LORENTZ NONINVARIANT NEUTRINO OSCILLATIONS

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DPF '11
11Aug2011

• Review of neutrino oscillations due to neutrino mass
• The Standard Model Extension (SME): Lorentz and $CPT$ violation in neutrinos
• Bicycle model
• General direction-independent Lorentz-violating (LV) models with 3 massless neutrinos
• Conclusions
Neutrino oscillations due to neutrino mass

- Mass eigenstates propagate in time: $e^{-iEt/\hbar} \rightarrow e^{-iEL}$ for relativistic $\nu$'s ($\hbar = c = 1$)
- $E = \sqrt{p^2 + m^2} \simeq p + \frac{m^2}{2p} + ... \implies$ different mass eigenstates acquire different phases
- Mass eigenstates ($\nu_i$) $\neq$ flavor eigenstates ($\nu_\alpha$) due to mixing

$$\nu_\alpha = \sum_j U_{\alpha j} \nu_j \quad (U \text{ is } 3 \times 3, \text{ unitary})$$

- $\nu$'s created in flavor eigenstates, $\nu_\alpha(0) = \sum_j U_{\alpha j} \nu_j$, so after time $t$

$$\nu_\alpha(t) = \sum_j U_{\alpha j} e^{-iE_j L} \nu_j = \sum_\beta \left( \sum_j U_{\alpha j} e^{-iE_j L} U_{j \beta}^\dagger \right) \nu_\beta$$

- Oscillation probability is $|\langle \nu_\beta | \nu_\alpha \rangle|^2$, or

$$P(\nu_\alpha \rightarrow \nu_\beta) = \delta_{\alpha \beta} - \sum_{j<k} \left[ 4 \Re(U_{\alpha j} U_{\alpha k}^* U_{\beta j}^* U_{\beta k}) \sin^2\left(\frac{1}{2}\Delta_{jkL}\right) - 2 \Im(U_{\alpha j} U_{\alpha k}^* U_{\beta j}^* U_{\beta k}) \sin(\Delta_{jkL}) \right]$$

$$\Delta_{jk} \equiv E_j - E_k = \frac{\delta m^2_{jk}}{2E}, \quad \delta m^2_{jk} \equiv m^2_j - m^2_k$$
- $U$ described by 3 mixing angles ($\theta_{12}$, $\theta_{13}$, $\theta_{23}$) and a $CP$-violating phase ($\delta$)
- Two independent $\delta m^2$ ($\delta m^2_{21}$, $\delta m^2_{31}$)
- $\delta m^2_{21} \simeq 7.6 \times 10^{-5}$ eV$^2$ and $\sin^2 2\theta_{12} \simeq 0.87$ fit solar $\nu_e \rightarrow \nu_e$ and KamLAND reactor $\bar{\nu}_e \rightarrow \bar{\nu}_e$ data
- $\delta m^2_{31} \simeq 2.4 \times 10^{-3}$ eV$^2$ and $\sin^2 2\theta_{13} \simeq 1.0$ fit atmospheric and K2K, MINOS long-baseline data (predominantly $\nu_\mu \rightarrow \nu_\tau$)

- $\nu$ mass provides only complete description of all data (except LSND/MiniBooNE)
- $\Longrightarrow$ strong evidence for $\nu$ mass
Standard Model Extension (SME) (Colladay & Kostelecky)

- *Particle* Lorentz transformations that leave background vevs unchanged may be affected
- SME: all Lorentz symmetry-breaking terms that preserve $SU(3) \times SU(2) \times U(1)$
- Corresponding Hamiltonian for $\nu_L$ propagation in SME:

$$(h_{e f f})_{ij} = \left| \vec{p} \right| \delta_{ij} + \frac{(m^2)_{ij}}{2\left| \vec{p} \right|} + \frac{1}{\left| \vec{p} \right|} \left[ a^{\mu} p_{\mu} - c^{\mu \nu} p_{\mu} p_{\nu} \right]_{ij}, \quad i, j = e, \mu, \tau$$

- For $\bar{\nu}$, $a \rightarrow -a$ (*CPT* violation)
- Choose direction-independent terms ($\mu = \nu = 0$); for relativistic $\nu$'s, $\left| \vec{p} \right| \simeq E$ and $p^\mu \simeq (E, -E \hat{p})$

$$(h_{e f f})_{ij} = \left| \vec{p} \right| \delta_{ij} + \frac{(m^2)_{ij}}{2E} + a_{ij} + c_{ij} E$$

- $a$ terms are energy-independent ($a$ has dimensions of $E$)
- $c$ terms proportional to $E$ ($c$ dimensionless)
Bicycle Model (Kostelecky & Mewes)

\[
h_{\text{eff}} = \begin{pmatrix}
-2cE & \frac{1}{\sqrt{2}}a & \frac{1}{\sqrt{2}}a \\
\frac{1}{\sqrt{2}}a & 0 & 0 \\
\frac{1}{\sqrt{2}}a & 0 & 0
\end{pmatrix},
\]

- Eigenvalues \( \lambda_i = 0, -cE \pm \sqrt{(cE)^2 + a^2} \)
  \( \lambda_i - \lambda_j = \Delta_{ij} = \frac{1}{2E}(\delta m_{\text{eff}}^2)_{ij} \)

- must reproduce \( 1/E \) behavior at high \( E \) from \( cE, a \) terms

  In bicycle model there is a see-saw mechanism in large \( E \) limit \( (cE \gg a) \)

  \( \Delta_{32} \simeq a^2/(2cE) \) has correct \( E \) dependence!

  \( \delta m_{\text{eff}}^2 = a^2/c \) for atmospheric and long-baseline (LBL) \( \nu \)'s
• General bicycle model (BMW)
  
  Allow direction dependent and/or direction independent terms
  
  Add $a_{ee}$ term to adjust solar osc. prob. at low $E$
  
  Does not fit all data simultaneously

• Tandem model (Katori, Kostelecky & Tayloe)
  
  Lorentz invariance violation and neutrino mass
  
  Solar and KamLAND data fit by neutrino mass terms
  
  Atmospheric and LBL data fit by LV terms (like bicycle model)

• Puma model (Diaz & Kostelecky) see next talk
General Direction-Independent LV Models with 3 Massless $\nu$'s

- 16 independent parameters: $c_{ij}E + a_{ij}$ with $c$ and $a$ traceless ($i, j =$ flavors)
- Rather than a random sampling of parameter space, we searched for structures that lead to $1/E$ behavior of at least one $\Delta_{ij}$ at high energy (to mimic see-saw mechanism of bicycle model)
- Method:

  Require $c_{ij}E \gg a_{ij}$ at high $E$ ($\geq 1$ GeV)

  For given structure, expand eigenvalues of $h_{eff}$ in powers of $E$

  $$\lambda = \alpha_1 E + \alpha_0 + \alpha_{-1}E^{-1} + ...$$

- Oscillation argument is $\Delta_{j,k} = \lambda_j - \lambda_k$

  $\implies 1/E$ behavior requires degeneracy at leading order ($E^1$) and next-to-leading order ($E^0$)

- $\alpha_1$ values determined by eigenvalues of $c$ matrix – degeneracy at leading order $\implies$ constraints on $c_{ij}$

- Constraints on $a_{ij}$ from requiring degeneracy at next-to-leading order

- Some constraints are “natural”, others require fine tuning
• Classify models by $c_{ij}$ structure

3 real $c_{ii}$, 3 complex $c_{ij}$

$\implies 2^6 = 64$ textures

19 non-equivalent classes after allowing for flavor permutations

• Examples:

2B $D_iO_{ij} \implies \begin{pmatrix} c_{ee} & c_{e\mu} & 0 \\ c_{e\mu} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

2C $D_iO_{jk} \implies \begin{pmatrix} c_{ee} & 0 & 0 \\ 0 & 0 & c_{\mu\tau} \\ 0 & c_{\mu\tau} & 0 \end{pmatrix}$

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<th>Structure</th>
<th>Number of flavor permutations</th>
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Class 1A

\[ h_{\text{eff}} = \begin{pmatrix}
    c_{ee}E + a_{ee} & a_{e\mu} & a_{e\tau} \\
    a_{e\mu}^* & a_{\mu\mu} & a_{\mu\tau} \\
    a_{e\tau}^* & a_{\mu\tau}^* & a_{\tau\tau}
\end{pmatrix} \]

- \( \lambda_1 \approx cE + a_{ee} \), \( \lambda_2, \lambda_3 \approx \frac{1}{2} \left[ a_{\mu\mu} + a_{\tau\tau} \mp \sqrt{(a_{\mu\mu} - a_{\tau\tau})^2 + 4|a_{\mu\tau}|^2} \right] \) to order \( E^0 \)
- Degeneracy to order \( E^0 \) requires \( a_{\mu\mu} = a_{\tau\tau} \) and \( a_{\mu\tau} = 0 \)
- Reduces to general bicycle model

Class 1B

\[ h_{\text{eff}} = \begin{pmatrix}
    a_{ee} & c_{e\mu}E + a_{e\mu} & a_{e\tau} \\
    c_{e\mu}E + a_{e\mu}^* & a_{\mu\mu} & a_{\mu\tau} \\
    a_{e\tau}^* & a_{\mu\tau}^* & a_{\tau\tau}
\end{pmatrix} \]

- \( \lambda_1, \lambda_3 \approx \mp cE, \quad \lambda_2 = 0 \) to order \( E^0 \)
- Degeneracy to order \( E^0 \) not possible
• Can subtract off piece proportional to identity – does not affect oscillations
• Many cases reduce to others when degeneracy constraints applied
• Only 1A, 2C, 3B, 3F, 4D, 5B give $1/E$ dependence at high $E$

Class 2C

$$h_{\text{eff}} = \begin{pmatrix} a_{ee} & a_{e\mu} & c_{e\tau}E + a_{e\tau} \\ a_{e\mu}^* & c_{\mu\mu}E & a_{\mu\tau} \\ c_{e\tau}E + a_{e\tau}^* & a_{\mu\tau}^* & a_{\tau\tau} \end{pmatrix}$$

• Atmospheric and LBL neutrinos have

$$P(\nu_\mu \to \nu_e) = P(\nu_\mu \to \nu_\tau) = \frac{1}{2} \sin^2 2\theta \sin^2 \left( \frac{\delta m_{\text{eff}}^2 L}{2E} \right)$$

• Ruled out since $\nu_\mu$ oscillates predominantly to $\nu_\tau$
Class 3F

\[
 h_{\text{eff}} = \begin{pmatrix}
 a_{ee} & c_{e\mu} E + a_{e\mu} & c_{e\tau} E + a_{e\tau} \\
 c_{e\mu} E + a_{e\mu}^* & a_{\mu\mu} & c_{\mu\tau} E + a_{\mu\tau} \\
 c_{e\tau} E + a_{e\tau}^* & c_{\mu\tau} E + a_{\mu\tau}^* & a_{\tau\tau}
\end{pmatrix}
\]

- Atmospheric and LBL neutrinos have

\[
P(\nu_\mu \rightarrow \nu_\mu) = \frac{5}{9} - 4|U_{\mu 2}|^2 \left( \frac{2}{3} - |U_{\mu 2}|^2 \right) \sin^2 \left( \frac{\delta m_{\text{eff}}^2 L}{2E} \right)
\]

- Ruled out since downward atmospheric \( \nu_\mu \) are not depleted

Only three classes also have correct oscillation amplitude for atmospheric and LBL \( \nu \)'s (3B, 4D, 5B)
Class 3B

\[ h_{\text{eff}} = \begin{pmatrix} c_{ee}E + a_{ee} & a_{e\mu} & c_{e\tau}E + a_{e\tau} \\ a_{e\mu}^* & 0 & a_{\mu\tau} \\ c_{e\tau}E + a_{e\tau}^* & a_{\mu\tau}^* & c_{\tau\tau}E + a_{\tau\tau} \end{pmatrix} \]

- Degeneracy at order \( E \) requires \( c_{\tau\tau} = rc_{e\tau} = r^2c_{ee} \)

\[
P(\nu_\mu \to \nu_\mu) = 1 - \sin^2 2\theta \sin^2 \left( \frac{\delta m^2_{\text{eff}} L}{2E} \right)
\]
\[
P(\nu_\mu \to \nu_e) = \sin^2 \phi \sin^2 2\theta \sin^2 \left( \frac{\delta m^2_{\text{eff}} L}{2E} \right)
\]
\[
P(\nu_e \to \nu_e) \approx 1 - \sin^2 2\phi \sin^2 \left( \frac{(1 + r^2)cEL}{2} \right) - \sin^4 \phi \sin^2 2\theta \sin^2 \left( \frac{\delta m^2_{\text{eff}} L}{2E} \right)
\]

\[
\tan \phi \equiv r
\]

- \( \theta \approx \pi/4 \)

- Lack of large \( \nu_\mu \to \nu_e \) oscillation in K2K, MINOS and T2K implies small \( \phi \) \( (r < 0.43) \)
• At lower energies (solar, KamLAND) large $E$ limit does not apply
• Try to fit to KamLAND and solar (including matter effect)

Fit to KamLAND not as good as with neutrino mass

Resulting oscillation probability for solar $\nu$’s too large at high energy

KamLAND

Solar osc. prob. using KamLAND parameters
Best fit to solar data alone still not good at high energies

Can only fit solar data with $r \geq 1$

$(\sin^2 2\theta_{13}^{eff} \geq 0.50)$
Class 4D

\[
h_{eff} = \begin{pmatrix}
    c_{ee}E + a_{ee} & c_{e\mu}E + a_{e\mu} & c_{e\tau}E + a_{e\tau} \\
    c_{e\mu}E + a_{e\mu}^* & a_{\mu\mu} & c_{\mu\tau}E + a_{\mu\tau} \\
    c_{e\tau}E + a_{e\tau}^* & c_{\mu\tau}E + a_{\mu\tau}^* & a_{\tau\tau}
\end{pmatrix}
\]

- After degeneracy imposed at order $E$ and rotation in $\mu$-$\tau$ sector, $h_{eff}$ has same form as Class 3B

KamLAND fit

Solar osc. prob. using KamLAND parameters

Solar fit
Class 5B

\[ h_{\text{eff}} = \begin{pmatrix} c_{ee}E + a_{ee} & c_{e\mu}E + a_{e\mu} & c_{e\tau}E + a_{e\tau} \\ c_{e\mu}E + a_{e\mu}^* & 0 & c_{\mu\tau}E + a_{\mu\tau} \\ c_{e\tau}E + a_{e\tau}^* & c_{\mu\tau}E + a_{e\tau}^* & c_{\tau\tau}E + a_{\tau\tau} \end{pmatrix} \]

- Nearly maximal \( \nu_\mu \rightarrow \nu_\tau \), small \( \nu_\mu \rightarrow \nu_e \implies c_{\mu\tau}^2 \ll c_{e\mu}^2 \ll c_{e\tau}^2 \)

KamLAND fit
Solar osc. prob. using KamLAND parameters
Solar fit
Conclusions

• SME allows new terms in effective Hamiltonian for $\nu$ propagation that have different energy dependence from ordinary oscillations ($cE$, $a$ vs. $m^2/E$)

• A number of cases without neutrino mass can reproduce $1/E$ dependence at high $E$ for atmospheric and LBL neutrinos

• None can also successfully fit solar and KamLAND data

• Standard Model with just neutrino masses is very robust (modulo LSND/MiniBooNE)

• Apparently neutrino mass is needed to explain some (if not all) oscillation data