QCD Theory

DPF 2011

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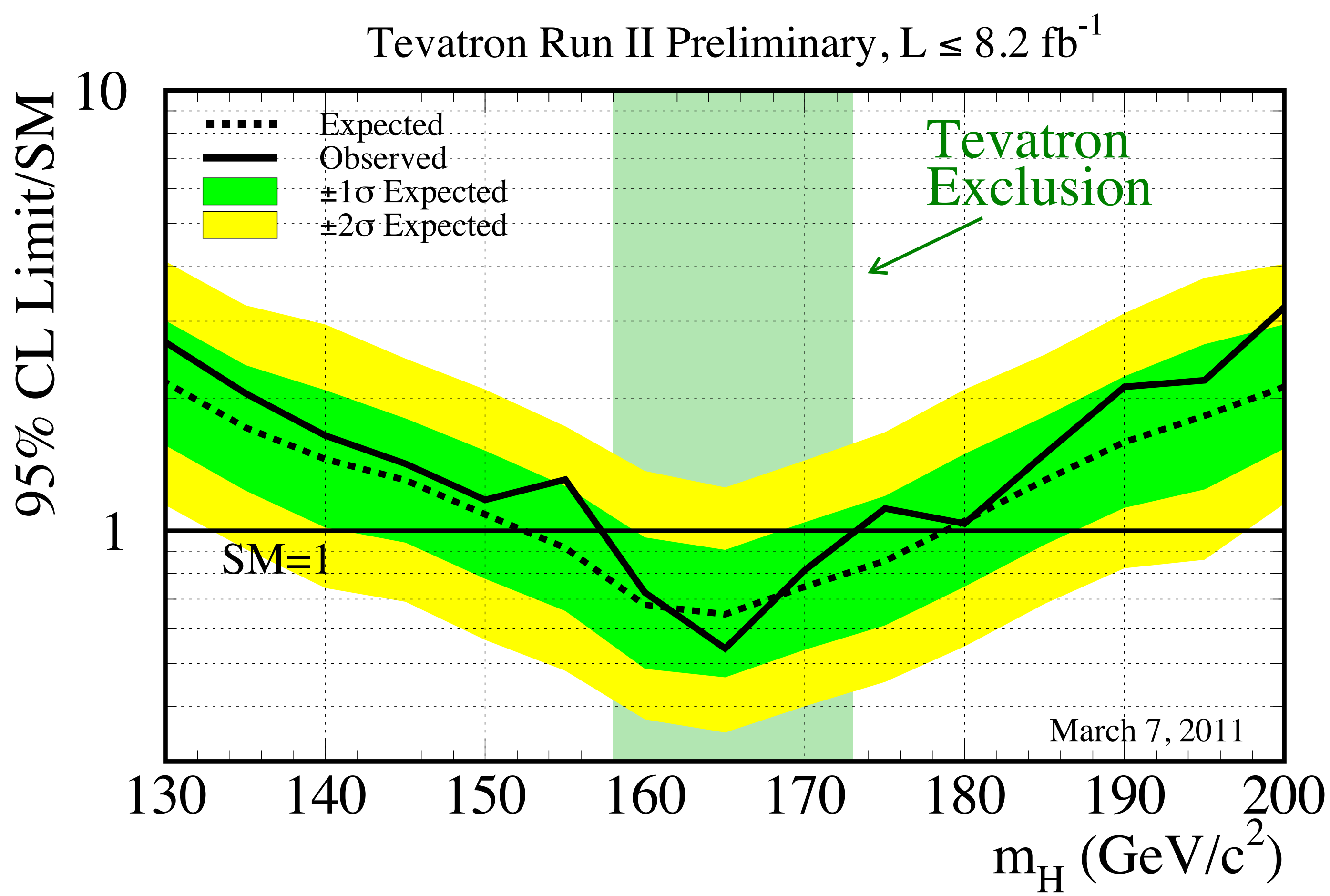
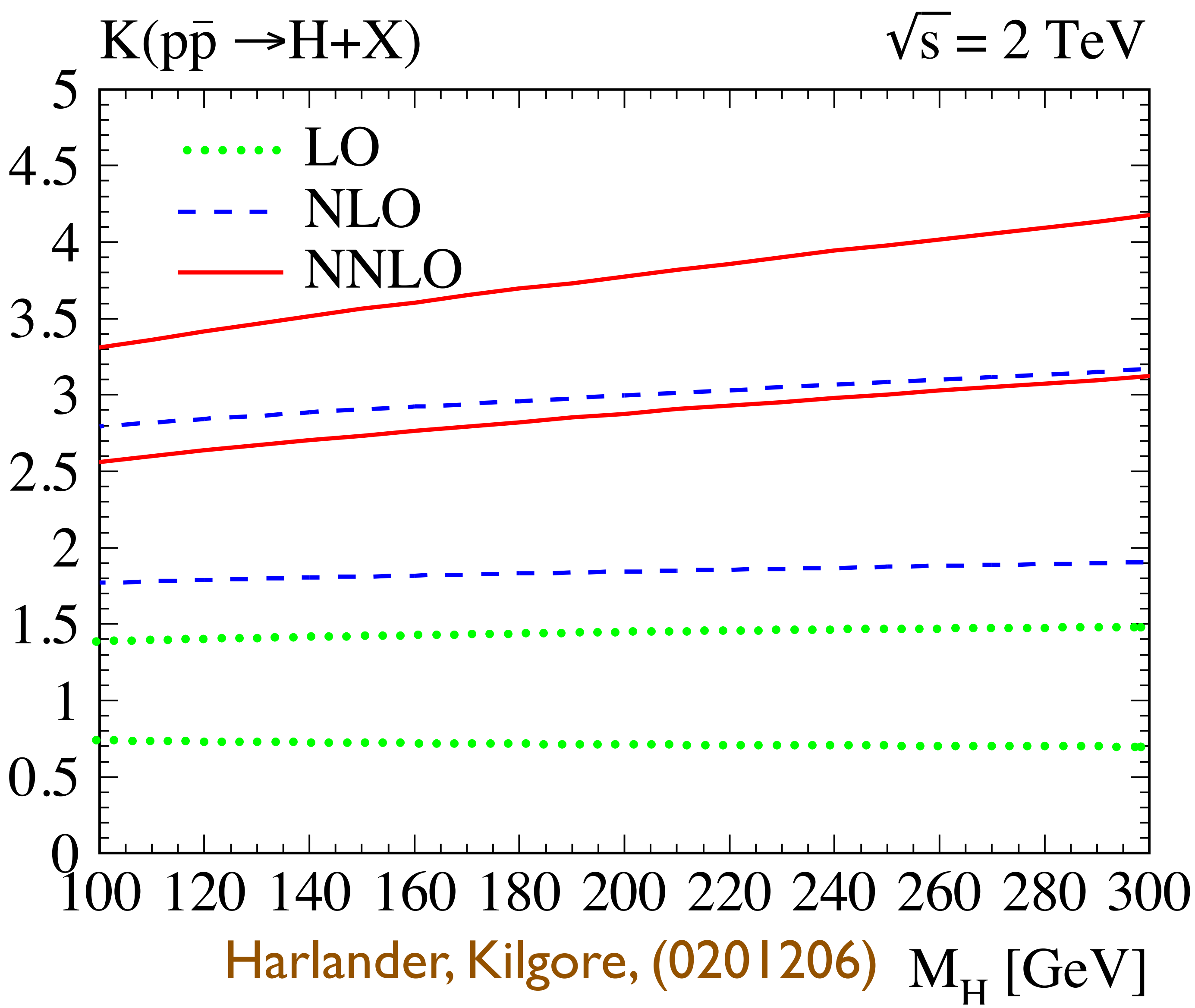
Some important lessons to remember...

... using Higgs search as example



Fixed order calculations

Cross-section for Higgs production known for long time



Harlander, Kilgore, (0201206)

LO order result completely underestimates central value and uncertainties

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Lesson 1:

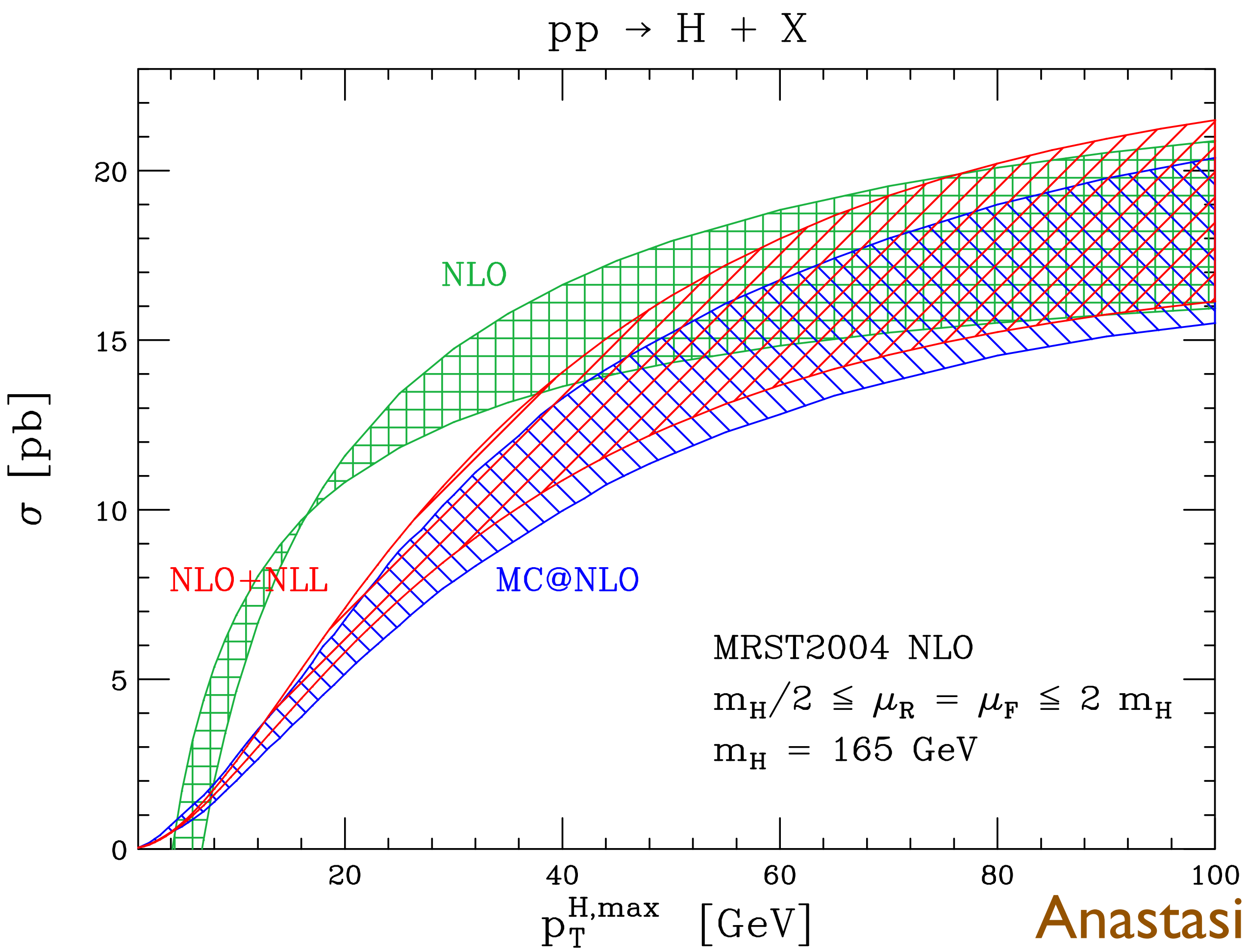
For reliable calculations, need at least NLO results

Large logarithms

Ratio of scales appear as logarithms in perturbative results

For each power of α_s, typically get two powers of logarithms

If large ratios of scales present, should sum these logarithms for well behaved perturbation theory



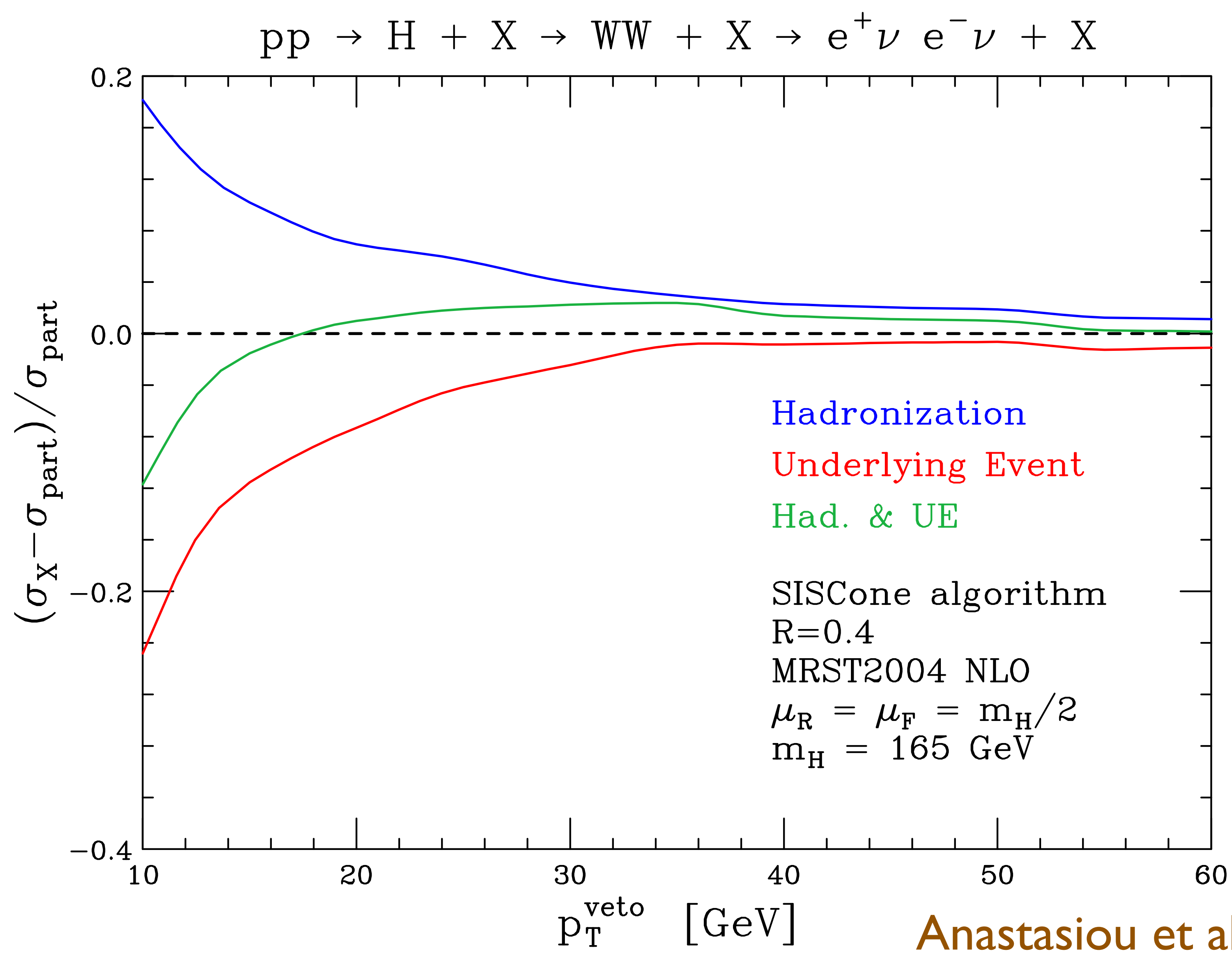
Anastasiou et al (0801.2682)

Lesson 2:

Large ratios of scales lead to logarithms that should be resummed

Hadronic effects

Hadronization and underlying event affects the cross sections



Anastasiou et al (0801.2682)

Can only be estimated using parton shower algorithms

Lesson 3:

Need parton shower simulations to understand long distance hadronic physics

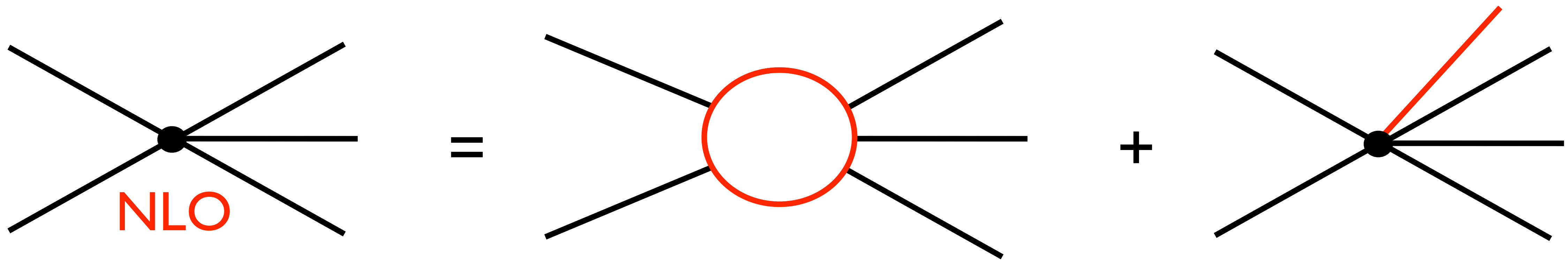
Outline

- Recent progress in NLO calculations
- Large logarithmic terms in perturbative expressions
- Merging fixed order calculations with parton showers

NLO calculations

General Idea

Need to combine one-loop with the real emission



Different complications for loop and real emissions diagrams:

- Loop diagrams require integrals over loop momenta
  - These integrals are divergent
- Real diagrams require integral over extra phase space
  - These integrals are divergent as well

At least part of the integrals have to be performed analytically

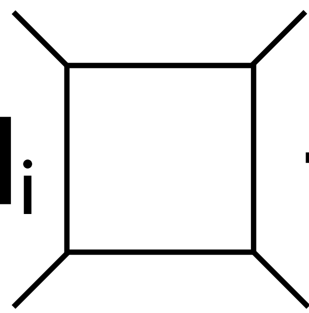
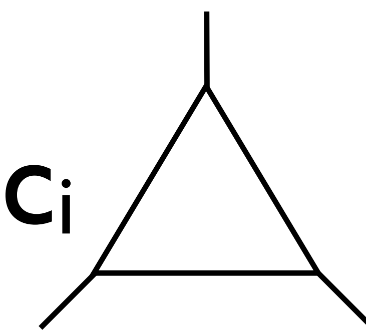
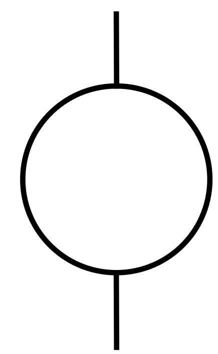
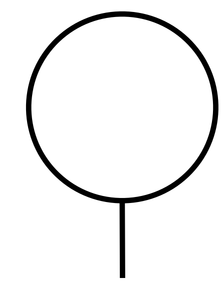
This issue has been solved for arbitrary real diagrams

Frixione, Kunzst, Signer, 9512328; Catani, Seymour, 9605323

Integrating loop diagrams

Generic loop integrals can get very complicated, so impossible to integrate by brute force

Strategy: Reduce to a set of master integrals

$A = \sum d_i$  $+ \sum c_i$  $+ \sum b_i$  $+ \sum a_i$  $+ R$

All master integrals have been calculated analytically

t'Hooft, Veltman (1979); Oldenborg, Vermaseren (19990); Beenakker, Denner, (1990); Bern, Dixon, Kosower (9306240); Duplancic, Nizic, (0006249); ...

All results presented in Ellis, Zanderighi (0712.1851) and available online at http://qcdloop.fnal.gov

Need to determine the coefficients d_i, c_i, b_i, a_i, and R

Obtaining the coefficients

Two different techniques available

Tensor reduction:

Decompose numerator into generic tensor structures, work out coefficients

$$C_1^\mu(p_1,p_2) = \int \frac{d^dk}{(2\pi)^d}\frac{k^\mu}{D_3(p_1,p_2)} \quad = \quad C_1^1 p_1^\mu + C_1^2 p_2^\mu$$

tHooft, Veltman, (1979); Passarino, Veltman, (1979); Dittmaier, Denner, (0509141); ...

Gets complicated for high rank tensors, since spurious divergences give numerical instabilities

Tools using this method:

GOLEM, ...

Generalized unitarity

Use analytical structure on loop amplitudes to work out coefficients

$$\mathcal{A}_1|_{4-cut_i} \quad = \quad d_i\, D0_i|_{4-cut}$$

Bern, Dixon, Forde, Kosower, et al.; Ellis, Giele, Kuzst, Melnikov, Zanderighi; Ossala, Papadopolous, Pittau; Britto, Cachazo, Feng

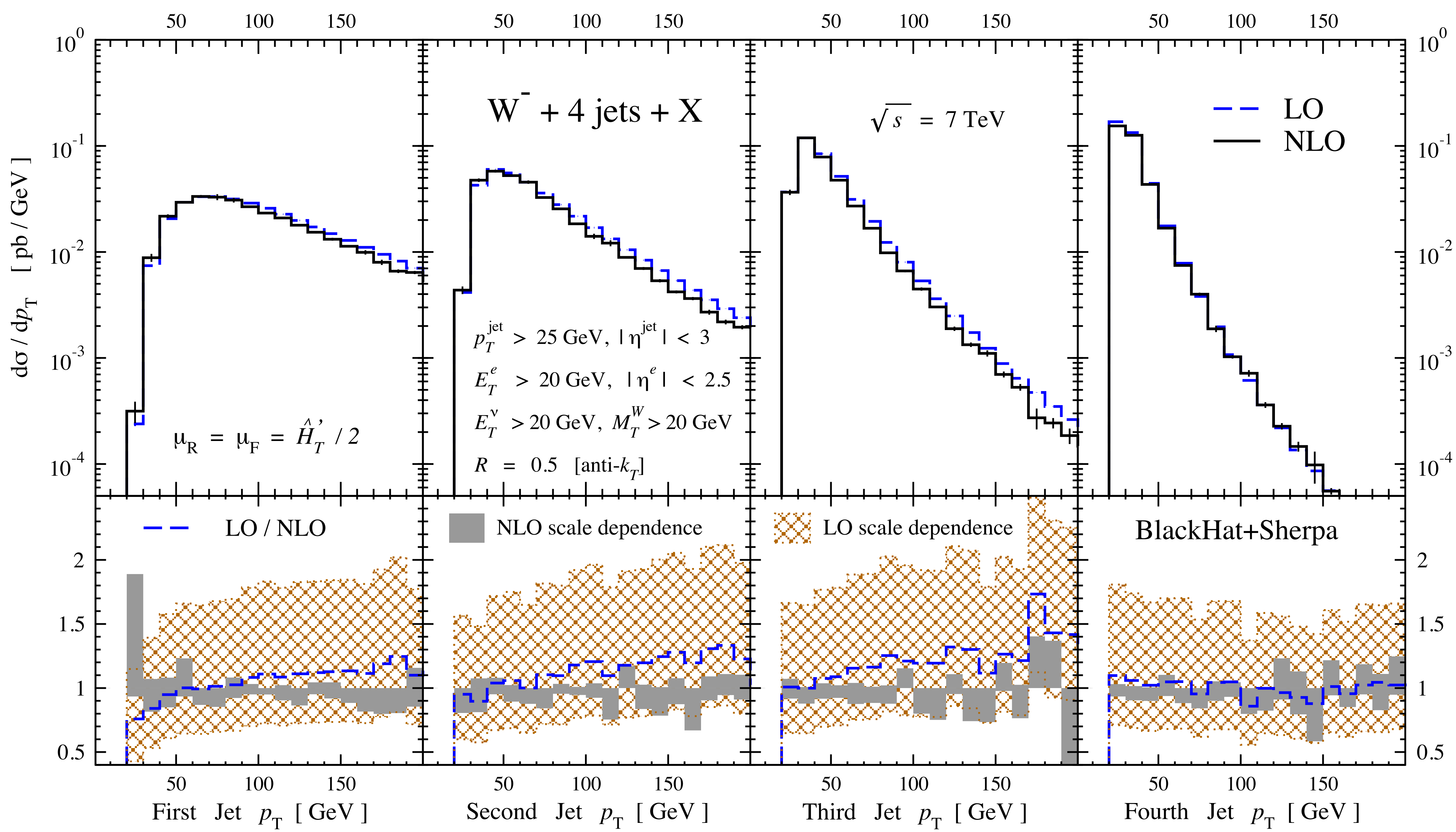
Lends itself to automatization, and this approach has allowed many of the recent new advances

Tools using this method:

Blackhat, Rocket, MadLoop, ...

Recent unitarity result

W+4 jet, first pp→5 process



Blackhat,(1009.2338)

Nobody would have imagined that this is possible 5 years ago

Automatization of NLO

Madgraph is tool that creates code calculating arbitrary LO results

Alwall, Herquet, Maltoni, Mattelaer, Stelzer, ...

	Process	μ	n_{lf}	Cross section (pb) LO	NLO
a.1	$pp \to t\bar{t}$	m_{top}	5	123.76 ± 0.05	162.08 ± 0.12
a.2	$pp \to tj$	m_{top}	5	34.78 ± 0.03	41.03 ± 0.07
a.3	$pp \to tjj$	m_{top}	5	11.851 ± 0.006	13.71 ± 0.02
a.4	$pp \to t\bar{b}j$	$m_{top}/4$	4	25.62 ± 0.01	30.96 ± 0.06
a.5	$pp \to t\bar{b}jj$	$m_{top}/4$	4	8.195 ± 0.002	8.91 ± 0.01
b.1	$pp \to (W^+ \to)e^+\nu_e$	m_W	5	5072.5 ± 2.9	6146.2 ± 9.8
b.2	$pp \to (W^+ \to)e^+\nu_e\, j$	m_W	5	828.4 ± 0.8	1065.3 ± 1.8
b.3	$pp \to (W^+ \to)e^+\nu_e\, jj$	m_W	5	298.8 ± 0.4	300.3 ± 0.6
b.4	$pp \to (\gamma^*/Z \to)e^+e^-$	m_Z	5	1007.0 ± 0.1	1170.0 ± 2.4
b.5	$pp \to (\gamma^*/Z \to)e^+e^-\, j$	m_Z	5	156.11 ± 0.03	203.0 ± 0.2
b.6	$pp \to (\gamma^*/Z \to)e^+e^-\, jj$	m_Z	5	54.24 ± 0.02	56.69 ± 0.07
c.1	$pp \to (W^+ \to)e^+\nu_e b\bar{b}$	$m_W + 2m_b$	4	11.557 ± 0.005	22.95 ± 0.07
c.2	$pp \to (W^+ \to)e^+\nu_e t\bar{t}$	$m_W + 2m_{top}$	5	0.009415 ± 0.000003	0.01159 ± 0.00001
c.3	$pp \to (\gamma^*/Z \to)e^+e^- b\bar{b}$	$m_Z + 2m_b$	4	9.459 ± 0.004	15.31 ± 0.03
c.4	$pp \to (\gamma^*/Z \to)e^+e^- t\bar{t}$	$m_Z + 2m_{top}$	5	0.0035131 ± 0.0000004	0.004876 ± 0.000002
c.5	$pp \to \gamma t\bar{t}$	$2m_{top}$	5	0.2906 ± 0.0001	0.4169 ± 0.0003
d.1	$pp \to W^+W^-$	$2m_W$	4	29.976 ± 0.004	43.92 ± 0.03
d.2	$pp \to W^+W^-\, j$	$2m_W$	4	11.613 ± 0.002	15.174 ± 0.008
d.3	$pp \to W^+W^+\, jj$	$2m_W$	4	0.07048 ± 0.00004	0.1377 ± 0.0005
e.1	$pp \to HW^+$	$m_W + m_H$	5	0.3428 ± 0.0003	0.4455 ± 0.0003
e.2	$pp \to HW^+\, j$	$m_W + m_H$	5	0.1223 ± 0.0001	0.1501 ± 0.0002
e.3	$pp \to HZ$	$m_Z + m_H$	5	0.2781 ± 0.0001	0.3659 ± 0.0002
e.4	$pp \to HZ\, j$	$m_Z + m_H$	5	0.0988 ± 0.0001	0.1237 ± 0.0001
e.5	$pp \to Ht\bar{t}$	$m_{top} + m_H$	5	0.08896 ± 0.00001	0.09869 ± 0.00003
e.6	$pp \to Hb\bar{b}$	$m_b + m_H$	4	0.16510 ± 0.00009	0.2099 ± 0.0006
e.7	$pp \to Hjj$	m_H	5	1.104 ± 0.002	1.036 ± 0.002

Madloop attempts same @ NLO

R. Frederix, S. Frixione, M.V. Garzelli, V. Hirschi, F. Maltoni, R. Pittau, (1103.0621)

- Attempts to bring same easy of use as Madgraph
- Combined with MadFKS to deal with IR divergences
- Currently limited to relatively low number of legs
- No public code available yet

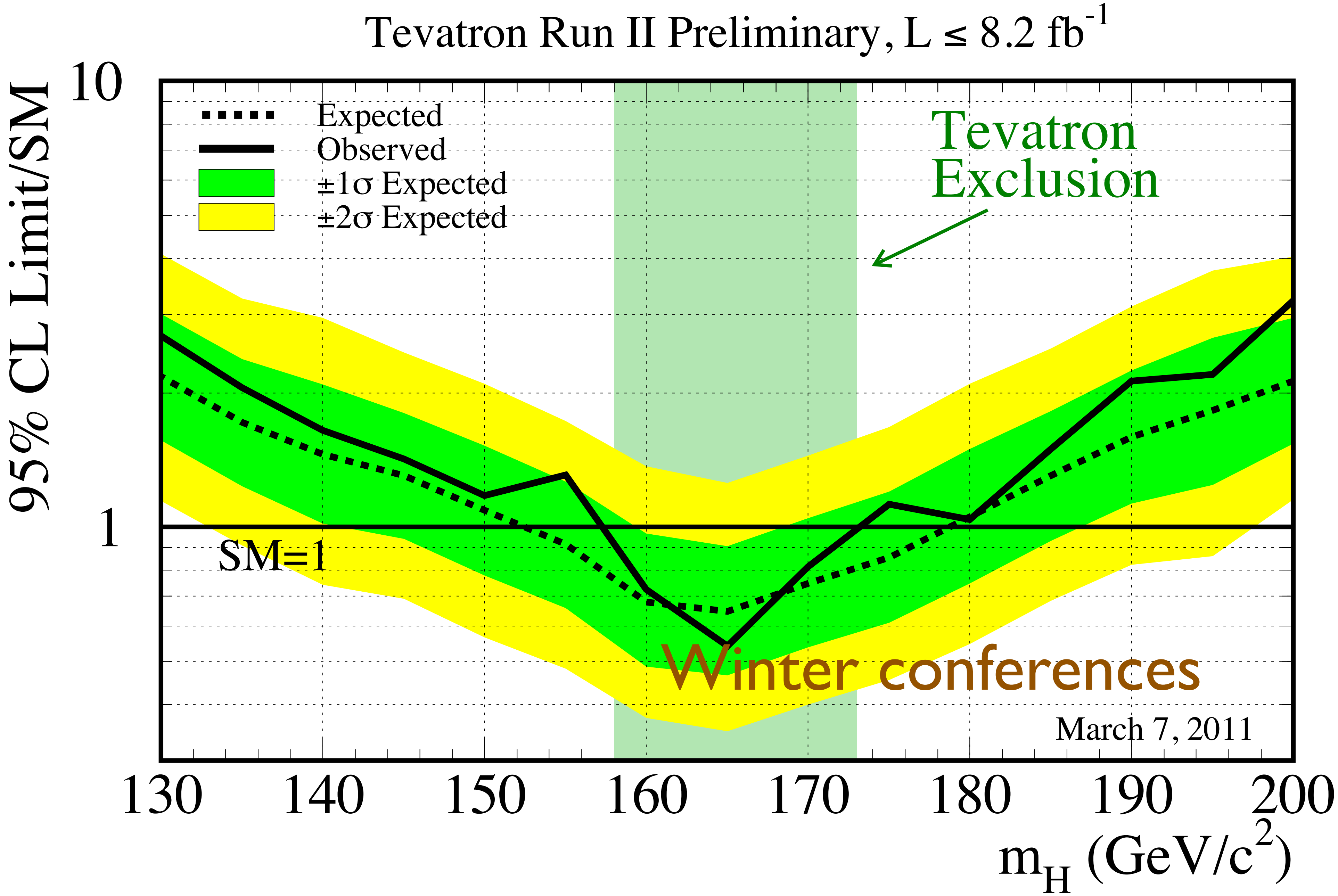
NLO Conclusions

- For any reasonable precision, need at least NLO
- While procedure to calculate well known, we have hit the limit on analytical calculability
- Numerical calculations face issues of divergent integrals
- Tremendous progress over last few years for numerical techniques
- Has allowed results for previously unimaginable processes
- Much progress on complete automatization of NLO

Large logarithms

Searching for the Higgs

Tevatron has sensitivity to Higgs, mainly through $gg \to H \to WW$



Clearly, exclusion depends on the uncertainties assumed

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pp→H→WW→lνlν

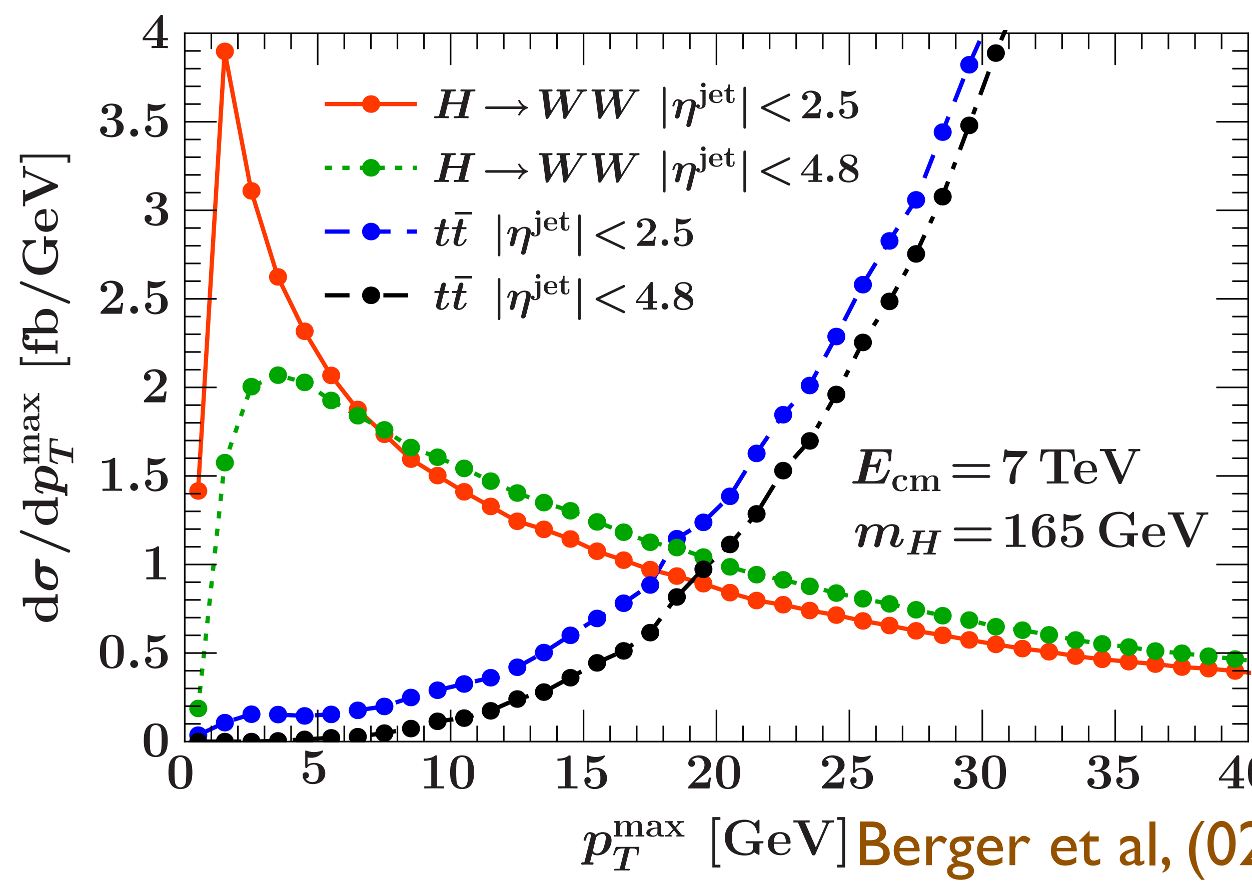
$\sigma_{\rm inc}$ [fb]	LO	NLO	NNLO	$K^{\rm NLO}$	$K^{\rm NNLO}$
$\mu = m_{\rm H}/2$	1.998 ± 0.003	4.288 ± 0.004	5.252 ± 0.016	2.149 ± 0.008	2.629 ± 0.009
$\mu = m_{\rm H}$	1.398 ± 0.001	3.366 ± 0.003	4.630 ± 0.010	2.412 ± 0.002	3.312 ± 0.008
$\mu = 2\,m_{\rm H}$	1.004 ± 0.001	2.661 ± 0.002	4.012 ± 0.007	2.651 ± 0.008	3.996 ± 0.008

Scale variation at NNLO 14%

Anastasiou et al (0801.2682)

Large bkgr is tt production
pp→tt→bbWW→bblνlν

Bottom quarks give rise to extra jet activity



Berger et al, (0201206)

Different amounts of jet activity should be treated differently

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Fixed order predictions

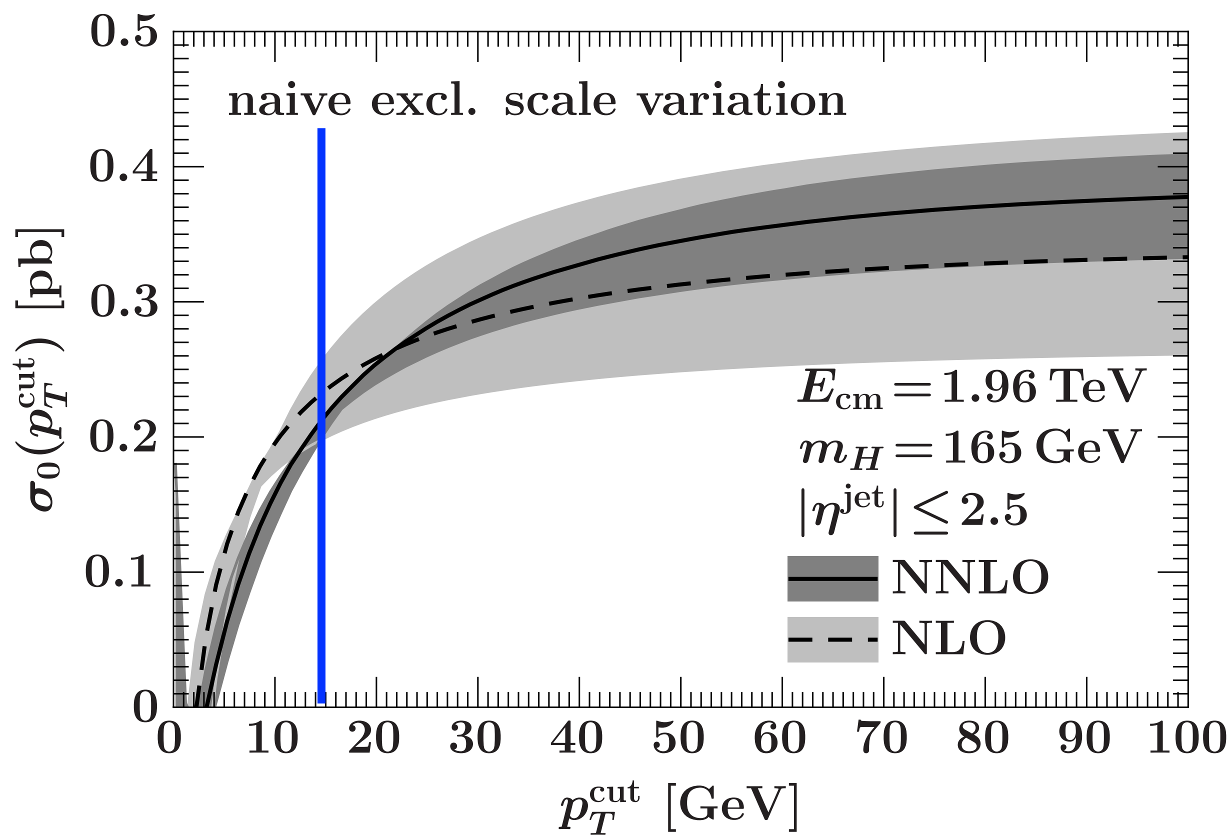
Results used by Tevatron winter analysis

σ [fb]	LO (pdfs, α_s)	NLO (pdfs, α_s)	NNLO (pdfs, α_s)
0-jets	$3.452^{+7\%}_{-10\%}$	$2.883^{+4\%}_{-9\%}$	$2.707^{+5\%}_{-9\%}$
1-jet	$1.752^{+30\%}_{-26\%}$	$1.280^{+24\%}_{-23\%}$	$1.165^{+24\%}_{-22\%}$
$\geq$ 2-jets	$0.336^{+91\%}_{-44\%}$	$0.221^{+81\%}_{-42\%}$	$0.196^{+78\%}_{-41\%}$

Anastasiou et al (0801.2682)

$$\frac{\Delta\sigma_{\text{total}}}{\sigma_{\text{total}}} = 66.5\% \times \begin{pmatrix} +5\% \\ -9\% \end{pmatrix} + 28.6\% \times \begin{pmatrix} +24\% \\ -22\% \end{pmatrix} + 4.9\% \times \begin{pmatrix} +78\% \\ -41\% \end{pmatrix} = \begin{pmatrix} +14\% \\ -14\% \end{pmatrix}.$$

How much can we trust uncertainties in individual channels?



Exclusive x-section has smaller uncertainties than inclusive one?

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The perturbative expansion

Stewart, Tackmann, (1107.2117)

Inclusive cross section has form

$$\sigma_{\rm total} = (0.15\,{\rm pb})\left[1 + 9.0\,\alpha_s + 34\,\alpha_s^2 + \mathcal{O}(\alpha_s^3)\right]$$

poorly behaved series
☐ large K factor

Zero jet cross section has form

$$\sigma_0\,(15\,{\rm GeV}) = (0.15\,{\rm pb})\,\left[1 + 3.7\alpha_s - 2\alpha_s^2\right]$$

seems like much better behaved series

Should remember a simple relation

$$\sigma_0\left(p^{\rm cut}\right) = \sigma_{\rm total} - \sigma_{\geq 1}\left(p^{\rm cut}\right)$$

$$\sigma_{\rm total} \simeq \sigma_B\left[1 + \alpha_s + \alpha_s^2\right]$$

$$\begin{aligned}\sigma_{\geq 1}(p^{\rm cut}) \simeq \sigma_B\big[&\alpha_s(L^2 + L + 1)\\ &+ \alpha_s^2(L^4 + L^3 + L^2 + L + 1)\big]\end{aligned}$$

$$L^2 = \ln^2(p^{\rm cut}/Q)$$

Estimating uncertainties

Stewart, Tackmann, (1107.2117)

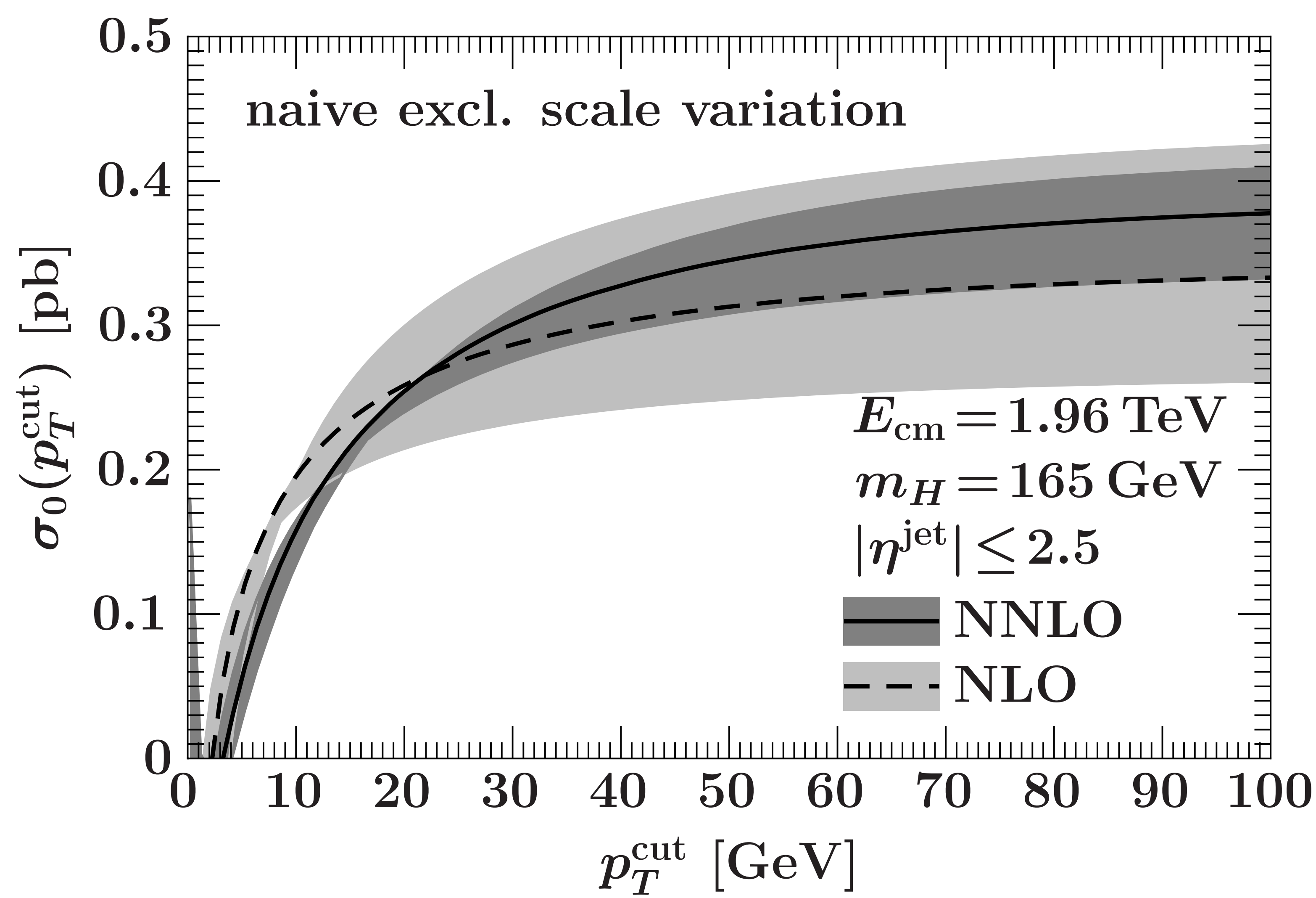
More conservative: Treat σ_{total} and $\sigma_{\geq 1}$ as uncorrelated, with separate uncertainties Δ_{total} and $\Delta_{\geq 1}$

Gives covariance matrix

$$\{\sigma_{\text{total}}, \sigma_{\geq 1}\}$$
$$\begin{pmatrix} \Delta^2_{\text{total}} & 0 \\ 0 & \Delta^2_{\geq 1} \end{pmatrix}$$

which implies that

$$\{\sigma_{\text{total}}, \sigma_0, \sigma_{\geq 1}\}$$
$$\begin{pmatrix} \Delta^2_{\text{total}} & \Delta^2_{\text{total}} & 0 \\ \Delta^2_{\text{total}} & \Delta^2_{\geq 1} + \Delta^2_{\text{total}} & -\Delta^2_{\geq 1} \\ 0 & -\Delta^2_{\geq 1} & \Delta^2_{\geq 1} \end{pmatrix}$$



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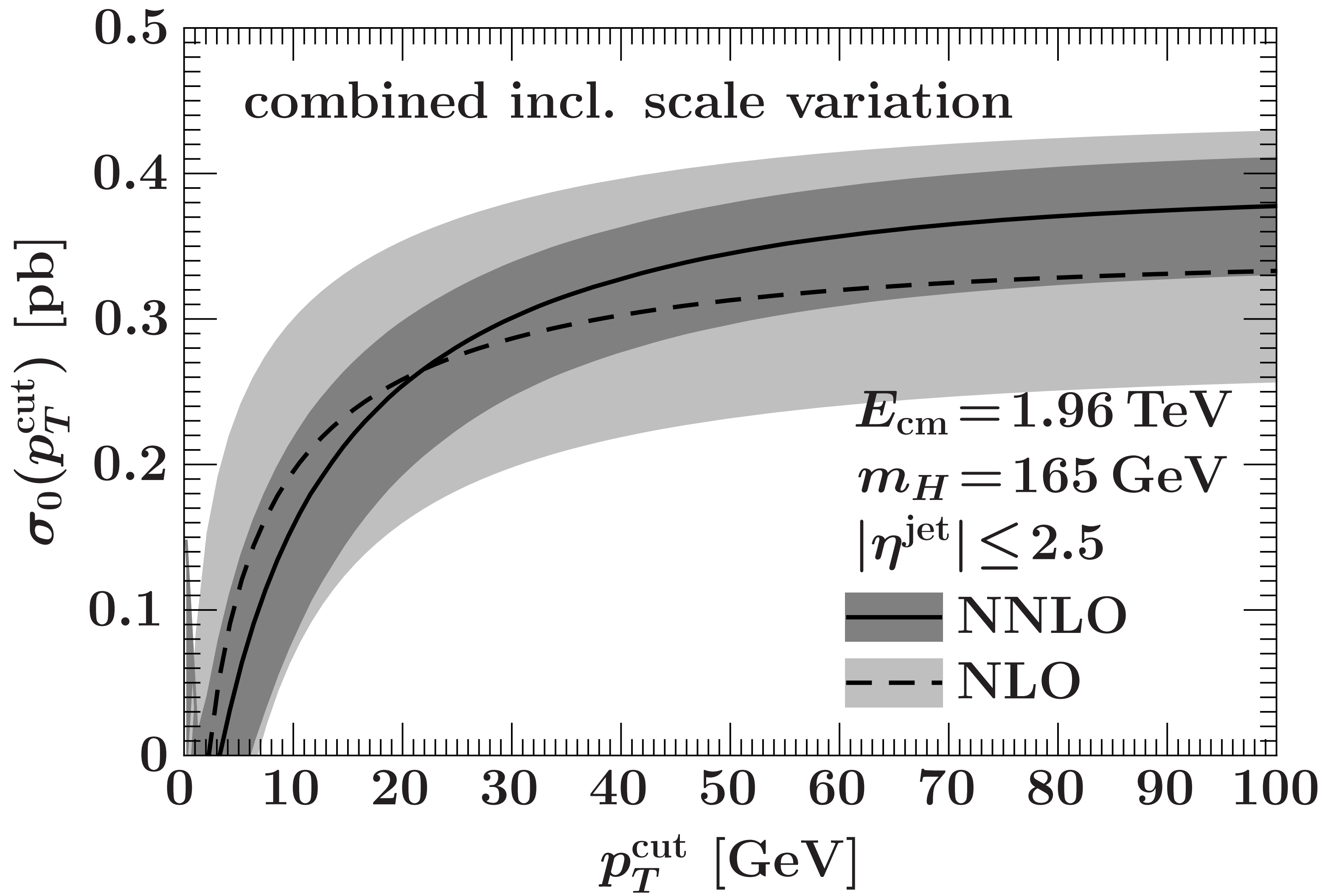
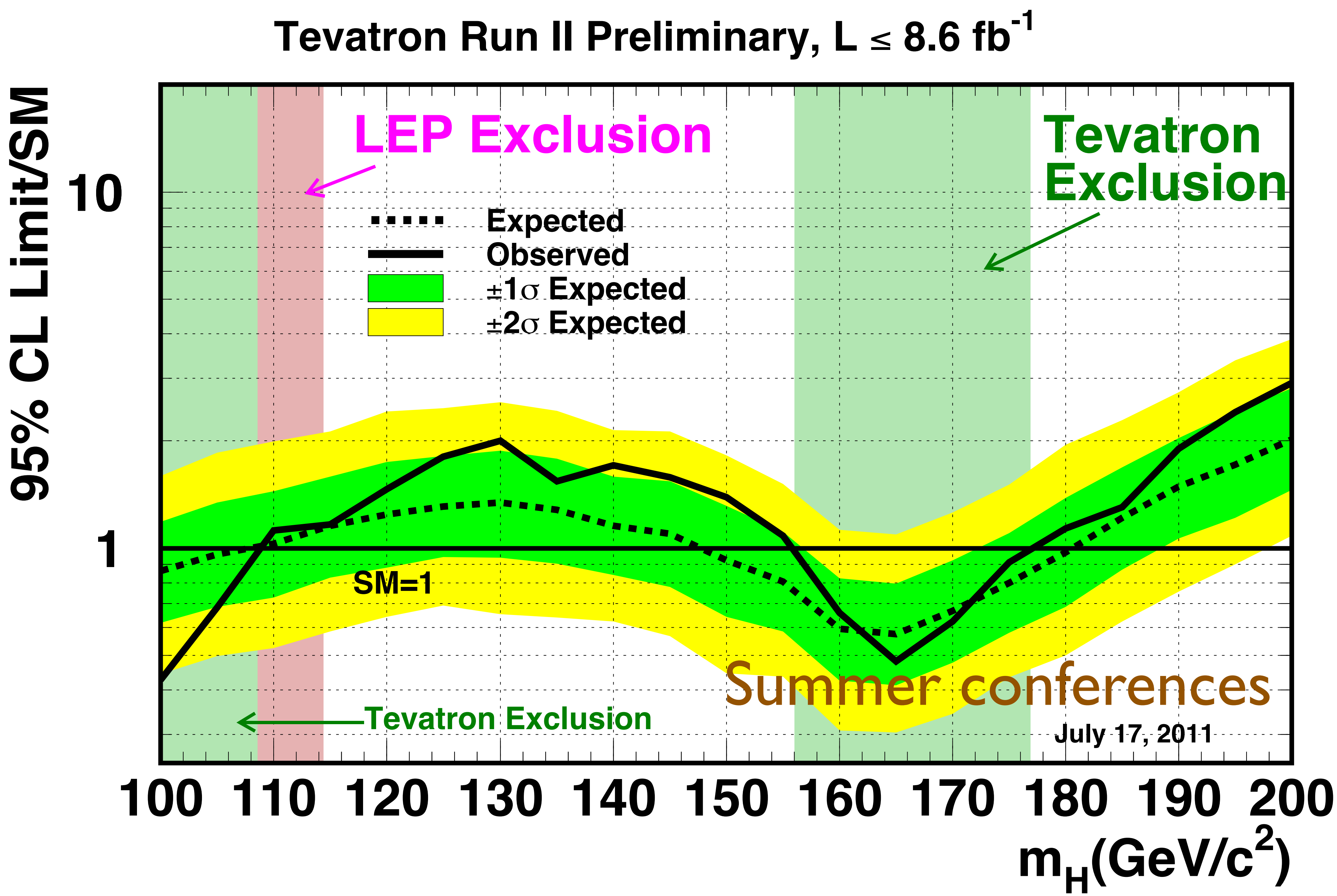


Effect on Tevatron analysis?

Procedure is used in latest Tevatron combinations



Used smaller overall uncertainties for inclusive cross-section

Log Conclusions

- Large ratios of scales ⇒ large logs in perturbative series
- Poorer convergence of series
- Gives rise to larger perturbative uncertainties
- Important to treat this effect carefully

Merging of fixed order calculations with parton showers

The need for merging

To combine predictions @ short distances with important long distance physics, need to merge fixed order calcs and parton showers

Problem is solved for LO accuracy ...

Catani, Krauss, Kuhn, Webber (0109231)

... and implemented in many computer codes

Alpgen, Madgraph, Sherpa, ...

Has been completely automated

For many analyses this is now standard theoretical tool

Idea behind merging

Φ	
$\vdots$	
n+3-jets	
n+2-jets	
n+1-jets	
n-jets	

Idea behind merging

Φ	
$\vdots$	
n+3-jets	
n+2-jets	
n+1-jets	
n-jets	FO

Idea behind merging

Φ	
$\vdots$	$\vdots$
n+3-jets	PS
n+2-jets	PS
n+1-jets	PS
n-jets	FO

Idea behind merging

Φ	S_1
⋮	⋮
n+3-jets	PS
n+2-jets	PS
n+1-jets	PS
n-jets	FO

Idea behind merging

Φ	S_1	
⋮	⋮	
n+3-jets	PS	
n+2-jets	PS	
n+1-jets	PS	FO
n-jets	FO	

Idea behind merging

Φ	S_1	
⋮	⋮	⋮
n+3-jets	PS	PS
n+2-jets	PS	PS
n+1-jets	PS	FO
n-jets	FO	

Idea behind merging

Φ	S_1	S_2
⋮	⋮	⋮
n+3-jets	PS	PS
n+2-jets	PS	PS
n+1-jets	PS	FO
n-jets	FO	

Idea behind merging

Φ	S_1	S_2	S_3
⋮	⋮	⋮	⋮
n+3-jets	PS	PS	PS
n+2-jets	PS	PS	FO
n+1-jets	PS	FO	
n-jets	FO		

Idea behind merging

Φ	S_1	S_2	S_3	S_4
⋮	⋮	⋮	⋮	⋮
n+3-jets	PS	PS	PS	FO
n+2-jets	PS	PS	FO	
n+1-jets	PS	FO		
n-jets	FO			

Idea behind merging

Φ	S_1	S_2	S_3	S_4
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
n+3-jets	PS	PS	PS	FO
n+2-jets	PS	PS	FO	
n+1-jets	PS	FO		
n-jets	FO			

Want to combine all samples together

Idea behind merging

Φ	S_1	S_2	S_3	S_4
⋮	⋮	⋮	⋮	⋮
n+3-jets	PS	PS	PS	FO
n+2-jets	PS	PS	FO	
n+1-jets	PS	FO		
n-jets	FO			

Want to combine all samples together

Main Issue:

Avoid double counting:
Ensure that sum of all samples
S_i still gives right answer for m>n jets

Two different implementations possible...

Idea behind merging

Φ	S_1	S_2	S_3	S_4
⋮	⋮	⋮	⋮	⋮
n+3-jets	PS	PS	PS	FO
n+2-jets	PS	PS	FO	
n+1-jets	PS	FO		
n-jets	FO			

Want to combine all samples together

Main Issue:

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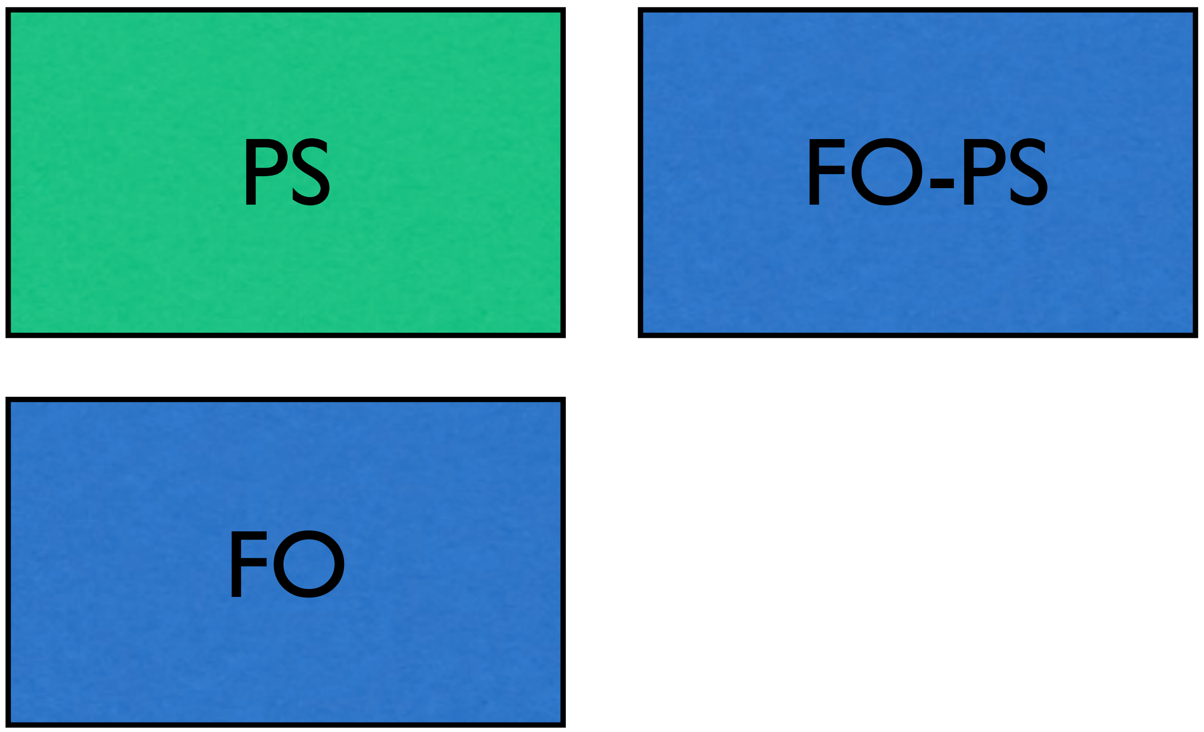
Two different implementations possible...

Two main choices

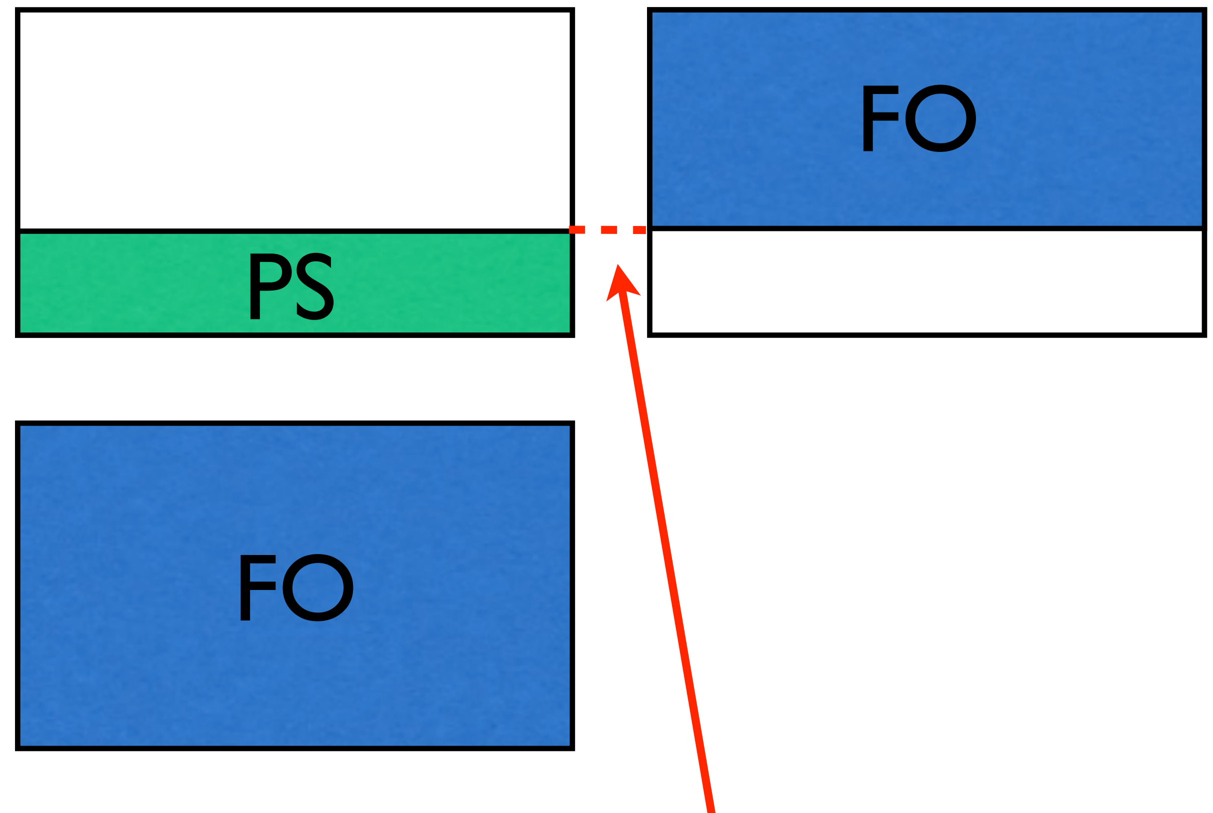
Subtraction



Need to know analytical expressions for parton shower

LO-PS can be negative!

Phase space separation



Need to be independent of arbitrary phase space separation

More difficult to sum correct logs

Merging at LO

As already mentioned, worked out at LO 10 years ago

Catani, Krauss, Kuhn, Webber (0109231); Alpgen, Madgraph, Sherpa,

Φ	S_1	S_2	S_3	S_4
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
n+3-jets	PS	PS	PS	LO
n+2-jets	PS	PS	LO	
n+1-jets	PS	LO		
n-jets	LO			

All approaches use phase space separation

Merging at NLO

NLO methods available since

Φ	S_1	S_2
$\vdots$	$\vdots$	$\vdots$
n+3-jets	PS	PS
n+2-jets	PS	PS
n+1-jets	PS	LO
n-jets	NLO	

<u>Subtraction:</u>

MC@NLO

<u>Phase space separation:</u>

POWHEG

Both Subtraction and phase space separation used

General framework to implement new processes (POWHEG Box)

Ongoing work to completely automate process (aMC@NLO)

Merging at LO

Recent ideas allow to combine this with LO at higher multiplicity

Φ	S_1	S_2	S_3	S_4
⋮	⋮	⋮	⋮	⋮
n+3-jets	PS	PS	PS	LO
n+2-jets	PS	PS	LO	
n+1-jets	PS	LO		
n-jets	NLO			

CWB, Tackmann, Thaler (0801.4026),
Hamilton, Nason (2010)
Höche, Krauss, Schönherr, Siegert (2010)

Merging at LO

Ongoing work to have NLO for all multiplicities

CWB, with Geneva collaboration

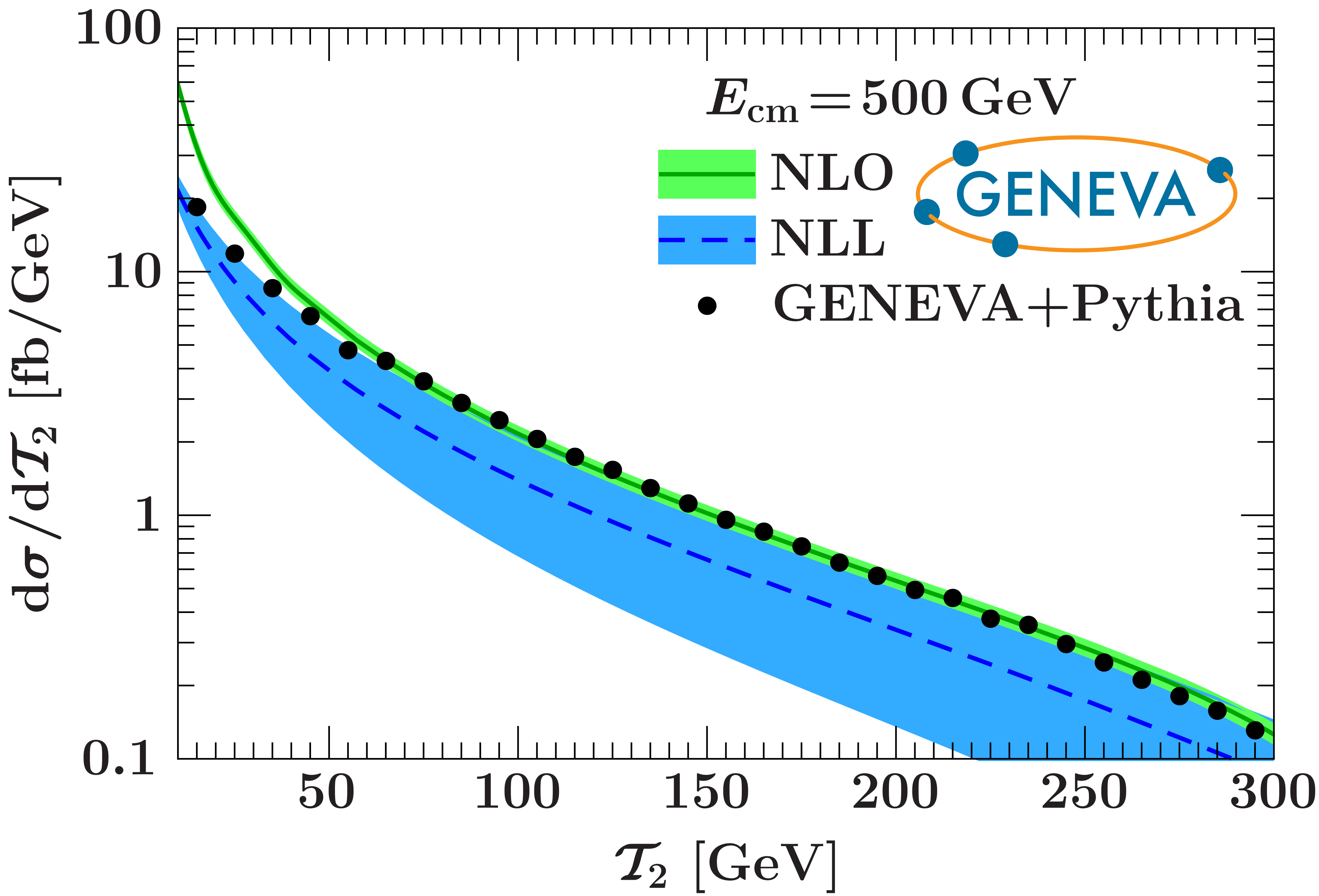
Φ	S_1	S_2	S_3	S_4
⋮	⋮	⋮	⋮	⋮
n+3-jets	PS	PS	PS	NLO
n+2-jets	PS	PS	NLO	
n+1-jets	PS	NLO		
n-jets	NLO			

Made possible through applications of SCET to event generation

CWB, Fleming, Luke, Pirjol Stewart;
CWB, Schwartz (0607296)

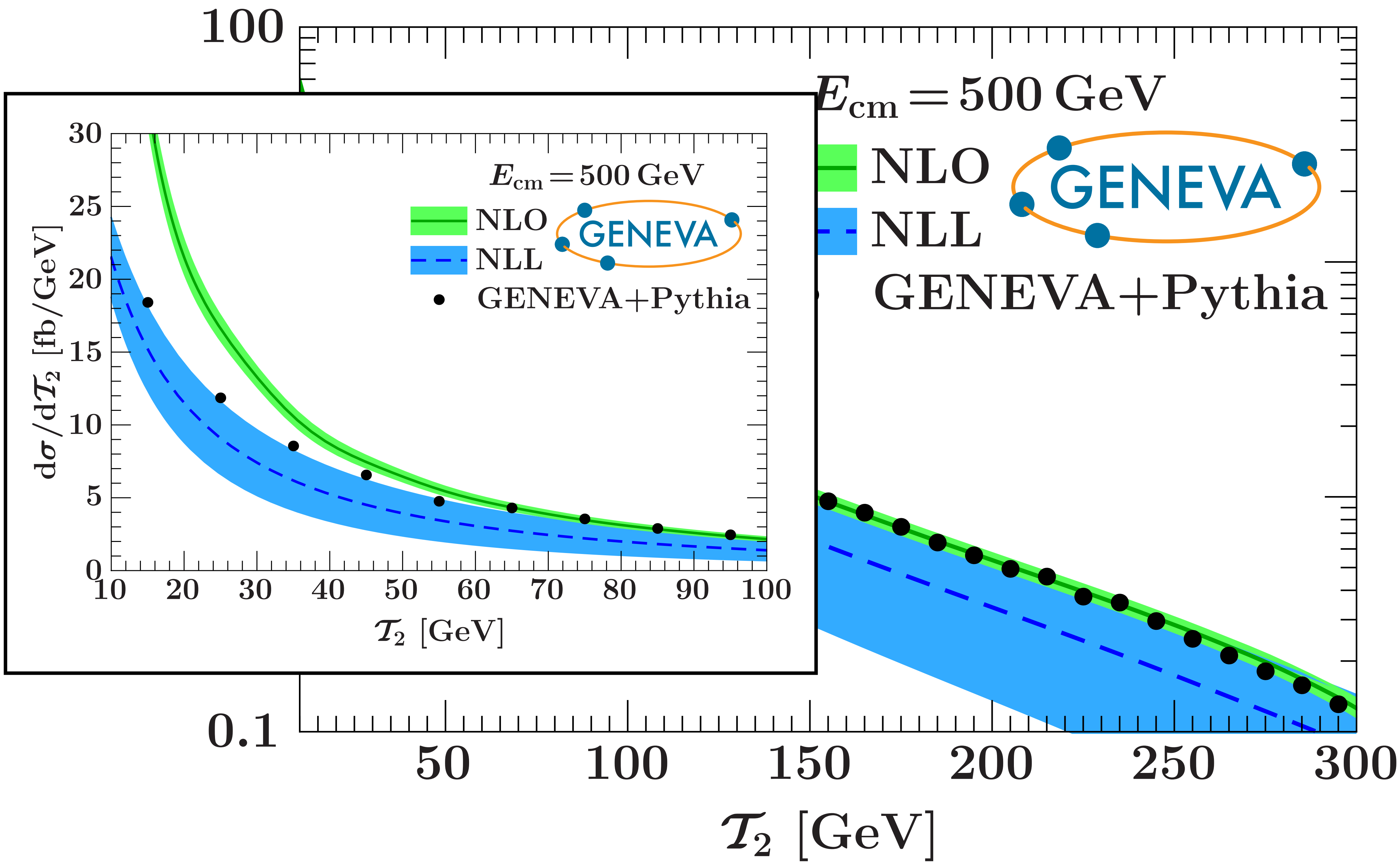
Preliminary Geneva

Event shape in $e^+e^- \to$jets, with both 2j and 3j final states correct to NLO



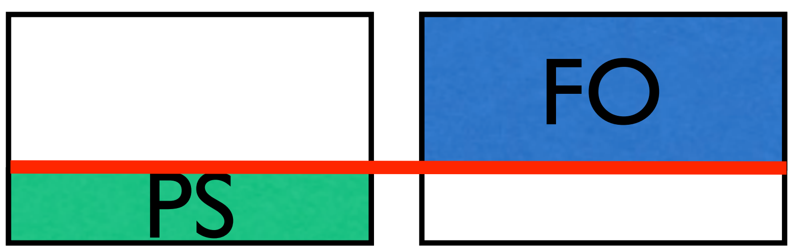
Preliminary Geneva

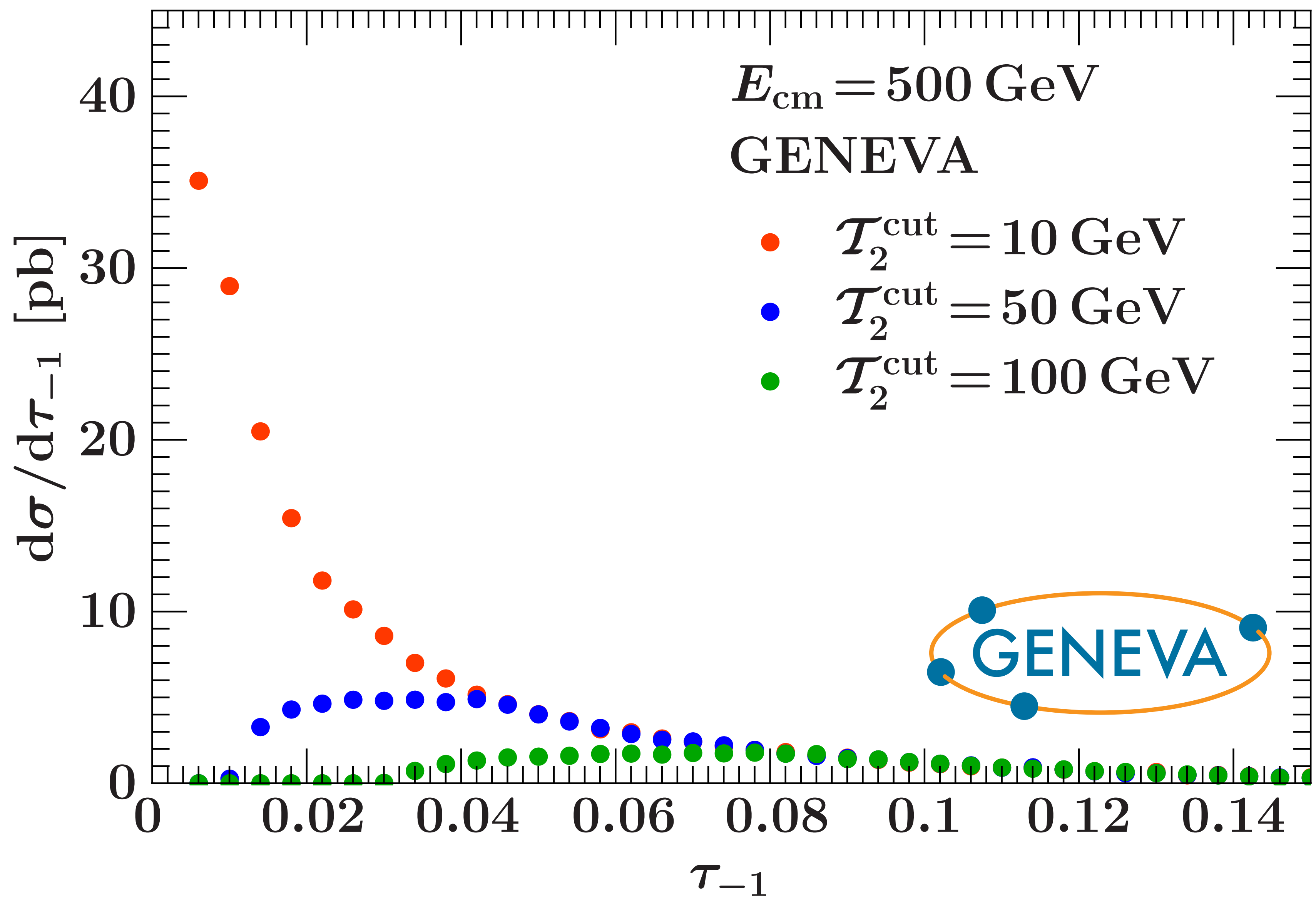
Event shape in $e^+e^- \to$ jets, with both 2j and 3j final states correct to NLO



Preliminary Geneva

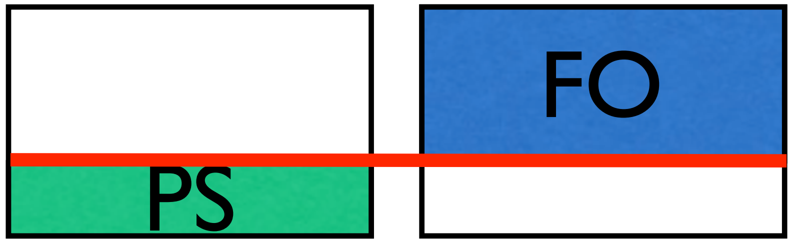
Can also study the cancellation of the dependence on merging scale

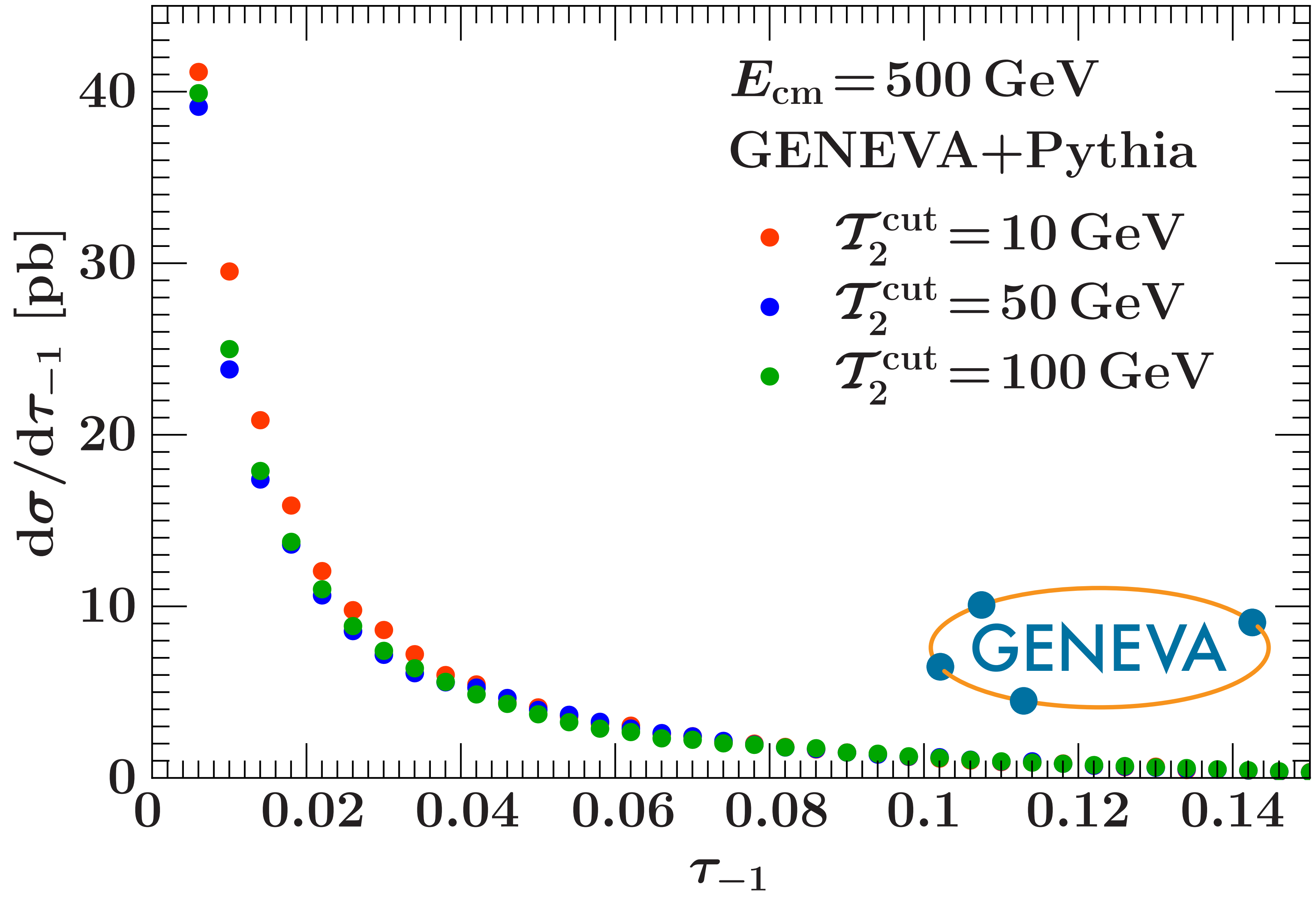




Preliminary Geneva

Can also study the cancellation of the dependence on merging scale





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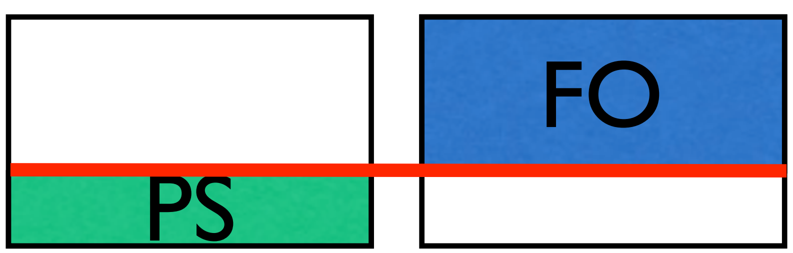
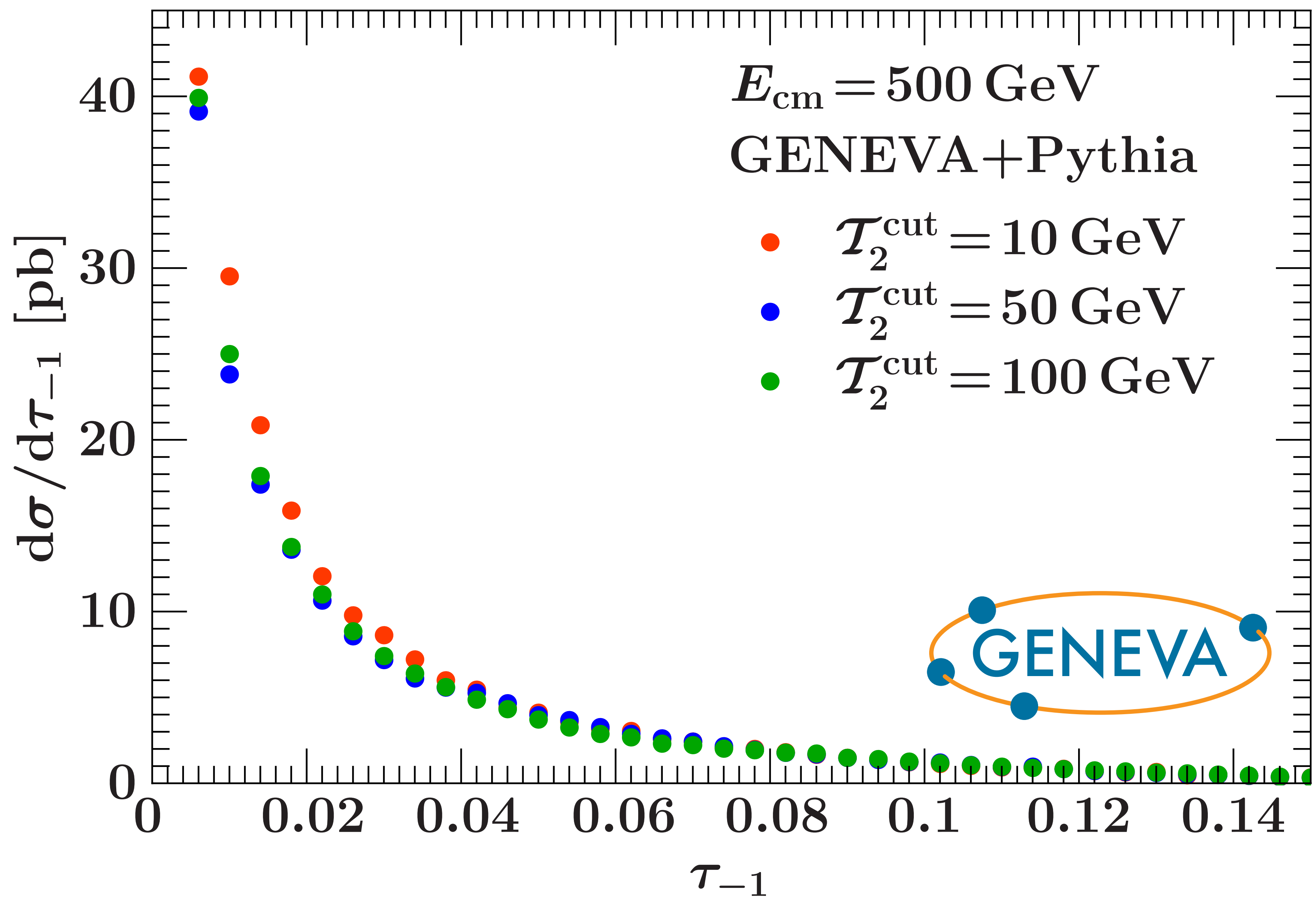


Proper cancellation once PS is added



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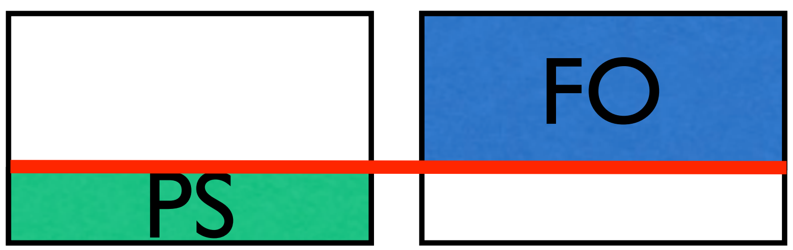

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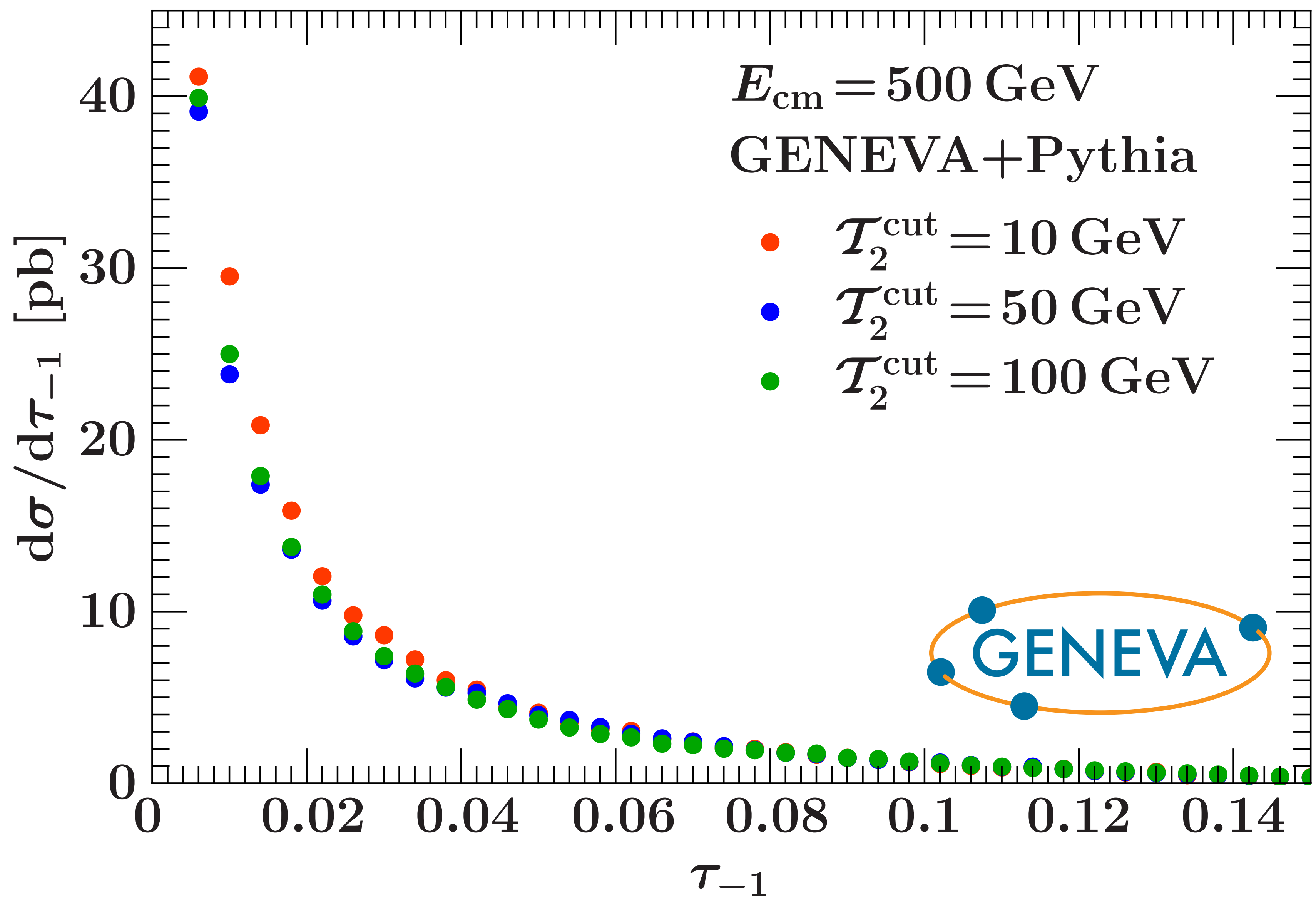

All above easily generalized to pp collisions, in particular W+0 and W+1 jet production

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Merging Conclusions

- Much progress in recent years on merging fixed order calculations with parton shower algorithms
- Merging of LO calculations completely standard
- Merging of NLO calculation getting quite mature
- Next goal is to find general way to combine multiple NLO calculations

Final Thoughts

QCD is a fascinating theory, which needs many different theoretical approaches

The LHC has given theorists a new problems to think about

Have only been able to show you a small selection of progress made in past years