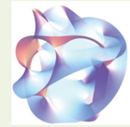
NS5 Branes on the Resolved Cone over $Y^{p,q}$



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Introduction

Purpose: The AdS/CFT correspondence provides a powerful tool to attack very important questions of strong coupling dynamics using gravitational duals. The Klebanov-Stassler prototype has a large family of duals that contain $\mathcal{N}$=1 SYM. A new and distinct family of supergravity solutions containing a sector dual to $\mathcal{N}$=1 SYM might be related to the resolved cone over Einstein-Sasaki spaces. In this work we extend the construction of five brane solutions on the resolved cone over $Y^{p,q}$ spaces by expanding the generalizations of the complex deformations in the context of the warped resolved deformed conifold. This work augments recent work which established the existence of supersymmetric five branes solutions wrapped on two-cycles of the resolved cone over $Y^{p,q}$ in the probe limit. We present an ansatz and the corresponding equations of motion. Here we attempt to solve the field equations and give explicit solutions with the expected properties for a theory related to strongly coupled Yang-Mills theories.

We Want:

- String theory dual to $\mathcal{N}$=1, super Yang Mills (SYM) (standard model)
- broken supersymmetry at low energy scales
- confinement
- stable moduli fields

Compactification Strategy:

- Compactifying with background fluxes using branes and string theory – gauge theory duals can give us everything we want

String Theory – Gauge Theory Duals

○ Simplest example: Type IIB superstring theory in $AdS_5 \times S^5$ background ⟷ $\mathcal{N}$=4,D=4, SYM, gauge group SU(N)

○ Sasaki-Einstein spaces (Einstein spaces where cones over must be noncompact Calabi-Yau) ⟷ $\mathcal{N}$=1 conformal

- Simple Example: $AdS_5 \times T^{1,1}$
- Our Work: $AdS_5 \times Y^{p,q}$
- Future work: $AdS_5 \times L^{p,q,r}$

Review

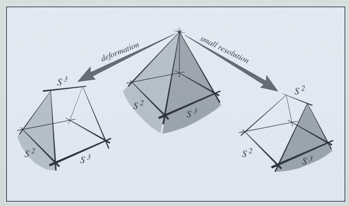
Cone over X_{n-1}

Consider the n dimensional metric on Y_n:

$$h_{mn}dx^m dx^n = dr^2 + r^2 g_{ij} dx^i dx^j$$

where g_{ij} is a metric on an X_{n-1} manifold

- Y_n is a "cone" over X_{n-1}



Deformed and Resolved Cones

[1] Smooth out singularity at tip of cone:

- Deformed Cone: near apex of cone, S^2 shrinks to zero size while S^3 remains with finite radius
- Resolved Cone: S^3 shrinks while S^2 remains nonvanishing

Family of Conifold Strategies

I. D_3 branes on tip of conifold ⟷ $\mathcal{N}$=1, SU(N) x SU(N) conformal, singular in IR **(Klebanov-Witten)**[2]

II. a) N D_3 + M D_5 on 2-cycles on conifold ⟷ $\mathcal{N}$=1, SU(N) x SU(M) non-conformal, singular in IR **(Klebanov-Tseytlin)**[3]
b) N D_3 + M D_5 on 2-cycles on deformed conifold ⟷ $\mathcal{N}$=1 non-conformal, runs log, smooth near IR **(Klebanov-Strassler)**[4]

III. D_5 wrapping 2-cycle on resolved conifold ⟷ $\mathcal{N}$=1 SYM, SU(N), non-conformal, smooth in IR **(Maldacena-Nunez)**[5]

All of these supergravity solutions form one family of geometries.

But no NEW family of supergravity solutions dual to $\mathcal{N}$=1 SYM has been found, but there is hope with $Y^{p,q}$ spaces and branes...

Current Work

- **Cones over $Y^{p,q}$ do not admit complex deformations, but do admit resolutions**
- **Our technique: use NS5 branes on a resolution of the cone over Ypq- only needs SU(3) structure**
- Existence of supersymmetric five brane solutions wrapped on two-cycles of resolved cone over $Y^{p,q}$ in probe limit already established [6]
- Now we use a more general Ansatz and attempt to solve the field equations
- Solution currently in progress…

Geometry

We use the sechsbein $ds^2 = \delta_{ab} e^a e^b$

where the coordinate frames are given to the right. $A = -\frac{Cos[\theta]d[\phi]}{2}$

The string frame metric contains 6 deformation factors dependent on x and y

$$e_1 = \frac{1}{2}d(y)\sqrt{\frac{y-x}{\mathrm{Y}(y)}}\mathrm{a}(x,y) + \frac{1}{2}\sqrt{(1-x)(1-y)}(d(\theta)\cos(2(\tau+\psi)) - d(\phi)\sin(\theta)\sin(2(\tau+\psi))),$$

$$e_2 = \sqrt{\frac{\mathrm{Y}(y)}{y-x}}\mathrm{a}(x,y)((1-x)(A+d(\psi))+d(\tau)) + \frac{1}{2}\sqrt{(1-x)(1-y)}(d(\phi)\sin(\theta)\cos(2(\tau+\psi)) + d(\theta)\sin(2(\tau+\psi))),$$

$$e_3 = \sqrt{\frac{\mathrm{X}(x)}{y-x}}((1-y)(A+d(\psi))+d(\tau)),$$

$$e_4 = -\frac{1}{2}d(y)\sqrt{\frac{y-x}{\mathrm{Y}(y)}},$$

$$e_5 = \sqrt{\frac{\mathrm{Y}(y)}{y-x}}(-(1-x)(A+d(\psi))-d(\tau)),$$

$$e_6 = -\frac{1}{2}d(x)\sqrt{\frac{y-x}{\mathrm{X}(x)}}$$

2-form J

$$J = j(a,b)e_a \wedge e_b$$

Almost Complex Structure Constraint

$$J^n_m J^p_n = -\delta^p_m$$

Holomorphic 3-form Ω

$$\Omega = o_r(a,b,c)e_a \wedge e_b \wedge e_c + i o_i(a,b,c)e_a \wedge e_b \wedge e_c$$

Closed Flux H_3

$$H_3 = H_{closed} + H_{exact} \qquad H_{exact} = dB_2$$

$$dH_3 = 0 \qquad dH_{closed} = 0$$

Constraints

Algebraic (Topological) Constraints

$$J \wedge \Omega = 0$$

$$\Omega \wedge \bar{\Omega} = -\frac{4i}{3}J^3$$

Differential Constraints "Calibrating Conditions"

$$d(e^{-2\phi}\Omega) = 0$$

$$e^{2\phi}d(e^{-2\phi}J) = -\star_6 H_3$$

$$d(e^{-2\phi}J \wedge J) = 0 \quad [7]$$

References

[1] Image taken from *Calabi-Yau Manifolds—A Bestiary for Physicists*, Tristan Hübsch: *World Scientific*, Singapore 1992, [2] hep-th/9807080, [3] hep-th/0012034, [4] hep-th/0007191, [5] hep-th/0008001, [6] E.Caceres, M.N. Mahato, L.A. Pando Zayas, and V.G.J Rodgers, *Toward NS5 Branes on the Resolved Cone over $Y^{p,q}$*, [7] L. Martucci, *D-branes on general N=1 backgrounds: Superpotentials and D-terms,* JHEP 06 (2006) 033, hep-th/0602129