Physics Cases For Muon Colliders

Tao Han
University of Pittsburgh

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(Brown University, Aug. 11, 2011)
“Who ordered that?” (I. I. Rabi)
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Although sharing the same EW interactions, it isn’t another electron:

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\begin{align*}
    m_\mu &\approx 207 \; m_e \\
    \tau(\mu \to e\bar{\nu}_e\nu_\mu) &\approx 2.2 \; \mu s \\
    c\tau &\approx 660 \; m.
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$$c\tau \approx 660 \ m.$$ 

It is these features: heavy mass, short lifetime that dictate the physics.
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\[ \Delta E \sim \gamma^4 = \left( \frac{E}{m_\mu} \right)^4 \]
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which allows a higher energy and much smaller machine:*  

- **LHC**  
  - Protons (P-P)  
  - (1.5 TeV)  

- **ILC**  
  - $e^+e^-$  
  - (0.5 TeV)  

- **CLIC**  
  - $e^+e^-$  
  - (3 TeV)  

- **Mu-Mu**  
  - (4 TeV)  

*Palmer
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(3). Challenges: “Never play with an unstable thing!”

(see Ron Lipton’s talk next)

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Physics at Muon Colliders
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Most unique of all at a muon collider: the $s$-channel scalar resonance.

†Barger, Berger, Gunion, Han
A Higgs Factory

The $s$-channel resonant production:

$$
\sigma(\mu^+\mu^- \rightarrow H, A \rightarrow X) = \frac{4\pi\Gamma(H, A \rightarrow \mu^+\mu^-) \Gamma(H, A \rightarrow X)}{(s - M_H^2)^2 + \Gamma_H M_H^2} \epsilon_E \ll \Gamma_H \Rightarrow \frac{4\pi\Gamma(H, A \rightarrow \mu^+\mu^-) \Gamma(H, A \rightarrow X)}{\Gamma_H^2 M_H^2}.
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$$

$$
\overline{\sigma}(s) = \int d\sqrt{s} \sigma(\mu^+\mu^- \to H, A \to X) \frac{dL}{d\sqrt{s}}
$$

$$
\delta E \ll \Gamma_H \Rightarrow \frac{4\pi \Gamma(H, A \to \mu^+\mu^-) \Gamma(H, A \to X)}{\Gamma_H^2 M_H^2}.
$$

Higgs Total Widths

Effective Cross Sections: $m_h=110$ GeV
Heavy Higgs degenerate as $M_A$ large: $\delta M \approx \frac{M_Z^2}{2M_A} \sin^2 2\beta$. 

400 GeV Higgses resolved! 900 GeV Higgses not resolvable.‡

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Even at high mass, there is sufficient info understanding the Higgs sector:

$$\sigma_{\text{measured}}(b\bar{b}, t\bar{t}, \tau\tau) \Rightarrow \frac{4\pi \Gamma(H, A \rightarrow \mu^+\mu^-) \Gamma(H, A \rightarrow X)}{\Gamma_{\text{tot}}^2 M_H^2}.$$

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- $M_H$: peak, accurate!  
- $\Gamma_{\text{tot}}$: profile, accurate by scanning!  
- $\sigma^{\text{measured}}$: $(b\bar{b})/(t\bar{t}) \approx (m_b^2/m_t^2) \tan^4 \beta$, $(b\bar{b})/(\tau\tau) \approx 3m_b^2/m_\tau^2$ upto radiative corrections.  
- $\sigma^{\text{tot}} = (b\bar{b}) + (t\bar{t}) + \text{(smaller ones)} \Rightarrow \Gamma(\mu^+\mu^-)$ upto missing channels.

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- $\sigma_{\text{tot}} = (b\bar{b}) + (t\bar{t}) + ($smaller ones$) \Rightarrow \Gamma(\mu^+\mu^-)$ upto missing channels.
- Compare with theory: $\Gamma(H, A \rightarrow \mu^+\mu^-)$, learn how many $H, A$'s contributing.
- If $t\bar{t}, \tau\tau$ decay kinematics reconstructed, hope to see CP violation!

‡Gunion, Han
Or any resonance $R$, that couples to a muon

$$\sigma(\mu^+\mu^- \to R \to X) \approx \frac{4(2J + 1)\pi \Gamma(R \to \text{initial}) \Gamma(R \to X)}{\Gamma^2 M^2}.$$
A $Z'$ Factory

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for various couplings and spins (1, 2).

Comment: The LHC will have the full coverage upto $M_{Z'} \sim 4 - 6$ TeV, which will soon motivate/define the machine needs (or otherwise).

Hewett, Rizzo
(1). At LHC, $h_{SM}$ fully covered, but $H, A, H^\pm$ may not.

At $\sqrt{s} = 14$ GeV, still a large hole, especially $M_{H,A} > 500$ GeV.

Significance contours for SUSY Higgses

Regions of the MSSM parameter space ($m_A, \tan\beta$) explorable through various SUSY Higgs channels

- 5 $\sigma$ significance contours
- two-loop / RGE-improved radiative corrections
- $m_{\text{top}} = 175$ GeV, $m_{\text{SUSY}} = 1$ TeV, no stop mixing

$\Delta M_{\tau\tau}$

$\tan\beta$

$m_A$ (GeV)

CMS, $3 \cdot 10^4$ pb$^{-1}$

$H^\pm \to \tau\nu$

$10^4$ pb$^{-1}$

$A, H, h \to \tau\tau \to e + \mu + X$

$A, H \to \tau\tau \to \ell^\pm + \tau \text{jet + } X$

$A, H \to \tau\tau \to h^+ + h^- + X$

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$\tau\tau$

$\gamma\gamma$

LEP II $\sqrt{s} = 200$ GeV

Denegri
(2). At LHC, heavy EW pairs are difficult to search for
\[ \mu^+ \mu^- \rightarrow H_1 H_2, \tilde{H}^+ \tilde{H}^-, \tilde{H}^0 \tilde{H}^0, \tilde{\ell} \tilde{\ell}. \]

IF no help from colored states \[ \tilde{g} \rightarrow q\bar{q} \rightarrow qq' \tilde{\chi}^0,\pm \ldots \]
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At lepton colliders, pair production rather robust:
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\textbf{At lepton colliders, pair production rather robust:}

Once crossing the pair threshold, observation straightforward.
(rather model-independent, like in Two-Higgs Doublet model etc.)
(3). Dark Matter connection:
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Energy edges in chain decays:

$$\mu^+ \mu^- \rightarrow \tilde{\mu}^+ \tilde{\mu}^- \rightarrow \mu^+ \mu^- + E_{\text{miss}} (\tilde{\chi}_0 \tilde{\chi}_0)$$

$$E_{\text{max, min}} = \frac{\sqrt{s}}{4} (1 - \frac{M^2_{\tilde{\chi}_0}}{M^2_{\tilde{\mu}}}) (1 \pm \beta), \quad \beta = (1 - \frac{4M^2_{\tilde{\mu}}}{s})^{1/2}.$$
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Comment: very difficult at LHC due to under-constrained kinematics.

\[ \text{Data, Kong, Matchev} \]
Strong Electroweak Dynamics

$W_L W_L$ scattering

The scattering amplitude behaves as

$$A \sim \begin{cases} 
  m_H^2/v^2 & \text{if light Higgs,} \\
  s_{WW}/v^2 & \text{if no light Higgs.}
\end{cases}$$

Partial wave unitarity implies: $m_H$ or $\sqrt{s_{WW}} \leq 1.2$ TeV.

$$\Rightarrow \sqrt{s_{\mu\mu}} \sim (4)\sqrt{s_{WW}} \geq 4 \text{ TeV.}$$
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\[ \Rightarrow \sqrt{s_{\mu\mu}} \sim (4)\sqrt{s_{WW}} \gtrsim 4 \text{ TeV}. \]

For model connections:

\[ \frac{\sigma(W_L^+W_L^- \rightarrow W_L^+W_L^-)}{\sigma(W_L^+W_L^- \rightarrow Z_LZ_L)} \sim 2 \quad \text{scalar } H^0, \]

\[ \gg 1 \quad \text{vector } \rho_0^{TC}, \]

\[ \sim 2/3 \quad \text{LET } \sqrt{s} \ll M. \]
Consider $\mu^+\mu^- \rightarrow \nu \nu W^+ W^-$, $\nu \nu ZZ$ and $\nu \nu t \bar{t}$ via $H$, $V$ or non-resonance at $\sqrt{s} = 4$ TeV.

Comment: This would be very challenging to test at the LHC.**

**Bagger et al.
Benchmark Processes

Recent meeting at Telluride, CO, *Muon Collider 2011*: 
http://conferences.fnal.gov/muon11
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(1) Higgs, scalar resonances
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(6) Contact interactions
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The main difference between (s)LHC and lepton colliders:

1. LHC: more channels accessible (energy threshold, color, spin).
2. LHC: much larger SM backgrounds.
3. LHC: less constrained kinematics.
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Need to take advantage of the complementarity!