Higgs Tansverse Momentum Distributions at the LHC

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Aug 10, 2011/ DPF Meeting

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Outline

In the literature...

QCD calculations with resummation in the literature:

1. Collins, Soper, and Sterman (1985)

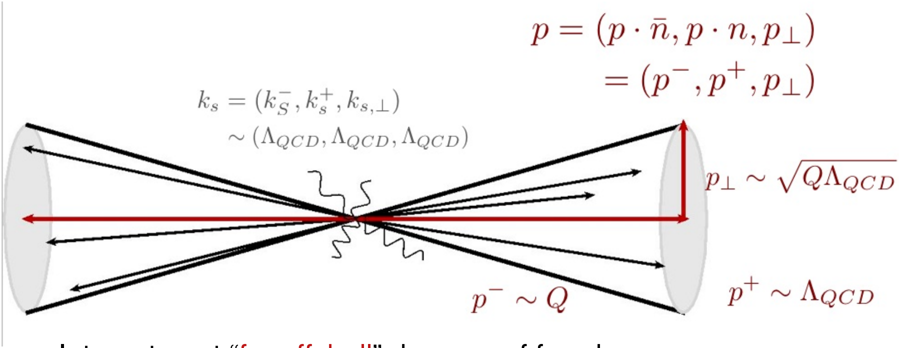
SCET resummation for p_T distribution

2. Gao, Li, Liu (2005)
3. Idilbi, Ji, Yuan (2005)
4. Mantry, Petriello (2010)
5. Becher, Bell and Neubert (2010)

Soft Collinear Effective Theory (SCET I)

(Luke, Bauer, Fleming, Pirjol, Stewart)

- Describe interactions between energetic particles $E \sim Q$.
- Fluctuations, (Λ_{QCD} or other low energy scales, about light cone coordinate $n = (1, 0, 0, 1)$

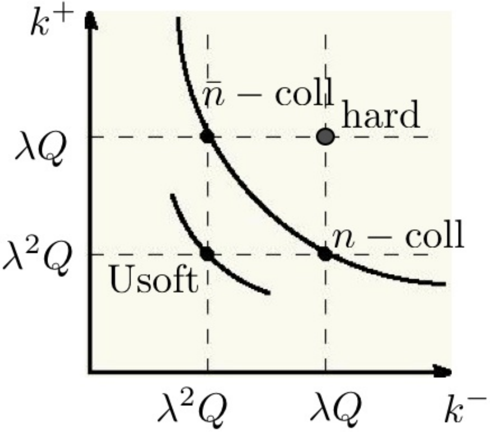


- Integrate out “far offshell” degrees of freedom.
  - soft-collinear decoupling

SCET degrees of freedom (modes)

$$p^\mu = (p^-, p^+, p^\perp); \qquad p^2 = p^+p^- + p_\perp^2$$

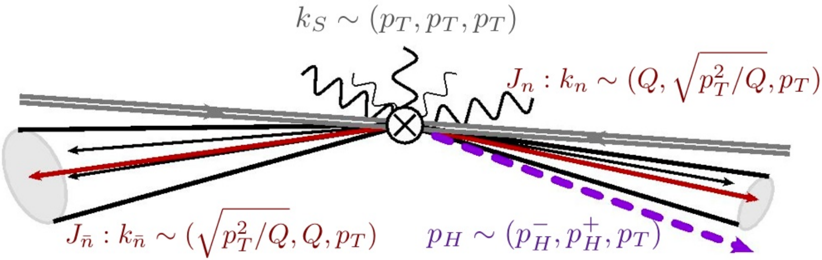
- Light Cone Coordinates:
  $n = (1, \vec{n}) \sim (1, 0, 0, 1)$
- power counting parameter
  $\lambda \equiv \frac{\Lambda_{QCD}}{Q}$
- hard modes: $p^2 \sim Q^2$
  integrated out
- n-collinear
  $p^\mu \sim Q(1, \lambda^2, \lambda)$
- $\bar{n}$-collinear
  $p^\mu \sim Q(\lambda^2, 1, \lambda)$
- usoft ($p^2 \sim Q^2\lambda^4$)
  $p^\mu = Q(\lambda^2, \lambda^2, \lambda^2)$



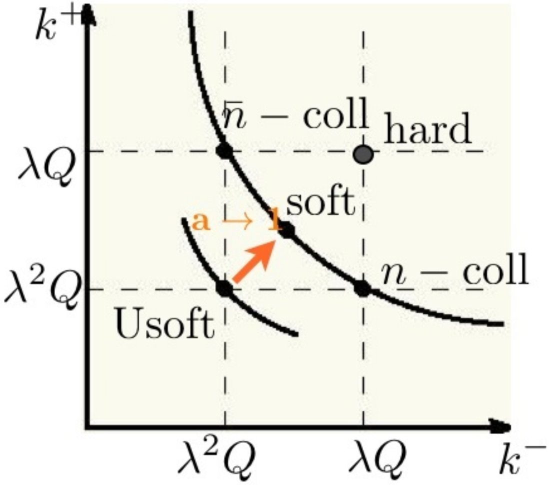
When observing p_T (or related observables)

- When measuring thurst distribution or ,
  $|p_\perp^s| \ll |p_\perp^n| \sim |p_\perp^{\bar{n}}|$ and $|\eta^s| \ll |\eta^n| \sim |\eta^{\bar{n}}|$.
  Ultra-Soft radiation decoupled from the collinear radiation.

- When observing transverse momentum (p_T) distribution or jet broadening event shape (B_T), $k_{S,\perp} \sim k_{n,\perp} \sim k_{\bar{n},\perp}$, both soft and collinear radiation contribute to the transverse momentum at the same order.



SCET Modes



k^+
$\bar{n}$ − coll
hard
λQ
soft
a → 1
$\lambda^2 Q$
n − coll
Usoft
$\lambda^2 Q$
λQ
k^-

p_T Resummation in SCET

- Idilbi, Ji, Yuan (2005)
  - Calculation using SCET, no factorization theorem derived
- Mantry and Petriello (2009, 2010)
  - Factorization theorem derived in SCET
  - Keep residual momentum, and thus power suppressed terms for each sector to be well regularized.
- Becher and Neubert (2010)
  - Absence of soft function
  - Analytic regulator break factorization

Higgs p_T distribution in SCET$_{II}$

with η-regulator and ν-RG

$$\frac{d\sigma}{dQ^2 dp_T^2 dy} \propto \int \frac{dx_1}{x_1} \int \frac{dx_2}{x_2} \delta(\vec{p}_T^H + \vec{p}_{n,t} + \vec{p}_{\bar{n},t} + \vec{p}_{s,t}) g^{\perp}_{\alpha\sigma} g^{\perp}_{\beta\omega}$$
$$\times \quad H(m_H^2, \mu) \mathcal{S}\left(\vec{p}_{s,t}; \mu_s, \mu; \nu_s, \nu\right)$$
$$\times \quad \mathcal{F}^{\alpha\beta}_{n;g}\left(x_1, \vec{p}_{n,t}; \mu_B, \mu; \nu_B, \nu\right)$$
$$\times \quad \mathcal{F}^{\sigma\omega}_{\bar{n};g}\left(x_2, \vec{p}_{\bar{n},t}; \mu_B, \mu; \nu_B, \nu\right)$$

$$f_{g/p}(\frac{\omega_a}{P^-}, \mu) = -\sum_{\text{spins}} \theta(\omega_a)\omega_a \langle p_n | B^{c\mu}_{n\perp}(0) \delta(\frac{\omega_a}{P^-} - \bar{P}_n) B^{c}_{n\perp\mu}(0) | p_n \rangle$$

$$\mathcal{F}^{\alpha\beta}_g(\frac{\omega_a}{P^-}, \vec{p}_t, \mu) = -\sum_{\text{spins}} \int d^2\vec{b}_t e^{-i\vec{b}_t\cdot\vec{p}_t} \theta(\omega_a) \langle p_n | B^{c\alpha}_{n\perp}(\vec{b}_t) \delta(\frac{\omega_a}{P^-} - z) B^{c\beta}_{n\perp}(0) | p_n \rangle$$
$$= \sum_i \frac{1}{z} \int_z^1 \frac{dz'}{z'} \int d^2\vec{b}_t e^{-i\vec{b}_t\cdot\vec{p}_t} I^{\alpha\beta}_{gi}(\frac{\omega_a}{z' P^-}, \vec{b}_t, \mu) f_i(z', \mu)$$

Naive Calculation with Pure Dim-Reg

- Bare Beam Function at one-loop:

$$F^{(1)\alpha\beta}_{g\leftarrow g}(z,\vec{p}_t) = \frac{G^2 C_A \pi^{-\epsilon}}{(2\pi)^{1-2\epsilon}}\Gamma(1+\epsilon)\frac{\mu^{2\epsilon}}{(p_t^2)^{(1+\epsilon)}} \left[\left(2z-3-\frac{1+z}{1-z}\right)g_t^{\alpha\beta}+4(1-\epsilon^2)\frac{1-z}{z^2}\frac{p_t^\alpha p_t^\beta}{p_t^2}\right]$$

  - No plus prescription for $1\to z$
  - Leftover $\frac{1}{\epsilon}$ divergence multiplies non-zero (1-z) terms that virtual diagrams, which are always proportional to $\delta(1-z)$, cannot cancel.
  - Traditional dim-reg regulating the $\vec{p}_t$ part of the real radiation dose not regulate the phase space integral while p_T is fixed.

- Bare Soft Function at one-loop:

$$S^{(1)}(\vec{p}_t) = \delta(1-z)\frac{2C_A G^2(\pi)^{-\epsilon}}{(2\pi)^{1-2\epsilon}}\Gamma(1+\epsilon)\frac{\mu^{2\epsilon}}{(p_t^2)^{1+\epsilon}}\int_0^\infty \frac{dk^-}{k^-}$$

  - Un-regulated k^- integral

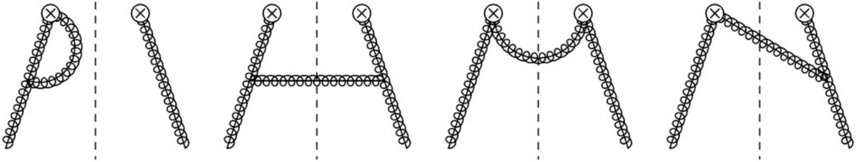
New Regulator and ν-Renomalization Group

- Goal:
  - Multiplicatively Renormalizable
  - In the spirit of dimensional regularization
  - Does not introduce new dimensionful scales in the integrants, and maintains manifest power counting in the effective theory.
- η-regulator

$$W_n = \left[\sum_{\text{perm}} \exp\left(\frac{-g}{\bar{n}\cdot\hat{P}} \left[\frac{|\bar{n}\cdot\hat{P}|^{-\eta}}{\nu^{-\eta}} \bar{n}\cdot A_{n,q}(0) \right] \right) \right]$$

$$S_n = \left[\sum_{\text{perm}} \exp\left(\frac{-g}{n\cdot\hat{P}} \left[\frac{|2\hat{P}^3|^{-\eta/2}}{\nu^{-\eta/2}} n\cdot A_{s,q}(0) \right] \right) \right]$$

Beam Function Calculation with η-regulator



Total beam function in n-direction including real and virtual yields

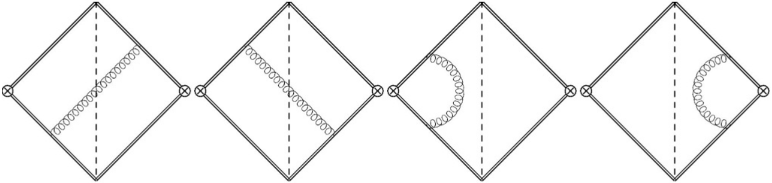
$$\mathcal{F}^{\mu\nu}_{g\leftarrow g}(z,\vec{p}_t) \propto \Gamma(1+\epsilon)\frac{\mu^{2\epsilon}}{(p_t^2)^{(1+\epsilon)}}\Big[g_t^{\mu\nu}\frac{\delta(1-z)}{\eta}\Big(\frac{\nu}{\omega_a}\Big)^{\eta} + p_{gg}\left(\frac{1}{z}\right) g_t^{\mu\nu}$$
$$+4\frac{(1-z)}{z^2}\Big(\Big(\frac{p_t^\mu p_t^\nu}{p_t^2}+\frac{1}{2}g_t^{\mu\nu}\Big)+2\epsilon\Big(\frac{p_t^\mu p_t^\nu}{p_t^2}-\frac{1}{2\epsilon}g^{\mu\nu}_{-2\epsilon}\Big)\Big)\Big]$$

- splitting function

$$p_{gg}(z)=\frac{1+(1-z)^4+z^4}{[1-z]_+ z}$$

- $(\mu_B,\nu_B)=(p_t,\omega)$

Soft Function Calculation with η-regulator



Soft

$$\mathcal{S}(\vec{p}_t) \propto \Gamma\left(1+\epsilon+\frac{\eta}{2}\right)\Gamma\left(\frac{\eta}{2}\right)\left(\frac{\mu^{2\epsilon}}{(p_t^2)^{(1+\epsilon)}}\right)\left(\frac{\nu^{\eta}}{(p_t^2)^{\eta/2}}\right)$$

- $(\mu_s, \nu_s) = (p_t, p_t)$

μ and ν RG similar to the previous case

- The rapidity divergences for the jet and soft functions introduce a new set of anomalous dimension $(\gamma_B^\nu, \gamma_S^\nu)$:

$$\nu \frac{\mathrm{d}}{\mathrm{d}\nu} \mathcal{S}^{\mathrm{R}}(\mu,\nu) = \gamma_S^\nu \, \mathcal{S}^{\mathrm{R}}(\mu,\nu), \; \nu \frac{\mathrm{d}}{\mathrm{d}\nu} \mathcal{B}_n^{\mathrm{R}}(\mu,\nu) = \gamma_B^\nu \, \mathcal{B}_n^{\mathrm{R}}(\mu,\nu)$$

Just like the traditional μ anomalous dimension:

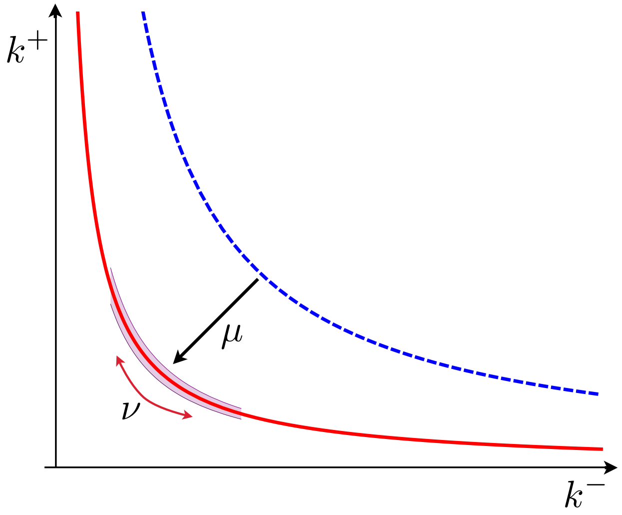
$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} \mathcal{S}^{\mathrm{R}}(\mu,\nu) = \gamma_S^\mu \, \mathcal{S}^{\mathrm{R}}(\mu,\nu), \text{ and } \mu \frac{\mathrm{d}}{\mathrm{d}\mu} \mathcal{B}_n^{\mathrm{R}}(\mu,\nu) = \gamma_B^\mu \, \mathcal{B}_n^{\mathrm{R}}(\mu,\nu)$$

- Since the cross-section is invariant under μ and ν variation, and that the hard function itself is free from rapidity divergence (and therefore $\gamma_H^\nu = 0$), we must have the relations

$$\gamma_H^\mu + \gamma_{B_n}^\mu + \gamma_{B_{\bar{n}}}^\mu + \gamma_S^\mu = 0, \text{ and } \gamma_{B_n}^\nu + \gamma_{B_{\bar{n}}}^\nu + \gamma_S^\nu = 0.$$

- Recover CSS formula by μ and ν RG

Running Strategy: 2-Parameter RG



Renormalization Group Equations

- η-divergences and ν-anomalous dimensions cancels when we sum up the contributions from the jet and soft fuctions.
- Individual J and S are multiplicatively renormalizable.
- η-divergence are absorbed in the renormalization constants, $Z_{B,S}$, such that

$$B_n^{(0)} B_{\bar{n}}^{(0)} S^{(0)} = \left[Z_{B_n}(\mu,\nu) B_n^{\mathrm{R}}(\mu,\nu)\right] \left[Z_{B_{\bar{n}}}(\mu,\nu) B_{\bar{n}}^{\mathrm{R}}(\mu,\nu)\right] \left[Z_S(\mu,\nu) S^{\mathrm{R}}(\mu,\nu)\right],$$

where

$$Z_{B_n}(\mu,\nu) Z_{B_{\bar{n}}}(\mu,\nu) Z_S(\mu,\nu) = Z_H^{-1}(\mu).$$

Renormalization Group Equations

- The rapidity divergences for the jet and soft functions introduce a new set of anomalous dimension $(\gamma_J^\nu, \gamma_S^\nu)$:

$$\nu \frac{\mathrm{d}}{\mathrm{d}\nu} \mathcal{S}^{\mathrm{R}}(\mu,\nu) = \gamma_S^\nu \, \mathcal{S}^{\mathrm{R}}(\mu,\nu),\ \nu \frac{\mathrm{d}}{\mathrm{d}\nu} B_n^{\mathrm{R}}(\mu,\nu) = \gamma_B^\nu \, B_n^{\mathrm{R}}(\mu,\nu)$$

  Just like the traditional μ anomalous dimension:

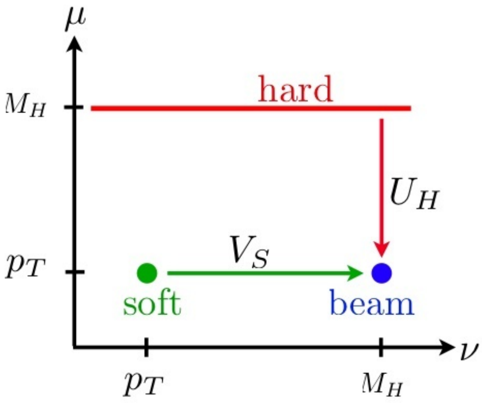
$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} \mathcal{S}^{\mathrm{R}}(\mu,\nu) = \gamma_S^\mu \, \mathcal{S}^{\mathrm{R}}(\mu,\nu), \text{ and } \mu \frac{\mathrm{d}}{\mathrm{d}\mu} B_n^{\mathrm{R}}(\mu,\nu) = \gamma_B^\mu \, B_n^{\mathrm{R}}(\mu,\nu)$$

- Since the cross-section is invariant under μ and ν variation, and that the hard function itself is free from rapidity divergence (and therefore $\gamma_H^\nu = 0$), we must have the relations

$$\gamma_H^\mu + \gamma_{B_n}^\mu + \gamma_{B_{\bar{n}}}^\mu + \gamma_S^\mu = 0, \text{ and } \gamma_{B_n}^\nu + \gamma_{B_{\bar{n}}}^\nu + \gamma_S^\nu = 0.$$

Running Strategy

- Natural scales:
  - hard function: independent of ν, $\mu_H = M_H$
  - soft function $(\nu_S, \mu_S) = (p_T, p_T)$
  - beam functions $(\nu_B, \mu_J) = (M_H, p_T)$
- Running
  - In μ: Evolve hard function from high scale $\mu_H = M_H$ to common low scale μ_B=μ_S=p_T
  - In ν: Evolve soft function from $\nu_S = p_T$ to jet scale $\nu_B = M_H$



Solution to the μ-RGE for the hard function

$$H(s,\mu) = U_H(M_H;\mu_H,\mu)H(M_H;\mu_H)$$

with

$$U_H(M_H;\mu_H,\mu) = \left| e^{K_H(\mu_H,\mu)} \left(\frac{-M_H^2 - i0}{\mu_H^2} \right)^{\eta_H(\mu_H,\mu)} \right|^2$$

Solution to the ν-RGE for the soft function

$$\mathcal{S}(\mu,\nu) = V_s\left(\mu, \frac{\nu}{\nu_s}\right) \otimes \mathcal{S}(\mu,\nu_S)$$

with

$$V_s\left(p_t;\omega_s,\mu,\nu\right) = e^{-2\gamma_E\omega_s}\frac{\Gamma(1-\omega_s)}{\Gamma(1+\omega_s)}\left[\frac{\omega_s}{\mu}\left[\frac{1}{(\frac{p_t}{\mu})^{1-\omega_s}}\right]_+ + \delta(p_t)\right]$$

and $\omega_s\left(\mu, \frac{\nu}{\nu_s}\right) = 2\Gamma_{cusp}[\alpha_s(\mu)]\log\frac{\nu}{\nu_s}$.

Resummed cross-section up to NLL

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}p_T} = U_H(M_H;\mu_H,\mu = p_T)\, V_s\left(p_t;\omega_s,\mu = p_T,\nu = M_H\right)$$

Comparison with literature...

QCD calculations with resummation in the literature:

1. Collins, Soper, and Sterman (1985)

SCET resummation for p_T distribution

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3. Idilbi, Ji, Yuan (2005)
   - SCET-like calculation, no factorization theorem derived
   - log hidden in phase space
4. Mantry, Petriello
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Higgs Production in SM vs MSSM

- SUSY imposes that H_1 (H_2) couples exclusively to down- (up-) type fermions:

$$\mathcal{L}_{Yuk} = -\lambda_u \left[\bar{u} P_L u H_2^0 - \bar{u} P_L d H_2^+\right] - \lambda_d \left[\bar{d} P_L d H_1^0 - \bar{d} P_L u H_1^-\right] + h.c.$$

$$\lambda_u = \frac{\sqrt{2} m_u}{v_2} = \frac{\sqrt{2} m_u}{v_{SM} \sin\beta}$$

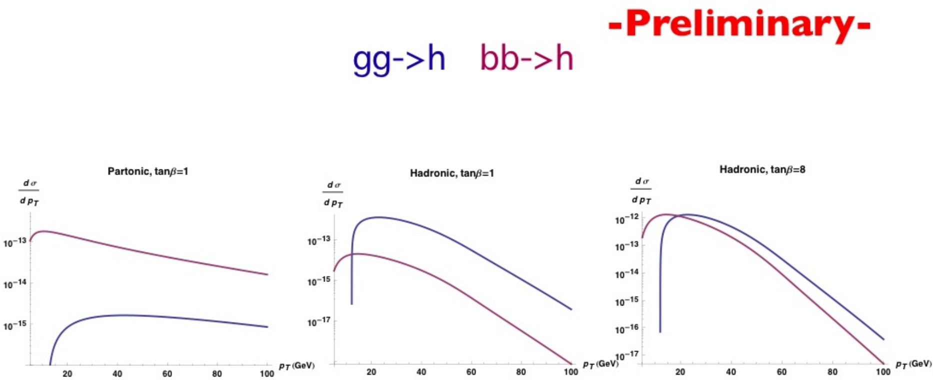
$$\lambda_d = \frac{\sqrt{2} m_d}{v_1} = \frac{\sqrt{2} m_d}{v_{SM} \cos\beta}$$

- Enhanced coupling for down-type fermions for large tanβ
- For tan$\beta \gtrsim 7$, $b\bar{b}$ production mode can be comparable to gg
- Distinct p_T spectrum at high energy due to large logarithms
  - Feature captured by resummation

Numerics

-Preliminary-

- For $\tan\beta \gtrsim 7$, $b\bar{b}$ production mode can be comparable to gg
- bb initiated process dominate at small p_T



Other Applications?

Rapidity divergences do not only appear when observing Jet Broadening...

- Jet Broadening Event Shape Observable and other angularity observables for $0 \ll a < 2$.
  - JC's talk in pQCD session.
- TMD-PDF, Generalized Parton Distribution
- Electroweak corrections to high energy process at the LHC
- ...
- In general, processes or observables involving collinear and soft mode with similar transverse momentum or off-shellness.

Conclusion

- Distinct higgs p_T spectrum between gluon fusion and b-quark fusion initiated processes due to large logarithms at small p_T.
- Resummation is important to capture the turning back at low p_T
- When measuring transverse momentum related observables...
  - Soft contributions are important
  - Uncanceled divergences remain in each sector, rapidity divergence.
  - New kind of logarithms to resum, yet related to the cups angle (the high scale).
- There are other cases with rapidity divergence such as, jet broadening event shape, and electroweak corrections to LHC processes.
- Rapidity RG making use of the η-regulator provides controllable form to divergences, and a way to resum the log systematically, for p_T dependent observables.