Recent Progress in Field and String Theory

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Some Disclaimers

- My research for the last decade has focused almost entirely on something called the AdS/CFT correspondence. All the "progress" that I describe today has some not always explicit connection with AdS/CFT.
- There has been great progress this year in understanding supersymmetric gauge theories. Omitted due to time constraints and the interests of my audience.
- There exist active communities trying to derive the standard model from string theory and trying to understand the physics of the early universe using string theory. Omitted due to my ignorance of their work.
- My apologies to those of you who feel I should have included different topics.

The Four Things I will Talk About

- Progress in scattering amplitudes which some hope may eventually lead to a reformulation of perturbation theory.
- c-like theorems and the irreversibility of renormalization group flows.
- Using AdS/CFT to understand strongly interacting field theories with real world applications to superconductors, superfluids, quark-gluon plasma, ...
- Higher spin theories and a better understanding of AdS/CFT itself.

The slogan: $2011 = 1986 + 25$

Scattering Amplitudes

Figure: Toyota S Matrix '11

What's the big deal about scattering amplitudes?

During the past few years, an enormous effort has gone into calculating the S-matrix of maximally supersymmetric Yang-Mills theory ($\mathcal{N} = 4$ SYM) in 3+1 dimensions. Why?

- Because of its large symmetry group and its dual description as a string theory via AdS/CFT, some argue that it is the harmonic oscillator of field theories.
- New methods exist that make amplitude calculations far easier than with old fashioned Feynman diagrams. Some argue that we are on the verge of a better formulation of perturbation theory.
- These same methods can be and have been extended to other possibly more interesting theories, for example $\mathcal{N} = 8$ supergravity and QCD.

Some notation

As noticed originally by Parke and Taylor (1986), gluon scattering amplitudes are more compactly expressed in a spinorial, color stripped, on-shell form.

$$\mathcal{A}(p_i, \epsilon_i) \to \mathcal{A}(\lambda_i, \tilde{\lambda}_i, h_i) \quad , \quad h_i = \pm \ .$$

Positive and negative chirality spinors: $\lambda^a, \tilde{\lambda}^{\dot{a}}$ where $a, \dot{a} = 1, 2$.
Momentum:

$$\lambda_a \tilde{\lambda}_{\dot{a}} = p_{a\dot{a}} = \sigma^{\mu}_{a\dot{a}} p_\mu \qquad \text{where} \qquad p_\mu p^\mu = \det(p_{a\dot{a}}) \ .$$

Positive and negative helicity polarization tensors:

$$\epsilon_{a\dot{a}} = \frac{\lambda_a \tilde{\mu}_{\dot{a}}}{[\tilde{\lambda}, \tilde{\mu}]} \qquad \text{and} \qquad \tilde{\epsilon}_{a\dot{a}} = \frac{\mu_a \tilde{\lambda}_{\dot{a}}}{\langle \mu, \lambda \rangle} \ .$$

$$\langle 12 \rangle = \langle \lambda_1, \lambda_2 \rangle = \epsilon_{ab} \lambda_1^a \lambda_2^b \ ,$$

$$[12] = [\tilde{\lambda}_1, \tilde{\lambda}_2] = \epsilon_{\dot{a}\dot{b}} \tilde{\lambda}_1^{\dot{a}} \tilde{\lambda}_2^{\dot{b}} \ .$$

The simplifications continue

- Before the spinor notation, tree level gluon scattering amplitudes took pages and pages to write down.
- With spinor notation, things got much simpler

$$\mathcal{A}(+-+-) = \frac{\langle 24 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 41 \rangle} \ .$$

- Using these and more recent developments, the tree level S-matrix for $\mathcal{N} = 4$ SYM has now been calculated (Drummond and Henn, 2008).
- With the unitarity cut construction, we can sew together tree amplitudes to get on-shell loop amplitudes.
- By a specialization of the Kawai, Llewellyn, and Tye (1986) results for open and closed string scattering amplitudes, supergravity amplitudes are essentially squares of SYM amplitudes.

What happened this year for scattering

- Continuing improvements in methodology
  - Tree level scattering amplitudes as volumes of polytopes (Arkani-Hamed, Bourjaily, Cachazo, Hodges, Trnka)
  - Recursion relations for planar loop integrands (Arkani-Hamed, Boels, Bourjaily, Cachazo, Caron-Huot, Trnka)
  - Simplification of loop amplitudes using high tech identities for polylogs (Goncharov's symbol) (Goncharov, Spradlin, Vergu, Volovich)
- $\mathcal{N} = 8$ supergravity is finite at $L < 7$ loops because of an $E_{7(7)}$ symmetry. (Beisert, Elvang, Freedman, Kiermaier, Morales, Stieberger). Could it be perturbatively finite?

Renormalization Group Flows and c-like Theorems

The renormalization group plays a central role in physics.
Can an RG flow be reversible, periodic?

Zamolodchikov's (1986) c-theorem states that for QFTs in two dimensions, there exists a positive definite real function $c(g)$ on the space of couplings g_i, which satisfies the following three properties:

- $c(g)$ is monotonically decreasing along RG flows.
- $c(g)$ is stationary at RG fixed points.
- At RG fixed points, $c(g)$ equals the central charge of the corresponding CFT.

RG flows in two dimensions are irreversible.
Is there an analog statement in higher dimensions?

The a-theorem in $d = 4$

- In 2d, one way of defining c is through the trace anomaly of the stress tensor of the CFT

$$\langle T^i{}_i \rangle = -\frac{c}{12} R \ .$$

- In $d = 4$, we have instead

$$\begin{aligned} \langle T^i{}_i \rangle &= \frac{c}{16\pi^2} W_{ijkl} W^{ijkl} \\ &\quad - \frac{a}{16\pi^2} (R_{ijkl} R^{ijkl} - 4 R_{ij} R^{ij} + R^2) - \frac{a'}{16\pi^2} \nabla^2 R \ . \end{aligned}$$

  W_{ijkl} and R_{ijkl} are the Weyl and Riemann curvature tensors respectively.

- Cardy (1988) conjectured a should have similar properties in $d = 4$. Komargodski and Schwimmer have now produced a proof.

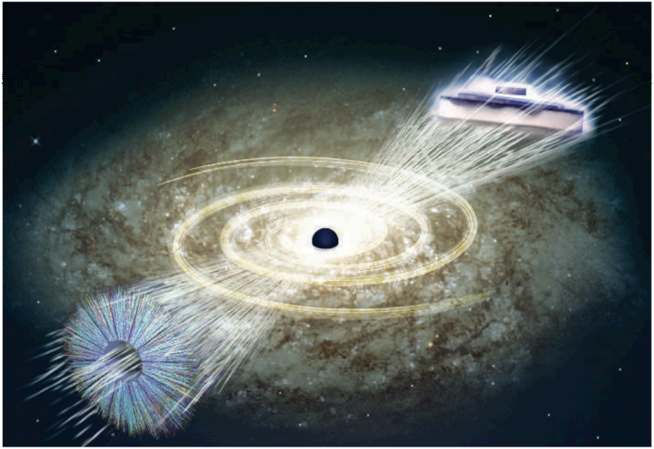
What happens in $d = 3$?

- There is no trace anomaly in odd dimensions.
- Myers and Sinha have proposed that entanglement entropy may play the role of c.
- Jafferis, Klebanov, Pufu, Safdi have proposed an F-theorem, that

  $$F = -\log |Z_{S^3}|$$

  may play the role of c. Casini, Huerta, and Myers have demonstrated this proposal is equivalent to the entanglement entropy proposal.
- No proof exists yet, but many checks have been carried out (e.g. Amariti and Siani or Gulotta, Herzog, and Pufu).
- For $d > 4$ and d non-integer, the situation remains less clear.

AdS/CFT and the Real World



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AdS/CFT and the Real World

There continues to be a large effort to use the AdS/CFT correspondence to understand strongly interacting systems from their dual classical gravity descriptions. In the past few years, targets have broadened from QCD like theories to include systems in condensed matter physics.

- AdS/CMT: superconductors, superfluids, heavy fermions, quantum Hall effect, ...
- AdS/QCD: confinement, chiral symmetry breaking, quark-gluon plasma, ...

Let me review some facts about AdS/CFT and string theory first.

Bednorz and Müller discovered the first high T_c superconductor in 1986.

Basic facts about string theory

- There are open strings (strings with end points) and closed strings (loops).
- Strings may split with a likelihood g_s.
- Strings have a tension $T_0 = 1/\ell_s^2$ where ℓ_s is the string scale.
- The modes on a string have mass proportional to $1/\ell_s$. ℓ_s is usually taken to be so tiny that only the lowest lying modes are accessible.

The original correspondence

maximally supersymmetric $SU(N)$ Super Yang Mills in 3+1 dimensions	$\sim$	type IIB closed string theory on $AdS_5 \times S^5$
$g_{YM}^2 N \equiv \lambda$	$=$	L^4/ℓ_s^4
g_{YM}^2	$=$	$4\pi g_s$

AdS_5: Five dimensional anti-de Sitter space. A hyperboloid with a time direction and a boundary.
S^5: Five dimensional sphere.

CFT: This Yang-Mills theory is conformal for all values of λ.

Classical strings: Take $N \to \infty$ with λ fixed means $g_s \to 0$. Strings don't split.
Supergravity: Take λ large. The radius of curvature is large compared to the string scale.

New Transport Coefficients

- Calculation of transport coefficients for a strongly interacting system using AdS/CFT.
- A great success was the discovery that the viscosity takes a universally low value for all field theories with classical gravity duals

$$\eta = \frac{\hbar s}{4\pi k_B} \ ,$$

  where s is the entropy density (Kovtun, Son, Starinets 2004).
- This low value has been used to "explain" the measured low viscosity of the quark-gluon plasma and cold fermion gases at a Feshbach resonance.

A Result this Year in AdS/CFT Transport Coefficients

- Can we calculate other transport coefficients using AdS/CFT? Are these other coefficients also universal?
- For example, superfluids have several viscosities associated with the relative motion of the normal and superfluid components.
- Herzog, Lisker, Surowka, Yarom and independently Minwalla, Bhattacharya, Bhattacharyya computed the third bulk viscosity of a relativistic conformal superfluid.

Higher Spin Theories

Higher Spin Theories

Why are string theorists excited about higher spin theories? The short answer is AdS/CFT.

- Through the correspondence, higher spin theories may let us learn something about string theory from the non-interacting limit of certain field theories.
- Higher spin theories may lead us to a more precise formulation (or even proof) of the AdS/CFT correspondence.

What Higher Spin Theory Means

- In the previous section, to get classical gravity, we took $\lambda = (L/\ell_s)^4$ to be large.
- The higher spin theories come from taking λ small.
- When λ is small, $L/\ell_s \to 0$ and the modes of the string become very close together. An infinite tower of "higher spin fields" must be included in an effective field theory description of the strings in AdS_5.
- A formalism for this type of higher spin theory was worked out by Vasiliev (1987) years ago.

The Year in Higher Spins

- Giombi and Yin; Jevicki, Jin, et al.: map of critical and free O(N) vector model to a higher spin theory in AdS_4, confirming a conjecture of Klebanov and Polyakov (2002).
- Gaberdiel, Gopakumar, Hartman, Raju, et al.: map of the higher spin theory in AdS_3 to a W_N minimal model
- Douglas, Mazzucato, and Razamat: map of a free field theory to a higher spin theory.

To Recap: $2011 = 1986 + 25$

- Scattering amplitudes
- c-like theorems and renormalization group flows
- AdS/CFT and the real world
- Higher spin theories