}{\pi} \frac{\sqrt{\pi}\,\Gamma(\eta) e^{\epsilon\gamma_E}}{2^{2\eta}\Gamma(1/2+\eta)\Gamma(1-\epsilon)} \left(\frac{\mu}{Q}\right)^{2\epsilon} \left(\frac{\nu}{Q}\right)^{2\eta} \left[\frac{1}{e_s^{1+2\epsilon+2\eta}}\right]_+^{(+\infty)}$$
$$\times \Big\{ \delta(p_{1t} - Qe_s)\delta(p_{2t}) + \delta(p_{1t})\delta(p_{2t} - Qe_s) \Big\}.$$

$$V_s(e, \vec{p}, \mu, \nu, \nu_0) = (4\pi)\theta(e)\theta(eQ - p_t) \frac{\zeta e^{\gamma_E \zeta}}{\Gamma(-\zeta)} \frac{Q}{\mu (eQ)^2} \left(\frac{\mu e Q}{e^2 Q^2 - p_t^2}\right)^{1+\zeta}$$

Jet and Soft Function Contributions to the cross-section

For integrated cross-section with a cut on jet broadening, define

$$\Sigma(e_c \ll 1) \equiv \int_0^{e_c} de \frac{d\sigma}{de}.$$

We have

- Contribution from jet functions

$$\frac{1}{\sigma_0}\Sigma_{jet}(e_c) = -\frac{\alpha_s C_F}{2\pi} \ln\left(\frac{\sqrt{s}e_c}{2\mu}\right)\left[3 + 4\ln\left(\frac{\nu}{Q^\pm}\right)\right]$$

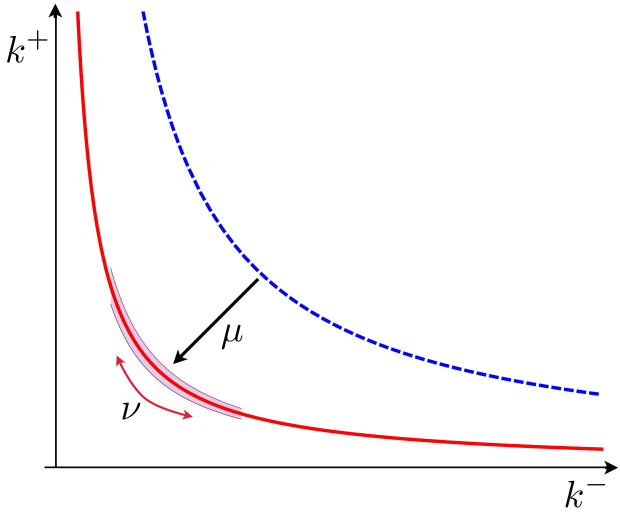
  - To minimize the log, the natural scales for jet functions are: $\mu_J = \frac{1}{2}\sqrt{s}e_c$, and $\nu_J = Q^\pm$.

- Contribution from soft function

$$\frac{1}{\sigma_0}\Sigma_{soft}(e_c) = \frac{\alpha_s C_F}{\pi}\left[-2\ln^2\left(\frac{\sqrt{s}e_c}{2\mu}\right) + 4\ln\frac{\nu}{\mu}\ln\left(\frac{\sqrt{s}e_c}{2\mu}\right)\right]$$

  - Natural scales for soft functions are: $\nu_s = \mu_s = \frac{1}{2}\sqrt{s}e_c$

Running Strategy: 2-Parameter RG



Renormalization Group Equations

- η-divergences and ν-anomalous dimensions cancels when we sum up the contributions from the jet and soft fuctions.
- Individual J and S are multiplicatively renormalizable.
- η-divergence are absorbed in the renormalization constants, $Z_{J,S}$, such that

$$J_n^{(0)} J_{\bar{n}}^{(0)} S^{(0)} = \left[Z_{J_n}(\mu,\nu) J_n^{\mathrm{R}}(\mu,\nu)\right] \left[Z_{J_{\bar{n}}}(\mu,\nu) J_{\bar{n}}^{\mathrm{R}}(\mu,\nu)\right] \left[Z_S(\mu,\nu) S^{\mathrm{R}}(\mu,\nu)\right],$$

where

$$Z_{J_n}(\mu,\nu) Z_{J_{\bar{n}}}(\mu,\nu) Z_S(\mu,\nu) = Z_H^{-1}(\mu).$$

Renormalization Group Equations

- The rapidity divergences for the jet and soft functions introduce a new set of anomalous dimension $(\gamma_J^\nu, \gamma_S^\nu)$:

$$\nu \frac{\mathrm{d}}{\mathrm{d}\nu} \mathcal{S}^{\mathrm{R}}(\mu, \nu) = \gamma_S^\nu \, \mathcal{S}^{\mathrm{R}}(\mu, \nu), \; \nu \frac{\mathrm{d}}{\mathrm{d}\nu} J_n^{\mathrm{R}}(\mu, \nu) = \gamma_J^\nu \, J_n^{\mathrm{R}}(\mu, \nu)$$

  Just like the traditional μ anomalous dimension:

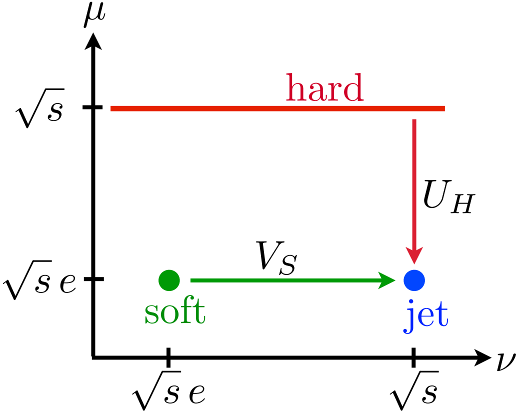
$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} \mathcal{S}^{\mathrm{R}}(\mu, \nu) = \gamma_S^\mu \, \mathcal{S}^{\mathrm{R}}(\mu, \nu), \text{ and } \mu \frac{\mathrm{d}}{\mathrm{d}\mu} J_n^{\mathrm{R}}(\mu, \nu) = \gamma_J^\mu \, J_n^{\mathrm{R}}(\mu, \nu)$$

- Since the cross-section is invariant under μ and ν variation, and that the hard function itself is free from rapidity divergence (and therefore $\gamma_H^\nu = 0$), we must have the relations

$$\gamma_H^\mu + \gamma_{J_n}^\mu + \gamma_{J_{\bar{n}}}^\mu + \gamma_S^\mu = 0, \text{ and } \gamma_{J_n}^\nu + \gamma_{J_{\bar{n}}}^\nu + \gamma_S^\nu = 0.$$

Running Strategy

- Natural scales:
  - hard function: independent of ν, $\mu_H = \sqrt{s}$
  - soft function $(\nu_S, \mu_S) = (\sqrt{s}\,e, \sqrt{s}\,e)$
  - jet functions $(\nu_J, \mu_J) = (\sqrt{s}, \sqrt{s}\,e)$
- Running
  - In μ:
    Evolve hard function from high scale $\mu_H = \sqrt{s}$ to common low scale μ_J=μ_S=$\sqrt{s}e$
  - In ν:
    Evolve soft function from $\nu_S = \sqrt{s}e$ to jet scale $\nu_J = \sqrt{s}$



Solution to the μ-RGE for the hard function

$$H(s,\mu) = U_H(s;\mu_H,\mu)H(s;\mu_H)$$

with

$$U_H(s;\mu_H,\mu) = \left| e^{K_H(\mu_H,\mu)} \left(\frac{-s-i0}{\mu_H^2}\right)^{\eta_H(\mu_H,\mu)} \right|^2$$

Solution to the ν-RGE for the soft function

$$\mathcal{S}(\mu,\nu) = V_s\left(\mu,\frac{\nu}{\nu_s}\right) \otimes \mathcal{S}(\mu,\nu_S)$$

with

$$V_s(e,\vec{p},\mu,\nu) = (4\pi)\theta(e)\theta(eQ-p_t)\frac{-\omega_S e^{-\gamma_E\omega_S}}{\Gamma(-\zeta)}\frac{Q}{\mu(eQ)^2}\left(\frac{\mu eQ}{e^2Q^2-p^2}\right)^{1-\omega_S}$$

and $\omega_s\left(\mu,\frac{\nu}{\nu_s}\right) = 2\Gamma_{cusp}[\alpha_s(\mu)]\log\frac{\nu}{\nu_s}$.

NLO singular cross-section ($e \equiv 2B_T$)

$$\frac{1}{\sigma_0}\frac{d\sigma}{de} = \frac{1}{e}\frac{\alpha_s(\mu)C_F}{\pi}(-3 - 4\log\frac{e}{2})$$

Resummed cross-section up to NLL

$$\frac{1}{\sigma_0}\frac{d\sigma}{de} = \frac{1}{e}\frac{U_H(s,\mu_H,\mu)}{\Gamma(2\omega_S)4e^{2\gamma_E\omega_S}}\left(\frac{eQ}{\mu}\right)^{2\omega_S}\left[1 - {}_2F_1(1,1;1+\omega_S;-1)\right]^2$$

Comparison with literature...

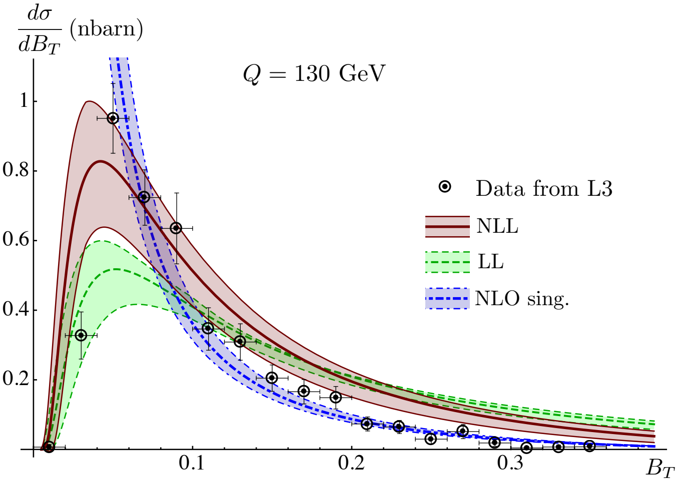
QCD calculations with resummation in the literature:

1. Catani, Turnock and Webber (1992) $\Rightarrow$ Correct up to NLL. Agree with our NLL result in ref.
2. Dokshitzer, Lucenti, Marchesini and Salam (1998) $\Rightarrow$ Corrections to previous result in [1] at NLL' or NNLL

SCET resummation for B_T after our letter...

3. Becher, Bell and Neubert (2011)
   $\Rightarrow$ Claim to agree with [2] for all logs at α_s^2 order.
   $\Rightarrow$ Factorization broken

Comparison with data



Other Applications?

Rapidity divergences do not only appear when observing Jet Broadening...

- p_T spectrum for Higgs/Drell-Yan Production
  - JC's talk in the Higgs session
- TMD-PDF, Generalized Parton Distribution
- Electroweak corrections to high energy process
- ...
- In general, processes or observables involving collinear and soft mode with similar transverse momentum or off-shellness.

Conclusion

- When measuring transverse momentum related observables...
  - Soft contributions are important
  - Uncanceled divergences remain in each sector, rapidity divergence.
  - New kind of logarithms to resum, yet related to the cups angle (the high scale).
- There are other cases with rapidity divergence such as, higgs p_T distribution, and electroweak corrections to LHC processes.
- Rapidity RG making use of the η-regulator provides controllable form to divergences, and a way to resum the log systematically.