Neutrinos Physics: Theory

Lisa L. Everett
U. Wisconsin, Madison

Meeting of the Division of Particles and Fields
of the American Physical Society
August 9-13, 2011
Main Theme

Discovery of Neutrino Oscillations:

$$P_{\nu_\alpha \rightarrow \nu_\beta} (L) = \sum_{ij} U_{i\alpha} U^*_{i\beta} U^*_{j\alpha} U_{j\beta} e^{-i\Delta m^2_{ij} L / 2E}$$

surprises, confusion, excitement for beyond SM physics theory!

“Reference Picture”:

data (except LSND) consistent with 3ν mixing picture
intriguing pattern of masses, mixings: paradigm shift for SM flavor puzzle

Challenges to the Reference Picture

LSND, Recent results from MINOS, MiniBooNE, reactor neutrino anomaly
differences b/w ν, \bar{\nu} modes! suggestions of sterile neutrinos, NSI’s,...

If robust, potentially profound implications...
The Reference Picture: Neutrino Masses and Mixings

Assume: 3 neutrino mixing

<table>
<thead>
<tr>
<th>parameter</th>
<th>best fit ±1σ</th>
<th>2σ</th>
<th>3σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta m_{21}^2$ [10^{-5}eV^2]</td>
<td>7.59±0.20_{-0.18}</td>
<td>7.24–7.99</td>
<td>7.09–8.19</td>
</tr>
<tr>
<td>$\Delta m_{31}^2$ [10^{-3}eV^2]</td>
<td>2.45 ± 0.09</td>
<td>2.28 – 2.64</td>
<td>2.18 – 2.73</td>
</tr>
<tr>
<td></td>
<td>−(2.34±0.10_{-0.09})</td>
<td>−(2.17 – 2.54)</td>
<td>−(2.08 – 2.64)</td>
</tr>
<tr>
<td>$\sin^2 \theta_{12}$</td>
<td>0.312±0.017</td>
<td>0.28–0.35</td>
<td>0.27–0.36</td>
</tr>
<tr>
<td>$\sin^2 \theta_{23}$</td>
<td>0.51 ± 0.06</td>
<td>0.41–0.61</td>
<td>0.39–0.64</td>
</tr>
<tr>
<td></td>
<td>0.52 ± 0.06</td>
<td>0.42–0.61</td>
<td></td>
</tr>
<tr>
<td>$\sin^2 \theta_{13}$</td>
<td>0.010±0.009_{-0.006}</td>
<td>≤ 0.027</td>
<td>≤ 0.035</td>
</tr>
<tr>
<td></td>
<td>0.013±0.009_{-0.007}</td>
<td>≤ 0.031</td>
<td>≤ 0.039</td>
</tr>
</tbody>
</table>

Normal Hierarchy (upper line)  Inverted Hierarchy (lower line)

$3 \quad 2 \quad 1 \quad 3$

Cosmology (WMAP): $\sum_i m_i < 0.7 \text{ eV}$
The Reference Picture: Lepton Mixing

\[ U_{\text{MNSP}} = R_1(\theta_{23})R_2(\theta_{13}, \delta_{\text{MNSP}})R_3(\theta_{12})P \]

Rewriting...

- **Solar:** \( \theta_{12} = 34.0^\circ \pm 1.0 \) (2 large)
- **Atmospheric:** \( \theta_{23} = 45.6^\circ \pm 3.5 \)
- **Reactor:** \( \theta_{13} = 5.7^\circ + 2.2^\circ - 2.1^\circ \) (1 small)

No constraints on CP violation

**Hints for nonzero \( \theta_{13} \) !**

- **~2008:** slight tension noted b/w datasets in global fits
  
- **June 2011:** new results for \( \nu_\mu \to \nu_e \)
  
  - **T2K** \( 0.03(0.04) < \sin^2 2\theta_{13} < 0.28(0.34) \) (90% c.l.)
    - disfavor zero reactor angle at \( 2.5\sigma \)
  
  - **MINOS**
    - disfavor zero reactor angle at \( 1.7\sigma \)

- updated global fit: claims evidence now at \( > 3\sigma \)
  
  - Fogli et al., '11,
For Comparison: Quark Mixing

\[ \mathcal{U}_{\text{CKM}} = R_1(\theta_{23}^{\text{CKM}})R_2(\theta_{13}^{\text{CKM}}, \delta_{\text{CKM}})R_3(\theta_{12}^{\text{CKM}}) \]

Mixings:

\[ \begin{align*}
\theta_{12}^{\text{CKM}} &= 13.0^\circ \pm 0.1^\circ \\
\theta_{23}^{\text{CKM}} &= 2.4^\circ \pm 0.1^\circ \\
\theta_{13}^{\text{CKM}} &= 0.2^\circ \pm 0.1^\circ 
\end{align*} \]

CP violation:

\[ J \equiv \text{Im}(U_{\alpha i}U_{\beta j}U_{\beta i}^*U_{\alpha j}^*) \]

\[ J_{\text{CP}}^{(\text{CKM})} \approx \sin 2\theta_{12}^{\text{CKM}} \sin 2\theta_{23}^{\text{CKM}} \sin 2\theta_{13}^{\text{CKM}} \sin \delta_{\text{CKM}} \]

\[ J \sim 10^{-5} \quad \delta_{\text{CKM}} = 60^\circ \pm 14^\circ \]

0(1) CP-violating phase
Challenge to the Reference Picture: MiniBooNE

(exclusion region)

(MiniBooNE allowed regions)
Challenge to the Reference Picture: MINOS

\[ \Delta m^2_{32} = 2.35^{+0.11}_{-0.08} \times 10^{-3} \text{ eV}^2 \]

\[ \Delta \bar{m}^2_{32} = 3.36^{+0.45}_{-0.40} \times 10^{-3} \text{ eV}^2 \]

\[ \sin^2 2\theta_{23} > 0.91 \]

\[ \sin^2 2\bar{\theta}_{23} = 0.86 \pm 0.11 \]

90% c.l.

Challenge to the Reference Picture: reactor $\nu$ anomaly

recent improvement to calculation of reactor $\bar{\nu}_e$ flux

new prediction is 3% higher

Mueller et al., 1101.2663

deficit of $\bar{\nu}_e$ at all oscillation searches at reactors!
Theoretical Implications: Reference Picture

Shifts in the paradigm for addressing SM flavor puzzle:

- **Suppression of neutrino mass scale**

- **Mixing Angles** quarks small, leptons 2 large, 1 small

Strikingly different flavor patterns for quarks and leptons!

implications for quark-lepton unification?
**Mass Generation**

**Quarks, Charged Leptons**

“natural” mass scale tied to electroweak scale
Dirac mass terms, parametrized by Yukawa couplings

\[ Y_{ij} H \cdot \bar{\psi}_L \psi_R \]

- t quark: \( O(1) \) Yukawa coupling
- rest: suppression (flavor symmetry)

**Neutrinos**

beyond physics of Yukawa couplings!

Options: **Dirac** or **Majorana**
Majorana first:

**advantages:** naturalness, leptogenesis, $0\nu\beta\beta$

SM at NR level: Weinberg dim 5 operator

$$\frac{\lambda_{ij}}{\Lambda} L_i H L_j H$$

(if $\lambda \sim O(1)$, $\Lambda \gg m \sim O(100 \text{ GeV})$ ... but a priori unknown)

Underlying mechanism: examples

- **Type I seesaw** $\nu_R$ (fermion singlet)
- **Type II seesaw** $\Delta$ (scalar triplet)
- **Type III seesaw** $\Sigma$ (fermion triplet)

+ variations
Prototype: Type I seesaw

right-handed neutrinos:

\[
Y_{ij} L_i \nu_{Rj} H + M_{Rij} \nu_{Ri} \nu_{Rj}^c
\]

\[
\mathcal{M}_\nu = \begin{pmatrix} 0 & m \\ m & M \end{pmatrix}
\]

\[m \sim \mathcal{O}(100 \text{ GeV})\]

\[M \gg m\]

\[m_1 \sim \frac{m^2}{M}\]

\[m_2 \sim M \gg m_1\]

\[\nu_{1,2} \sim \nu_{L,R} + \frac{m}{M} \nu_{R,L}\]

advantages: naturalness, connection to grand unification, leptogenesis,...

disadvantage: testability (even at low scales)

Different in Type II, III: new EW charged states, may be visible at LHC

see e.g. Fileviez Perez, Han et al., '08
Many other ideas for Majorana neutrino masses...

more seesaws (double, inverse,...),
loop-induced masses (Babu-Zee, ...),
SUSY with R-parity violation, RS models,
higher-dimensional (>5) operators,...

What about Dirac masses?
more difficult in general,
but suppression mechanisms exist.
e.g. large/warped extra dimensions, extra gauge
symms (non-singlet $\nu_R$), SUSY breaking,...

General themes:
Trade-off b/w naturalness and testability. Much richer than quark
and charged lepton sectors. Everyone has a favorite scenario.
Lepton (and Quark) Mixing Angle Generation

Standard paradigm: spontaneously broken flavor symmetry

\[ Y_{ij} H \cdot \overline{\psi}_{Li} \psi_{Rj} \rightarrow \left( \frac{\varphi}{M} \right)^{n_{ij}} H \cdot \overline{\psi}_{Li} \psi_{Rj} \]

**Quarks:**
- hierarchical masses, small mixings: continuous family symmetries
- CKM matrix: small angles and/or alignment of left-handed mixings

\[ U_{CKM} = U_u U_d^\dagger \sim 1 + \mathcal{O}(\lambda) \]

\[ \lambda \sim \frac{\varphi}{M} \]

Wolfenstein parametrization: \( \lambda \equiv \sin \theta_c = 0.22 \)

suggests Cabibbo angle (or some power) as a flavor expansion parameter
**Leptons:** Observed 2 large, 1 small pattern in $\mathbf{U}_{\text{MNSP}} = \mathbf{U}_e \mathbf{U}_{\nu}^\dagger$

most intriguing possibility (for 3-family mixing)

Handwave a bit: in diagonal charged lepton basis

<table>
<thead>
<tr>
<th>Pattern</th>
<th>Condition</th>
<th>Complexity</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 small angles</td>
<td>$\sim$ diagonal $\mathbf{M}_\nu$</td>
<td>(easy)</td>
</tr>
<tr>
<td>1 large, 2 small</td>
<td>$\sim$ Rank$\mathbf{M}_\nu &lt; 3$</td>
<td>(easy)</td>
</tr>
<tr>
<td>3 large angles</td>
<td>“anarchical” $\mathbf{M}_\nu$</td>
<td>(easy)</td>
</tr>
<tr>
<td>2 large, 1 small</td>
<td>fine-tuning, non-Abelian</td>
<td>(harder)</td>
</tr>
</tbody>
</table>

Also suggests new focus: **discrete (non-Abelian)** family symmetries

good for lepton sector, not obviously ideal for quarks...
Proceed by noting that in some limit of flavor symmetry:

\[ \mathcal{U}_{\text{MNSP}} = \mathcal{U}_e \mathcal{U}^\dagger_\nu \sim \mathcal{W} + \mathcal{O}(\lambda') \]

“bare” mixing angles \((\theta_{12}^0, \theta_{13}^0, \theta_{23}^0)\) perturbation

**Main theme: many theoretical starting points!**

Perturbations: useful (and well-motivated in many scenarios) to take

\[ \lambda' = \lambda \equiv \sin \theta_c \]

ideas of “Cabibbo haze” and quark-lepton complementarity (more shortly)

Datta, Everett, Ramond ’05

Raidal ’04, Minakata+Smirnov ’04, many, many others...

within the framework of quark-lepton unification, Cabibbo-sized effects can “leak” into lepton sector
So in the lepton sector, classify models by \( \mathcal{W}(\theta_{12}^0, \theta_{13}^0, \theta_{23}^0) \)

Choose: \( \theta_{23}^0 = 45^\circ \quad \theta_{13}^0 = 0^\circ \) (reasonable*)

Choices for “bare” solar angle \( \theta_{12}^0 \)

- “bimaximal” mixing requires large perturbations \( \theta_{12} = \theta_{12}^0 + \mathcal{O}(\lambda) \)
- “tri-bimaximal” mixing exact, or are there corrections?
- “hexagonal” mixing need moderate perturbations \( \theta_{12} = \theta_{12}^0 + \mathcal{O}(\lambda^2) \)
- “golden ratio” mixing

All obtainable from discrete non-Abelian family symmetries

Recent overview: Albright, Dueck, Rodejohann 1004.2798 (ADR)

Further enumeration of schemes: Rodejohann, Zhang, Zhou ’11

*question: how will perturbations affect reactor angle \( \theta_{13} \)
The dominant paradigm:

**Tri-bimaximal (HPS) Mixing**

"bare" solar angle \( \tan \theta_{12}^0 = \frac{1}{\sqrt{2}} \) \( \theta_{12}^0 = 35.26^\circ \)

\[
U^{(HPS)}_{\text{MNSP}} = \begin{pmatrix}
\sqrt{\frac{2}{3}} & -\frac{1}{\sqrt{3}} & 0 \\
\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}}
\end{pmatrix}
\]

(~Clebsch-Gordan coeffs!)

Meshkov; Zee,...

Readily obtained within many discrete subgroups of \( SO(3), SU(3) \)

\( A_4, S_4, T', \Delta(3n^2), \ldots \)

(100s of papers. Some key players: Ma, Altarelli and Feruglio, King, and many, many, many others

why? HPS via further breakdown to (simple) coset space
Bimaximal Mixing

“bare” solar angle \( \theta_{12}^0 = 45^{\circ} \) \( \tan \theta_{12}^0 = 1 \)

\[
U_{\text{MNSP}}^{(BM)} = \begin{pmatrix}
\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\
\frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\
\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}}
\end{pmatrix}
\]

\( \theta_{12} = \theta_{12}^0 + \mathcal{O}(\lambda) \sim \frac{\pi}{4} - \theta_c \)

“quark-lepton complementarity”

Raidal; Minakata, Smirnov; Frampton, Mohapatra; Xing; Ferrandis, Pakvasa; King; Ramond; Rodejohann, many, many others...

Also obtainable in discrete non-Abelian family symmetry framework.

Large perturbations good for larger reactor angle (T2K hint)?

Recent resurgence in literature. Predict it will continue!!
Other intriguing schemes

Hexagonal Mixing:  

\( \mathcal{U}_{\text{MNSP}}^{(\text{HM})} = \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \)  

\( \theta_{12}^0 = \frac{\pi}{6} \)

dihedral symmetry  
\( \mathcal{D}_{12} \quad \mathcal{D}_6 \quad \text{ADR '10} \)

Golden Ratio Mixing:  

\( \phi = \frac{(1 + \sqrt{5})}{2} \)

GR1. “bare” solar angle  \( \tan \theta_{12} = \phi^{-1} \quad \theta_{12} = 31.72^\circ \)

GR2. “bare” solar angle  \( \cos \theta_{12} = \frac{\phi}{2} \quad \theta_{12} = 36^\circ \)

\( \mathcal{U}_{\text{MNSP}}^{(\text{GR1})} = \begin{pmatrix} \sqrt{\frac{\phi}{\sqrt{5}}} & -\sqrt{\frac{1}{\sqrt{5}\phi}} & 0 \\ \frac{1}{\sqrt{2}} \sqrt{\frac{1}{\sqrt{5}\phi}} & \frac{1}{\sqrt{2}} \sqrt{\frac{\phi}{\sqrt{5}}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \sqrt{\frac{1}{\sqrt{5}\phi}} & \frac{1}{\sqrt{2}} \sqrt{\frac{\phi}{\sqrt{5}}} & \frac{1}{\sqrt{2}} \end{pmatrix} \)

\( \mathcal{U}_{\text{MNSP}}^{(\text{GR2})} = \begin{pmatrix} \frac{\phi}{2} & -\frac{1}{2} \sqrt{\frac{5}{\phi}} & 0 \\ \frac{1}{2} \sqrt{\frac{5}{2\phi}} & \frac{\phi}{2\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} \sqrt{\frac{5}{2\phi}} & \frac{\phi}{2\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \)

GR1: Datta, Ling, Ramond ’05; Kajiyama, Raidal, Strumia ’07; Everett, Stuart ’08, ’11; Feruglio ’11  
GR2: Adulpravitchai, Blum, Rodejohann ’09
Beyond the Reference Picture

Question: theoretical implications of distinct oscillation patterns for $\nu, \bar{\nu}$?

Ideas proposed in previous contexts (e.g. LSND):

- CPT violation (CPTV)
- Lorentz violation (LV)

  effective CPTV via enhanced matter effects due to nonstandard interactions/sterile neutrinos

  effective LV (extra dimensions)

  (decaying) sterile neutrino quantum decoherence...

  Nelson, Walsh ’07,...

  Murayama, Yanagida ’00
  Barenboim et al. ’01,
  Skaudhage ’01, Bilenky et al., ’01, Barger et al. ’03,
  Kostelecky et al ’03, Jacobson, Ohlsson ’03,...

  Pas, Pakvasa, Weiler ’05
  Palomares-Ruiz, Pascoli, Schwetz ’05
  Farzan, Schwetz, Smirnov ’08,...
Beyond the Reference Picture: CPT no longer a fundamental symmetry

CPTV → Lorentz Violation

much attention paid recently to this exciting possibility

e.g. MiniBooNe 1008.0906, Diaz, Kostelecky 1012.5985, many others, lots of press/blog attention,...

Challenge: confining LV to neutrino sector

e.g. braneworld w/ bulk neutrinos + ghost condensation

Mukohyama, Park 1009.1251

question: theoretical motivation?
Beyond the Reference Picture: Effective CPTV

Non-Standard Interactions (NSI) w/ or w/o Sterile $\nu$

Recent overview of NSIs w/o sterile neutrinos:
Kopp, Machado, Parke 1009.0014

Bottom line: can accommodate data, but some tension
NSI interactions must be $\sim$electroweak
must avoid charged lepton NSI's (affects theory embedding)

Sterile neutrinos in eV range plus NSI's:

1+3 scheme +NSI  
Akhmedov and Schwetz 1007.4171

2+3 scheme favored  
Kopp, Maltoni, Schwetz 1103.4570

Can address new reactor neutrino anomaly. challenge for BBN?
New long-range forces as source of NSI’s

Class of models with a **new ultralight Abelian gauge boson** that is **very weakly coupled** (fifth force constraints)

sign of matter effect differs for $\nu, \bar{\nu}$

**gauged symmetry: several examples**

\[
B - L + \text{eV-scale sterile } \nu
\]

\[
L_\mu - L_\tau
\]

\[
B - L_e - 2L_\tau
\]

gauge boson extremely light and weakly coupled, e.g. for last 2

\[
\alpha \sim 10^{-50} \quad M_{Z'} \sim 10^{-18} \text{ eV}
\]

motivation? tension with atmospheric $\nu$
Theoretical Implications of eV-scale Sterile Neutrinos

suggested by LSND, MiniBooNE, reactor neutrino anomaly

\[ n_s \text{ sterile neutrinos: } 3(n_s + 1) \text{ mixing angles} \]
\[ 2n_s + 1 \text{ Dirac phases} \]
\[ n_s + 2 \text{ Majorana phases} \]

Global fits:

\[ n_s = 1 \] “2+2” strongly disfavored, “3+1” tension w/cosmology (3 at eV scale)
1+3 (1 at eV scale) better, but no possibility of CP violation in SBL

\[ n_s = 2 \] “3+2” tension w/cosmology (3 at eV scale), “2+3,” “1+3+1” better
allows for CPV in SBL experiments

Implications:

even 1 eV-scale sterile neutrino can have important impact on \( 0\nu\beta\beta \)
can be implemented within Type I seesaw framework
relatively straightforward to incorporate in non-Abelian flavor models
Conclusions

Neutrino data has taken beyond SM physics theory on a wild ride, with no signs of stopping (may even get wilder!)

Bottom Line:

A number of ways to generate masses/mixings, all with advantages/disadvantages. “Favorites” are not the only options.

Improved data (esp. reactor angle) will greatly aid these efforts!

Challenges have emerged to the reference picture, suggesting new (and perhaps quite exotic) physics. Only hints now, but potentially very exciting if the hints remain with more data. New ideas needed!

Stay tuned!