Higgs Transverse Momentum Distributions at the LHC

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Aug 10, 2011/ DPF Meeting

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Outline

1. Ressumation for Small $p_T$

2. Soft Collinear Effective Field Theory (SCET)
   - Higgs $p_T$ distribution

3. $\eta$-Regulator and $\nu$-Renormalization Group

4. SM vs MSSM

5. Numerics

6. Other Applications

7. Conclusion
In the literature...

QCD calculations with resummation in the literature:

SCET resummation for $p_T$ distribution
2. Gao, Li, Liu (2005)
3. Idilbi, Ji, Yuan (2005)
Soft Collinear Effective Theory (SCET I)
(Luke, Bauer, Fleming, Pirjol, Stewart)

- Describe interactions between energetic particles $E \sim Q$.
- Fluctuations, ($\Lambda_{QCD}$ or other low energy scales, about light cone coordinate $n = (1, 0, 0, 1)$

$$p = (p \cdot \bar{n}, p \cdot n, p_{\perp})$$

$$= (p^-, p^+, p_{\perp})$$

$$k_s = (k_s^-, k_s^+, k_s_{\perp})$$

$$\sim (\Lambda_{QCD}, \Lambda_{QCD}, \Lambda_{QCD})$$

- Integrate out “far offshell” degrees of freedom.
  - soft-collinear decoupling
SCET degrees of freedom (modes)

\[ p^\mu = (p^-, p^+, p_\perp); \quad p^2 = p^+ p^- + p_{\perp}^2 \]

- Light Cone Coordinates:
  \[ n = (1, \bar{n}) \sim (1, 0, 0, 1) \]

- power counting parameter
  \[ \lambda \equiv \frac{\Lambda_{QCD}}{Q} \]

- hard modes: \( p^2 \sim Q^2 \) integrated out

- \( n \)-collinear
  \[ p^\mu \sim Q(1, \lambda^2, \lambda) \]

- \( \bar{n} \)-collinear
  \[ p^\mu \sim Q(\lambda^2, 1, \lambda) \]

- usoft (\( p^2 \sim Q^2 \lambda^4 \))
  \[ p^\mu = Q(\lambda^2, \lambda^2, \lambda^2) \]
When observing $p_T$ (or related observables)

- When measuring thrust distribution or $|p_s^\perp| \ll |p_n^\perp| \sim |p_{\bar{n}}^\perp|$ and $|\eta_s^s| \ll |\eta_n| \sim |\eta_{\bar{n}}|$. Ultra-Soft radiation decoupled from the collinear radiation.

- When observing transverse momentum ($p_T$) distribution or jet broadening event shape ($B_T$), $k_{S,\perp} \sim k_{n,\perp} \sim k_{\bar{n},\perp}$, both soft and collinear radiation contribute to the transverse momentum at the same order.
SCET Modes

![Diagram showing SCET Modes](image)
$p_T$ Resummation in SCET

- Idilbi, Ji, Yuan (2005)
  - Calculation using SCET, no factorization theorem derived

  - Factorization theorem derived in SCET
  - Keep residual momentum, and thus power suppressed terms for each sector to be well regularized.

- Becher and Neubert (2010)
  - Absence of soft function
  - Analytic regulator break factorization
Higgs $p_T$ distribution in SCET$_{\parallel}$

with $\eta$-regulator and $\nu$-RG

$$\frac{d\sigma}{dQ^2 dp_T^2 dy} \propto \int \frac{dx_1}{x_1} \int \frac{dx_2}{x_2} \delta(\vec{p}_T^H + \vec{p}_n,t + \vec{p}_{\bar{n}},t + \vec{p}_s,t)g_{\alpha\sigma}g_{\beta\omega}$$

$$\times H(m_H^2, \mu)S(\vec{p}_{s,t}; \mu_s, \mu; \nu_s, \nu)$$

$$\times \mathcal{F}_{\alpha\beta}^{\gamma} (x_1, \vec{p}_{n,t}; \mu_B, \mu; \nu_B, \nu)$$

$$\times \mathcal{F}_{\sigma\omega}^{\gamma} (x_2, \vec{p}_{\bar{n},t}; \mu_B, \mu; \nu_B, \nu)$$

$$f_{g/p}(\frac{\omega_a}{P_-}, \mu) = - \sum_{\text{spins}} \theta(\omega_a) \omega_a \langle p_n | B_{n\perp}^{c\mu}(0) \delta(\frac{\omega_a}{P_-} - \vec{P}_n)B_{n\perp\mu}^{c}(0) | p_n \rangle$$

$$\mathcal{F}_{g}^{\alpha\beta}(\frac{\omega_a}{P_-}, \vec{p}_t, \mu) = - \sum_{\text{spins}} \int d^2 \vec{b}_t e^{-i \vec{b}_t \cdot \vec{p}_t} \theta(\omega_a) \langle p_n | B_{n\perp}^{c\alpha}(\vec{b}_t) \delta(\frac{\omega_a}{P_-} - z)B_{n\perp}^{c\beta}(0) | p_n \rangle$$

$$= \sum_i \frac{1}{z} \int \frac{dz'}{z} \int d^2 \vec{b}_t e^{-i \vec{b}_t \cdot \vec{p}_t} \mathcal{I}_{i\beta}^{\alpha\beta}(\frac{\omega_a}{z' P_-}, \vec{b}_t, \mu) f_i(z', \mu)$$
Naive Calculation with Pure Dim-Reg

Bare Beam Function at one-loop:

\[ F_{\gamma \rightarrow g}^{(1)}(z, \vec{p}_t) = \frac{G^2 C_A \pi^{-\epsilon}}{(2\pi)^{1-2\epsilon}} \Gamma(1 + \epsilon) \frac{\mu^{2\epsilon}}{(p_t^2)^{1+\epsilon}} \left[ \left( 2z - 3 - \frac{1 + z}{1 - z} \right) g_\alpha^\beta + 4(1 - \epsilon^2) \frac{1 - z}{z^2} \frac{p_t^\alpha}{p_t^\beta} \right] \]

- No plus prescription for \( 1 \rightarrow z \)
- Leftover \( \frac{1}{\epsilon} \) divergence multiplies non-zero \((1-z)\) terms that virtual diagrams, which are always proportional to \( \delta(1 - z) \), cannot cancel.
- Traditional dim-reg regulating the \( \vec{p}_t \) part of the real radiation dose not regulate the phase space integral while \( p_T \) is fixed.

Bare Soft Function at one-loop:

\[ S^{(1)}(\vec{p}_t) = \delta(1 - z) \frac{2C_A G_\pi^{2\epsilon}}{(2\pi)^{1-2\epsilon}} \Gamma(1 + \epsilon) \frac{\mu^{2\epsilon}}{(p_t^2)^{1+\epsilon}} \int_0^\infty \frac{dk^-}{k^-} \]

- Un-regulated \( k^- \) integral
New Regulator and $\nu$-Renormalization Group

- Goal:
  - Multiplicatively Renormalizable
  - In the spirit of dimensional regularization
  - Does not introduce new dimensionful scales in the integrants, and maintains manifest power counting in the effective theory.

- $\eta$-regulator

\[
W_n = \left[ \sum_{\text{perm}} \exp \left( -\frac{g}{\bar{n} \cdot \hat{P}} \left[ \frac{|\bar{n} \cdot \hat{P}|^{-\eta}}{\nu^{-\eta}} \bar{n} \cdot A_{n,q(0)} \right] \right) \right]
\]

\[
S_n = \left[ \sum_{\text{perm}} \exp \left( -\frac{g}{n \cdot \hat{P}} \left[ \frac{|2\hat{P}^3|^{-\eta/2}}{\nu^{-\eta/2}} n \cdot A_{s,q(0)} \right] \right) \right]
\]
Total beam function in \( n \)-direction including real and virtual yields

\[
\mathcal{F}_{g \leftarrow g}^\mu\nu(z, \vec{p_t}) \propto \Gamma(1 + \epsilon) \frac{\mu^{2\epsilon}}{(p_t^2)^{(1+\epsilon)}} \left[ g_t^{\mu\nu} \frac{\delta(1 - z)}{\eta} \left( \frac{\nu}{\omega_a} \right)^\eta + p_{gg} \left( \frac{1}{z} \right) g_t^{\mu\nu} \right. \\
\left. + 4 \frac{1 - z}{z^2} \left( \frac{p_t^{\mu} p_t^{\nu}}{p_t^2} + \frac{1}{2} g_t^{\mu\nu} \right) + 2 \epsilon \left( p_t^{\mu} p_t^{\nu} \right) - \frac{1}{2 \epsilon} g_t^{\mu\nu} \right] 
\]

- splitting function

\[
p_{gg}(z) = \frac{1 + (1 - z)^4 + z^4}{[1 - z]_+ z}
\]

- \((\mu_B, \nu_B) = (p_t, \omega)\)
Soft Function Calculation with $\eta$-regulator

$S(\vec{p}_t) \propto \Gamma \left(1 + \epsilon + \frac{\eta}{2}\right) \Gamma \left(\frac{\eta}{2}\right) \left(\frac{\mu^{2\epsilon}}{(p^2_t)^{(1+\epsilon)}}\right) \left(\frac{\nu^\eta}{(p^2_t)^{\eta/2}}\right)$

\( (\mu_s, \nu_s) = (p_t, p_t) \)
\( \mu \) and \( \nu \) RG similar to the previous case

- The rapidity divergences for the jet and soft functions introduce a new set of anomalous dimension \((\gamma^\nu_B, \gamma^\nu_S)\):

\[
\nu \frac{d}{d\nu} S^R(\mu, \nu) = \gamma^\nu_S S^R(\mu, \nu), \quad \nu \frac{d}{d\nu} B_n^R(\mu, \nu) = \gamma^\nu_B B_n^R(\mu, \nu)
\]

Just like the traditional \( \mu \) anomalous dimension:

\[
\mu \frac{d}{d\mu} S^R(\mu, \nu) = \gamma^\mu_S S^R(\mu, \nu), \quad \text{and} \quad \mu \frac{d}{d\mu} B_n^R(\mu, \nu) = \gamma^\mu_B B_n^R(\mu, \nu)
\]

- Since the cross-section is invariant under \( \mu \) and \( \nu \) variation, and that the hard function itself is free from rapidity divergence (and therefore \( \gamma^\nu_H = 0 \)), we must have the relations

\[
\gamma^\mu_H + \gamma^\mu_B + \gamma^\mu_B + \gamma^\mu_S = 0, \quad \text{and} \quad \gamma^\nu_B + \gamma^\nu_B + \gamma^\nu_S = 0.
\]

- Recover CSS formula by \( \mu \) and \( \nu \) RG
Running Strategy: 2-Parameter RG
Renormalization Group Equations

- $\eta$-divergences and $\nu$-anomalous dimensions cancels when we sum up the contributions from the jet and soft functions.
- Individual $J$ and $S$ are multiplicatively renormalizable.
- $\eta$-divergence are absorbed in the renormalization constants, $Z_{B,S}$, such that

\[
B_n^{(0)} B_{\bar{n}}^{(0)} S^{(0)} = \left[ Z_{B_n}(\mu, \nu) B_n^R(\mu, \nu) \right] \left[ Z_{B_{\bar{n}}}(\mu, \nu) B_{\bar{n}}^R(\mu, \nu) \right] \left[ Z_S(\mu, \nu) S^R(\mu, \nu) \right],
\]

where

\[
Z_{B_n}(\mu, \nu) Z_{B_{\bar{n}}}(\mu, \nu) Z_S(\mu, \nu) = Z_H^{-1}(\mu).
\]
Renormalization Group Equations

- The rapidity divergences for the jet and soft functions introduce a new set of anomalous dimension \((\gamma^J, \gamma^S)\):

\[
\frac{d}{d\nu} S^R(\mu, \nu) = \gamma^S S^R(\mu, \nu), \quad \frac{d}{d\nu} B^R_n(\mu, \nu) = \gamma^B B^R_n(\mu, \nu)
\]

Just like the traditional \(\mu\) anomalous dimension:

\[
\frac{d}{d\mu} S^R(\mu, \nu) = \gamma^\mu S^R(\mu, \nu), \quad \frac{d}{d\mu} B^R_n(\mu, \nu) = \gamma^\mu B^R_n(\mu, \nu)
\]

- Since the cross-section is invariant under \(\mu\) and \(\nu\) variation, and that the hard function itself is free from rapidity divergence (and therefore \(\gamma^H = 0\)), we must have the relations

\[
\gamma^H + \gamma^\mu_{B_n} + \gamma^\mu_{B_n} + \gamma^S = 0, \quad \text{and} \quad \gamma^\nu_{B_n} + \gamma^\nu_{B_n} + \gamma^S = 0.
\]
Running Strategy

- **Natural scales:**
  - hard function: independent of $\nu$, $\mu_H = M_H$
  - soft function $(\nu_S, \mu_S) = (p_T, \rho_T)$
  - beam functions $(\nu_B, \mu_J) = (M_H, p_T)$

- **Running**
  - In $\mu$:
    Evolve hard function from high scale $\mu_H = M_H$ to common low scale $\mu_B = \mu_S = \rho_T$
  - In $\nu$:
    Evolve soft function from $\nu_S = p_T$ to jet scale $\nu_B = M_H$
Solution to the $\mu$-RGE for the hard function

$$H(s, \mu) = U_H(M_H; \mu_H, \mu)H(M_H; \mu_H)$$

with

$$U_H(M_H; \mu_H, \mu) = \left| e^{K_H(\mu_H, \mu)} \left( \frac{-M_H^2 - i0}{\mu_H^2} \right) \eta_H(\mu_H, \mu) \right|^2$$

Solution to the $\nu$-RGE for the soft function

$$S(\mu, \nu) = V_s \left( \mu, \frac{\nu}{\nu_s} \right) \otimes S(\mu, \nu_S)$$

with

$$V_s(p_t; \omega_s, \mu, \nu) = e^{-2\gamma_E \omega_s} \Gamma(1 - \omega_s) \Gamma(1 + \omega_s) \left[ \frac{\omega_s}{\mu} \left[ \frac{1}{(\mu/p_t)^{1-\omega_s}} + \delta(p_t) \right] \right]$$

and

$$\omega_s \left( \mu, \frac{\nu}{\nu_s} \right) = 2\Gamma_{cusp}[\alpha_s(\mu)] \log \frac{\nu}{\nu_s}.$$ 

Resummed cross-section up to NLL

$$\frac{1}{\sigma_0} \frac{d\hat{\sigma}}{dp_T} = U_H(M_H; \mu_H, \mu = p_T)V_s(p_t; \omega_s, \mu = p_T, \nu = M_H)$$
Comparison with literature...

QCD calculations with resummation in the literature:


SCET resummation for $p_T$ distribution

2. Gao, Li, Liu (2005)
3. Idilbi, Ji, Yuan (2005)
   - SCET-like calculation, no factorization theorem derived
   - log hidden in phase space
4. Mantry, Petriello
   - Factorization theorem derived in SCET
   - Keep residual momentum, and thus power suppressed terms for each sector to be well regularized.
   - Absence of soft function
   - Analytic regulator break factorization
Higgs Production in SM vs MSSM

- SUSY imposes that $H_1$ ($H_2$) couples exclusively to down- (up-) type fermions:

$$\mathcal{L}_{Yuk} = -\lambda_u \left[ \bar{u} P_L u H_2^0 - \bar{u} P_L d H_2^+ \right] - \lambda_d \left[ \bar{d} P_L d H_1^0 - \bar{d} P_L u H_1^- \right] + h.c.$$ 

$$\lambda_u = \frac{\sqrt{2} m_u}{v_2} = \frac{\sqrt{2} m_u}{v_{SM} \sin \beta}$$ 

$$\lambda_d = \frac{\sqrt{2} m_d}{v_1} = \frac{\sqrt{2} m_d}{v_{SM} \cos \beta}$$

- Enhanced coupling for down-type fermions for large $\tan \beta$
- For $\tan \beta \gtrsim 7$, $b \bar{b}$ production mode can be comparable to $gg$
- Distinct $p_T$ spectrum at high energy due to large logarithms
  - Feature captured by resummation
For $\tan\beta \gtrsim 7$, $b\bar{b}$ production mode can be comparable to $gg$ initiated process dominate at small $p_T$.

$gg \rightarrow h$, $bb \rightarrow h$
Other Applications?

Rapidity divergences do not only appear when observing Jet Broadening...

- Jet Broadening Event Shape Observable and other angularity observables for $0 \ll a < 2$.
  - JC’s talk in pQCD session.
- TMD-PDF, Generalized Parton Distribution
- Electroweak corrections to high energy process at the LHC
- ...

In general, processes or observables involving collinear and soft mode with similar transverse momentum or off-shellness.
Conclusion

- Distinct higgs $p_T$ spectrum between gluon fusion and b-quark fusion initiated processes due to large logarithms at small $p_T$.
- Resummation is important to capture the turning back at low $p_T$.
- When measuring transverse momentum related observables...
  - Soft contributions are important
  - Uncanceled divergences remain in each sector, rapidity divergence.
  - New kind of logarithms to resum, yet related to the cups angle (the high scale).
- There are other cases with rapidity divergence such as, jet broadening event shape, and electroweak corrections to LHC processes.
- Rapidity RG making use of the $\eta$-regulator provides controllable form to divergences, and a way to resum the log systematically, for $p_T$ dependent observables.