# Measurement of the Drell-Yan differential cross section $d\sigma/dM$ at $\sqrt{s}$ = 7 TeV

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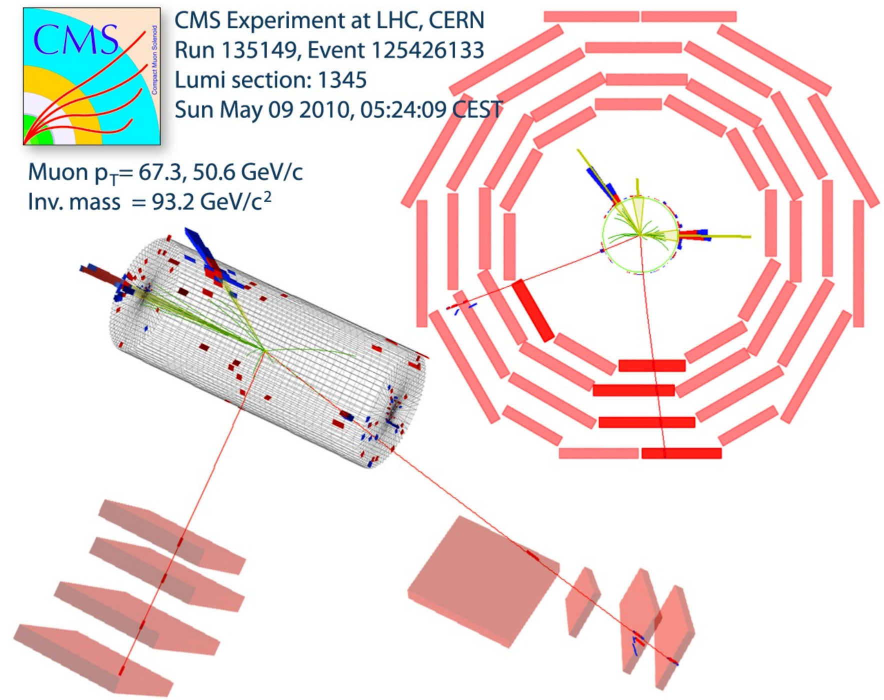
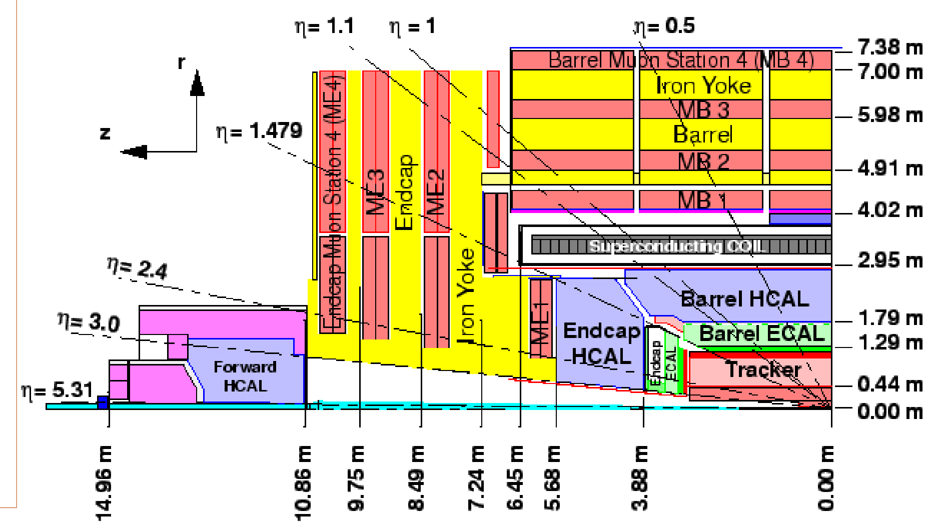
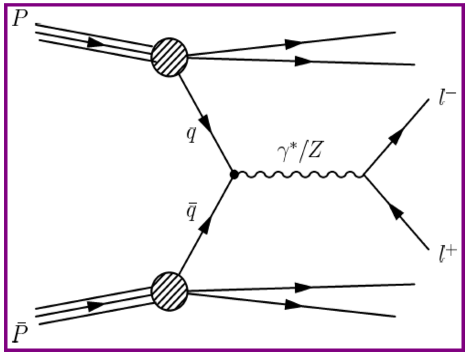


# Motivation and theoretical expectations

**Drell-Yan (DY) process**




- Important SM benchmark process
- Can be used to constraint PDFs and test pQCD
- Important background for BSM searches

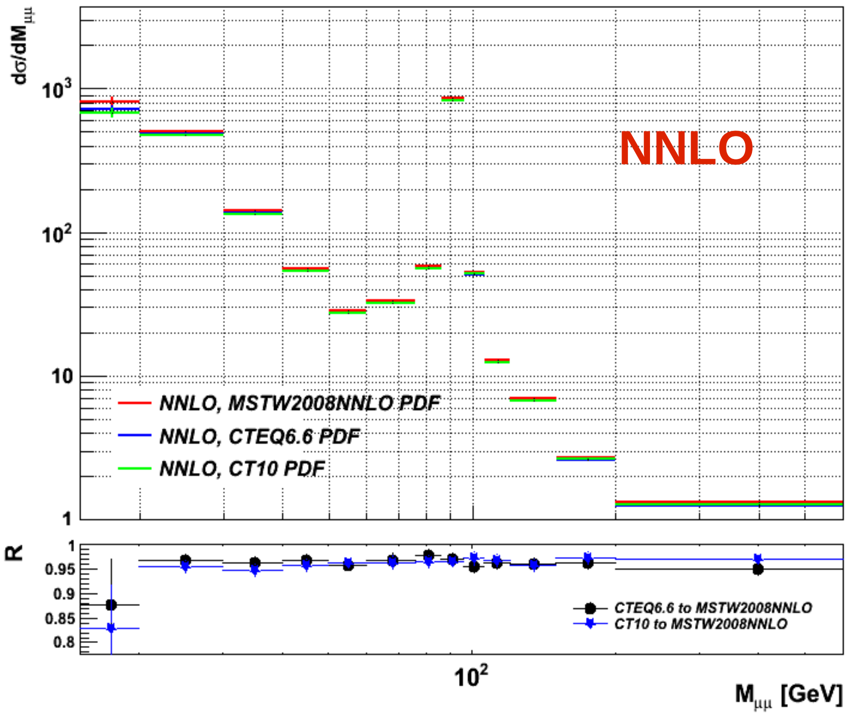
Our goal is to measure the differential $(1/\sigma_{ll})d\sigma/dM(ll)$ cross section.

We normalize to the the cross section around the Z peak, 60-120 GeV ($\sigma_{ll}$), which cancels part of the systematic uncertainties.

We calculate the theoretical expectations up to NNLO using (updated) FEWZ (2.0)
http://gate.hep.anl.gov/fpetriello/FEWZ.html, arXiv:1011.3540v1

Three PDF sets employed:
MSTW2008, Eur. Phys. J. C63 (2009)
CTEQ66, Phys. Rev. D78, 013004 (2008)
CT10, Phys. Rev. D82, 074024 (2010)

EWK corrections considered by HORACE, no QED final state radiation (FSR) in the model.



# Analysis procedure

To measure R = $(1/\sigma_{ll})d\sigma/dM(ll)$ we use the formula:

$$\sigma_i = \frac{N_i^U}{A_i \epsilon_i \rho_i L_{int}}$$

i – mass bin, M(ll) – invariant mass
$N^U$ – unfolded
(and background corrected) yield
A – acceptance
$\epsilon$ - efficiency
$\rho$ – efficiency (and FSR) correction
$L_{int}$ – integrated luminosity

## Procedure:

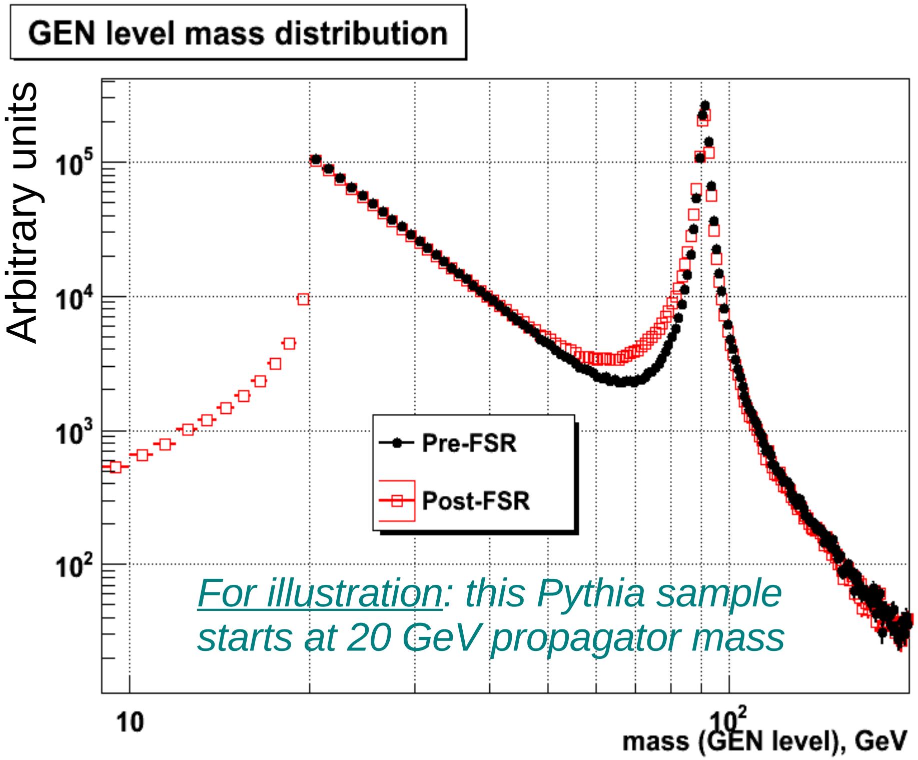
- Event selection
- Background subtraction
  - data-driven methods for dominant sources
- Unfolding
  - correcting for resolution effects
  - based on migration matrices from MC
- Acceptance and efficiency calculation using (NNLO matched) MC
- Efficiency correction using data-driven methods
- FSR correction (for the pre-FSR measurements)
  - based on the (NNLO matched) MC
- Cross section shape calculations (NORM refers to 60-120 GeV ):

$$R_i = \frac{N_i^U}{A_i \epsilon_i \rho_i} / \frac{N_{NORM}^U}{A_{NORM} \epsilon_{NORM} \rho_{NORM}} \qquad r_i = \frac{R_i}{\Delta M_i}$$

| Inv. mass binning (GeV) |
|---|
| 15 - 20 |
| 20 - 30 |
| 30 - 40 |
| 40 - 50 |
| 50 - 60 |
| 60 - 76 |
| 76 - 86 |
| 86 - 96 |
| 96 - 106 |
| 106 - 120 |
| 120 - 150 |
| 150 – 200 |
| 200 – 600 |

(resolution and statistics driven)

# Analysis – types of results




FSR changes the observed spectra.

We make both FSR corrected and FSR not corrected measurements.

Thus we report four different types of results.

- **Direct measurement**: post-FSR cross-section within the detector acceptance
- **Measurement with acceptance corrections**: post-FSR measurement in the full phase space
- **Measurements with FSR corrections**: pre-FSR measurements within the detector acceptance and in the full phase space
  - FSR corrections from Pythia
  - makes possible comparisons to models which do not have FSR

# Data and MC samples

The measurements are based on 2010 data recorded by CMS:
total integrated luminosity: $35.9 \pm 1.4\ pb^{-1}$

We use data triggered by single muons or electrons
with transverse momentum thresholds of < 15 GeV and < 17 GeV, respectively.

There is a special check made with double-muon triggers where thresholds are at 3 GeV.

Centrally produced MC samples:

DY signal: PYTHIA (v. 6.422) + POWHEG with CT10 PDF and Z2 tune

Backgrounds: PYTHIA / and MadGraph (v. 4.4.12)/

- QCD (genuine or mis-identified leptons)
- EWK (DY$\rightarrow\tau\tau$, W$\rightarrow$l$\nu$, diboson production)
- Top quark pairs

The signal sample corresponds to ~30 times the statistics in data.
Backgrounds correspond to at least few times the statistics in data.

# Baseline selections

## Muons

- Kinematic
  - two muons with opposite charges
  - $p_T(\mu_1)$>16 GeV, $|\eta(\mu_1)|$<2.1,
    $p_T(\mu_2)$>7 GeV, $|\eta(\mu_1)|$<2.4
  - at least one of the muons triggers
    ($p_T(\mu)$>16 GeV, $|\eta(\mu)|$<2.1)
- ID selection
  - minimal hits in the tracker
    to insure good pT measurement
  - minimal muon hits, maximal $\chi^2$/ndf
    to avoid bad reconstructed muons
  - impact parameter (beam spot)
    |dxy|<0.2 cm
- Isolation
  - relative isolation (no ECAL) in $\Delta R$<0.3

$$I_{rel}=\left(\sum p_T(tracks)+\sum E_T(had)\right)/P_T(\mu)<0.15$$

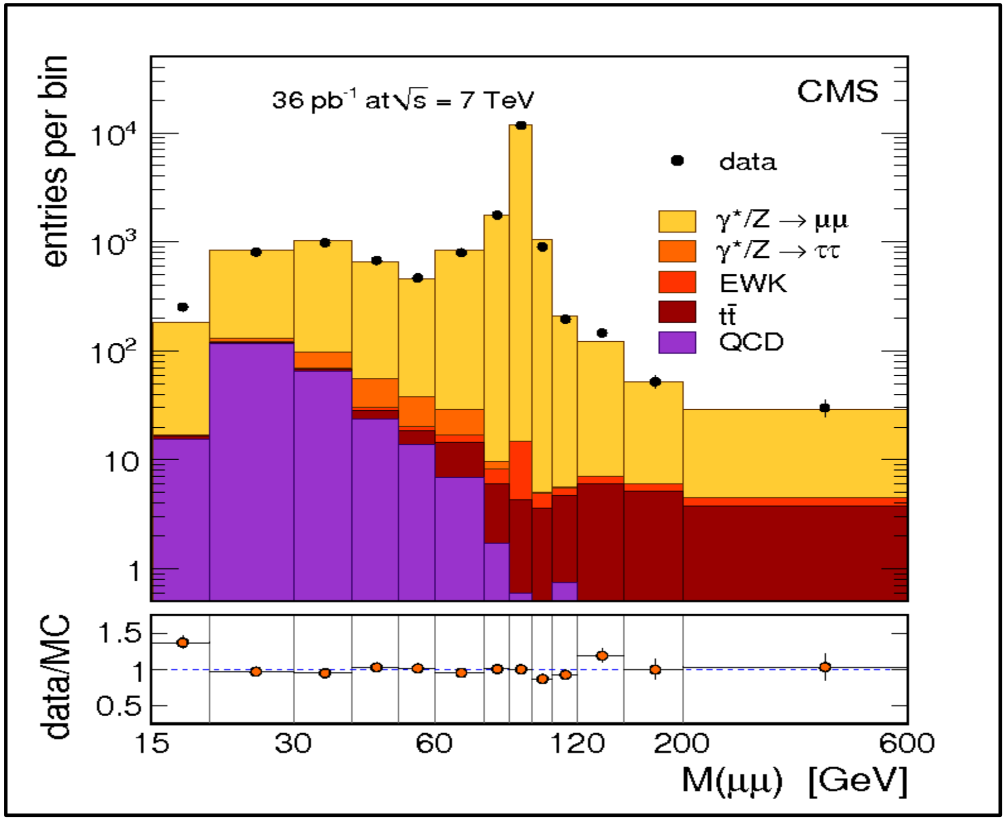
- Di-muon
  - di-muon vertex probability > 0.02
  - ($\pi$-) 3D angle between muons > 5 mrad
    to further suppress cosmic contamination

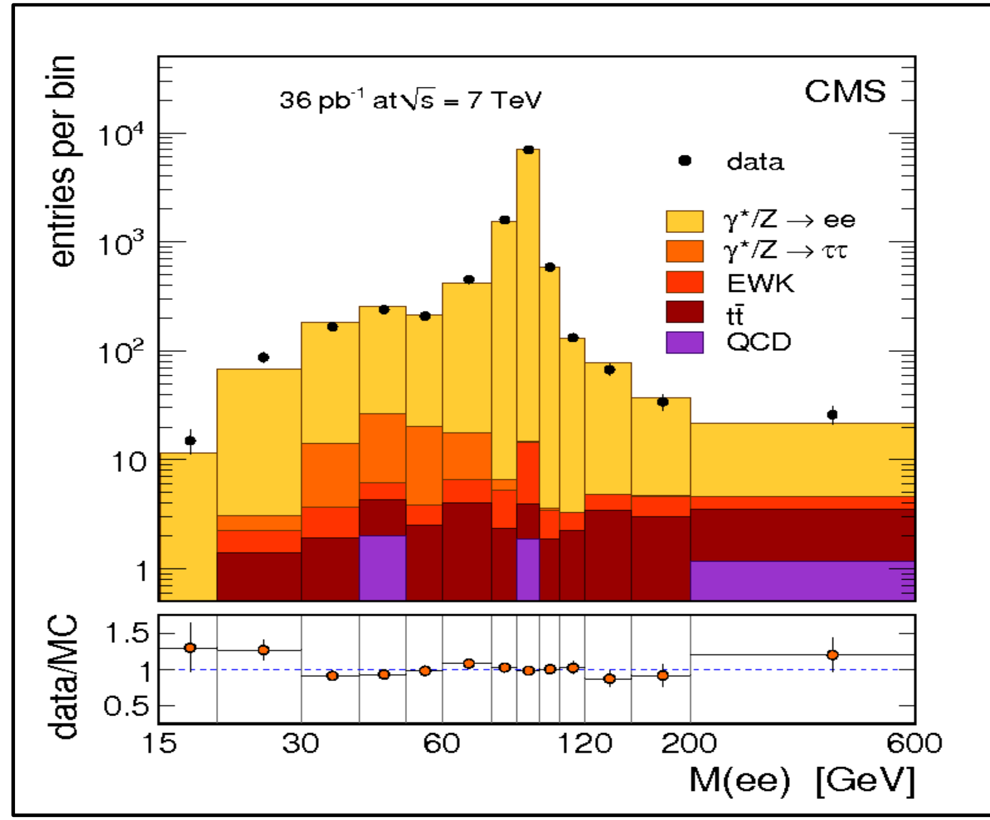
## Electrons

- Kinematic
  - two ECAL-driven electrons
  - $E_T(e_1)$>20 GeV, $E_T(e_2)$>10 GeV,
    $|\eta(e)|$<1.44 OR 1.57<$|\eta(e)|$<2.5
  - the leading electron matches the trigger
- ID selection
  - minimal hits in the tracker
  - track-ECAL cluster matching quality
  - HCAL energy fraction restriction
  - conversion removal
  - impact parameter (primary vertex)
    |dxy|<0.02cm, |dz|<1cm
- Isolation
  - relative isolation in $\Delta R$<0.3

$$\left(\sum p_T(tracks)+\sum E_T(em)+\sum E_T(had)\right)/P_T(\mu)<0.1$$

# Backgrounds and background estimation




- Muon channel
  - QCD upto the Z peak - dominant
    - opposite sign (OS)/same sign (SS) method
    - template fits (muon isolation based)
  - MC estimations for the rest
    - EWK backgrounds (in particular DY($\tau\tau$))
    - top quark pairs
    - QCD at higher masses only

- Electron channel
  - true di-electron (DY→$\tau\tau$, ttbar, WW, tW) - dominant : use data-driven e-mu method
  - fake electron backgrounds (QCD, W+jets) –relatively small : use data-driven “fake rate” method
  - true Z backgrounds (WZ, ZZ) –non-dominant : use MC prediction

Details - in the backup slides.

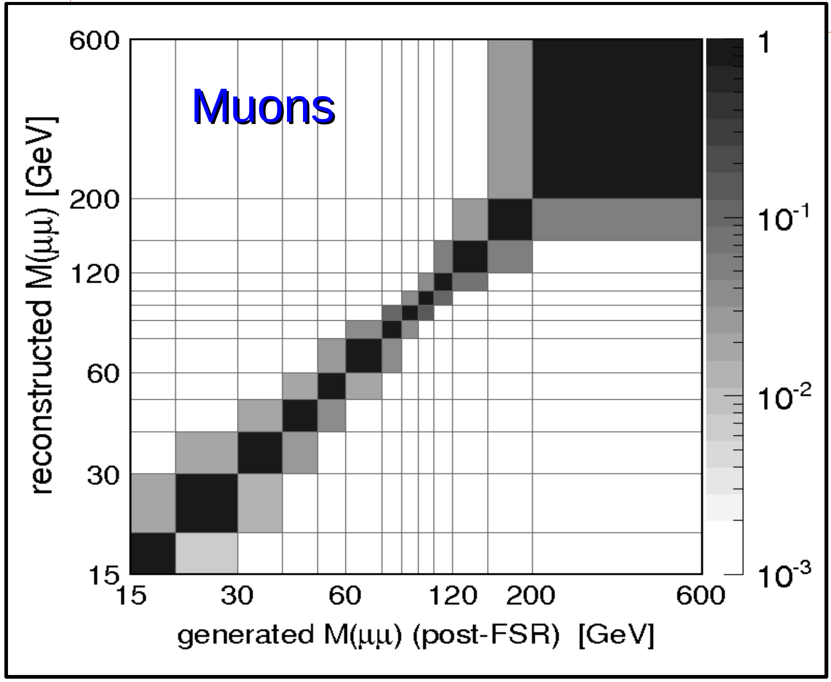
# Unfolding correction

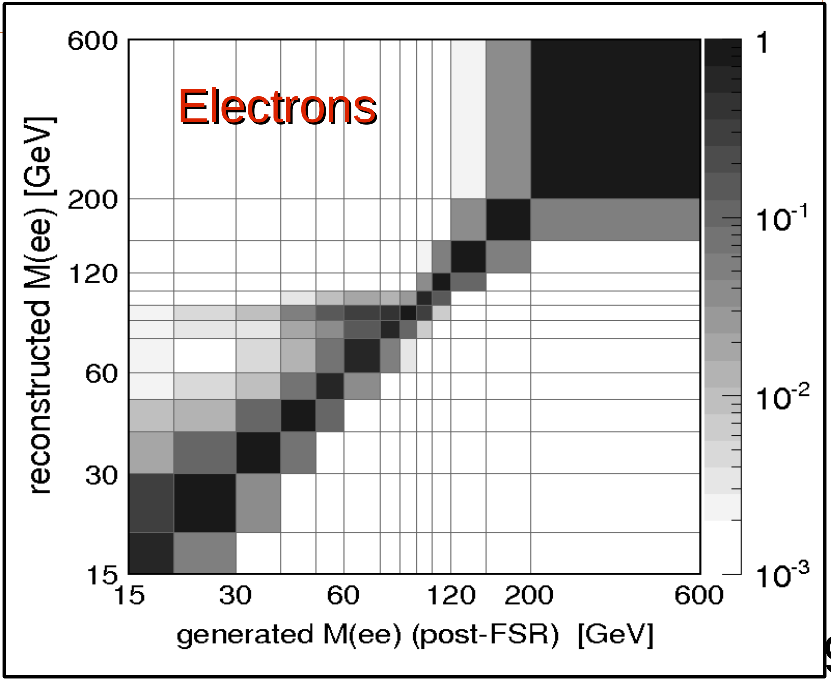
- The unfolding procedure "removes" the effects of the resolution on the mass spectrum
  - FSR correction is done at a different stage
- We use the technique of matrix inversion to unfold the spectrum
  - in the limit of no background, the "ideal" (infinitely good resolution) mass spectrum $N^{true}$ is related to the observed one, $N^{obs}$, by response matrix T: $N_i^{obs} = \sum_k T_{ik} N_k^{true}$

  with $T_{ik}$ –the probability that an event with true mass bin k is reconstructed in mass bin i, which is extracted from the signal MC sample

- Then by inverting the response matrix we can recover the initial spectrum :

$$N_k^{true} = \sum_k (T^{-1})_{ki} N_i^{obs}$$



# Yields

| Inv. mass bin (GeV) | Muons $N^{obs}$ | Muons $N^{obs}$-$N^{bg}$ | Muons $N^{unfolded}$ | Electrons $N^{obs}$ | Electrons $N^{obs}$-$N^{bg}$ | Electrons $N^{unfolded}$ |
|---|---|---|---|---|---|---|
| 15-20 | 253 ± 16 | 241 ± 18 | **243 ± 18** | 16 ± 4 | 16 ± 4 | **16 ± 6** |
| 20-30 | 809 ± 28 | 735 ± 36 | **736 ± 36** | 91 ±10 | 88 ±10 | **94 ± 12** |
| 30-40 | 986 ± 31 | 910 ± 36 | **907 ± 37** | 179 ±13 | 163 ±14 | **164 ± 17** |
| 40-50 | 684 ± 26 | 632 ± 29 | **631 ± 30** | 243 ±16 | 208 ±18 | **219 ± 22** |
| 50-60 | 471 ± 22 | 435 ± 24 | **436 ± 26** | 211 ±15 | 187 ±16 | **234 ± 25** |
| 60-76 | 797 ± 28 | 768 ± 29 | **752 ± 31** | 455 ±21 | 428 ±22 | **620 ± 45** |
| 76-86 | 1761 ± 42 | 1755 ± 42 | **1471 ± 49** | 1599 ±40 | 1588 ±40 | **1277 ± 89** |
| 86-96 | 11786±109 | 11761±109 | **12389±119** | 6998 ±84 | 6981 ±84 | **7182 ±117** |
| 96-106 | 909 ± 30 | 904 ± 30 | **591 ± 38** | 587 ±24 | 581 ±24 | **441 ± 36** |
| 106-120 | 194 ± 14 | 191 ± 30 | **178 ± 17** | 132 ±11 | 127 ±12 | **127 ± 15** |
| 120-150 | 145 ± 12 | 141 ± 12 | **142 ± 13** | 67 ± 8 | 57 ± 9 | **53 ± 10** |
| 150-200 | 53 ± 7 | 49 ± 8 | **47 ± 9** | 34 ± 6 | 27 ± 7 | **25 ± 7** |
| 200-600 | 30 ± 6 | 27 ± 6 | **28 ± 6** | 26 ± 5 | 22 ± 6 | **21 ± 5** |

# Modification of the original DY MC samples

- The DY samples we use are effectively ~NLO (Powheg + Pythia parton showers )
- At low invariant masses, the two high $p_T$ leptons in the analysis, indirectly impose the existence of a hard gluon in the process (in other words, the cross section in LO is vanishing for such a selection)
- The lepton kinematic distributions in this region are very sensitive to the exact description and the acceptance differs by ~50% (NLO vs NNLO) for the lowest invariant mass bin (and less than few % elsewhere)
- Thus for proper description of the low invariant mass region NNLO is mandatory!
- We have applied weights to the original MC samples determined from the ratio between the differential cross sections calculated at NNLO with FEWZ and at NLO with the Powheg MC
- We use this "corrected MC" for calculating acceptance, efficiency and FSR corrections
- Binning effects (limitations/validity of perturbative QCD + statatistical restrictions) are considered as an additional source of systematic uncertainty (~10% at lowest masses)

# Acceptance and efficiency

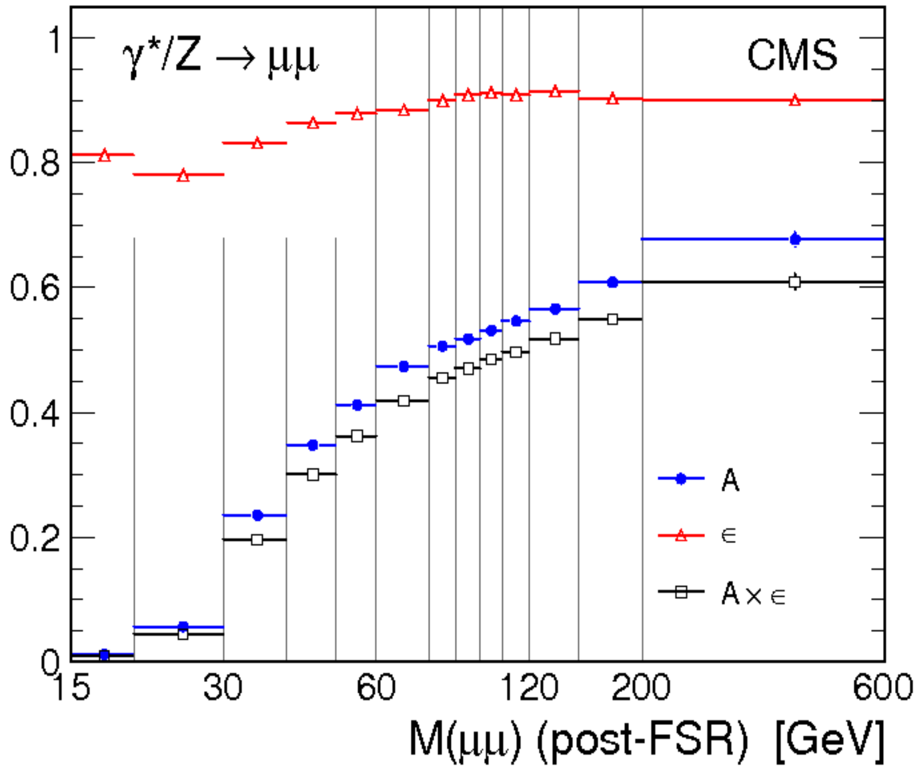
- Acceptance * efficiency is derived from simulation according to:

$$A * \epsilon = \frac{N_{ACC}}{N_{GEN}} \frac{N_{SEL}}{N_{ACC}} = \frac{N_{SEL}}{N_{GEN}} \qquad (\ \leq 1\ )$$

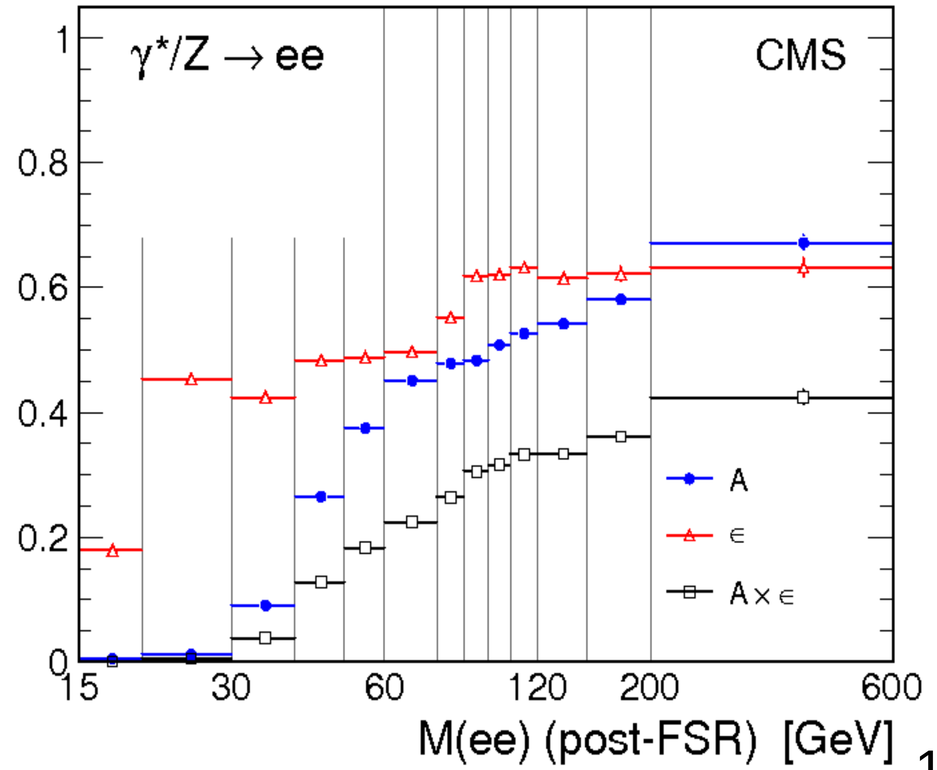
$N_X$ – number of generated events, with X:
GEN – initially generated
ACC – in the acceptance
SEL – (RECO) selected

- Post-FSR lepton quantities are used to calculate the di-lepton invariant mass and apply kinematic cuts
- The acceptance accounts for the $p_T$ and $\eta$ cuts, the efficiency reflects the full selection

Muons




Electrons



# Efficiency factorization and efficiency correction

We need to correct the MC efficiency to "match" the data. We factorize the event efficiency:

$$\epsilon(event)=\epsilon(l_1)\epsilon(l_2)\epsilon(dilepton|l_1,l_2)\epsilon(event,trigger|dilepton)$$

lepton selection

di-lepton selection

trigger selection

**Muons**:

$$\epsilon(event,trig|dimuon)=\epsilon(\mu_1,trig|\mu_1)+\epsilon(\mu_2,trig|\mu_2)-\epsilon(\mu_1,trig|\mu_1)\epsilon(\mu_2,trig|\mu_2)$$

$$\epsilon(\mu)=\epsilon(track)\epsilon(reco+id|track)\epsilon(iso|reco+id)$$

track efficiency

reco+id efficiency

isolation efficiency

**Electrons**:

$$\epsilon(event,trig|dielectron)=\epsilon(leading\ \ e,trig|e)$$

$$\epsilon(e)=\epsilon(reco|ECAL\,E\,deposit)\epsilon(ID|reco)$$

electron object efficiency

selection (id) efficiency

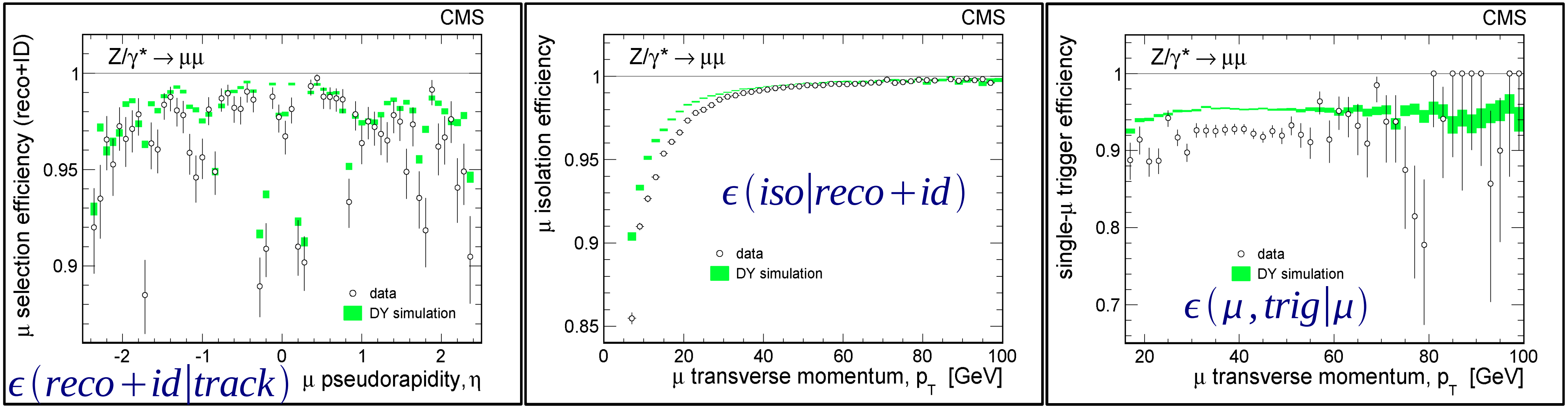
From the deviations observed between data and MC we extract correction factors:

$$\rho_{eff}(p_T,\eta)=\frac{\epsilon_{data}(p_T,\eta)}{\epsilon_{mc}(p_T\eta)}$$

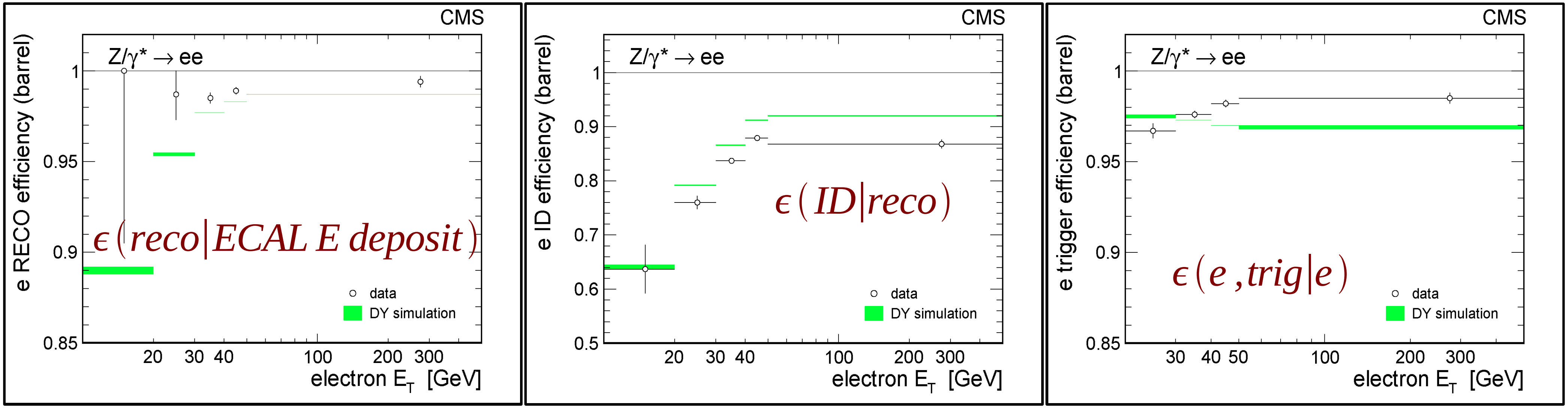
The correction factors are applied per lepton as weights in MC, following the efficiency factorization.

# Data-driven efficiency measurements

## Muons




## Electrons




The lepton efficiencies are estimated by a tag and probe method with the exception of the muon isolation efficiency where the LKTC ("random cones") algorithm is applied.

# Systematic uncertainties

- Energy scale: 2% uncertainty per electron, very significant effect on the mass shape
- Efficiency correction : statistically dominated, significant for lower energy electrons
- Backgrounds : more significant for lower masses (~4%), dominant for higher masses
  - based on data in the regions of significance
- Unfolding: significant around the Z region (~4%)
  - apart from error propagation, additional resolution effects contribute to the uncertainty; for muons, the small momentum scale uncertainty (~0.1%) is incorporated here
- FSR: significant below the Z peak (2%); irrelevant for electrons as other factors dominate
  - based on detailed comparisons between photon spectra properties in data and simulation
- Others: remaining non-dominant sources (pile-up, di-lepton selection)
- Acceptance: theory/PDF uncertainties on the ratio of acceptances (for a shape measurement with acceptance correction) – between 1 and 3% per invariant mass bin
  - based on a single PDF set, correlation between bins taken into account
  - theory uncertainties on the acceptance are small (typically below 1%)

Find the details in the backup slides.

# DY shape measurements and normalization

The DY measurements are normalized to the Z region (60 GeV <M(ll) < 120 GeV) for each of the measurements (so as $R^*_{NORM} \equiv 1$):

$$R_{det,i} = \frac{N_i^U}{\epsilon_i \rho_i} / \frac{N^U_{NORM}}{\epsilon_{NORM}\, \rho_{NORM}}$$

$$R_i = \frac{N_i^U}{A_i \epsilon_i \rho_i} / \frac{N^U_{NORM}}{A_{NORM}\, \epsilon_{NORM}\, \rho_{NORM}}$$

Separately,
with and without FSR corrections
(four measurements per channel)

(pre-FSR) $$r_i = \frac{R_i}{\Delta M_i}$$

We combine the two lepton channels for it and make direct comparisons with predictions (NNLO)

Correlations between bins and between channels do not lead to a significant difference.

# Normalized DY differential cross sections (muons)

| Inv. Mass bin (GeV) | $R_{det,post\text{-}FSR}$ ($10^{-3}$) | $R_{det}$ ($10^{-3}$) | $R_{post\text{-}FSR}$ ($10^{-3}$) | **R ($10^{-3}$)** |
|---|---|---|---|---|
| 15-20 | 18 ± 2 | 19 ± 2 | 772 ± 67 | **780 ± 69** |
| 20-30 | 58 ± 3 | 58 ± 3 | 528 ± 33 | **533 ± 34** |
| 30-40 | 67 ± 3 | 67 ± 3 | 147 ± 8 | **147 ± 8** |
| 40-50 | 44 ± 2 | 41 ± 2 | 66 ± 4 | **62 ± 4** |
| 50-60 | 30 ± 2 | 23 ± 2 | 37 ± 3 | **30 ± 2** |
| 60-76 | 51 ± 2 | 28 ± 1 | 55 ± 3 | **32 ± 2** |
| 76-86 | 97 ± 4 | 56 ± 3 | 98 ± 5 | **58 ± 3** |
| 86-96 | 803 ± 14 | 861 ± 15 | 799 ± 23 | **857 ± 26** |
| 96-106 | 38 ± 3 | 43 ± 3 | 37 ± 3 | **41 ± 3** |
| 106-120 | 12 ± 1 | 12 ± 1 | 11 ± 1 | **12 ± 1** |
| 120-150 | 9.2 ± 0.9 | 9.7 ± 1.0 | 8.4 ± 0.8 | **8.8 ± 0.9** |
| 150-200 | 3.1 ± 0.6 | 3.2 ± 0.7 | 2.6 ± 0.5 | **2.7 ± 0.6** |
| 200-600 | 1.8 ± 0.4 | 1.9 ± 0.5 | 1.4 ± 0.3 | **1.5 ± 0.4** |

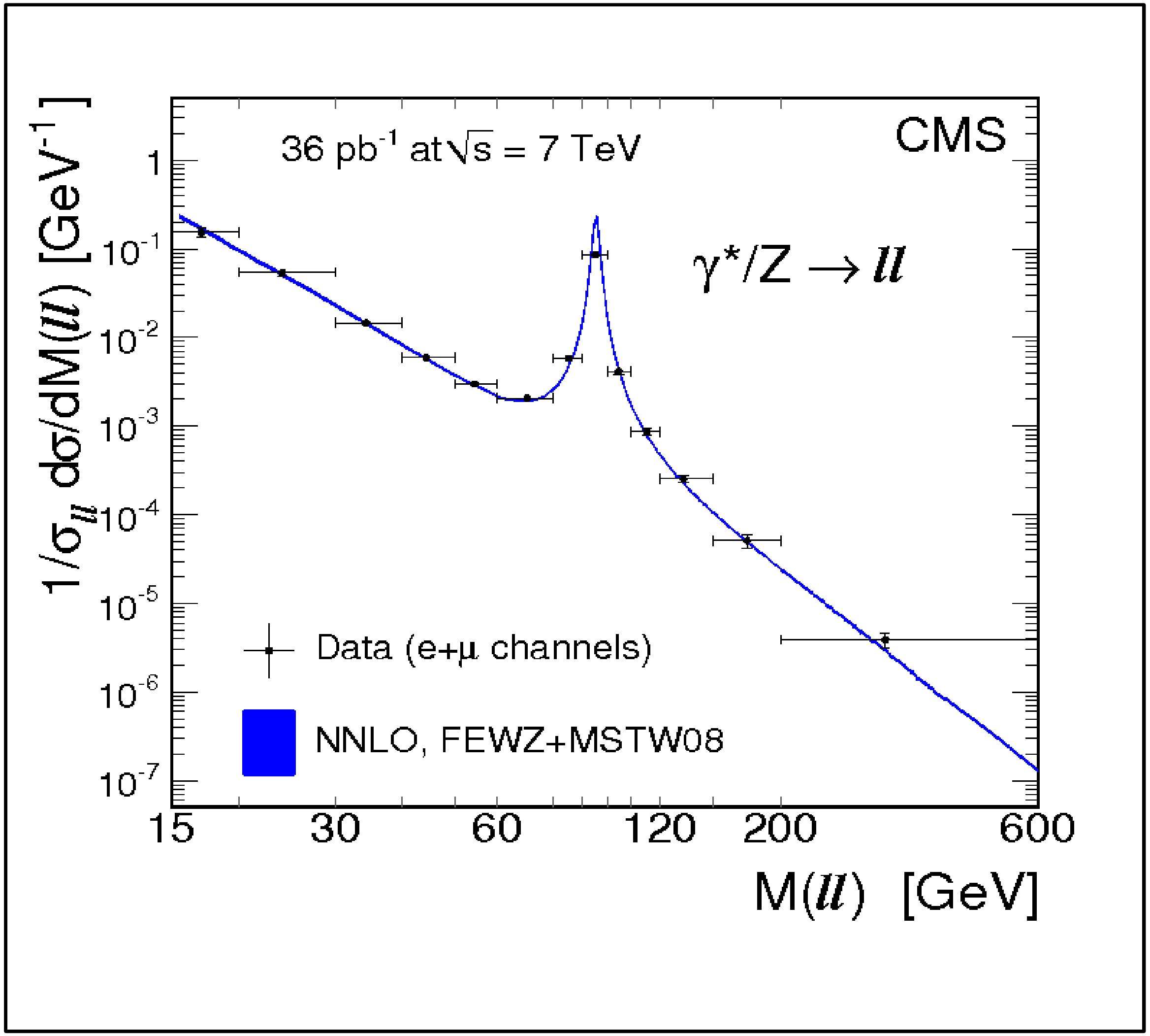
Only R (last column) is directly comparable between channels!

# Normalized DY differential cross sections (electrons)

| Inv. mass bin (GeV) | $R_{det,post\text{-}FSR}$ ($10^{-3}$) | $R_{det}$ ($10^{-3}$) | $R_{post\text{-}FSR}$ ($10^{-3}$) | **R ($10^{-3}$)** |
|---|---|---|---|---|
| 15-20 | 6 ± 3 | 6 ± 3 | 487 ± 230 | **508 ± 238** |
| 20-30 | 13 ± 2 | 13 ± 2 | 536 ± 96 | **559 ± 97** |
| 30-40 | 24 ± 4 | 22 ± 4 | 129 ± 22 | **131 ± 21** |
| 40-50 | 28 ± 4 | 24 ± 4 | 52 ± 8 | **47 ± 7** |
| 50-60 | 30 ± 5 | 19 ± 3 | 39 ± 6 | **27 ± 4** |
| 60-76 | 78 ± 12 | 30 ± 4 | 84 ± 13 | **36 ± 5** |
| 76-86 | 144 ± 60 | 61 ± 25 | 147 ± 60 | **64 ± 26** |
| 86-96 | 722 ± 62 | 839 ± 60 | 715 ± 62 | **834 ± 60** |
| 96-106 | 44 ± 21 | 55 ± 26 | 43 ± 20 | **53 ± 25** |
| 106-120 | 13 ± 3 | 15 ± 3 | 12 ± 2 | **14 ± 3** |
| 120-150 | 5.4 ± 1.2 | 6.0 ± 1.3 | 4.8 ± 1.1 | **5.4 ± 1.2** |
| 150-200 | 2.5 ± 0.8 | 2.8 ± 0.8 | 2.1 ± 0.6 | **2.3 ± 0.7** |
| 200-600 | 2.1 ± 0.6 | 2.4 ± 0.7 | 1.5 ± 0.5 | **1.7 ± 0.5** |

Only R (last column) is directly comparable between channels!

# Results – graphical representation




The vertical error bar indicates the experimental (statistical and systematic) uncertainties summed in quadrature with the theory uncertainty resulting from the model-dependent kinematic distributions inside each bin.

Each data point is located on the horizontal axis at the position where the theoretical function has a value equal to its mean value over the bin.

The measurements are in good agreement with the NNLO theoretical predictions, as computed with FEWZ.

# Conclusions

- The CMS Collaboration have measured the Drell-Yan differential cross section normalized to the Z region in the dilepton invariant mass range 15 GeV <M(ll)< 600 GeV
  - it is based on 36 $pb^{-1}$
- We present results both inside the detector acceptance and in the full phase space
- The effect of final state QED radiation on the results is reported as well
- A correct description of the measurements requires modeling to NNLO for dilepton invariant masses below about 30 GeV
- The measurements are in good agreement with the NNLO theoretical predictions, as computed with FEWZ

Find the paper in arXiv:1108.0566v1 [hep-ex], submitted to JHEP.

# Backup

FEWZ@NNLO, MSTW2008:

| Inv. mass bin (GeV) | **R ($10^{-3}$)** | PDF uncertainties (%) | Theory uncertainties (%) |
|---|---|---|---|
| 15-20 | 812 | +4.3 / -3.3 | +2.5 / -2.7 |
| 20-30 | 494 | +3.6 / -2.8 | +1.9 / -3.6 |
| 30-40 | 141 | +2.7 / -2.3 | +3.1 / -2.1 |
| 40-50 | 55 | +2.1 / -1.9 | +2.4 / -2.5 |
| 50-60 | 28 | +1.6 / -1.5 | +2.6 / -2.0 |
| 60-76 | 33 | +0.9 / -0.9 | +2.0 / -2.4 |
| 76-86 | 58 | +0.2 / -0.2 | +2.1 / -2.5 |
| 86-96 | 844 | +0.1 / -0.1 | +1.8 / -2.2 |
| 96-106 | 52 | +0.2 / -0.2 | +2.8 / -2.0 |
| 106-120 | 13 | +0.5 / -0.5 | +2.6 / -2.2 |
| 120-150 | 6.9 | +0.9 / -0.9 | +2.5 / -1.7 |
| 150-200 | 2.7 | +1.5 / -1.6 | +2.0 / -1.8 |
| 200-600 | 1.3 | +2.8 / -2.9 | +1.8 / -2.1 |

# Backup

Data-driven background estimation methods:

- OS/SS method
  - define 6 categories of dilepton events n|X, where n = 0,1,2 is the number of isolated muons and X is OS or SS pair (per mass bin)
  - estimate the ratio OS/SS for the different categories; note that in MC **1|OS / 1|SS = 2|OS / 2|SS** and assume it holds for data

$$N(2|OS) = N(2|SS)\frac{N(1|OS)}{N(1|SS)}$$

- Template fit method
  - use the isolation variable ($I_{rel}$) shape to discriminate signal from background
  - the background shape is extracted from SS di-muon events
  - the signal shape is extracted from muons in the Z peak given a very tight selection
  - the isolation distribution in the signal region is fitted in bins of muon $p_T$ and di-muon mass

- e-mu method
  - predict true ee (or $\mu\mu$) backgrounds using data e$\mu$ candidates
  - e$\mu$ sample is virtually signal free
  - e$\mu$ $\rightarrow$ ee extrapolation : use acceptance ratio from MC and 2x from branching fraction

All of them agree with the MC predictions within uncertainties.

# Backup

Background estimations, per channel, per source

Electrons Muons

| Inv. mass bin (GeV) | genuine $e^+e^-$ | mis-id electrons | | QCD | EWK, ttbar |
|---|---|---|---|---|---|
| 15-20 | 0.0 ± 0.2 | 0.4 ± 0.7 | | 11 ± 8 | 1 ± 1 |
| 20-30 | 2.5 ± 1.7 | 0.9 ± 1.1 | | 59 ± 21 | 15 ± 4 |
| 30-40 | 14.3 ± 4.6 | 1.5 ± 1.4 | | 46 ± 15 | 30 ± 6 |
| 40-50 | 31.4 ± 6.9 | 3.7 ± 2.7 | | 22 ± 8 | 30 ± 6 |
| 50-60 | 19.9 ± 5.2 | 3.9 ± 2.8 | | 11 ± 7 | 25 ± 6 |
| 60-76 | 22.4 ± 5.3 | 4.9 ± 3.3 | | 7 ± 6 | 22 ± 5 |
| 76-86 | 8.5 ± 2.8 | 2.5 ± 2.1 | | - | 6 ± 3 |
| 86-96 | 12.5 ± 1.8 | 4.4 ± 3.1 | | - | 25 ± 6 |
| 96-106 | 3.5 ± 1.8 | 2.1 ± 1.8 | | - | 5 ± 3 |
| 106-120 | 3.2 ± 1.9 | 1.5 ± 1.4 | | - | 3 ± 2 |
| 120-150 | 7.8 ± 3.1 | 2.0 ± 1.7 | | - | 4 ± 3 |
| 150-200 | 5.5 ± 2.5 | 1.6 ± 1.4 | | - | 4 ± 3 |
| 200-600 | 3.0 ± 1.9 | 1.4 ± 1.4 | | - | 3 ± 2 |

Different challenges suggest different approaches for the two channels.

# Backup

**Systematic uncertainties (Muons / Electrons)_In % :**

| Inv. mass bin (GeV) | Energy scale | Efficiency correction | Back-grounds | Unfolding | FSR | Others | **Total** |
|---|---|---|---|---|---|---|---|
| 15-20 | - / 23.4 | 1.1 / 9.2 | 3.6 / 6.2 | 0.4 / 8.7 | 1.5 / - | 1.0 / - | **4.2 / 27.3** |
| 20-30 | - / 3.6 | 1.1 / 8.5 | 3.1 / 2.8 | 0.2 / 2.1 | 1.1 / - | 1.0 / - | **3.6 / 9.9** |
| 30-40 | - / 2.7 | 1.2 / 9.4 | 1.9 / 4.0 | 0.1 / 1.5 | 0.7 / - | 1.0 / - | **2.6 / 10.6** |
| 40-50 | - / 3.3 | 1.2 / 7.5 | 1.7 / 5.2 | 0.2 / 1.4 | 0.7 / - | 1.0 / - | **2.4 / 9.9** |
| 50-60 | - / 3.3 | 0.8 / 5.2 | 2.1 / 4.6 | 0.2 / 1.9 | 0.5 / - | 0.5 / - | **2.4 / 7.9** |
| 60-76 | - / 10.3 | 0.6 / 3.3 | 1.0 / 2.2 | 0.2 / 2.0 | 1.4 / - | 0.5 / - | **1.9 / 11.2** |
| 76-86 | - / 39.5 | 0.4 / 2.5 | 0.2 / 0.8 | 1.7 / 3.1 | 2.0 / - | 0.5 / - | **2.7 / 39.7** |
| 86-96 | - / 3.9 | 0.3 / 1.9 | 0.05/ 0.2 | 0.2 / 0.6 | 0.5 / - | 0.5 / - | **0.8 / 4.4** |
| 96-106 | - / 45.6 | 0.3 / 2.0 | 0.4 / 0.9 | 3.8 / 3.6 | 0.5 / - | 0.5 / - | **3.9 / 45.8** |
| 106-120 | - / 13.2 | 0.3 / 2.1 | 1.4 / 2.6 | 0.7 / 2.4 | 0.5 / - | 3.0 / - | **3.4 / 13.9** |
| 120-150 | - / 6.0 | 1.1 / 2.4 | 2 / 8.2 | 0.4 / 2.6 | 0.5 / - | 1.0 / - | **2.6 / 10.8** |
| 150-200 | - / 5.7 | 2.1 / 2.8 | 6 / 12.9 | 0.9 / 2.4 | 0.5 / - | 1.0 / - | **6.5 / 14.5** |
| 200-600 | - / 4.6 | 2.1 / 3.2 | 10 / 11.8 | 0.1 / 1.6 | 0.5 / - | 1.0 / - | **10.3 / 13.1** |

# Backup

In the full phase space, in %:

| Inv. mass bin (GeV) | ($\mu$) FSR correction | (e) FSR correction |
|---|---|---|
| 15-20 | 97.28±0.02 | 93.8±0.1 |
| 20-30 | 97.28±0.02 | 93.9±0.2 |
| 30-40 | 98.43±0.02 | 96.8±0.3 |
| 40-50 | 104.0±0.1 | 107.7±0.6 |
| 50-60 | 120.2±0.3 | 139.3±1.0 |
| 60-76 | 166.4±0.5 | 230.7±1.4 |
| 76-86 | 167.1±0.4 | 224.1±1.0 |
| 86-96 | 91.63±0.03 | 83.9±0.1 |
| 96-106 | 88.0±0.1 | 78.5±0.5 |
| 106-120 | 91.3±0.2 | 83.9±1.0 |
| 120-150 | 93.2±0.3 | 87.9±1.4 |
| 150-200 | 94.3±0.4 | 89.1±2.2 |
| 200-600 | 92.8±0.7 | 87.5±3.2 |

# Backup

- Examine data and MC simulation with respect to the FSR
  - sum energy in a cone of $\Delta R<0.3$ around the muon
  - difference in R between the muon and photon with various cuts on the photon energy
- No significant systematic bias
  - the FSR modeling in MC shows a remarkable agreement
  - possible systematic effects estimated by proper statistical variations of the fraction of FSR events as well as the energy and angular distributions of FSR photons