LORENTZ NONINVARIANT NEUTRINO OSCILLATIONS WITHOUT NEUTRINO MASSES†

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- Review of neutrino oscillations due to neutrino mass
- The Standard Model Extension (SME): Lorentz and CPT violation in neutrinos
- Bicycle model
- General direction-independent Lorentz-violating (LV) models with 3 massless neutrinos
- Conclusions

† With V. Barger, J. Liao and D. Marfatia, arXiv:1106.6023 [hep-ph]

Neutrino oscillations due to neutrino mass

- Mass eigenstates propagate in time: $e^{-iE_i t/\hbar} \longrightarrow e^{-iEL}$ for relativistic ν's ($\hbar = c = 1$)
- $E = \sqrt{p^2 + m^2} \simeq p + \frac{m^2}{2p} + \ldots \Longrightarrow$ different mass eigenstates acquire different phases
- Mass eigenstates (ν_i) $\neq$ flavor eigenstates (ν_α) due to mixing

$$\nu_\alpha = \sum_j U_{\alpha j} \nu_j \qquad (U \text{ is } 3 \times 3, \text{ unitary})$$

- ν's created in flavor eigenstates, $\nu_\alpha(0) = \sum_j U_{\alpha j} \nu_j$, so after time t

$$\nu_\alpha(t) = \sum_j U_{\alpha_j} e^{-iE_j L} \nu_j = \sum_\beta \left(\sum_j U_{\alpha j} e^{-iE_j L} U^\dagger_{j\beta} \right) \nu_\beta$$

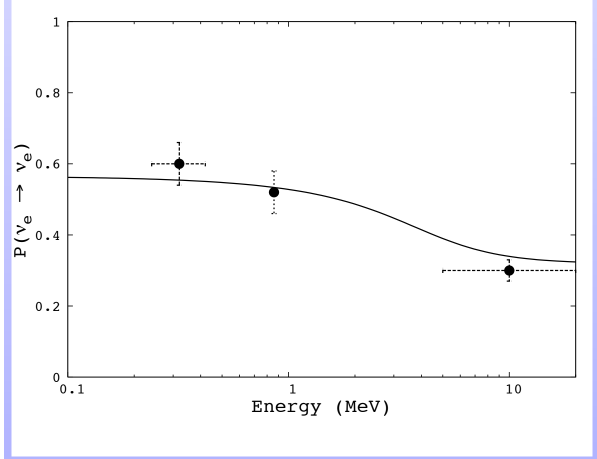
- Oscillation probability is $|\langle \nu_\beta | \nu_\alpha \rangle|^2$, or

$$P(\nu_\alpha \to \nu_\beta) \;=\; \delta_{\alpha\beta} - \sum_{j<k} \left[4 \,\mathrm{Re}(U_{\alpha j} U^*_{\alpha k} U^*_{\beta j} U_{\beta k}) \sin^2(\frac{1}{2}\Delta_{jk} L) - 2 \,\mathrm{Im}(U_{\alpha j} U^*_{\alpha k} U^*_{\beta j} U_{\beta k}) \sin(\Delta_{jk} L) \right]$$

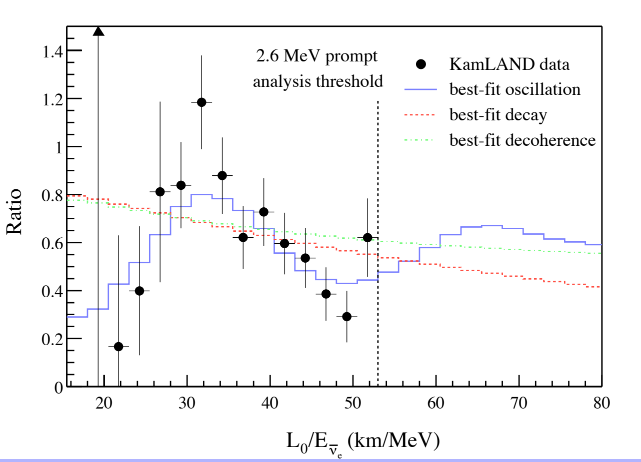
$$\Delta_{jk} \;\equiv\; E_j - E_k = \frac{\delta m^2_{jk}}{2E}, \qquad \delta m^2_{jk} \equiv m^2_j - m^2_k$$

- U described by 3 mixing angles (θ_{12}, θ_{13}, θ_{23}) and a CP-violating phase (δ)
- Two independent δm^2 (δm^2_{21}, δm^2_{31})
- $\delta m^2_{21} \simeq 7.6 \times 10^{-5}$ eV^2 and $\sin^2 2\theta_{12} \simeq 0.87$ fit solar $\nu_e \to \nu_e$ and KamLAND reactor $\bar{\nu}_e \to \bar{\nu}_e$ data
- $\delta m^2_{31} \simeq 2.4 \times 10^{-3}$ eV^2 and $\sin^2 2\theta_{13} \simeq 1.0$ fit atmospheric and K2K, MINOS long-baseline data (predominantly $\nu_\mu \to \nu_\tau$)

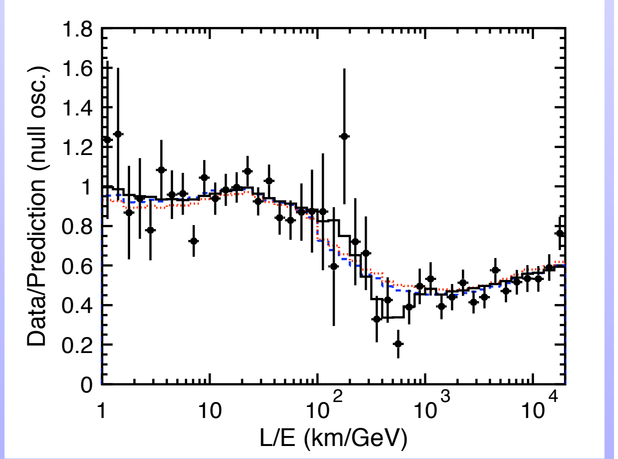
Solar



KamLAND



Atmospheric



- ν mass provides only complete description of all data (except LSND/MiniBooNE)
- $\Longrightarrow$ strong evidence for ν mass

Standard Model Extension (SME) (Colladay & Kostelecky)

- *Particle* Lorentz transformations that leave background vevs unchanged may be affected
- SME: all Lorentz symmetry-breaking terms that preserve $SU(3) \times SU(2) \times U(1)$
- Corresponding Hamiltonian for ν_L propagation in SME:

$$(h_{eff})_{ij} = |\vec{p}|\ \delta_{ij} + \frac{(m^2)_{ij}}{2|\vec{p}|} + \frac{1}{|\vec{p}|}\left[a^\mu p_\mu - c^{\mu\nu} p_\mu p_\nu\right]_{ij}, \qquad i,j = e,\mu,\tau$$

- For $\bar{\nu}$, $a \to -a$ (CPT violation)
- Choose direction-independent terms ($\mu = \nu = 0$); for relativistic ν's, $|\vec{p}| \simeq E$ and $p^\mu \simeq (E, -E\hat{p})$

$$(h_{eff})_{ij} = |\vec{p}|\ \delta_{ij} + \frac{(m^2)_{ij}}{2E} + a_{ij} + c_{ij}E$$

- a terms are energy-independent (a has dimensions of E)
- c terms proportional to E (c dimensionless)

Bicycle Model (Kostelecky & Mewes)

$$h_{eff} = \begin{pmatrix} -2cE & \frac{1}{\sqrt{2}}a & \frac{1}{\sqrt{2}}a \\ \frac{1}{\sqrt{2}}a & 0 & 0 \\ \frac{1}{\sqrt{2}}a & 0 & 0 \end{pmatrix},$$

- Eigenvalues $\lambda_i = 0, -cE \pm \sqrt{(cE)^2 + a^2}$ $\qquad \lambda_i - \lambda_j = \Delta_{ij} = \frac{1}{2E}(\delta m^2_{eff})_{ij}$

- must reproduce $1/E$ behavior at high E from cE, a terms

  In bicycle model there is a see-saw mechanism in large E limit ($cE \gg a$)

  $\Delta_{32} \simeq a^2/(2cE)$ has correct E dependence!

  $\delta m^2_{eff} = a^2/c$ for atmospheric and long-baseline (LBL) ν's

- General bicycle model (BMW)

  Allow direction dependent and/or direction independent terms

  Add a_{ee} term to adjust solar osc. prob. at low E

  Does not fit all data simultaneously

- Tandem model (Katori, Kostelecky & Tayloe)

  Lorentz invariance violation *and* neutrino mass

  Solar and KamLAND data fit by neutrino mass terms

  Atmospheric and LBL data fit by LV terms (like bicycle model)

- Puma model (Diaz & Kostelecky) see next talk

General Direction-Independent LV Models with 3 Massless ν's

- 16 independent parameters: $c_{ij}E + a_{ij}$ with c and a traceless ($i, j =$ flavors)
- Rather than a random sampling of parameter space, we searched for structures that lead to $1/E$ behavior of at least one Δ_{ij} at high energy (to mimic see-saw mechanism of bicycle model)
- Method:

  Require $c_{ij}E \gg a_{ij}$ at high E (≥ 1 GeV)

  For given structure, expand eigenvalues of h_{eff} in powers of E

  $$\lambda = \alpha_1 E + \alpha_0 + \alpha_{-1} E^{-1} + ...$$

- Oscillation argument is $\Delta_{jk} = \lambda_j - \lambda_k$

  $\Longrightarrow$ $1/E$ behavior requires degeneracy at leading order (E^1) *and* next-to-leading order (E^0)

- α_1 values determined by eigenvalues of c matrix – degeneracy at leading order $\Longrightarrow$ constraints on c_{ij}
- Constraints on a_{ij} from requiring degeneracy at next-to-leading order
- Some constraints are "natural", others require fine tuning

- Classify models by c_{ij} structure

  3 real c_{ii}, 3 complex c_{ij}

  $\Longrightarrow \qquad 2^6 = 64$ textures

  19 non-equivalent classes after allowing for flavor permutations

- Examples:

$$\text{2B} \quad D_i O_{ij} \Longrightarrow \begin{pmatrix} c_{ee} & c_{e\mu} & 0 \\ c_{e\mu} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\text{2C} \quad D_i O_{jk} \Longrightarrow \begin{pmatrix} c_{ee} & 0 & 0 \\ 0 & 0 & c_{\mu\tau} \\ 0 & c_{\mu\tau} & 0 \end{pmatrix}$$

Number of nonzero c_L	Subclass	Structure	Number of flavor permutations
0	0	–	1
1	1A	D_i	3
	1B	O_{ij}	3
2	2A	$D_i D_j$	3
	2B	$D_i O_{ij}$	6
	2C	$D_i O_{jk}$	3
	2D	$O_{ij} O_{ik}$	3
3	3A	$D_i D_j D_k$	1
	3B	$D_i D_j O_{ij}$	3
	3C	$D_i D_j O_{ik}$	6
	3D	$D_i O_{ij} O_{ik}$	3
	3E	$D_j O_{ij} O_{ik}$	6
	3F	$O_{ij} O_{ik} O_{jk}$	1
4	4A	$D_i D_j D_k O_{ij}$	3
	4B	$D_i D_j O_{ij} O_{ik}$	6
	4C	$D_i D_j O_{ik} O_{jk}$	3
	4D	$D_i O_{ij} O_{ik} O_{jk}$	3
5	5A	$D_i D_j D_k O_{ij} O_{ik}$	3
	5B	$D_i D_j O_{ij} O_{ik} O_{jk}$	3
6	6	$D_i D_j D_k O_{ij} O_{ik} O_{jk}$	1

Class 1A

$$h_{eff} = \begin{pmatrix} c_{ee}E + a_{ee} & a_{e\mu} & a_{e\tau} \\ a_{e\mu}^* & a_{\mu\mu} & a_{\mu\tau} \\ a_{e\tau}^* & a_{\mu\tau}^* & a_{\tau\tau} \end{pmatrix}$$

- $\lambda_1 \approx cE + a_{ee}, \qquad \lambda_2, \lambda_3 \approx \frac{1}{2}\left[a_{\mu\mu} + a_{\tau\tau} \mp \sqrt{(a_{\mu\mu} - a_{\tau\tau})^2 + 4|a_{\mu\tau}|^2}\right]$ to order E^0
- Degeneracy to order E^0 requires $a_{\mu\mu} = a_{\tau\tau}$ and $a_{\mu\tau} = 0$
- Reduces to general bicycle model

Class 1B

$$h_{eff} = \begin{pmatrix} a_{ee} & c_{e\mu}E + a_{e\mu} & a_{e\tau} \\ c_{e\mu}E + a_{e\mu}^* & a_{\mu\mu} & a_{\mu\tau} \\ a_{e\tau}^* & a_{\mu\tau}^* & a_{\tau\tau} \end{pmatrix}$$

- $\lambda_1, \lambda_3 \approx \mp cE, \qquad \lambda_2 = 0$ to order E^0
- Degeneracy to order E^0 not possible

- Can subtract off piece proportional to identity – does not affect oscillations
- Many cases reduce to others when degneracy constraints applied
- Only 1A, 2C, 3B, 3F, 4D, 5B give $1/E$ dependence at high E

Class 2C

$$h_{eff} = \begin{pmatrix} a_{ee} & a_{e\mu} & c_{e\tau}E + a_{e\tau} \\ a^*_{e\mu} & c_{\mu\mu}E & a_{\mu\tau} \\ c_{e\tau}E + a^*_{e\tau} & a^*_{\mu\tau} & a_{\tau\tau} \end{pmatrix}$$

- Atmospheric and LBL neutrinos have

$$P(\nu_\mu \to \nu_e) = P(\nu_\mu \to \nu_\tau) = \frac{1}{2}\sin^2 2\theta \sin^2\left(\frac{\delta m^2_{eff} L}{2E}\right)$$

- Ruled out since ν_μ oscillates predominantly to ν_τ

Class 3F

$$h_{eff} = \begin{pmatrix} a_{ee} & c_{e\mu}E + a_{e\mu} & c_{e\tau}E + a_{e\tau} \\ c_{e\mu}E + a^*_{e\mu} & a_{\mu\mu} & c_{\mu\tau}E + a_{\mu\tau} \\ c^*_{e\tau}E + a^*_{e\tau} & c_{\mu\tau}E + a^*_{\mu\tau} & a_{\tau\tau} \end{pmatrix}$$

- Atmospheric and LBL neutrinos have

$$P(\nu_\mu \to \nu_\mu) = \frac{5}{9} - 4|U_{\mu 2}|^2 \left(\frac{2}{3} - |U_{\mu 2}|^2\right) \sin^2\left(\frac{\delta m^2_{eff} L}{2E}\right)$$

- Ruled out since downward atmospheric ν_μ are not depleted

Only three classes also have correct oscillation amplitude for atmospheric and LBL ν's (3B, 4D, 5B)

Class 3B

$$h_{eff} = \begin{pmatrix} c_{ee}E + a_{ee} & a_{e\mu} & c_{e\tau}E + a_{e\tau} \\ a^*_{e\mu} & 0 & a_{\mu\tau} \\ c_{e\tau}E + a^*_{e\tau} & a^*_{\mu\tau} & c_{\tau\tau}E + a_{\tau\tau} \end{pmatrix}$$

- Degeneracy at order E requires $c_{\tau\tau} = r c_{e\tau} = r^2 c_{ee}$

$$\begin{aligned} P(\nu_\mu \to \nu_\mu) &= 1 - \sin^2 2\theta \sin^2\left(\frac{\delta m^2_{eff} L}{2E}\right) \\ P(\nu_\mu \to \nu_e) &= \sin^2\phi \sin^2 2\theta \sin^2\left(\frac{\delta m^2_{eff} L}{2E}\right) \\ P(\nu_e \to \nu_e) &\approx 1 - \sin^2 2\phi \sin^2\left(\frac{(1+r^2)cEL}{2}\right) - \sin^4\phi \sin^2 2\theta \sin^2\left(\frac{\delta m^2_{eff} L}{2E}\right) \\ \tan\phi &\equiv r \end{aligned}$$

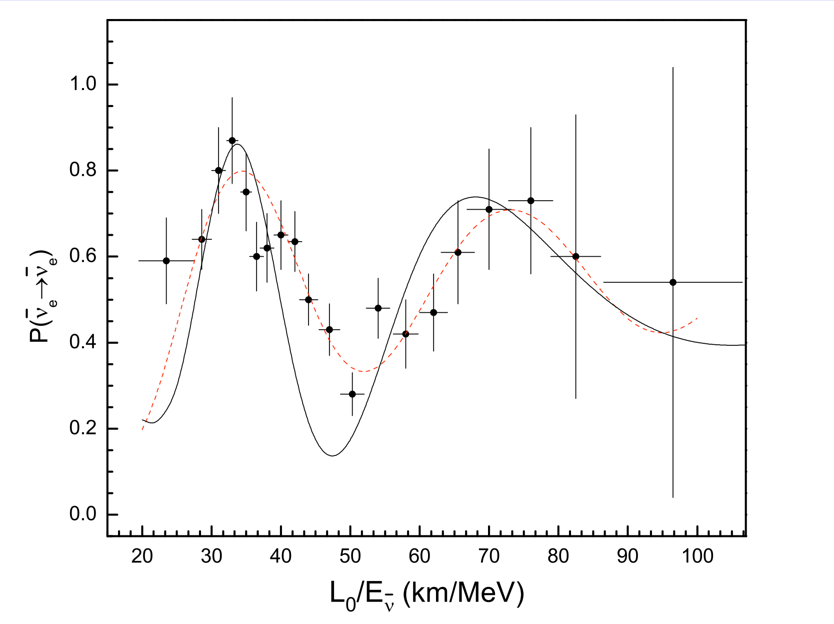
- $\theta \approx \pi/4$
- Lack of large $\nu_\mu \to \nu_e$ oscillation in K2K, MINOS and T2K implies small ϕ ($r < 0.43$)

- At lower energies (solar, KamLAND) large E limit does not apply
- Try to fit to KamLAND and solar (including matter effect)

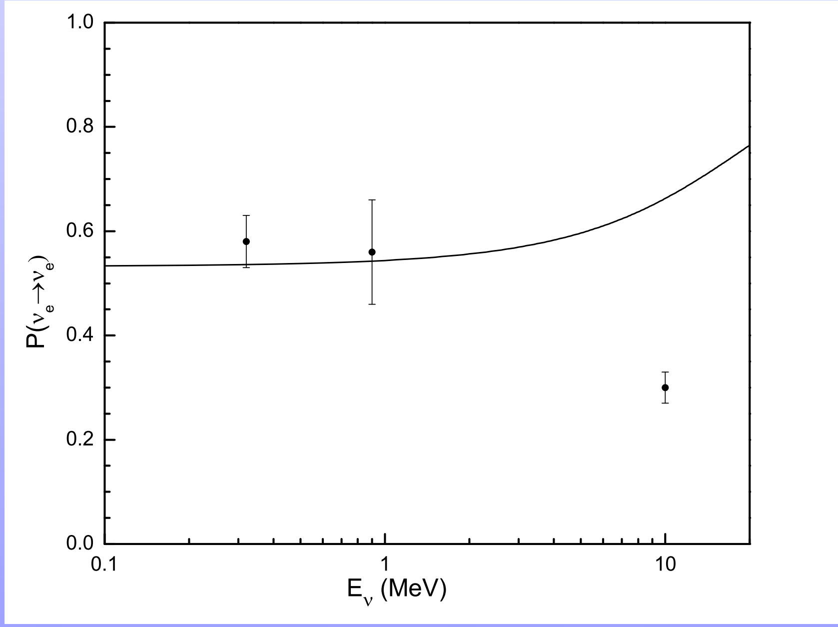
Fit to KamLAND not as good as with neutrino mass

Resulting oscillation probability for solar ν's too large at high energy

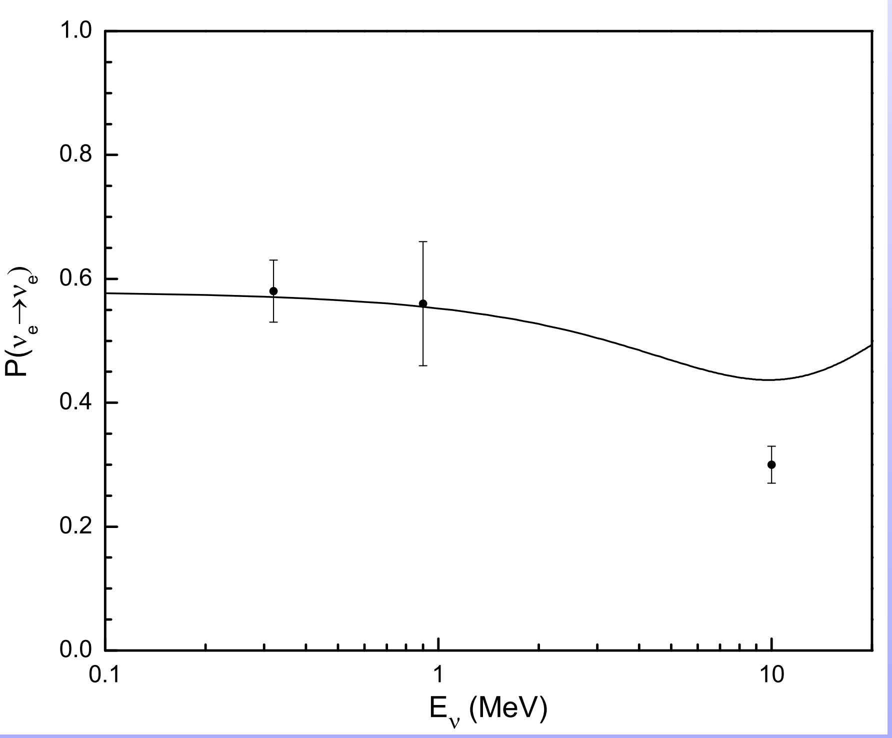
KamLAND



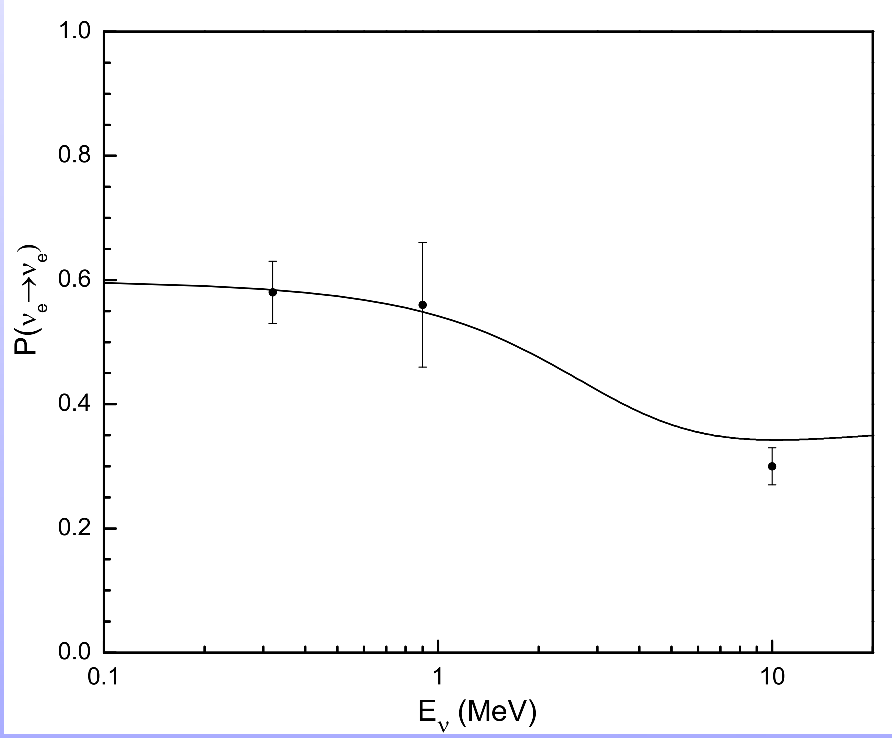
Solar osc. prob. using KamLAND parameters



Best fit to solar data alone still
not good at high energies



Can only fit solar data with $r \geq 1$
$(\sin^2 2\theta_{13}^{eff} \geq 0.50)$

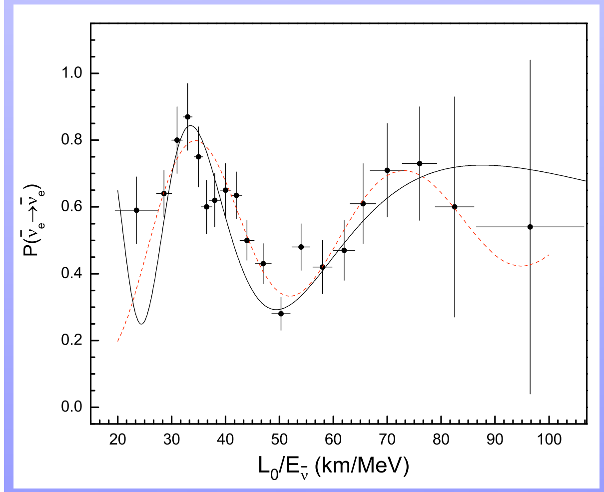


Class 4D

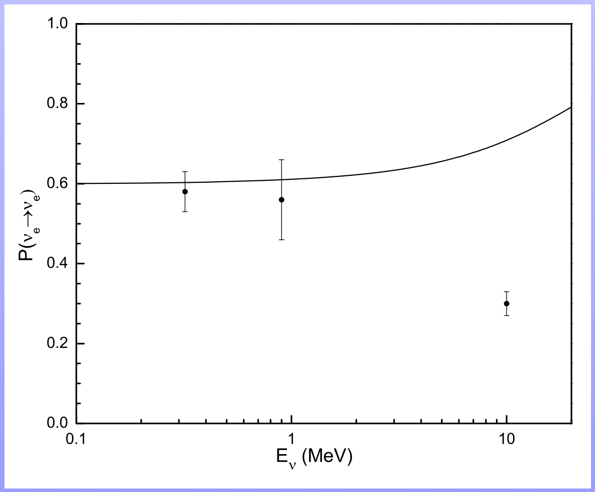
$$h_{eff} = \begin{pmatrix} c_{ee}E + a_{ee} & c_{e\mu}E + a_{e\mu} & c_{e\tau}E + a_{e\tau} \\ c_{e\mu}E + a^*_{e\mu} & a_{\mu\mu} & c_{\mu\tau}E + a_{\mu\tau} \\ c_{e\tau}E + a^*_{e\tau} & c_{\mu\tau}E + a^*_{\mu\tau} & a_{\tau\tau} \end{pmatrix}$$

- After degeneracy imposed at order E and rotation in μ-τ sector, h_{eff} has same form as Class 3B

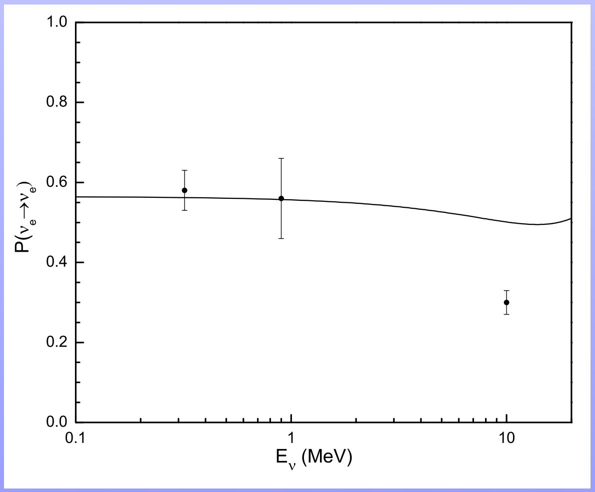
KamLAND fit



Solar osc. prob. using KamLAND parameters



Solar fit

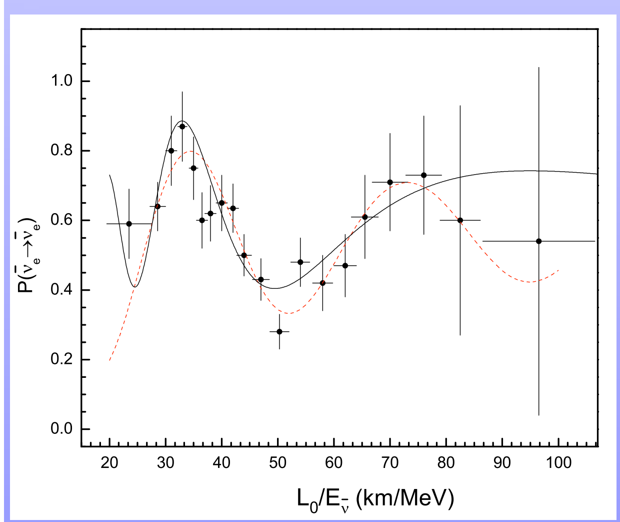


Class 5B

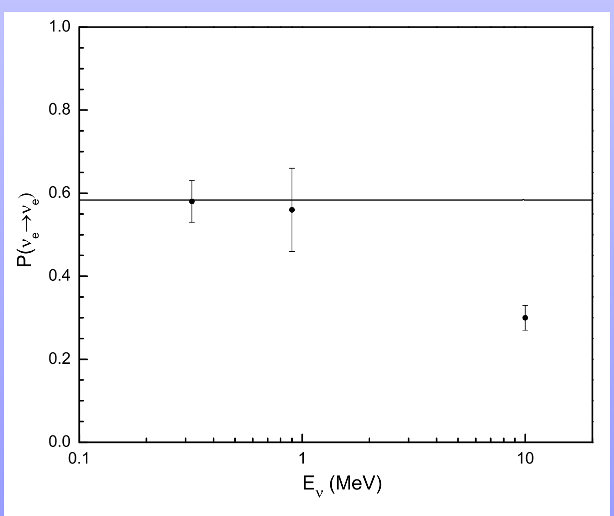
$$h_{eff} = \begin{pmatrix} c_{ee}E + a_{ee} & c_{e\mu}E + a_{e\mu} & c_{e\tau}E + a_{e\tau} \\ c_{e\mu}E + a^*_{e\mu} & 0 & c_{\mu\tau}E + a_{\mu\tau} \\ c_{e\tau}E + a^*_{e\tau} & c_{\mu\tau}E + a^*_{\mu\tau} & c_{\tau\tau}E + a_{\tau\tau} \end{pmatrix}$$

- Nearly maximal $\nu_\mu \to \nu_\tau$, small $\nu_\mu \to \nu_e \Longrightarrow c^2_{\mu\tau} \ll c^2_{e\mu} \ll c^2_{e\tau}$

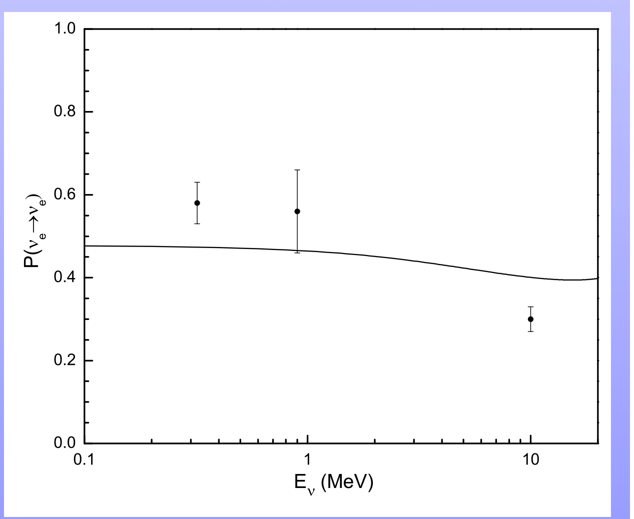
KamLAND fit



Solar osc. prob. using KamLAND parameters



Solar fit



Conclusions

- SME allows new terms in effective Hamiltonian for ν propagation that have different energy dependence from ordinary oscillations (cE, a vs. m^2/E)

- A number of cases without neutrino mass can reproduce $1/E$ dependence at high E for atmospheric and LBL neutrinos

- None can also successfully fit solar and KamLAND data

- Standard Model with just neutrino masses is very robust (modulo LSND/MiniBooNE)

- Apparently neutrino mass is needed to explain some (if not all) oscillation data