The Wake of a Quark Moving through Hot QCD vs. N=4 SYM Plasma

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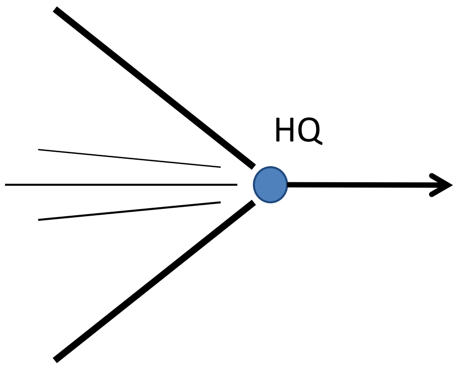
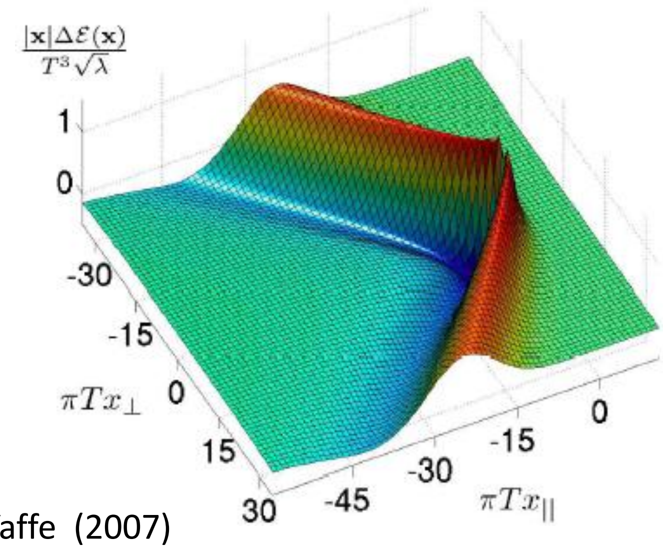
“Mach Cone”





P. Chesler , L. Yaffe (2007)

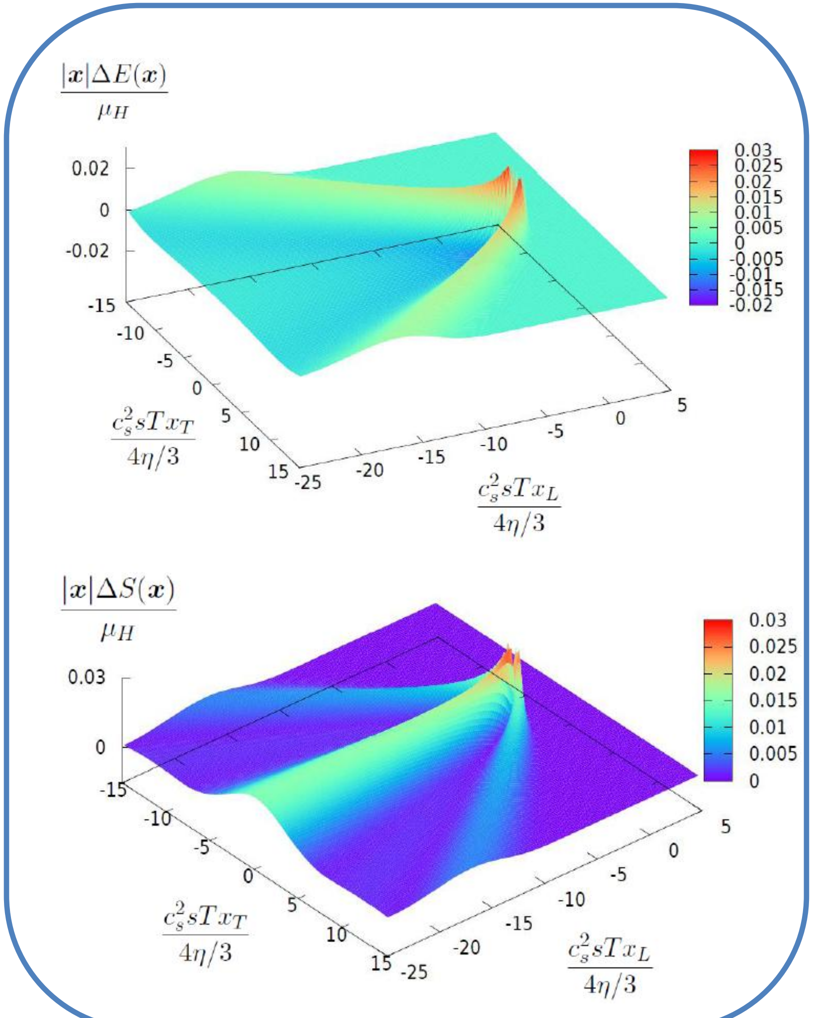
- Simulate the “Mach Cone” at Weak Coupling.

- Solve the Boltzmann Equation at Leading-Log Approximation :

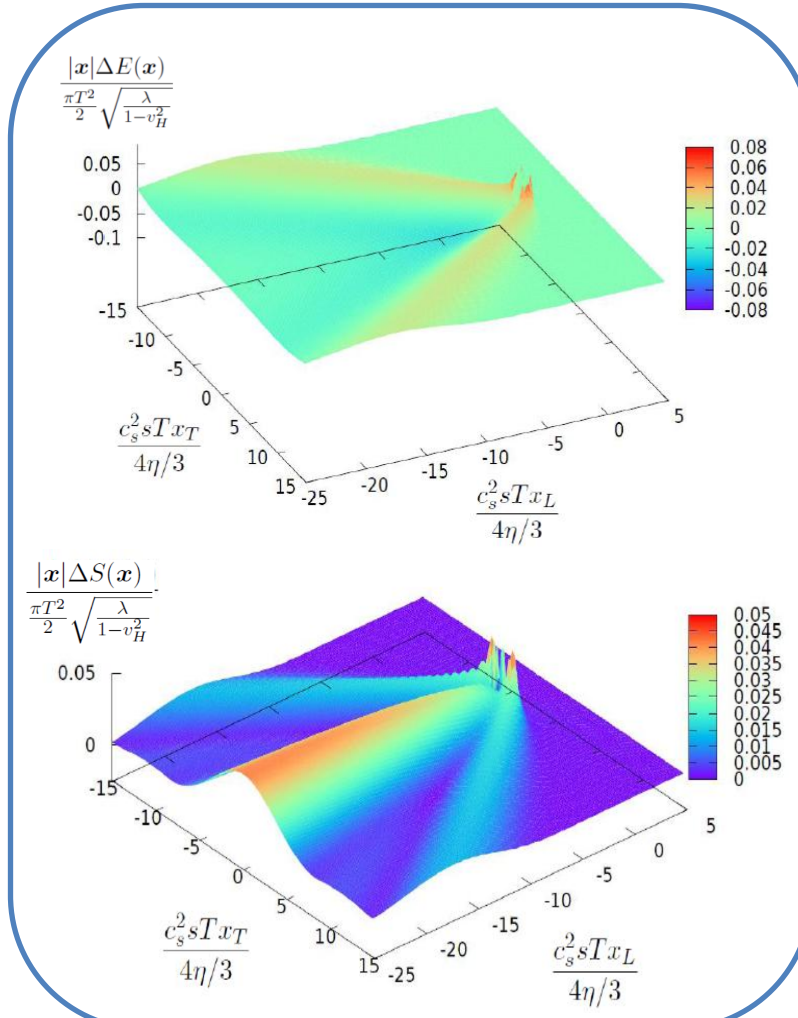
$$(\partial_t + \mathbf{v}_p \cdot \partial_x) f(\boldsymbol{p}, \boldsymbol{x}) = \mathrm{C}[f, \boldsymbol{p}]$$

Hot QCD vs. N=4 SYM

Boltzmann



AdS/CFT



Outline

- Boltzmann Equation
- Mach Cone
- Hydrodynamics

Boltzmann Equation

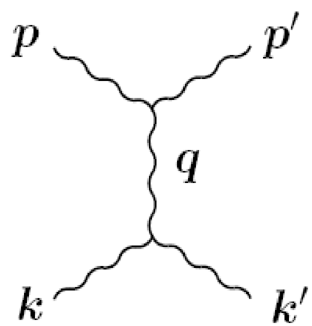
Boltzmann Equation

- Linearization :

$$(\partial_t + \mathbf{v}_p \cdot \partial_{\boldsymbol{x}}) f(\boldsymbol{p}, \boldsymbol{x}) = \mathrm{C}[f, \boldsymbol{p}]$$

$$f = n_p + \delta f \qquad n_p = \frac{1}{e^{p/T_o} - 1}$$

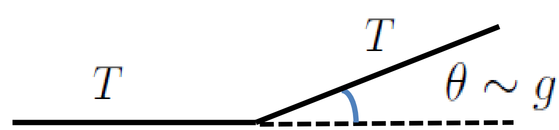
- In a Leading-Log Order : 2 → 2 Scattering



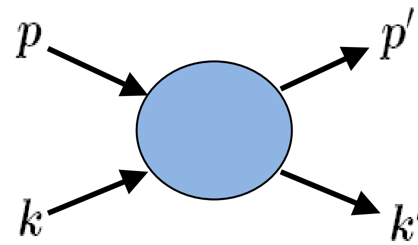
Hard $\boldsymbol{p}, \boldsymbol{k} \sim T$

Soft $\boldsymbol{q} \sim gT$

- Small Angle Scattering :



Boltzmann at Leading Log



- Collision Term :

$$C[f,\boldsymbol{p}] \sim \int_{\boldsymbol{k},\boldsymbol{p}',\boldsymbol{k}'} |M|^2 \delta^4(P_{tot}) \Big[f(\boldsymbol{p}) f(\boldsymbol{k}) [1+f(\boldsymbol{p}')][1+f(\boldsymbol{k}')] - f(\boldsymbol{p}') f(\boldsymbol{k}') [1+f(\boldsymbol{p})][1+f(\boldsymbol{k})] \Big]$$

$$\delta f(\boldsymbol{p}) \equiv n_p(1+n_p)\chi(\boldsymbol{p})$$

- Boltzmann with Loss/Gain :

loss term

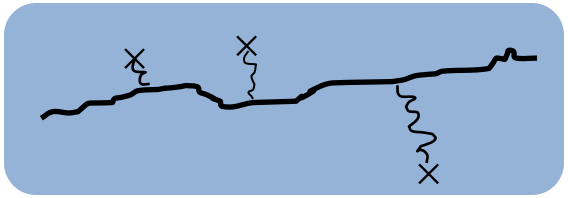
$$(\partial_t + \mathbf{v}_p \cdot \partial_{\mathbf{x}})\delta f = T\mu_A \frac{\partial}{\partial \boldsymbol{p}^i}\left[n_p(1+n_p)\frac{\partial \chi}{\partial \boldsymbol{p}^i} \right] + \text{gain terms}$$

$$\mu_A \equiv \frac{g^2 C_A m_D^2}{8\pi} \log\left(\frac{T}{m_D}\right)$$

J. Hong , D. Teaney (2010)

Energy/Momentum Conservation

- Loss : Random Walk of Hard Particles



$$\frac{\partial}{\partial t}\delta f = T\mu_A \frac{\partial}{\partial \boldsymbol{p}^i}\left[n_p(1+n_p)\frac{\partial \chi}{\partial \boldsymbol{p}^i}\right]$$

$$= -\frac{\partial}{\partial \boldsymbol{p}}\cdot\left[\hat{\boldsymbol{p}}\mu_A(1+2n_p)\delta f\right] + \nabla^2_{\boldsymbol{p}}\left[T\mu_A\delta f\right]$$

- Work on Excess Particles : $\frac{\mathrm{d}E}{\mathrm{d}t} \equiv -T\mu_A \int \frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3} n_p(1+n_p)\hat{\boldsymbol{p}}\cdot\frac{\partial \chi}{\partial \boldsymbol{p}}$

- Gain : Energy/Momentum Conservation

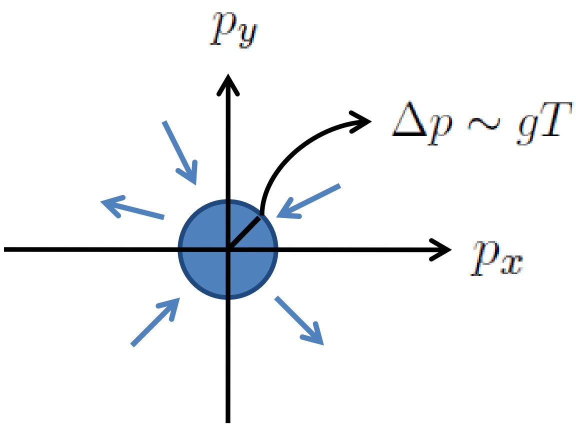
$$\text{gain terms} = \frac{1}{\xi_B}\left[\frac{1}{p^2}\frac{\partial}{\partial p}p^2 n_p(1+n_p)\right]\frac{\mathrm{d}E}{\mathrm{d}t} + \frac{1}{\xi_B}\left[\frac{\partial}{\partial \boldsymbol{p}}n_p(1+n_p)\right]\cdot\frac{\mathrm{d}\boldsymbol{P}}{\mathrm{d}t}$$

$$\xi_B \equiv \int \frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3} n_p(1+n_p) = \frac{T^3}{6}$$

Particle Number Change

- Emit/Absorb Soft Gluons at small p :

$$\int_{p=0}^{p=\Delta p} \frac{d^3\boldsymbol{p}}{(2\pi)^3} n_p(1+n_p)\chi(\boldsymbol{p}) \simeq \frac{T^2}{2\pi^2}\chi(\boldsymbol{0})\Delta p$$



- Absorptive Boundary Condition :

$$\chi(\boldsymbol{p})|_{\boldsymbol{p}\to 0} = 0$$

P. Arnold , C. Dogan, G. Moore (2006)

- Particle Number Change!

Transport Coefficients

- Use the Boltzmann Equation to Determine Transport Coefficients :

$$\eta\,,\ \zeta\,,\ \tau_\pi\,,\ \tau_\Pi\,,\ D\,,\ \tau_J\,,\ \cdots$$

J. Hong , D. Teaney (2010)

- Shear Viscosity :

$$\boxed{\frac{\eta}{sT} = 0.4613\frac{T}{\mu_A}} \qquad \mu_A \equiv \frac{g^2 C_A m_D^2}{8\pi}\log\left(\frac{T}{m_D}\right)$$

- Second Order Hydrodynamic Coefficient :

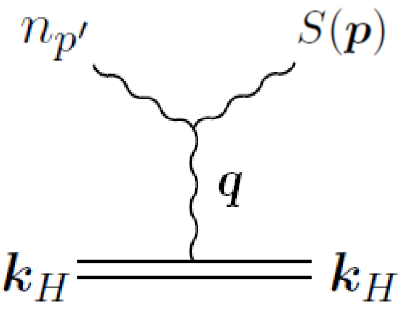
$$\boxed{\tau_\pi = 6.32\frac{\eta}{sT}}$$

Mach Cone

Heavy Quark Source

- Linearized Boltzmann Equation with Source :

$$(\partial_t + \mathbf{v}_p \cdot \partial_{\boldsymbol{x}})\, \delta f(t, \boldsymbol{x}, \boldsymbol{p}) = C[f, \boldsymbol{p}] + S(t, \boldsymbol{x}, \boldsymbol{p})$$



- Source : Collision Integral with Delta Function

$$S(t, \boldsymbol{x}, \boldsymbol{p}) = S(\boldsymbol{p})\delta^3(\boldsymbol{x} - \mathbf{v}_H t)$$

$$\sim \int_{\boldsymbol{k}, \mathbf{p}', \boldsymbol{k}'} |M|^2 \delta^4(P_{tot}) \Big[f(\boldsymbol{p}) f_H(\boldsymbol{k})[1 + f(\boldsymbol{p}')][1 + f(\boldsymbol{k}')] - f(\boldsymbol{p}') f(\boldsymbol{k}')[1 + f(\boldsymbol{p})][1 + f_H(\boldsymbol{k})] \Big]$$

$$f_H(\boldsymbol{k}) = (2\pi)^3 \delta^3(\boldsymbol{k} - \boldsymbol{k}_H)\delta^3(\boldsymbol{x} - \mathbf{v}_H t)$$

- $v_H = c$: $$S(\boldsymbol{p}) = \frac{\mu_H}{\xi_B} n_p (1 + n_p) \left[\left(-\frac{2T}{p} + 1 + 2n_p \right) + (1 + 2n_p) \hat{\boldsymbol{p}} \cdot \mathbf{v}_H \right]$$

Kinetic Theory

- Boltzmann Equation in Fourier Space :

$$(-i\omega + i\mathbf{v}_p \cdot \boldsymbol{k})\delta f(\omega, \boldsymbol{k}, \boldsymbol{p}) = C_{\boldsymbol{p},\mathbf{p}'}\delta f(\omega, \boldsymbol{k}, \mathbf{p}') + S(\boldsymbol{p})$$

- Kinetic Theory :

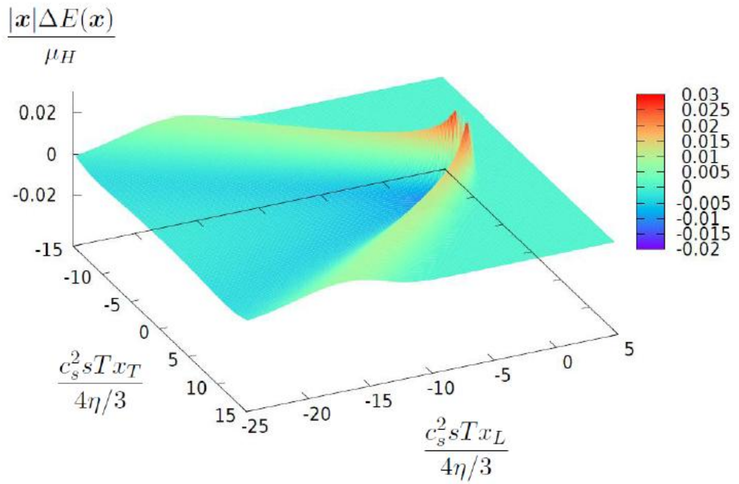
$$\delta T^{0\mu}(\omega, \boldsymbol{k}) = \nu_g \int_{\boldsymbol{p}} p^\mu \delta f(\omega, \boldsymbol{k}, \boldsymbol{p})$$

- Fourier Transform :

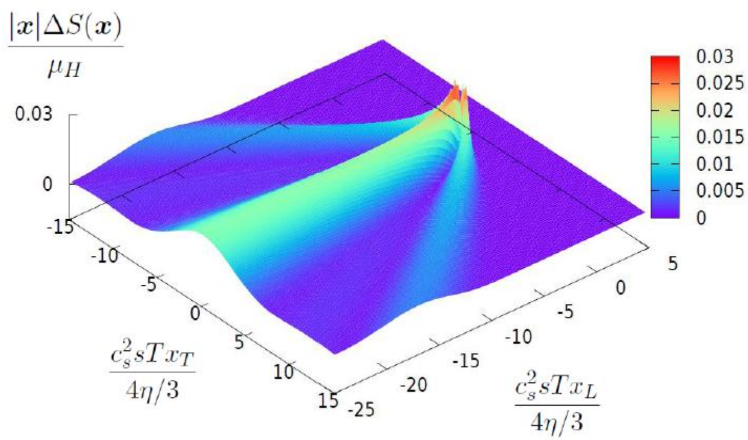
$$\delta T^{0\mu}(t, \boldsymbol{x}) = \int_{\omega,\boldsymbol{k}} e^{-i\omega + i\boldsymbol{k}\cdot\boldsymbol{x}} \delta T^{0\mu}(\omega, \boldsymbol{k})$$

Energy Density/Flux Distributions

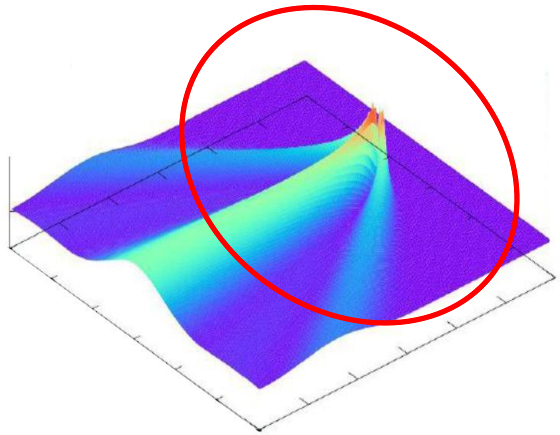
- Energy :

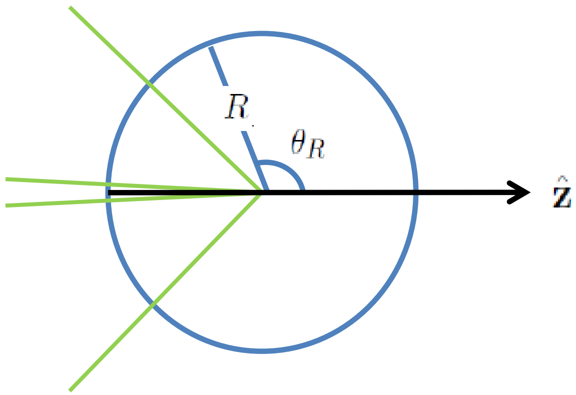


- Momentum :



Angular Distribution





$$\mathbf{R} = x_T\hat{x} + x_L\hat{z}$$

- Energy :

$$\Delta E \equiv \delta T^{00}$$

$$\frac{dE_R}{d\theta_R} = 4\pi R^2 \Delta E(\mathbf{R})$$

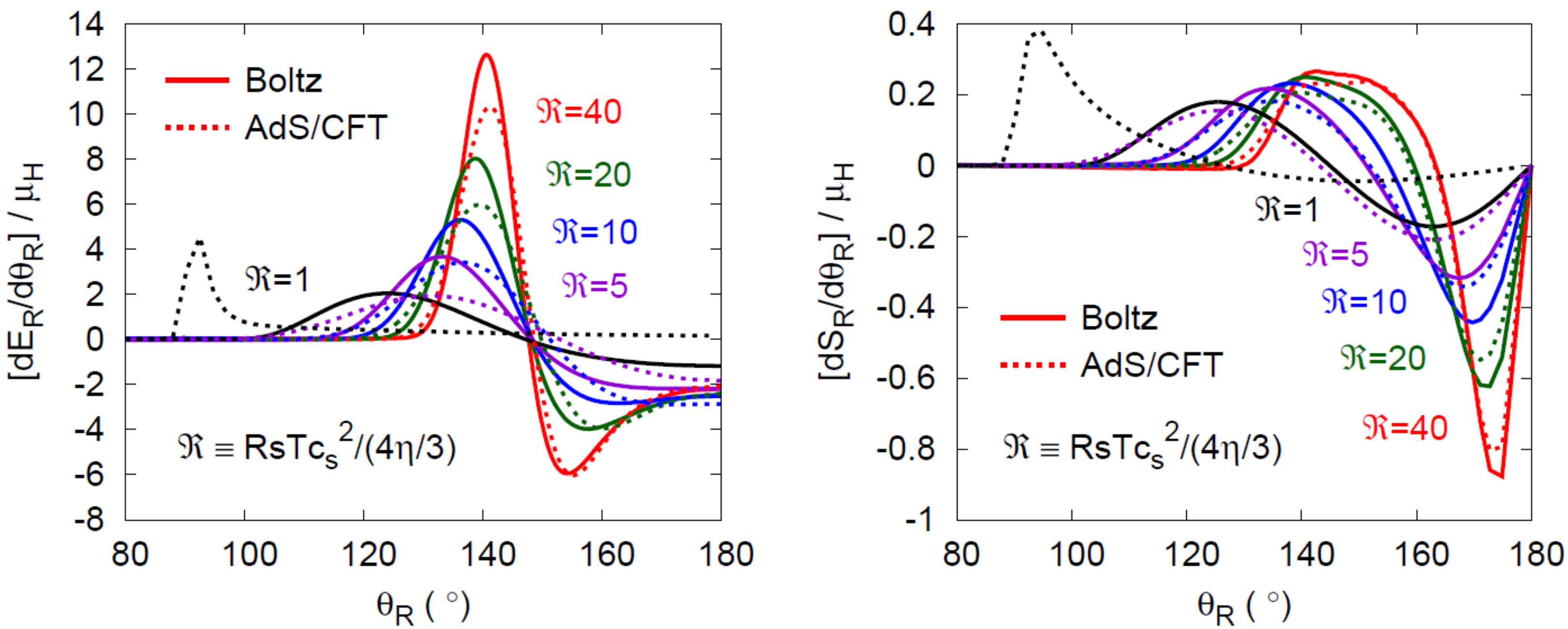
- Momentum :

$$\Delta S \equiv |\delta T^{0i}|$$

$$\frac{dS_R}{d\theta_R} = 2\pi R^2 \sin\theta_R \hat{\mathbf{R}} \cdot \Delta S(\mathbf{R})$$

Boltzmann vs. AdS/CFT

AdS/CFT by P. Chesler



- Sound Wave and Wake
- Shear Length Unit

Boltzmann : $\frac{\eta}{sT} = 0.4613\frac{T}{\mu_A}$, AdS/CFT : $\frac{\eta}{sT} = \frac{1}{4\pi T}$

- Zero Temperature Contribution in AdS/CFT (R=1)

Hydrodynamics

Hydrodynamics

- Constituent Relation in Conformal Limit (Static) :

$$T^{\mu\nu}_{hydro} = (e+\mathcal{P})u^\mu u^\nu + \mathcal{P}g^{\mu\nu} \underbrace{- 2\eta\,\langle\partial^\mu u^\nu\rangle}_{\text{1st}} \underbrace{- 2\eta\tau_\pi\langle\partial^\mu\partial^\nu \ln T\rangle}_{\text{2nd}}$$

R. Baier , P. Romatschke , D. T. Son , A. O. Starinets , M. A. Stephanov (2007)

- Equation of Motion : $\partial_\mu T^{\mu\nu} = F^\nu_{micro}$

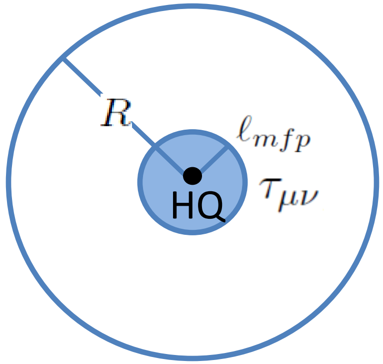
- Force : $F^\nu_{micro} = \dfrac{dp^\nu}{dt} = \displaystyle\int_{\boldsymbol{p}} p^\nu S(\boldsymbol{p})$

Source Correction

P. Chesler , L. Yaffe (2007)

$$\partial_\mu \left(T^{\mu\nu}_{hydro} + \boxed{\tau_{\mu\nu}} \right) = F^\nu$$

$$- i\omega T^{0j}_{hydro} + ik^i T^{ij}_{hydro} = F^j_{micro} \underbrace{- ik^i \tau^{ij}}_{\Delta\text{Source}}$$



- $\tau^{\mu\nu}$ Localized near Quark

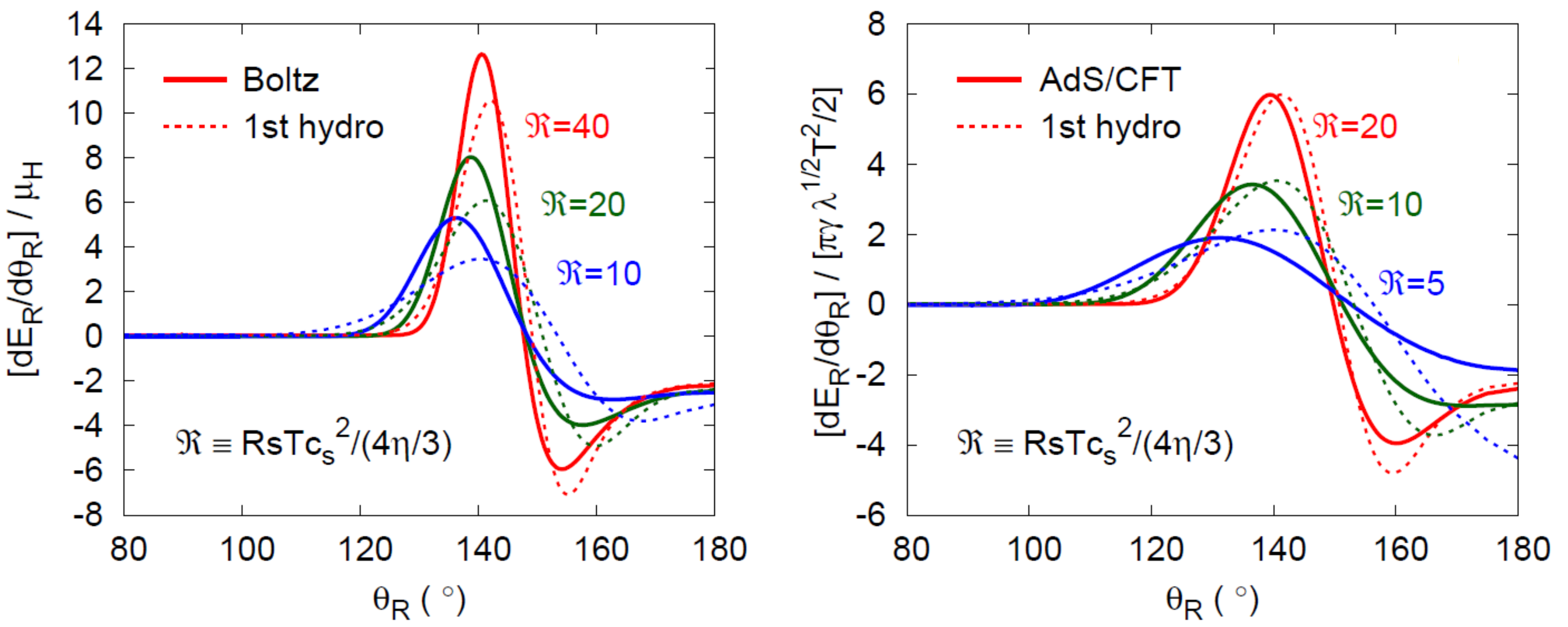
- Acts as a Source : $\Delta\text{Source} \sim \underbrace{\partial \left[\delta^3(\boldsymbol{x} - \mathbf{v}_H t) \right]}_{\text{1st}} + \underbrace{\partial\partial \left[\delta^3(\boldsymbol{x} - \mathbf{v}_H t) \right]}_{\text{2nd}}$

- $\tau^{\mu\nu}$ Determined by Comparison : $T^{\mu\nu}(\omega, \boldsymbol{k}) = T^{\mu\nu}_{hydro}(\omega, \boldsymbol{k}) + \tau^{\mu\nu}(\omega, \boldsymbol{k})$

- Expand in Powers of w , k : Determine #'s for Boltzmann, AdS/CFT.

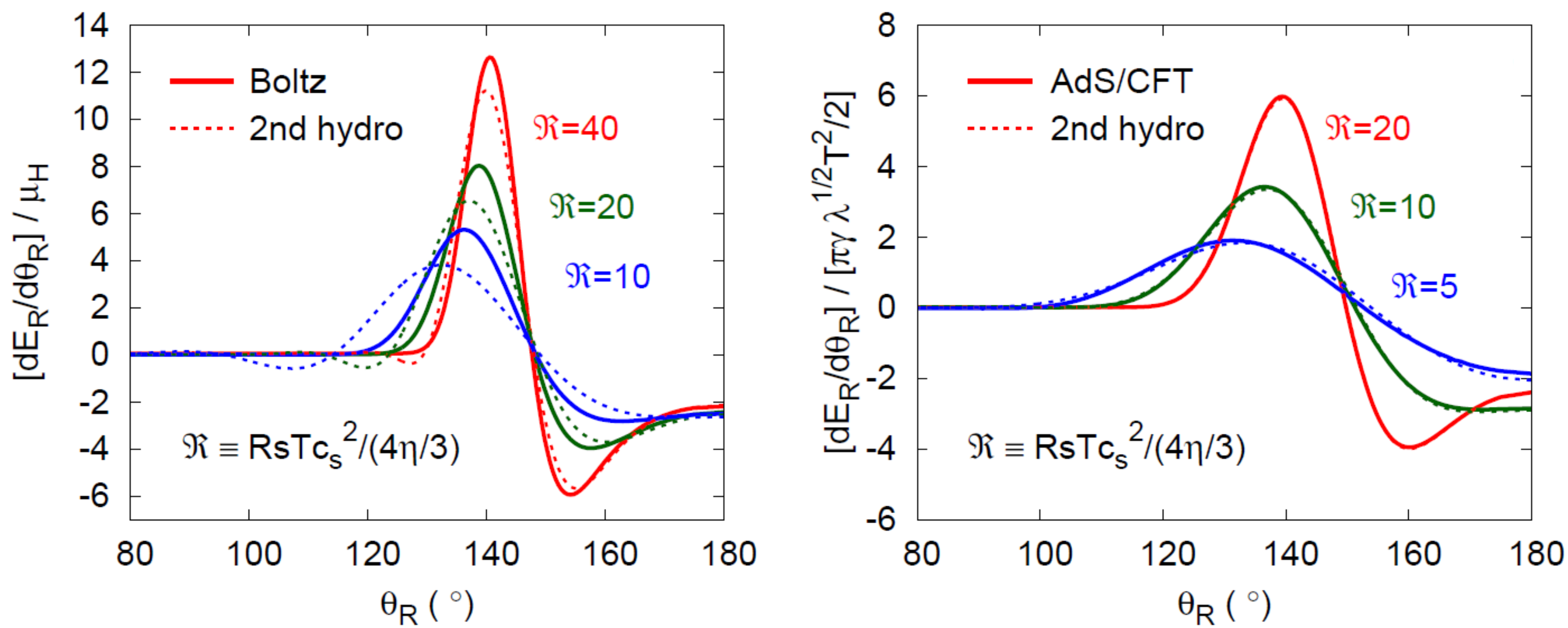
$$- ik^i \tau^{ij} = \underbrace{\# i\omega v^j_H + \# ik^j}_{\text{1st}} + \underbrace{\# \omega^2 v^j_H + \# k^2 v^j_H + \# \omega k^j}_{\text{2nd}} + \mathcal{O}(k^3)$$

1st Order Hydrodynamics



- Energy Density : Sound Mode
- Boltzmann : Hydrodynamics at Longer Distances

2^{nd} Order Hydrodynamics



- Boltzmann : Reactive to High Frequencies
- AdS/CFT is Amazing!
- Differences due to τ_π

Summary : Boltzmann vs. AdS/CFT

- Boltzmann : Slow Transition to Hydrodynamics

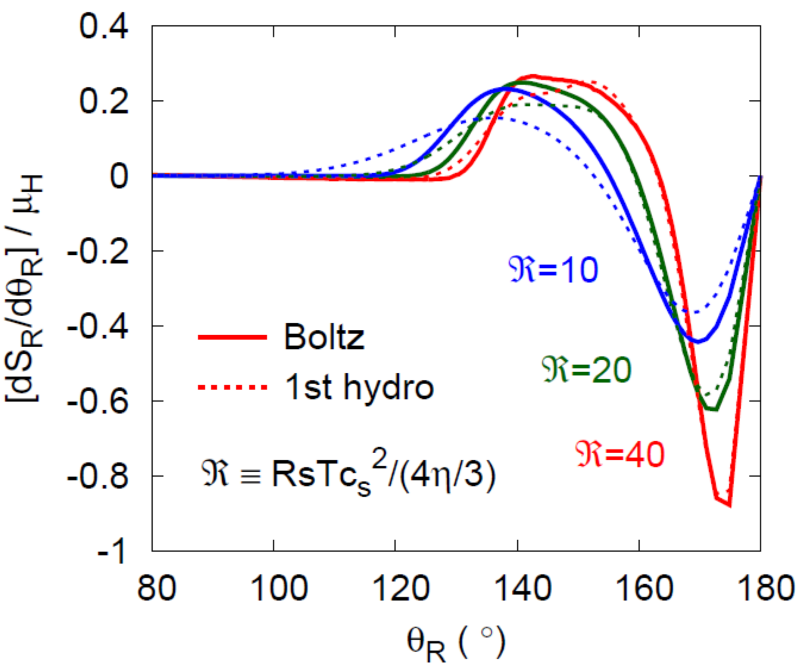
- Boltzmann :
$$\tau_\pi = \frac{6.32\eta}{sT} = 1.58\left[\frac{4\eta/3}{c_s^2 sT}\right]$$

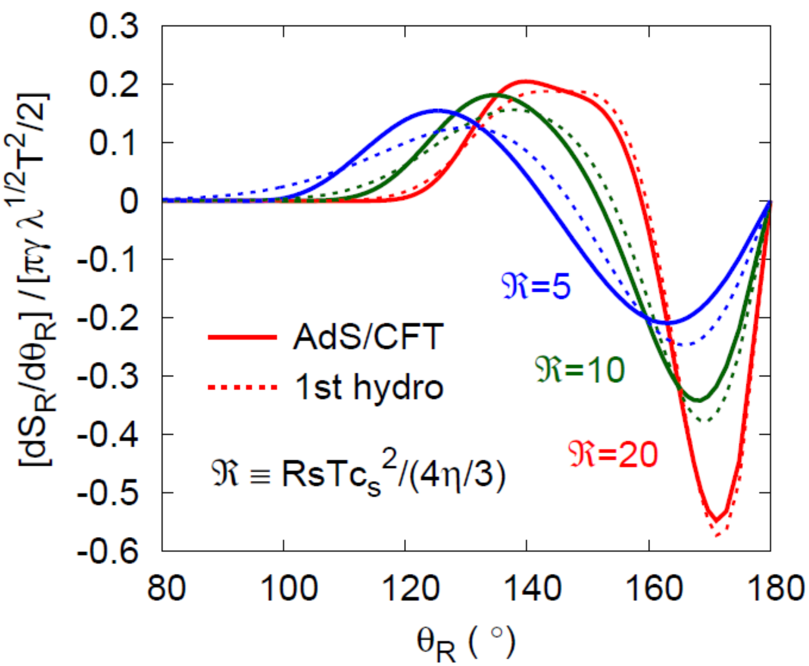
AdS/CFT :
$$\tau_\pi = \frac{2-\ln 2}{2\pi T} = 0.65\left[\frac{4\eta/3}{c_s^2 sT}\right]$$

- The Larger the Second Order Hydrodynamic Coefficient is,
The Longer it Takes to Reach the Hydrodynamic Regime.

Backup

Flux : 1st Order Hydrodynamics





Flux : 2nd Order Hydrodynamics