New Mathematics
for
Old Scattering Amplitudes

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DPF 2011

Many have contributed significantly to these developments including

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R. Boels
J. Bourjaily
N.E. Bjerrum-Bohr
A. Brandhuber
M. Bullimore
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S. Caron-Huot
J.J. Carrasco
V. Del Duca
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L. Dolan
C. Duhr
J. Drummond

B. Eden
H. Elvang
B. Feng
D. Freedman
D. Gaiotto
P. Goddard
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J. Henn
P. Heslop
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D. Kosower
L. Lipatov
F. Loebbert

D. Maitre
J. Maldacena
L. Mason
J. Plefka
R. Roiban
A. Sabio Vera
A. Sever
D. Skinner
A. Smirnov
E. Sokatchev
W. Spence
M. Staudacher
G. Travaglini
J. Trnka
C. Vergu
P. Vieira
A. Volovich
C. Wen
E. Witten

The goals of this research program are:

- to explore the rich mathematical structure of scattering amplitudes
- to exploit that structure to make previously impossible calculations trivial

The Appeal of Long-Lasting Vitality

From 2004 through 2011 (and beyond?) the

impossible → hard → trivial

phase transition often occurred on the timescale of months, not years.

Communal pride trumps (occasional) individual shame!

Why $\mathcal{N}=4$ Super-Yang Mills Theory?

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- Moreover it = quantum gravity in AdS space

Why $\mathcal{N}=4$ Super-Yang Mills Theory?

- A nontrivial but apparently solvable four-dimensional QFT.
- But not just some random QFT, it is a gauge theory, and hence a cousin of QCD.
- Moreover it = quantum gravity in AdS space
- Most importantly, because it is there and

YES WE CAN!

S-Matrix Theory Reloaded?

The spirit behind much recent progress is similar to that of "S-Matrix Theory."

If we have seen further than the giants of the past, it is because

S-Matrix Theory Reloaded?

The spirit behind much recent progress is similar to that of "S-Matrix Theory."

If we have seen further than the giants of the past, it is because we're focused squarely on the well-manicured lawn beneath our feet ($N=4$) rather than the dark jungle looming ahead (QCD).

A Completely Different Philosophy

Then, the idea was

(1) enumerate the principles
(locality, analyticity, etc.)

(2) Solve!

A Completely Different Philosophy

Then, the idea was

(1) enumerate the principles
(locality, analyticity, etc.)

(2) Solve!

Now, our aim is to

(1) write down the solution BY ANY MEANS NECESSARY

(2) deduce the principles from understanding its structure

BUCKLE
YOUR
SAFETY BELTS

I'll focus almost exclusively on planar, color-stripped partial amplitudes in $\mathcal{N}=4$ Yang-Mills theory.

$$\mathcal{A}(1,\dots,n)=\sum_{\sigma\in S_n} A(\sigma(1),\dots,\sigma(n))\,\mathrm{Tr}\left[T^{a_{\sigma(1)}}\cdots T^{a_{\sigma(n)}}\right]$$

color-ordered partial amplitude

In massless ϕ^3 theory, two representations of the tree-level four-particle amplitude are

$$A = -\frac{1}{s} - \frac{1}{t} - \frac{1}{u} \qquad \text{and}$$

$$A = \frac{10s^2}{tu} + \frac{20s}{t} - \frac{9s}{tu} + \frac{32s}{u} + \frac{10u}{t} - \frac{15}{t} + \frac{34t}{u} - \frac{15}{u} + 42 + \frac{22t}{s} - \frac{5u}{ts} + \frac{10u}{s} + \frac{12t^2}{us} - \frac{5t}{us} - \frac{11}{s}$$

They are equal, if you use

$$s + t + u = 0$$

A linear constraint $\Rightarrow$ switch to free variables

For particles with spin we need spinor helicity variables.

Solve $k^2=0$ via $k^\mu = \sigma^\mu_{a\dot{a}} \underbrace{\lambda^a \bar{\lambda}^{\dot{a}}}_{\text{2 2-component objects}}$

Notation $\langle ij\rangle = \epsilon_{ab}\lambda_i^a \lambda_j^b$ $\quad [ij] = \epsilon_{\dot{a}\dot{b}}\, \lambda_i^{\dot{a}} \lambda_j^{\dot{b}}$

$\langle i|j+k|\ell] = \langle ij\rangle[j\ell] + \langle ik\rangle[k\ell]$

A spin-1 particle has 2 physical states helicity $+1$ or -1

The n-particle MHV amplitude involves $\begin{matrix} n-2 & +\text{helicity} \\ 2 & -\text{helicity} \end{matrix}$ & particles

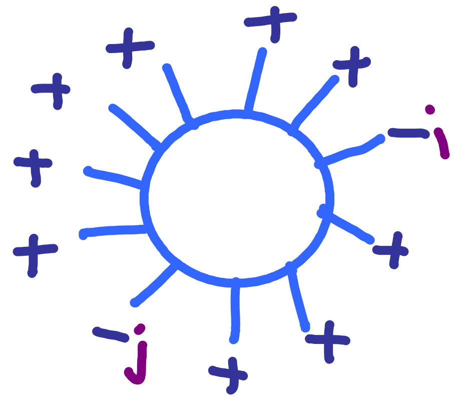
NMHV: $(n-3, 3)$

N^2MHV: $(n-4, 4)$

etc.

Amplitude
Representation
Spinor Helicity
MHV etc.
Parke-Taylor

The tree-level MHV amplitude is



$$= \frac{\langle ij\rangle^4}{\langle 12\rangle\langle 23\rangle\cdots\langle n1\rangle}$$

(Parke Taylor '86, Berends Giele '88)

This is, psychologically, the most important formula in our field.

It's message is ...

The Philosophy of Amplitudeology

1. Simplifications do not happen by accident.

Working in $\mathcal{N}=4$ superspace we write

$$A_{MHV} = \frac{\delta^8\left(\sum_{i=1}^{n} \lambda_i^a \eta_i^A\right)}{\langle 12\rangle\langle 23\rangle \cdots \langle n1\rangle}$$

(Nair '88)

supermomentum conservation

$$A = A_{MHV}\left[1 + P_{NMHV} + P_{NNMHV} + \cdots\right]$$

$O(\eta^8)$ $O(\eta^4)$ $O(\eta^8)$ etc.

Amplitude
Representation
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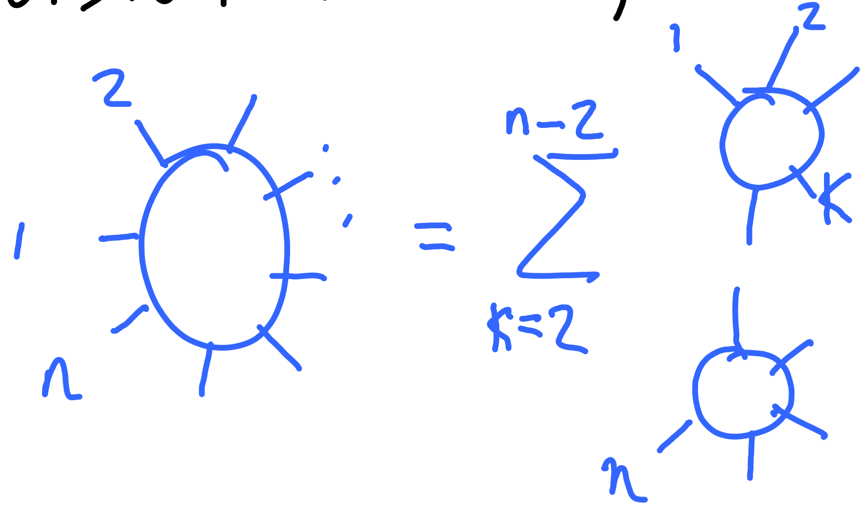
Parke-Taylor
Superamplitude
BCFW

BCFW is a quadratic on-shell recursion relation; schematically



It is important both conceptually and practically — for rapidly generating data.

There is no single "BCFW representation", different discrete choices give apparently different (but secretly =) results.

Example: The amplitude $A(1^-,2^-,3^-,4^+,5^+,6^+)$ may be expressed as either

$$\frac{[4|5+6|1\rangle^3}{[23][34]\langle 56\rangle\langle 61\rangle[2|3+4|5\rangle s_{234}} + \frac{[6|1+2|3\rangle^3}{[61][12]\langle 34\rangle\langle 45\rangle[2|3+4|5\rangle s_{612}}$$

or

$$\frac{s_{123}^3}{[12][23]\langle 45\rangle\langle 56\rangle[1|2+3|4\rangle[3|4+5|6\rangle} + \frac{\langle 12\rangle^3[45]^3}{[34]\langle 61\rangle[3|4+5|6\rangle[5|6+1|2\rangle s_{612}} + \frac{\langle 23\rangle^3[56]^3}{\langle 34\rangle[61][1|2+3|4\rangle[5|6+1|2\rangle s_{234}}$$

Equality of these two expressions can (essentially) only be checked by numerical experimentation.

[Richardson's theorem: there is no algorithm for determining if two mathematical expressions are equal.]

A. Hodges followed his nose and found that there is a <u>reason</u> why in this case (2 terms) = (3 terms).

Each expression computes the volume of a polytope in complex projective space, triangulated in different ways!

This is one of <u>many</u> examples showing that...

The Philosophy of Amplitudeology

1. Simplifications do not happen by accident.
2. This is an experimental science.
   (Get the answer first, by any means necessary, then analyze it.)

Amplitude
Representation
Spinor Helicity
MHV etc.
Parke-Taylor
Superamplitude
BCFW
A=Volumes
Twistor Space

"We can go to twistor space by carrying out a ½-fourier transform

$$A(\lambda_a, \mu^{\dot{a}}) = \int d^2\bar{\lambda}_{\dot{a}}\, e^{i\bar{\lambda}_{\dot{a}}\mu^{\dot{a}}} A(\lambda_a, \bar{\lambda}_{\dot{a}})$$ "

Since an overall rescaling of the 4 variables (λ, μ) leaves x^μ unchanged, they parameterize $\mathbb{CP}^3$.

points in $\mathbb{CP}^3 \Longleftrightarrow$ null rays in spacetime
lines in $\mathbb{CP}^3 \Longleftrightarrow$ points in spacetime

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Noting that

$$A_{MHV} = \overbrace{\int d^4x \, \exp\left(i x_{a\dot{a}} \sum_{i=1}^{n} \lambda_i^a \tilde{\lambda}_i^{\dot{a}}\right)}^{\text{momentum conserving delta function}} \frac{1}{\langle 12\rangle \cdots \langle n1\rangle}$$

Noting that

momentum conserving delta function

$$A_{MHV} = \int d^4x \overbrace{\exp\left(i x_{a\dot{a}} \sum_{i=1}^{n} \lambda_i^a \tilde{\lambda}_i^{\dot{a}}\right)}^{\text{momentum conserving delta function}} \frac{1}{\langle 12\rangle \cdots \langle n1\rangle}$$

measure on the moduli space of lines in $\mathbb{P}^3$

Witten invented a topological string theory on $\mathbb{P}^3$ which computes the tree-level N^kMHV amplitude as an integral over curves of degree $k+1$!

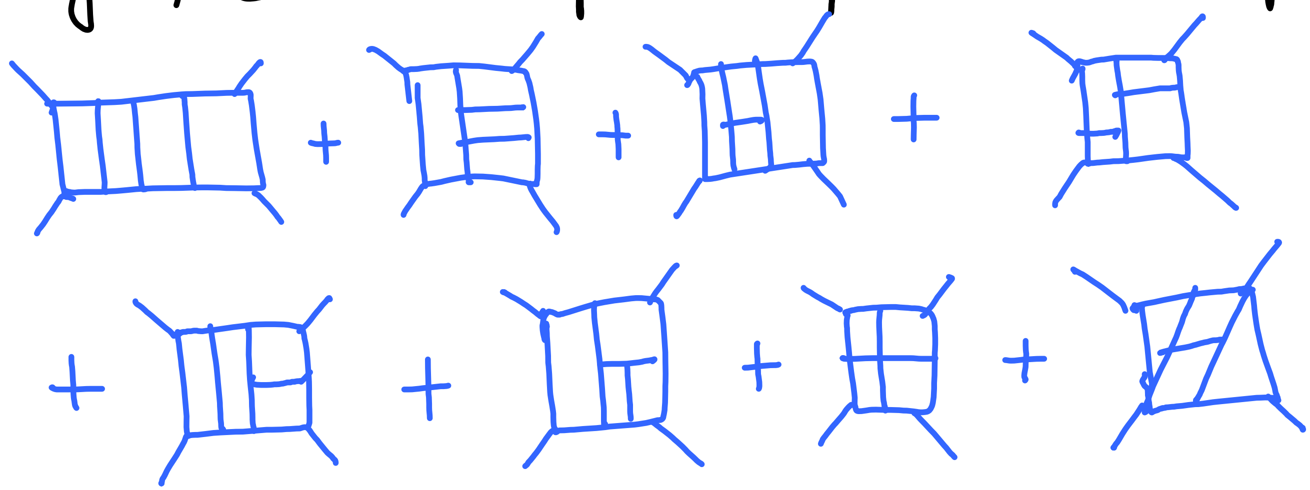
MOVING ON

TO

LOOP LEVEL

Union of the unitarity-based method (Bern, Dixon, Dunbar, Kosower) with more recent advances (Britto, Cachazo, Feng) allowed much more data to be collected

eg the four-loop four-particle amplitude



(Bern, Czakon, Dixon, Kosower, Smirnov)

A pattern in this data reveals a surprise...

What is Dual Conformal Symmetry?

A strange non-local symmetry under which scattering amplitudes are co-variant

- tree level gluon amplitudes in QCD
- all orders in SYM

$N_{color} = \infty$

No understanding at the level of the Lagrangian.

What is Dual Conformal Symmetry?

A strange non-local symmetry under which scattering amplitudes are co-variant

$$\frac{A_{n\text{-particle}}}{A^{\mathrm{MHV}}_{n\text{-particle}}} \text{ is invariant}$$

Dual conformal symmetry

(Drummond, Henn, Korchemsky, Sokatchev)

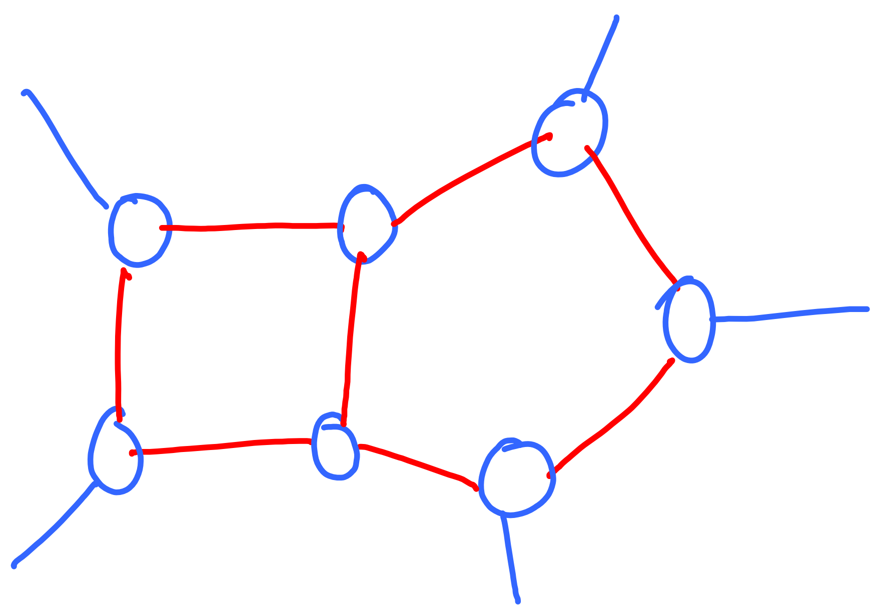
Parameterize

$$\left.\begin{aligned} k_1 &= x_1 - x_2 \\ k_2 &= x_2 - x_3 \\ &\vdots \\ k_n &= x_n - x_1 \end{aligned}\right\} \sum k_i = 0$$

$\Rightarrow$ Then amplitudes are invariant under conformal transformations on the x_i.

The L-loop integrand is a rational function with (by definition) only simple poles in $\mathbb{C}^{4L}$ (the loop momentum variables).

A leading singularity is a residue of the integrand @ one of these poles.



has a pole when all 8 propagators go on-shell

Yangian Symmetry

= the closure under commutation of
{ordinary conformal transformations,
dual conformal transformations}

R-invariants = a natural collection of Yangian invariant objects which constitute the building blocks for e.g. tree-level BCFW representations & leading singularities of loop integrands

(= δ-functions constraining five points in $\mathbb{CP}^{3|4}$ to lie inside a common $\mathbb{CP}^3$ subspace.)

Grassmannian Integral = A contour integral over the space of $k \times n$ matrices

$$\int [dC]_{k\times n} \frac{\delta^{4|4}(C_{ai} Z_i)}{(1\cdots k)(2\cdots k+1)\cdots(n+k-1)} \qquad Z_i = (\lambda_i, \mu_i)$$

minors of the matrix C

(Arkani-Hamed, Cachazo, Cheung, Kaplan)

which is a generating function for all n-particle N^kMHV Yangian invariants.

(It's residues are R-invariants.)

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Dual Conformal Inv.
The Integrand
Leading Sings.
Yangian Symmetry
R-Invariants
Grassmannian Int.
Strong Coupling

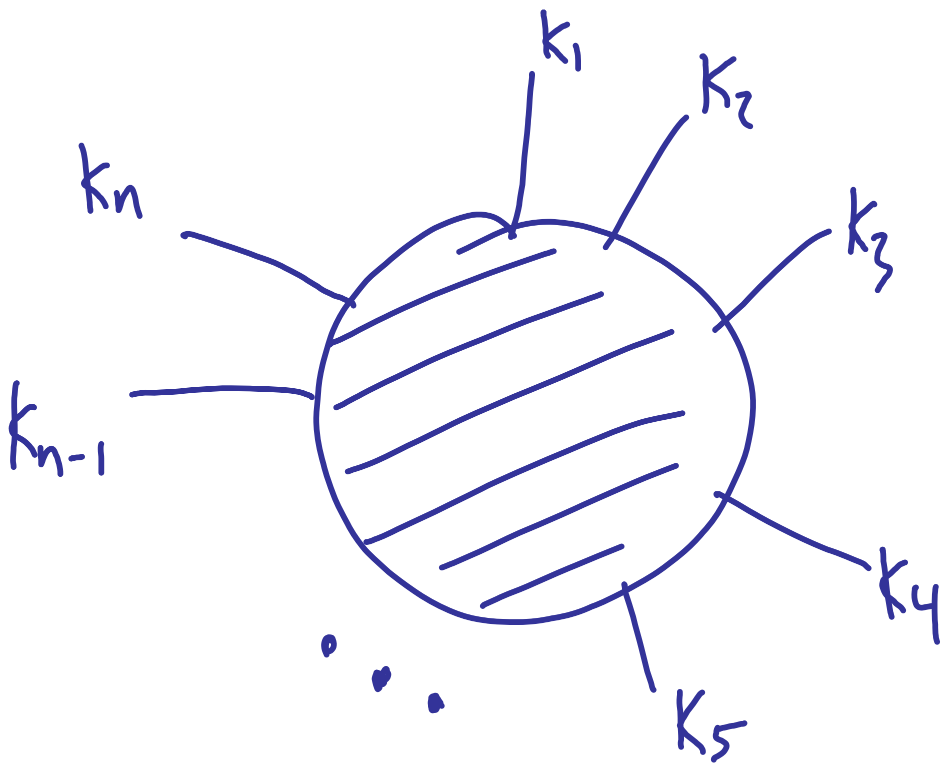
Using AdS/CFT, Alday & Maldacena showed that at strong coupling, amplitudes are computed as certain Wilson loops = minimal area surfaces in AdS_5.

Hence integrability methods can be brought to bear on the problem.

(Alday, Gaiotto, Maldacena)

Amplitudes and Wilson Loops

Another remarkable feature of $\mathcal{N}=4$ supersymmetric Yang-Mills is



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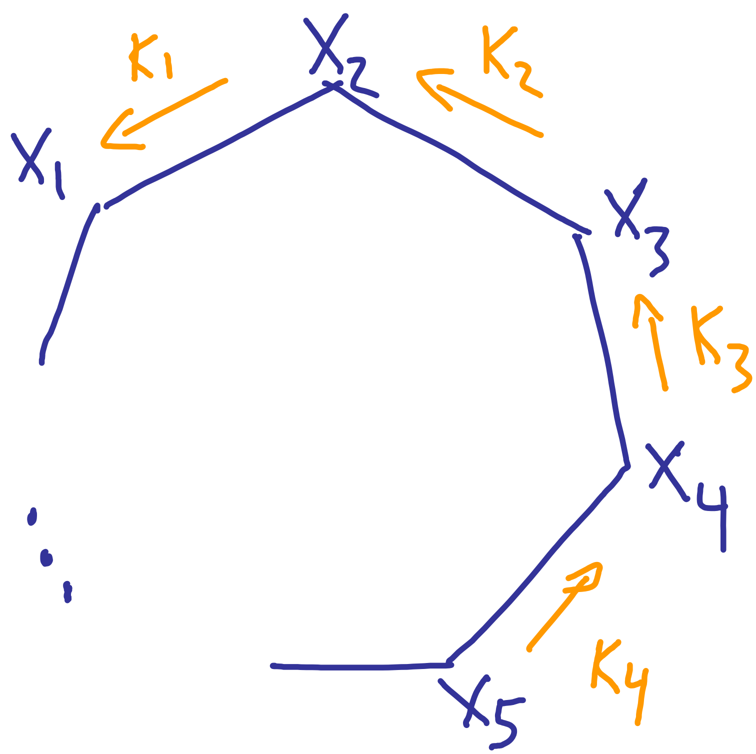

The MHV scattering amplitude, for all n and to all orders in perturbation theory, = the expectation value of a Wilson loop on an n-sided polygon with lightlike edges.

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Strong Coupling
A = Wilson Loop
A = Correlation Fn.

The integrand may in fact be computed yet another way, by correlation functions of the form

$$\Big\langle \underbrace{\mathcal{O}(x_1)\cdots\mathcal{O}(x_n)}_{\substack{\text{light like separated}\\ \text{1/2 BPS operators}}}\ \underbrace{\mathcal{L}(y_1)\cdots\mathcal{L}(y_L)}_{\text{Lagrangian insertions}} \Big\rangle$$

Eden, Heslop, Korchemsky, Sokatchev

Adamo, Bullimore, Mason, Skinner

You Say You Want a Revolution

One revolution has come to fruition

$$A = \int d^4p_1 \cdots d^4p_L \underbrace{\sum \text{Feynman diagrams}}$$

$\Downarrow$ exciting new technology

$$= \int d^4p_1 \cdots d^4p_L \text{ (relatively simple integrand)}$$

You Say You Want a Revolution

One revolution has come to fruition

$$A = \int d^4p_1 \cdots d^4p_L \underbrace{\sum \text{Feynman diagrams}}$$

$\Downarrow$ exciting new technology

$$= \underbrace{\int d^4p_1 \cdots d^4p_L}_{\text{still very hard to evaluate!}} \text{(relatively simple integrand)}$$

But another is desperately needed,

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BDS Remainder

The natural "observable" for loop-level MHV amplitudes is the BDS remainder function which is what is left after appropriate subtraction of IR divergences, leaving a finite & dual conformal invariant quantity.

The first nontrivial loop amplitude in $\mathcal{N}=4$ SYM is the

2-loop (1-loop is absorbed into BDS subtraction)

6-particle ($\nexists$ any cross-ratios for $n<6$)

MHV remainder function.

Computed analytically in late 2009 by Del Duca, Duhr & Smirnov

$$
\begin{aligned}
&R^{(2)}_{6,WL}(u_1,u_2,u_3) = \\
&\frac{1}{24}\pi^2 G\left(\frac{1}{1-u_1},\frac{u_2-1}{u_1+u_2-1};1\right)+\frac{1}{24}\pi^2 G\left(\frac{1}{u_1},\frac{1}{u_1+u_2};1\right)+\frac{1}{24}\pi^2 G\left(\frac{1}{u_1},\frac{1}{u_1+u_3};1\right)+\\
&\frac{1}{24}\pi^2 G\left(\frac{1}{1-u_2},\frac{u_3-1}{u_2+u_3-1};1\right)+\frac{1}{24}\pi^2 G\left(\frac{1}{u_2},\frac{1}{u_1+u_2};1\right)+\frac{1}{24}\pi^2 G\left(\frac{1}{u_2},\frac{1}{u_2+u_3};1\right)+\\
&\frac{1}{24}\pi^2 G\left(\frac{1}{1-u_3},\frac{u_1-1}{u_1+u_3-1};1\right)+\frac{1}{24}\pi^2 G\left(\frac{1}{u_3},\frac{1}{u_1+u_3};1\right)+\frac{1}{24}\pi^2 G\left(\frac{1}{u_3},\frac{1}{u_2+u_3};1\right)+\\
&\frac{3}{2}G\left(0,0,\frac{1}{u_1},\frac{1}{u_1+u_2};1\right)+\frac{3}{2}G\left(0,0,\frac{1}{u_1},\frac{1}{u_1+u_3};1\right)+\frac{3}{2}G\left(0,0,\frac{1}{u_2},\frac{1}{u_1+u_2};1\right)+\\
&\frac{3}{2}G\left(0,0,\frac{1}{u_2},\frac{1}{u_2+u_3};1\right)+\frac{3}{2}G\left(0,0,\frac{1}{u_3},\frac{1}{u_1+u_3};1\right)+\frac{3}{2}G\left(0,0,\frac{1}{u_3},\frac{1}{u_2+u_3};1\right)-\\
&\frac{1}{2}G\left(0,\frac{1}{u_1},0,\frac{1}{u_2};1\right)+G\left(0,\frac{1}{u_1},0,\frac{1}{u_1+u_2};1\right)-\frac{1}{2}G\left(0,\frac{1}{u_1},0,\frac{1}{u_3};1\right)+\\
&G\left(0,\frac{1}{u_1},0,\frac{1}{u_1+u_3};1\right)-\frac{1}{2}G\left(0,\frac{1}{u_1},\frac{1}{u_1},\frac{1}{u_1+u_2};1\right)-\frac{1}{2}G\left(0,\frac{1}{u_1},\frac{1}{u_1},\frac{1}{u_1+u_3};1\right)-\\
&\frac{1}{2}G\left(0,\frac{1}{u_1},\frac{1}{u_2},\frac{1}{u_1+u_2};1\right)-\frac{1}{2}G\left(0,\frac{1}{u_1},\frac{1}{u_3},\frac{1}{u_1+u_3};1\right)-\frac{1}{2}G\left(0,\frac{1}{u_2},0,\frac{1}{u_1};1\right)+\\
&G\left(0,\frac{1}{u_2},0,\frac{1}{u_1+u_2};1\right)-\frac{1}{2}G\left(0,\frac{1}{u_2},0,\frac{1}{u_3};1\right)+G\left(0,\frac{1}{u_2},0,\frac{1}{u_2+u_3};1\right)-\\
&\frac{1}{2}G\left(0,\frac{1}{u_2},\frac{1}{u_1},\frac{1}{u_1+u_2};1\right)-\frac{1}{2}G\left(0,\frac{1}{u_2},\frac{1}{u_2},\frac{1}{u_1+u_2};1\right)-\frac{1}{2}G\left(0,\frac{1}{u_2},\frac{1}{u_2},\frac{1}{u_2+u_3};1\right)-\\
&\frac{1}{2}G\left(0,\frac{1}{u_2},\frac{1}{u_3},\frac{1}{u_2+u_3};1\right)+\frac{1}{4}G\left(0,\frac{u_2-1}{u_1+u_2-1},0,\frac{1}{1-u_1};1\right)+\\
&\frac{1}{4}G\left(0,\frac{u_2-1}{u_1+u_2-1},\frac{1}{1-u_1},0;1\right)-\frac{1}{4}G\left(0,\frac{u_2-1}{u_1+u_2-1},\frac{1}{1-u_1},1;1\right)+\\
&\frac{1}{4}G\left(0,\frac{u_2-1}{u_1+u_2-1},\frac{1}{1-u_1},\frac{1}{1-u_1};1\right)-\frac{1}{4}G\left(0,\frac{u_2-1}{u_1+u_2-1},\frac{u_2-1}{u_1+u_2-1},\frac{1}{1-u_1};1\right)-\\
&\frac{1}{2}G\left(0,\frac{1}{u_3},0,\frac{1}{u_1};1\right)-\frac{1}{2}G\left(0,\frac{1}{u_3},0,\frac{1}{u_2};1\right)+G\left(0,\frac{1}{u_3},0,\frac{1}{u_1+u_3};1\right)+\\
&G\left(0,\frac{1}{u_3},0,\frac{1}{u_2+u_3};1\right)-\frac{1}{2}G\left(0,\frac{1}{u_3},\frac{1}{u_1},\frac{1}{u_1+u_3};1\right)-\frac{1}{2}G\left(0,\frac{1}{u_3},\frac{1}{u_2},\frac{1}{u_2+u_3};1\right)-\\
&\frac{1}{2}G\left(0,\frac{1}{u_3},\frac{1}{u_3},\frac{1}{u_1+u_3};1\right)-\frac{1}{2}G\left(0,\frac{1}{u_3},\frac{1}{u_3},\frac{1}{u_2+u_3};1\right)+\\
&\frac{1}{4}G\left(0,\frac{u_1-1}{u_1+u_3-1},0,\frac{1}{1-u_3};1\right)+\frac{1}{4}G\left(0,\frac{u_1-1}{u_1+u_3-1},\frac{1}{1-u_3},0;1\right)-\\
&\frac{1}{4}G\left(0,\frac{u_1-1}{u_1+u_3-1},\frac{1}{1-u_3},1;1\right)+\frac{1}{4}G\left(0,\frac{u_1-1}{u_1+u_3-1},\frac{1}{1-u_3},\frac{1}{1-u_3};1\right)-\\
&\frac{1}{4}G\left(0,\frac{u_1-1}{u_1+u_3-1},\frac{u_1-1}{u_1+u_3-1},\frac{1}{1-u_3};1\right)+\frac{1}{4}G\left(0,\frac{u_3-1}{u_2+u_3-1},0,\frac{1}{1-u_2};1\right)+\\
&\frac{1}{4}G\left(0,\frac{u_3-1}{u_2+u_3-1},\frac{1}{1-u_2},0;1\right)-\frac{1}{4}G\left(0,\frac{u_3-1}{u_2+u_3-1},\frac{1}{1-u_2},1;1\right)+\\
&\frac{1}{4}G\left(0,\frac{u_3-1}{u_2+u_3-1},\frac{1}{1-u_2},\frac{1}{1-u_2};1\right)-\frac{1}{4}G\left(0,\frac{u_3-1}{u_2+u_3-1},\frac{u_3-1}{u_2+u_3-1},\frac{1}{1-u_2};1\right)-\\
&\frac{1}{4}G\left(\frac{1}{1-u_1},1,\frac{1}{u_3},0;1\right)+\frac{1}{2}G\left(\frac{1}{1-u_1},\frac{1}{1-u_1},1,\frac{1}{1-u_1};1\right)+
\end{aligned}
\tag{H.1}
$$

$$
\begin{aligned}
&\frac{1}{4}G\left(\frac{1}{1-u_1},\frac{u_2-1}{u_1+u_2-1},0,1;1\right)-\frac{1}{4}G\left(\frac{1}{1-u_1},\frac{u_2-1}{u_1+u_2-1},0,\frac{1}{1-u_1};1\right)+\\
&\frac{1}{4}G\left(\frac{1}{1-u_1},\frac{u_2-1}{u_1+u_2-1},1,0;1\right)-\frac{1}{4}G\left(\frac{1}{1-u_1},\frac{u_2-1}{u_1+u_2-1},\frac{1}{1-u_1},0;1\right)+\\
&\frac{1}{4}G\left(\frac{1}{1-u_1},\frac{u_2-1}{u_1+u_2-1},\frac{1}{1-u_1},1;1\right)-\frac{1}{4}G\left(\frac{1}{1-u_1},\frac{u_2-1}{u_1+u_2-1},\frac{1}{1-u_1},\frac{1}{1-u_1};1\right)-\\
&\frac{1}{4}G\left(\frac{1}{1-u_1},\frac{u_2-1}{u_1+u_2-1},\frac{u_2-1}{u_1+u_2-1},1;1\right)+\\
&\frac{1}{4}G\left(\frac{1}{1-u_1},\frac{u_2-1}{u_1+u_2-1},\frac{u_2-1}{u_1+u_2-1},\frac{1}{1-u_1};1\right)-G\left(\frac{1}{u_1},0,0,\frac{1}{u_2};1\right)+\\
&\frac{1}{2}G\left(\frac{1}{u_1},0,0,\frac{1}{u_1+u_2};1\right)-G\left(\frac{1}{u_1},0,0,\frac{1}{u_3};1\right)+\frac{1}{2}G\left(\frac{1}{u_1},0,0,\frac{1}{u_1+u_3};1\right)-\\
&\frac{1}{4}G\left(\frac{1}{u_1},0,\frac{1}{u_1},\frac{1}{u_1+u_2};1\right)-\frac{1}{4}G\left(\frac{1}{u_1},0,\frac{1}{u_1},\frac{1}{u_1+u_3};1\right)-\frac{1}{4}G\left(\frac{1}{u_1},0,\frac{1}{u_2},\frac{1}{u_1+u_2};1\right)-\\
&\frac{1}{4}G\left(\frac{1}{u_1},0,\frac{1}{u_3},\frac{1}{u_1+u_3};1\right)-\frac{1}{4}G\left(\frac{1}{1-u_2},1,\frac{1}{u_1},0;1\right)+\\
&\frac{1}{2}G\left(\frac{1}{1-u_2},\frac{1}{1-u_2},1,\frac{1}{1-u_2};1\right)+\frac{1}{4}G\left(\frac{1}{1-u_2},\frac{u_3-1}{u_2+u_3-1},0,1;1\right)-\\
&\frac{1}{4}G\left(\frac{1}{1-u_2},\frac{u_3-1}{u_2+u_3-1},0,\frac{1}{1-u_2};1\right)+\frac{1}{4}G\left(\frac{1}{1-u_2},\frac{u_3-1}{u_2+u_3-1},1,0;1\right)-\\
&\frac{1}{4}G\left(\frac{1}{1-u_2},\frac{u_3-1}{u_2+u_3-1},\frac{1}{1-u_2},0;1\right)+\frac{1}{4}G\left(\frac{1}{1-u_2},\frac{u_3-1}{u_2+u_3-1},\frac{1}{1-u_2},1;1\right)-\\
&\frac{1}{4}G\left(\frac{1}{1-u_2},\frac{u_3-1}{u_2+u_3-1},\frac{1}{1-u_2},\frac{1}{1-u_2};1\right)-\\
&\frac{1}{4}G\left(\frac{1}{1-u_2},\frac{u_3-1}{u_2+u_3-1},\frac{u_3-1}{u_2+u_3-1},1;1\right)+\\
&\frac{1}{4}G\left(\frac{1}{1-u_2},\frac{u_3-1}{u_2+u_3-1},\frac{u_3-1}{u_2+u_3-1},\frac{1}{1-u_2};1\right)-G\left(\frac{1}{u_2},0,0,\frac{1}{u_1};1\right)+\\
&\frac{1}{2}G\left(\frac{1}{u_2},0,0,\frac{1}{u_1+u_2};1\right)-G\left(\frac{1}{u_2},0,0,\frac{1}{u_3};1\right)+\frac{1}{2}G\left(\frac{1}{u_2},0,0,\frac{1}{u_2+u_3};1\right)-\\
&\frac{1}{4}G\left(\frac{1}{u_2},0,\frac{1}{u_1},\frac{1}{u_1+u_2};1\right)-\frac{1}{4}G\left(\frac{1}{u_2},0,\frac{1}{u_2},\frac{1}{u_1+u_2};1\right)-\frac{1}{4}G\left(\frac{1}{u_2},0,\frac{1}{u_2},\frac{1}{u_2+u_3};1\right)-\\
&\frac{1}{4}G\left(\frac{1}{u_2},0,\frac{1}{u_3},\frac{1}{u_2+u_3};1\right)-\frac{1}{4}G\left(\frac{1}{1-u_3},1,\frac{1}{u_2},0;1\right)+\\
&\frac{1}{2}G\left(\frac{1}{1-u_3},\frac{1}{1-u_3},1,\frac{1}{1-u_3};1\right)+\frac{1}{4}G\left(\frac{1}{1-u_3},\frac{u_1-1}{u_1+u_3-1},0,1;1\right)-\\
&\frac{1}{4}G\left(\frac{1}{1-u_3},\frac{u_1-1}{u_1+u_3-1},0,\frac{1}{1-u_3};1\right)+\frac{1}{4}G\left(\frac{1}{1-u_3},\frac{u_1-1}{u_1+u_3-1},1,0;1\right)-\\
&\frac{1}{4}G\left(\frac{1}{1-u_3},\frac{u_1-1}{u_1+u_3-1},\frac{1}{1-u_3},0;1\right)+\frac{1}{4}G\left(\frac{1}{1-u_3},\frac{u_1-1}{u_1+u_3-1},\frac{1}{1-u_3},1;1\right)-\\
&\frac{1}{4}G\left(\frac{1}{1-u_3},\frac{u_1-1}{u_1+u_3-1},\frac{1}{1-u_3},\frac{1}{1-u_3};1\right)-\\
&\frac{1}{4}G\left(\frac{1}{1-u_3},\frac{u_1-1}{u_1+u_3-1},\frac{u_1-1}{u_1+u_3-1},1;1\right)-\frac{79\pi^4}{360}+\\
&\frac{1}{4}G\left(\frac{1}{1-u_3},\frac{u_1-1}{u_1+u_3-1},\frac{u_1-1}{u_1+u_3-1},\frac{1}{1-u_3};1\right)-G\left(\frac{1}{u_3},0,0,\frac{1}{u_1};1\right)-
\end{aligned}
$$

$$
\begin{aligned}
&G\left(\frac{1}{u_3},0,0,\frac{1}{u_2};1\right)+\frac{1}{2}G\left(\frac{1}{u_3},0,0,\frac{1}{u_1+u_3};1\right)+\frac{1}{2}G\left(\frac{1}{u_3},0,0,\frac{1}{u_2+u_3};1\right)-\\
&\frac{1}{4}G\left(\frac{1}{u_3},0,\frac{1}{u_1},\frac{1}{u_1+u_3};1\right)-\frac{1}{4}G\left(\frac{1}{u_3},0,\frac{1}{u_2},\frac{1}{u_2+u_3};1\right)-\frac{1}{4}G\left(\frac{1}{u_3},0,\frac{1}{u_3},\frac{1}{u_1+u_3};1\right)-\\
&\frac{1}{4}G\left(\frac{1}{u_3},0,\frac{1}{u_3},\frac{1}{u_2+u_3};1\right)-\frac{1}{24}\pi^2\mathcal{G}\left(\frac{1}{1-u_1},u_{123};1\right)+\frac{1}{8}\pi^2\mathcal{G}\left(\frac{1}{1-u_1},v_{123};1\right)+\\
&\frac{1}{8}\pi^2\mathcal{G}\left(\frac{1}{1-u_1},v_{132};1\right)-\frac{1}{24}\pi^2\mathcal{G}\left(\frac{1}{1-u_2},u_{231};1\right)+\frac{1}{8}\pi^2\mathcal{G}\left(\frac{1}{1-u_2},v_{213};1\right)+\\
&\frac{1}{8}\pi^2\mathcal{G}\left(\frac{1}{1-u_2},v_{231};1\right)-\frac{1}{24}\pi^2\mathcal{G}\left(\frac{1}{1-u_3},u_{312};1\right)+\frac{1}{8}\pi^2\mathcal{G}\left(\frac{1}{1-u_3},v_{312};1\right)+\\
&\frac{1}{8}\pi^2\mathcal{G}\left(\frac{1}{1-u_3},v_{321};1\right)-\frac{1}{4}\mathcal{G}\left(0,0,\frac{1}{1-u_1},v_{123};1\right)-\frac{1}{4}\mathcal{G}\left(0,0,\frac{1}{1-u_1},v_{132};1\right)-\\
&\frac{1}{4}\mathcal{G}\left(0,0,\frac{1}{1-u_2},v_{213};1\right)-\frac{1}{4}\mathcal{G}\left(0,0,\frac{1}{1-u_2},v_{231};1\right)-\frac{1}{4}\mathcal{G}\left(0,0,\frac{1}{1-u_3},v_{312};1\right)-\\
&\frac{1}{4}\mathcal{G}\left(0,0,\frac{1}{1-u_3},v_{321};1\right)-\frac{1}{4}\mathcal{G}\left(0,0,v_{123},\frac{1}{1-u_1};1\right)+\mathcal{G}\left(0,0,v_{132},0;1\right)-\\
&\frac{1}{4}\mathcal{G}\left(0,0,v_{132},\frac{1}{1-u_1};1\right)+\mathcal{G}\left(0,0,v_{213},0;1\right)-\frac{1}{4}\mathcal{G}\left(0,0,v_{213},\frac{1}{1-u_2};1\right)-\\
&\frac{1}{4}\mathcal{G}\left(0,0,v_{231},\frac{1}{1-u_2};1\right)-\frac{1}{4}\mathcal{G}\left(0,0,v_{312},\frac{1}{1-u_3};1\right)+\mathcal{G}\left(0,0,v_{321},0;1\right)-\\
&\frac{1}{4}\mathcal{G}\left(0,0,v_{321},\frac{1}{1-u_3};1\right)-\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_1},0,v_{123};1\right)-\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_1},0,v_{132};1\right)-\\
&\frac{1}{2}\mathcal{G}\left(0,\frac{1}{1-u_1},\frac{1}{1-u_1},v_{123};1\right)-\frac{1}{2}\mathcal{G}\left(0,\frac{1}{1-u_1},\frac{1}{1-u_1},v_{132};1\right)-\\
&\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_1},v_{123},1;1\right)-\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_1},v_{123},\frac{1}{1-u_1};1\right)-\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_1},v_{132},1;1\right)-\\
&\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_1},v_{132},\frac{1}{1-u_1};1\right)-\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_2},0,v_{213};1\right)-\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_2},0,v_{231};1\right)-\\
&\frac{1}{2}\mathcal{G}\left(0,\frac{1}{1-u_2},\frac{1}{1-u_2},v_{213};1\right)-\frac{1}{2}\mathcal{G}\left(0,\frac{1}{1-u_2},\frac{1}{1-u_2},v_{231};1\right)-\\
&\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_2},v_{213},1;1\right)-\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_2},v_{213},\frac{1}{1-u_2};1\right)-\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_2},v_{231},1;1\right)-\\
&\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_2},v_{231},\frac{1}{1-u_2};1\right)-\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_3},0,v_{312};1\right)-\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_3},0,v_{321};1\right)-\\
&\frac{1}{2}\mathcal{G}\left(0,\frac{1}{1-u_3},\frac{1}{1-u_3},v_{312};1\right)-\frac{1}{2}\mathcal{G}\left(0,\frac{1}{1-u_3},\frac{1}{1-u_3},v_{321};1\right)-\\
&\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_3},v_{312},1;1\right)-\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_3},v_{312},\frac{1}{1-u_3};1\right)-\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_3},v_{321},1;1\right)-\\
&\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_3},v_{321},\frac{1}{1-u_3};1\right)-\frac{1}{4}\mathcal{G}\left(0,u_{123},0,\frac{1}{1-u_1};1\right)-\frac{1}{4}\mathcal{G}\left(0,u_{123},\frac{1}{1-u_1},0;1\right)+\\
&\frac{1}{4}\mathcal{G}\left(0,u_{123},\frac{1}{1-u_1},1;1\right)-\frac{1}{4}\mathcal{G}\left(0,u_{123},\frac{1}{1-u_1},\frac{1}{1-u_1};1\right)-\\
&\frac{1}{4}\mathcal{G}\left(0,u_{123},\frac{u_2-1}{u_1+u_2-1},1;1\right)+\frac{1}{4}\mathcal{G}\left(0,u_{123},\frac{u_2-1}{u_1+u_2-1},\frac{1}{1-u_1};1\right)-\\
&\frac{1}{4}\mathcal{G}\left(0,u_{123},\frac{1}{u_3},0;1\right)-\frac{1}{4}\mathcal{G}\left(0,u_{231},0,\frac{1}{1-u_2};1\right)-\frac{1}{4}\mathcal{G}\left(0,u_{231},\frac{1}{u_1},0;1\right)-
\end{aligned}
$$

$$
\begin{aligned}
&\frac{1}{4}\mathcal{G}\left(0,u_{231},\frac{1}{1-u_2},0;1\right)+\frac{1}{4}\mathcal{G}\left(0,u_{231},\frac{1}{1-u_2},1;1\right)-\frac{1}{4}\mathcal{G}\left(0,u_{231},\frac{1}{1-u_2},\frac{1}{1-u_2};1\right)-\\
&\frac{1}{4}\mathcal{G}\left(0,u_{231},\frac{u_3-1}{u_2+u_3-1},1;1\right)+\frac{1}{4}\mathcal{G}\left(0,u_{231},\frac{u_3-1}{u_2+u_3-1},\frac{1}{1-u_2};1\right)-\\
&\frac{1}{4}\mathcal{G}\left(0,u_{312},0,\frac{1}{1-u_3};1\right)-\frac{1}{4}\mathcal{G}\left(0,u_{312},\frac{1}{u_2},0;1\right)-\frac{1}{4}\mathcal{G}\left(0,u_{312},\frac{1}{1-u_3},0;1\right)+\\
&\frac{1}{4}\mathcal{G}\left(0,u_{312},\frac{1}{1-u_3},1;1\right)-\frac{1}{4}\mathcal{G}\left(0,u_{312},\frac{1}{1-u_3},\frac{1}{1-u_3};1\right)-\\
&\frac{1}{4}\mathcal{G}\left(0,u_{312},\frac{u_1-1}{u_1+u_3-1},1;1\right)+\frac{1}{4}\mathcal{G}\left(0,u_{312},\frac{u_1-1}{u_1+u_3-1},\frac{1}{1-u_3};1\right)+\\
&\frac{1}{4}\mathcal{G}\left(0,v_{123},0,\frac{1}{1-u_1};1\right)-\frac{1}{2}\mathcal{G}\left(0,v_{123},1,\frac{1}{1-u_1};1\right)+\frac{1}{4}\mathcal{G}\left(0,v_{123},\frac{1}{1-u_1},0;1\right)-\\
&\frac{1}{2}\mathcal{G}\left(0,v_{123},\frac{1}{1-u_1},1;1\right)+\frac{1}{4}\mathcal{G}\left(0,v_{123},\frac{1}{1-u_1},\frac{1}{1-u_1};1\right)-\frac{1}{4}\mathcal{G}\left(0,v_{132},0,\frac{1}{1-u_1};1\right)-\\
&\frac{1}{4}\mathcal{G}\left(0,v_{132},\frac{1}{1-u_1},0;1\right)-\frac{1}{4}\mathcal{G}\left(0,v_{132},\frac{1}{1-u_1},\frac{1}{1-u_1};1\right)-\frac{1}{4}\mathcal{G}\left(0,v_{213},0,\frac{1}{1-u_2};1\right)-\\
&\frac{1}{4}\mathcal{G}\left(0,v_{213},\frac{1}{1-u_2},0;1\right)-\frac{1}{4}\mathcal{G}\left(0,v_{213},\frac{1}{1-u_2},\frac{1}{1-u_2};1\right)+\frac{1}{4}\mathcal{G}\left(0,v_{231},0,\frac{1}{1-u_2};1\right)-\\
&\frac{1}{2}\mathcal{G}\left(0,v_{231},1,\frac{1}{1-u_2};1\right)+\frac{1}{4}\mathcal{G}\left(0,v_{231},\frac{1}{1-u_2},0;1\right)-\frac{1}{2}\mathcal{G}\left(0,v_{231},\frac{1}{1-u_2},1;1\right)+\\
&\frac{1}{4}\mathcal{G}\left(0,v_{231},\frac{1}{1-u_2},\frac{1}{1-u_2};1\right)+\frac{1}{4}\mathcal{G}\left(0,v_{312},0,\frac{1}{1-u_3};1\right)-\frac{1}{2}\mathcal{G}\left(0,v_{312},1,\frac{1}{1-u_3};1\right)+\\
&\frac{1}{4}\mathcal{G}\left(0,v_{312},\frac{1}{1-u_3},0;1\right)-\frac{1}{2}\mathcal{G}\left(0,v_{312},\frac{1}{1-u_3},1;1\right)+\frac{1}{4}\mathcal{G}\left(0,v_{312},\frac{1}{1-u_3},\frac{1}{1-u_3};1\right)-\\
&\frac{1}{4}\mathcal{G}\left(0,v_{321},0,\frac{1}{1-u_3};1\right)-\frac{1}{4}\mathcal{G}\left(0,v_{321},\frac{1}{1-u_3},0;1\right)-\frac{1}{4}\mathcal{G}\left(0,v_{321},\frac{1}{1-u_3},\frac{1}{1-u_3};1\right)-\\
&\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},0,0,v_{123};1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},0,0,v_{132};1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},0,\frac{1}{1-u_1},v_{123};1\right)-\\
&\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},0,\frac{1}{1-u_1},v_{132};1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},0,v_{123},1;1\right)-\\
&\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},0,v_{123},\frac{1}{1-u_1};1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},0,v_{132},1;1\right)-\\
&\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},0,v_{132},\frac{1}{1-u_1};1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},\frac{1}{1-u_1},0,v_{123};1\right)-\\
&\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},\frac{1}{1-u_1},0,v_{132};1\right)-\frac{3}{4}\mathcal{G}\left(\frac{1}{1-u_1},\frac{1}{1-u_1},\frac{1}{1-u_1},v_{123};1\right)-\\
&\frac{3}{4}\mathcal{G}\left(\frac{1}{1-u_1},\frac{1}{1-u_1},\frac{1}{1-u_1},v_{132};1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},\frac{1}{1-u_1},v_{123},1;1\right)-\\
&\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},\frac{1}{1-u_1},v_{123},\frac{1}{1-u_1};1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},\frac{1}{1-u_1},v_{132},1;1\right)-\\
&\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},\frac{1}{1-u_1},v_{132},\frac{1}{1-u_1};1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},u_{123},0,1;1\right)+\\
&\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},u_{123},0,\frac{1}{1-u_1};1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},u_{123},1,0;1\right)+\\
&\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},u_{123},\frac{1}{1-u_1},0;1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},u_{123},\frac{1}{1-u_1},1;1\right)+
\end{aligned}
$$

$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},u_{123},\frac{1}{1-u_1},\frac{1}{1-u_1};1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},u_{123},\frac{u_2-1}{u_1+u_2-1},1;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},u_{123},\frac{u_2-1}{u_1+u_2-1},\frac{1}{1-u_1};1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},u_{123},\frac{1}{u_3},0;1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{123},0,0;1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{123},0,1;1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{123},0,\frac{1}{1-u_1};1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{123},1,0;1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},v_{123},1,\frac{1}{1-u_1};1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{123},\frac{1}{1-u_1},0;1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},v_{123},\frac{1}{1-u_1},1;1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{123},\frac{1}{1-u_1},\frac{1}{1-u_1};1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{132},0,0;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{132},0,1;1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{132},0,\frac{1}{1-u_1};1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{132},1,0;1\right)-$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},v_{132},1,\frac{1}{1-u_1};1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{132},\frac{1}{1-u_1},0;1\right)-$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},v_{132},\frac{1}{1-u_1},1;1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{132},\frac{1}{1-u_1},\frac{1}{1-u_1};1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},0,0,v_{213};1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},0,0,v_{231};1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},0,\frac{1}{1-u_2},v_{213};1\right)-$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},0,\frac{1}{1-u_2},v_{231};1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},0,v_{213},1;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},0,v_{213},\frac{1}{1-u_2};1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},0,v_{231},1;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},0,v_{231},\frac{1}{1-u_2};1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},\frac{1}{1-u_2},0,v_{213};1\right)-$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},\frac{1}{1-u_2},0,v_{231};1\right)-\frac{3}{4}\mathcal{G}\left(\frac{1}{1-u_2},\frac{1}{1-u_2},\frac{1}{1-u_2},v_{213};1\right)-$$
$$\frac{3}{4}\mathcal{G}\left(\frac{1}{1-u_2},\frac{1}{1-u_2},\frac{1}{1-u_2},v_{231};1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},\frac{1}{1-u_2},v_{213},1;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},\frac{1}{1-u_2},v_{213},\frac{1}{1-u_2};1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},\frac{1}{1-u_2},v_{231},1;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},\frac{1}{1-u_2},v_{231},\frac{1}{1-u_2};1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},u_{231},0,1;1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},u_{231},0,\frac{1}{1-u_2};1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},u_{231},1,0;1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},u_{231},\frac{1}{u_1},0;1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},u_{231},\frac{1}{1-u_2},0;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},u_{231},\frac{1}{1-u_2},1;1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},u_{231},\frac{1}{1-u_2},\frac{1}{1-u_2};1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},u_{231},\frac{u_3-1}{u_2+u_3-1},1;1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},u_{231},\frac{u_3-1}{u_2+u_3-1},\frac{1}{1-u_2};1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{213},0,0;1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{213},0,1;1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{213},0,\frac{1}{1-u_2};1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{213},1,0;1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},v_{213},1,\frac{1}{1-u_2};1\right)+$$

$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{213},\frac{1}{1-u_2},0;1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},v_{213},\frac{1}{1-u_2},1;1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{213},\frac{1}{1-u_2},\frac{1}{1-u_2};1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{231},0,0;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{231},0,1;1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{231},0,\frac{1}{1-u_2};1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{231},1,0;1\right)-$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},v_{231},1,\frac{1}{1-u_2};1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{231},\frac{1}{1-u_2},0;1\right)-$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},v_{231},\frac{1}{1-u_2},1;1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{231},\frac{1}{1-u_2},\frac{1}{1-u_2};1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},0,0,v_{312};1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},0,0,v_{321};1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},0,\frac{1}{1-u_3},v_{312};1\right)-$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},0,\frac{1}{1-u_3},v_{321};1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},0,v_{312},1;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},0,v_{312},\frac{1}{1-u_3};1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},0,v_{321},1;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},0,v_{321},\frac{1}{1-u_3};1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},\frac{1}{1-u_3},0,v_{312};1\right)-$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},\frac{1}{1-u_3},0,v_{321};1\right)-\frac{3}{4}\mathcal{G}\left(\frac{1}{1-u_3},\frac{1}{1-u_3},\frac{1}{1-u_3},v_{312};1\right)-$$
$$\frac{3}{4}\mathcal{G}\left(\frac{1}{1-u_3},\frac{1}{1-u_3},\frac{1}{1-u_3},v_{321};1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},\frac{1}{1-u_3},v_{312},1;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},\frac{1}{1-u_3},v_{312},\frac{1}{1-u_3};1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},\frac{1}{1-u_3},v_{321},1;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},\frac{1}{1-u_3},v_{321},\frac{1}{1-u_3};1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},u_{312},0,1;1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},u_{312},0,\frac{1}{1-u_3};1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},u_{312},1,0;1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},u_{312},\frac{1}{u_2},0;1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},u_{312},\frac{1}{1-u_3},0;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},u_{312},\frac{1}{1-u_3},1;1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},u_{312},\frac{1}{1-u_3},\frac{1}{1-u_3};1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},u_{312},\frac{u_1-1}{u_1+u_3-1},1;1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},u_{312},\frac{u_1-1}{u_1+u_3-1},\frac{1}{1-u_3};1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{312},0,0;1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{312},0,1;1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{312},0,\frac{1}{1-u_3};1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{312},1,0;1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},v_{312},1,\frac{1}{1-u_3};1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{312},\frac{1}{1-u_3},0;1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},v_{312},\frac{1}{1-u_3},1;1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{312},\frac{1}{1-u_3},\frac{1}{1-u_3};1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{321},0,0;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{321},0,1;1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{321},0,\frac{1}{1-u_3};1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{321},1,0;1\right)-$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},v_{321},1,\frac{1}{1-u_3};1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{321},\frac{1}{1-u_3},0;1\right)-$$

$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3}, v_{321}, \frac{1}{1-u_3}, 1; 1\right) + \frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3}, v_{321}, \frac{1}{1-u_3}, \frac{1}{1-u_3}; 1\right) +$$
$$\frac{1}{2}\mathcal{G}\left(v_{123}, 0, 1, \frac{1}{1-u_1}; 1\right) + \frac{1}{2}\mathcal{G}\left(v_{123}, 0, \frac{1}{1-u_1}, 1; 1\right) + \frac{1}{2}\mathcal{G}\left(v_{123}, 1, 0, \frac{1}{1-u_1}; 1\right) -$$
$$\frac{5}{4}\mathcal{G}\left(v_{123}, 1, 1, \frac{1}{1-u_1}; 1\right) + \frac{1}{2}\mathcal{G}\left(v_{123}, 1, \frac{1}{1-u_1}, 0; 1\right) - \frac{5}{4}\mathcal{G}\left(v_{123}, 1, \frac{1}{1-u_1}, 1; 1\right) +$$
$$\frac{1}{2}\mathcal{G}\left(v_{123}, 1, \frac{1}{1-u_1}, \frac{1}{1-u_1}; 1\right) + \frac{1}{2}\mathcal{G}\left(v_{123}, \frac{1}{1-u_1}, 0, 1; 1\right) + \frac{1}{2}\mathcal{G}\left(v_{123}, \frac{1}{1-u_1}, 1, 0; 1\right) -$$
$$\frac{5}{4}\mathcal{G}\left(v_{123}, \frac{1}{1-u_1}, 1, 1; 1\right) + \frac{1}{2}\mathcal{G}\left(v_{123}, \frac{1}{1-u_1}, 1, \frac{1}{1-u_1}; 1\right) +$$
$$\frac{1}{2}\mathcal{G}\left(v_{123}, \frac{1}{1-u_1}, \frac{1}{1-u_1}, 1; 1\right) - \frac{1}{4}\mathcal{G}\left(v_{132}, 1, 1, \frac{1}{1-u_1}; 1\right) - \frac{1}{4}\mathcal{G}\left(v_{132}, 1, \frac{1}{1-u_1}, 1; 1\right) -$$
$$\frac{1}{4}\mathcal{G}\left(v_{132}, \frac{1}{1-u_1}, 1, 1; 1\right) - \frac{1}{4}\mathcal{G}\left(v_{213}, 1, 1, \frac{1}{1-u_2}; 1\right) - \frac{1}{4}\mathcal{G}\left(v_{213}, 1, \frac{1}{1-u_2}, 1; 1\right) -$$
$$\frac{1}{4}\mathcal{G}\left(v_{213}, \frac{1}{1-u_2}, 1, 1; 1\right) + \frac{1}{2}\mathcal{G}\left(v_{231}, 0, 1, \frac{1}{1-u_2}; 1\right) + \frac{1}{2}\mathcal{G}\left(v_{231}, 0, \frac{1}{1-u_2}, 1; 1\right) +$$
$$\frac{1}{2}\mathcal{G}\left(v_{231}, 1, 0, \frac{1}{1-u_2}; 1\right) - \frac{5}{4}\mathcal{G}\left(v_{231}, 1, 1, \frac{1}{1-u_2}; 1\right) + \frac{1}{2}\mathcal{G}\left(v_{231}, 1, \frac{1}{1-u_2}, 0; 1\right) -$$
$$\frac{5}{4}\mathcal{G}\left(v_{231}, 1, \frac{1}{1-u_2}, 1; 1\right) + \frac{1}{2}\mathcal{G}\left(v_{231}, 1, \frac{1}{1-u_2}, \frac{1}{1-u_2}; 1\right) + \frac{1}{2}\mathcal{G}\left(v_{231}, \frac{1}{1-u_2}, 0, 1; 1\right) +$$
$$\frac{1}{2}\mathcal{G}\left(v_{231}, \frac{1}{1-u_2}, 1, 0; 1\right) - \frac{5}{4}\mathcal{G}\left(v_{231}, \frac{1}{1-u_2}, 1, 1; 1\right) + \frac{1}{2}\mathcal{G}\left(v_{231}, \frac{1}{1-u_2}, 1, \frac{1}{1-u_2}; 1\right) +$$
$$\frac{1}{2}\mathcal{G}\left(v_{231}, \frac{1}{1-u_2}, \frac{1}{1-u_2}, 1; 1\right) + \frac{1}{2}\mathcal{G}\left(v_{312}, 0, 1, \frac{1}{1-u_3}; 1\right) + \frac{1}{2}\mathcal{G}\left(v_{312}, 0, \frac{1}{1-u_3}, 1; 1\right) +$$
$$\frac{1}{2}\mathcal{G}\left(v_{312}, 1, 0, \frac{1}{1-u_3}; 1\right) - \frac{5}{4}\mathcal{G}\left(v_{312}, 1, 1, \frac{1}{1-u_3}; 1\right) + \frac{1}{2}\mathcal{G}\left(v_{312}, 1, \frac{1}{1-u_3}, 0; 1\right) -$$
$$\frac{5}{4}\mathcal{G}\left(v_{312}, 1, \frac{1}{1-u_3}, 1; 1\right) + \frac{1}{2}\mathcal{G}\left(v_{312}, 1, \frac{1}{1-u_3}, \frac{1}{1-u_3}; 1\right) + \frac{1}{2}\mathcal{G}\left(v_{312}, \frac{1}{1-u_3}, 0, 1; 1\right) +$$
$$\frac{1}{2}\mathcal{G}\left(v_{312}, \frac{1}{1-u_3}, 1, 0; 1\right) - \frac{5}{4}\mathcal{G}\left(v_{312}, \frac{1}{1-u_3}, 1, 1; 1\right) + \frac{1}{2}\mathcal{G}\left(v_{312}, \frac{1}{1-u_3}, 1, \frac{1}{1-u_3}; 1\right) +$$
$$\frac{1}{2}\mathcal{G}\left(v_{312}, \frac{1}{1-u_3}, \frac{1}{1-u_3}, 1; 1\right) - \frac{1}{4}\mathcal{G}\left(v_{321}, 1, 1, \frac{1}{1-u_3}; 1\right) - \frac{1}{4}\mathcal{G}\left(v_{321}, 1, \frac{1}{1-u_3}, 1; 1\right) -$$
$$\frac{1}{4}\mathcal{G}\left(v_{321}, \frac{1}{1-u_3}, 1, 1; 1\right) - \frac{3}{4}G\left(0, \frac{1}{u_1}, \frac{1}{u_1+u_2}; 1\right) H(0; u_1) -$$
$$\frac{3}{4}G\left(0, \frac{1}{u_1}, \frac{1}{u_1+u_3}; 1\right) H(0; u_1) - \frac{1}{4}G\left(0, \frac{1}{u_2}, \frac{1}{u_1+u_2}; 1\right) H(0; u_1) -$$
$$\frac{1}{4}G\left(0, \frac{1}{u_3}, \frac{1}{u_1+u_3}; 1\right) H(0; u_1) - \frac{1}{4}G\left(0, \frac{u_1-1}{u_1+u_3-1}, \frac{1}{1-u_3}; 1\right) H(0; u_1) +$$
$$\frac{1}{4}G\left(0, \frac{u_3-1}{u_2+u_3-1}, \frac{1}{1-u_2}; 1\right) H(0; u_1) - \frac{3}{4}G\left(\frac{1}{u_1}, 0, \frac{1}{u_1+u_2}; 1\right) H(0; u_1) -$$
$$\frac{3}{4}G\left(\frac{1}{u_1}, 0, \frac{1}{u_1+u_3}; 1\right) H(0; u_1) + \frac{1}{2}G\left(\frac{1}{u_1}, \frac{1}{u_1}, \frac{1}{u_1+u_2}; 1\right) H(0; u_1) +$$
$$\frac{1}{2}G\left(\frac{1}{u_1}, \frac{1}{u_1}, \frac{1}{u_1+u_3}; 1\right) H(0; u_1) + \frac{1}{4}G\left(\frac{1}{u_1}, \frac{1}{u_2}, \frac{1}{u_1+u_2}; 1\right) H(0; u_1) +$$
$$\frac{1}{4}G\left(\frac{1}{u_1}, \frac{1}{u_3}, \frac{1}{u_1+u_3}; 1\right) H(0; u_1) - \frac{1}{4}G\left(\frac{1}{1-u_2}, 1, \frac{1}{u_1}; 1\right) H(0; u_1) +$$

$$\frac{1}{4}G\left(\frac{1}{1-u_2}, \frac{u_3-1}{u_2+u_3-1}, 1; 1\right) H(0; u_1) -$$
$$\frac{1}{4}G\left(\frac{1}{1-u_2}, \frac{u_3-1}{u_2+u_3-1}, \frac{1}{1-u_2}; 1\right) H(0; u_1) + \frac{1}{2}G\left(\frac{1}{u_2}, 0, \frac{1}{u_1}; 1\right) H(0; u_1) -$$
$$\frac{1}{4}G\left(\frac{1}{u_2}, 0, \frac{1}{u_1+u_2}; 1\right) H(0; u_1) + \frac{1}{4}G\left(\frac{1}{u_2}, \frac{1}{u_1}, \frac{1}{u_1+u_2}; 1\right) H(0; u_1) +$$
$$\frac{1}{4}G\left(\frac{1}{1-u_3}, \frac{u_1-1}{u_1+u_3-1}, 0; 1\right) H(0; u_1) + \frac{1}{4}G\left(\frac{1}{1-u_3}, \frac{u_1-1}{u_1+u_3-1}, \frac{1}{1-u_3}; 1\right) H(0; u_1) -$$
$$\frac{1}{4}G\left(\frac{1}{1-u_3}, \frac{u_1-1}{u_1+u_3-1}, \frac{u_1-1}{u_1+u_3-1}; 1\right) H(0; u_1) + \frac{1}{2}G\left(\frac{1}{u_3}, 0, \frac{1}{u_1}; 1\right) H(0; u_1) -$$
$$\frac{1}{4}G\left(\frac{1}{u_3}, 0, \frac{1}{u_1+u_3}; 1\right) H(0; u_1) + \frac{1}{4}G\left(\frac{1}{u_3}, \frac{1}{u_1}, \frac{1}{u_1+u_3}; 1\right) H(0; u_1) +$$
$$\frac{1}{4}\mathcal{G}\left(0, \frac{1}{1-u_1}, v_{123}; 1\right) H(0; u_1) + \frac{1}{4}\mathcal{G}\left(0, \frac{1}{1-u_1}, v_{132}; 1\right) H(0; u_1) +$$
$$\frac{1}{4}\mathcal{G}\left(0, \frac{1}{1-u_2}, v_{213}; 1\right) H(0; u_1) - \frac{1}{4}\mathcal{G}\left(0, \frac{1}{1-u_2}, v_{231}; 1\right) H(0; u_1) +$$
$$\frac{1}{4}\mathcal{G}\left(0, \frac{1}{1-u_3}, v_{312}; 1\right) H(0; u_1) - \frac{1}{4}\mathcal{G}\left(0, \frac{1}{1-u_3}, v_{321}; 1\right) H(0; u_1) -$$
$$\frac{1}{4}\mathcal{G}\left(0, u_{231}, \frac{1}{u_1}; 1\right) H(0; u_1) - \frac{1}{4}\mathcal{G}\left(0, u_{231}, \frac{1}{1-u_2}; 1\right) H(0; u_1) +$$
$$\frac{1}{4}\mathcal{G}\left(0, u_{312}, \frac{1}{1-u_3}; 1\right) H(0; u_1) - \frac{1}{4}\mathcal{G}\left(0, u_{312}, \frac{u_1-1}{u_1+u_3-1}; 1\right) H(0; u_1) +$$
$$\frac{1}{4}\mathcal{G}\left(0, v_{123}, \frac{1}{1-u_1}; 1\right) H(0; u_1) + \frac{1}{4}\mathcal{G}\left(0, v_{132}, \frac{1}{1-u_1}; 1\right) H(0; u_1) -$$
$$\frac{1}{2}\mathcal{G}\left(0, v_{231}, \frac{1}{1-u_2}; 1\right) H(0; u_1) + \frac{1}{2}\mathcal{G}\left(0, v_{312}, \frac{1}{1-u_3}; 1\right) H(0; u_1) +$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1}, 0, v_{123}; 1\right) H(0; u_1) + \frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1}, 0, v_{132}; 1\right) H(0; u_1) +$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1}, \frac{1}{1-u_1}, v_{123}; 1\right) H(0; u_1) + \frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1}, \frac{1}{1-u_1}, v_{132}; 1\right) H(0; u_1) +$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1}, v_{123}, 1; 1\right) H(0; u_1) + \frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1}, v_{123}, \frac{1}{1-u_1}; 1\right) H(0; u_1) +$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1}, v_{132}, 1; 1\right) H(0; u_1) + \frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1}, v_{132}, \frac{1}{1-u_1}; 1\right) H(0; u_1) +$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2}, 0, v_{213}; 1\right) H(0; u_1) - \frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2}, 0, v_{231}; 1\right) H(0; u_1) +$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2}, \frac{1}{1-u_2}, v_{213}; 1\right) H(0; u_1) - \frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2}, \frac{1}{1-u_2}, v_{231}; 1\right) H(0; u_1) -$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2}, u_{231}, 1; 1\right) H(0; u_1) + \frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2}, u_{231}, \frac{1}{u_1}; 1\right) H(0; u_1) +$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2}, u_{231}, \frac{1}{1-u_2}; 1\right) H(0; u_1) + \frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2}, v_{213}, 0; 1\right) H(0; u_1) +$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2}, v_{213}, \frac{1}{1-u_2}; 1\right) H(0; u_1) - \frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2}, v_{231}, 0; 1\right) H(0; u_1) -$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2}, v_{231}, \frac{1}{1-u_2}; 1\right) H(0; u_1) + \frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3}, 0, v_{312}; 1\right) H(0; u_1) -$$

$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},0,v_{321};1\right)H(0;u_1)+\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},\frac{1}{1-u_3},v_{312};1\right)H(0;u_1)-$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},\frac{1}{1-u_3},v_{321};1\right)H(0;u_1)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},u_{312},0;1\right)H(0;u_1)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},u_{312},\frac{1}{1-u_3};1\right)H(0;u_1)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},u_{312},\frac{u_1-1}{u_1+u_3-1};1\right)H(0;u_1)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{312},0;1\right)H(0;u_1)+\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},v_{312},\frac{1}{1-u_3};1\right)H(0;u_1)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{321},0;1\right)H(0;u_1)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},v_{321},\frac{1}{1-u_3};1\right)H(0;u_1)+$$
$$\frac{1}{4}\mathcal{G}\left(v_{123},1,\frac{1}{1-u_1};1\right)H(0;u_1)+\frac{1}{4}\mathcal{G}\left(v_{123},\frac{1}{1-u_1},1;1\right)H(0;u_1)+$$
$$\frac{1}{4}\mathcal{G}\left(v_{132},1,\frac{1}{1-u_1};1\right)H(0;u_1)+\frac{1}{4}\mathcal{G}\left(v_{132},\frac{1}{1-u_1},1;1\right)H(0;u_1)+$$
$$\frac{1}{4}\mathcal{G}\left(v_{213},1,\frac{1}{1-u_2};1\right)H(0;u_1)+\frac{1}{4}\mathcal{G}\left(v_{213},\frac{1}{1-u_2},1;1\right)H(0;u_1)-$$
$$\frac{3}{4}\mathcal{G}\left(v_{231},1,\frac{1}{1-u_2};1\right)H(0;u_1)-\frac{3}{4}\mathcal{G}\left(v_{231},\frac{1}{1-u_2},1;1\right)H(0;u_1)+$$
$$\frac{3}{4}\mathcal{G}\left(v_{312},1,\frac{1}{1-u_3};1\right)H(0;u_1)+\frac{3}{4}\mathcal{G}\left(v_{312},\frac{1}{1-u_3},1;1\right)H(0;u_1)-$$
$$\frac{1}{4}\mathcal{G}\left(v_{321},1,\frac{1}{1-u_3};1\right)H(0;u_1)-\frac{1}{4}\mathcal{G}\left(v_{321},\frac{1}{1-u_3},1;1\right)H(0;u_1)-$$
$$\frac{1}{4}G\left(0,\frac{1}{u_1},\frac{1}{u_1+u_2};1\right)H(0;u_2)-\frac{3}{4}G\left(0,\frac{1}{u_2},\frac{1}{u_1+u_2};1\right)H(0;u_2)-$$
$$\frac{3}{4}G\left(0,\frac{1}{u_2},\frac{1}{u_2+u_3};1\right)H(0;u_2)-\frac{1}{4}G\left(0,\frac{u_2-1}{u_1+u_2-1},\frac{1}{1-u_1};1\right)H(0;u_2)-$$
$$\frac{1}{4}G\left(0,\frac{1}{u_3},\frac{1}{u_2+u_3};1\right)H(0;u_2)+\frac{1}{4}G\left(0,\frac{u_1-1}{u_1+u_3-1},\frac{1}{1-u_3};1\right)H(0;u_2)+$$
$$\frac{1}{4}G\left(\frac{1}{1-u_1},\frac{u_2-1}{u_1+u_2-1},0;1\right)H(0;u_2)+\frac{1}{4}G\left(\frac{1}{1-u_1},\frac{u_2-1}{u_1+u_2-1},\frac{1}{1-u_1};1\right)H(0;u_2)-$$
$$\frac{1}{4}G\left(\frac{1}{1-u_1},\frac{u_2-1}{u_1+u_2-1},\frac{u_2-1}{u_1+u_2-1};1\right)H(0;u_2)+\frac{1}{2}G\left(\frac{1}{u_1},0,\frac{1}{u_2};1\right)H(0;u_2)-$$
$$\frac{1}{4}G\left(\frac{1}{u_1},0,\frac{1}{u_1+u_2};1\right)H(0;u_2)+\frac{1}{4}G\left(\frac{1}{u_1},\frac{1}{u_2},\frac{1}{u_1+u_2};1\right)H(0;u_2)-$$
$$\frac{3}{4}G\left(\frac{1}{u_2},0,\frac{1}{u_1+u_2};1\right)H(0;u_2)-\frac{3}{4}G\left(\frac{1}{u_2},0,\frac{1}{u_2+u_3};1\right)H(0;u_2)+$$
$$\frac{1}{4}G\left(\frac{1}{u_2},\frac{1}{u_1},\frac{1}{u_1+u_2};1\right)H(0;u_2)+\frac{1}{2}G\left(\frac{1}{u_2},\frac{1}{u_2},\frac{1}{u_1+u_2};1\right)H(0;u_2)+$$
$$\frac{1}{2}G\left(\frac{1}{u_2},\frac{1}{u_2},\frac{1}{u_2+u_3};1\right)H(0;u_2)+\frac{1}{4}G\left(\frac{1}{u_2},\frac{1}{u_3},\frac{1}{u_2+u_3};1\right)H(0;u_2)-$$
$$\frac{1}{4}G\left(\frac{1}{1-u_3},1,\frac{1}{u_2};1\right)H(0;u_2)+\frac{1}{4}G\left(\frac{1}{1-u_3},\frac{u_1-1}{u_1+u_3-1},1;1\right)H(0;u_2)-$$
$$\frac{1}{4}G\left(\frac{1}{1-u_3},\frac{u_1-1}{u_1+u_3-1},\frac{1}{1-u_3};1\right)H(0;u_2)+\frac{1}{2}G\left(\frac{1}{u_3},0,\frac{1}{u_2};1\right)H(0;u_2)-$$
$$\frac{1}{4}G\left(\frac{1}{u_3},0,\frac{1}{u_2+u_3};1\right)H(0;u_2)+\frac{1}{4}G\left(\frac{1}{u_3},\frac{1}{u_2},\frac{1}{u_2+u_3};1\right)H(0;u_2)+$$

$$\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_1},v_{123};1\right)H(0;u_2)-\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_1},v_{132};1\right)H(0;u_2)+$$
$$\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_2},v_{213};1\right)H(0;u_2)+\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_2},v_{231};1\right)H(0;u_2)-$$
$$\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_3},v_{312};1\right)H(0;u_2)+\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_3},v_{321};1\right)H(0;u_2)+$$
$$\frac{1}{4}\mathcal{G}\left(0,u_{123},\frac{1}{1-u_1};1\right)H(0;u_2)-\frac{1}{4}\mathcal{G}\left(0,u_{123},\frac{u_2-1}{u_1+u_2-1};1\right)H(0;u_2)-$$
$$\frac{1}{4}\mathcal{G}\left(0,u_{312},\frac{1}{u_2};1\right)H(0;u_2)-\frac{1}{4}\mathcal{G}\left(0,u_{312},\frac{1}{1-u_3};1\right)H(0;u_2)+$$
$$\frac{1}{2}\mathcal{G}\left(0,v_{123},\frac{1}{1-u_1};1\right)H(0;u_2)+\frac{1}{4}\mathcal{G}\left(0,v_{213},\frac{1}{1-u_2};1\right)H(0;u_2)+$$
$$\frac{1}{4}\mathcal{G}\left(0,v_{231},\frac{1}{1-u_2};1\right)H(0;u_2)-\frac{1}{2}\mathcal{G}\left(0,v_{312},\frac{1}{1-u_3};1\right)H(0;u_2)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},0,v_{123};1\right)H(0;u_2)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},0,v_{132};1\right)H(0;u_2)+$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},\frac{1}{1-u_1},v_{123};1\right)H(0;u_2)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},\frac{1}{1-u_1},v_{132};1\right)H(0;u_2)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},u_{123},0;1\right)H(0;u_2)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},u_{123},\frac{1}{1-u_1};1\right)H(0;u_2)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},u_{123},\frac{u_2-1}{u_1+u_2-1};1\right)H(0;u_2)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{123},0;1\right)H(0;u_2)+$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},v_{123},\frac{1}{1-u_1};1\right)H(0;u_2)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{132},0;1\right)H(0;u_2)-$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},v_{132},\frac{1}{1-u_1};1\right)H(0;u_2)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},0,v_{213};1\right)H(0;u_2)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},0,v_{231};1\right)H(0;u_2)+\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},\frac{1}{1-u_2},v_{213};1\right)H(0;u_2)+$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},\frac{1}{1-u_2},v_{231};1\right)H(0;u_2)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{213},1;1\right)H(0;u_2)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{213},\frac{1}{1-u_2};1\right)H(0;u_2)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{231},1;1\right)H(0;u_2)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{231},\frac{1}{1-u_2};1\right)H(0;u_2)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},0,v_{312};1\right)H(0;u_2)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},0,v_{321};1\right)H(0;u_2)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},\frac{1}{1-u_3},v_{312};1\right)H(0;u_2)+$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},\frac{1}{1-u_3},v_{321};1\right)H(0;u_2)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},u_{312},1;1\right)H(0;u_2)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},u_{312},\frac{1}{u_2};1\right)H(0;u_2)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},u_{312},\frac{1}{1-u_3};1\right)H(0;u_2)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{312},0;1\right)H(0;u_2)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},v_{312},\frac{1}{1-u_3};1\right)H(0;u_2)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{321},0;1\right)H(0;u_2)+\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},v_{321},\frac{1}{1-u_3};1\right)H(0;u_2)+$$
$$\frac{3}{4}\mathcal{G}\left(v_{123},1,\frac{1}{1-u_1};1\right)H(0;u_2)+\frac{3}{4}\mathcal{G}\left(v_{123},\frac{1}{1-u_1},1;1\right)H(0;u_2)-$$

$$\frac{1}{4}\mathcal{G}\left(v_{132},1,\frac{1}{1-u_1};1\right)H(0;u_2)-\frac{1}{4}\mathcal{G}\left(v_{132},\frac{1}{1-u_1},1;1\right)H(0;u_2)+$$
$$\frac{1}{4}\mathcal{G}\left(v_{213},1,\frac{1}{1-u_2};1\right)H(0;u_2)+\frac{1}{4}\mathcal{G}\left(v_{213},\frac{1}{1-u_2},1;1\right)H(0;u_2)+$$
$$\frac{1}{4}\mathcal{G}\left(v_{231},1,\frac{1}{1-u_2};1\right)H(0;u_2)+\frac{1}{4}\mathcal{G}\left(v_{231},\frac{1}{1-u_2},1;1\right)H(0;u_2)-$$
$$\frac{3}{4}\mathcal{G}\left(v_{312},1,\frac{1}{1-u_3};1\right)H(0;u_2)-\frac{3}{4}\mathcal{G}\left(v_{312},\frac{1}{1-u_3},1;1\right)H(0;u_2)+$$
$$\frac{1}{4}\mathcal{G}\left(v_{321},1,\frac{1}{1-u_3};1\right)H(0;u_2)+\frac{1}{4}\mathcal{G}\left(v_{321},\frac{1}{1-u_3},1;1\right)H(0;u_2)+$$
$$\frac{1}{4}G\left(\frac{1}{u_1},\frac{1}{u_1+u_2};1\right)H(0;u_1)H(0;u_2)+\frac{1}{4}G\left(\frac{1}{u_2},\frac{1}{u_1+u_2};1\right)H(0;u_1)H(0;u_2)+$$
$$\frac{1}{4}G\left(\frac{1}{1-u_3},\frac{u_1-1}{u_1+u_3-1};1\right)H(0;u_1)H(0;u_2)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},u_{312};1\right)H(0;u_1)H(0;u_2)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{312};1\right)H(0;u_1)H(0;u_2)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{321};1\right)H(0;u_1)H(0;u_2)+\frac{5}{24}\pi^2H(0;u_1)H(0;u_2)-$$
$$\frac{1}{4}G\left(0,\frac{1}{u_1},\frac{1}{u_1+u_3};1\right)H(0;u_3)-\frac{1}{4}G\left(0,\frac{1}{u_2},\frac{1}{u_2+u_3};1\right)H(0;u_3)+$$
$$\frac{1}{4}G\left(0,\frac{u_2-1}{u_1+u_2-1},\frac{1}{1-u_1};1\right)H(0;u_3)-\frac{3}{4}G\left(0,\frac{1}{u_3},\frac{1}{u_1+u_3};1\right)H(0;u_3)-$$
$$\frac{3}{4}G\left(0,\frac{1}{u_3},\frac{1}{u_2+u_3};1\right)H(0;u_3)-\frac{1}{4}G\left(0,\frac{u_3-1}{u_2+u_3-1},\frac{1}{1-u_2};1\right)H(0;u_3)-$$
$$\frac{1}{4}G\left(\frac{1}{1-u_1},1,\frac{1}{u_3};1\right)H(0;u_3)+\frac{1}{4}G\left(\frac{1}{1-u_1},\frac{u_2-1}{u_1+u_2-1},1;1\right)H(0;u_3)-$$
$$\frac{1}{4}G\left(\frac{1}{1-u_1},\frac{u_2-1}{u_1+u_2-1},\frac{1}{1-u_1};1\right)H(0;u_3)+\frac{1}{2}G\left(\frac{1}{u_1},0,\frac{1}{u_3};1\right)H(0;u_3)-$$
$$\frac{1}{4}G\left(\frac{1}{u_1},0,\frac{1}{u_1+u_3};1\right)H(0;u_3)+\frac{1}{4}G\left(\frac{1}{u_1},\frac{1}{u_3},\frac{1}{u_1+u_3};1\right)H(0;u_3)+$$
$$\frac{1}{4}G\left(\frac{1}{1-u_2},\frac{u_3-1}{u_2+u_3-1},0;1\right)H(0;u_3)+\frac{1}{4}G\left(\frac{1}{1-u_2},\frac{u_3-1}{u_2+u_3-1},\frac{1}{1-u_2};1\right)H(0;u_3)-$$
$$\frac{1}{4}G\left(\frac{1}{1-u_2},\frac{u_3-1}{u_2+u_3-1},\frac{u_3-1}{u_2+u_3-1};1\right)H(0;u_3)+\frac{1}{2}G\left(\frac{1}{u_2},0,\frac{1}{u_3};1\right)H(0;u_3)-$$
$$\frac{1}{4}G\left(\frac{1}{u_2},0,\frac{1}{u_2+u_3};1\right)H(0;u_3)+\frac{1}{4}G\left(\frac{1}{u_2},\frac{1}{u_3},\frac{1}{u_2+u_3};1\right)H(0;u_3)-$$
$$\frac{3}{4}G\left(\frac{1}{u_3},0,\frac{1}{u_1+u_3};1\right)H(0;u_3)-\frac{3}{4}G\left(\frac{1}{u_3},0,\frac{1}{u_2+u_3};1\right)H(0;u_3)+$$
$$\frac{1}{4}G\left(\frac{1}{u_3},\frac{1}{u_1},\frac{1}{u_1+u_3};1\right)H(0;u_3)+\frac{1}{4}G\left(\frac{1}{u_3},\frac{1}{u_2},\frac{1}{u_2+u_3};1\right)H(0;u_3)+$$
$$\frac{1}{2}G\left(\frac{1}{u_3},\frac{1}{u_3},\frac{1}{u_1+u_3};1\right)H(0;u_3)+\frac{1}{2}G\left(\frac{1}{u_3},\frac{1}{u_3},\frac{1}{u_2+u_3};1\right)H(0;u_3)-$$
$$\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_1},v_{123};1\right)H(0;u_3)+\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_1},v_{132};1\right)H(0;u_3)-$$
$$\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_2},v_{213};1\right)H(0;u_3)+\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_2},v_{231};1\right)H(0;u_3)+$$

$$\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_3},v_{312};1\right)H(0;u_3)+\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_3},v_{321};1\right)H(0;u_3)-$$
$$\frac{1}{4}\mathcal{G}\left(0,u_{123},\frac{1}{1-u_1};1\right)H(0;u_3)-\frac{1}{4}\mathcal{G}\left(0,u_{123},\frac{1}{u_3};1\right)H(0;u_3)+$$
$$\frac{1}{4}\mathcal{G}\left(0,u_{231},\frac{1}{1-u_2};1\right)H(0;u_3)-\frac{1}{4}\mathcal{G}\left(0,u_{231},\frac{u_3-1}{u_2+u_3-1};1\right)H(0;u_3)-$$
$$\frac{1}{2}\mathcal{G}\left(0,v_{123},\frac{1}{1-u_1};1\right)H(0;u_3)+\frac{1}{2}\mathcal{G}\left(0,v_{231},\frac{1}{1-u_2};1\right)H(0;u_3)+$$
$$\frac{1}{4}\mathcal{G}\left(0,v_{312},\frac{1}{1-u_3};1\right)H(0;u_3)+\frac{1}{4}\mathcal{G}\left(0,v_{321},\frac{1}{1-u_3};1\right)H(0;u_3)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},0,v_{123};1\right)H(0;u_3)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},0,v_{132};1\right)H(0;u_3)-$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},\frac{1}{1-u_1},v_{123};1\right)H(0;u_3)+\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},\frac{1}{1-u_1},v_{132};1\right)H(0;u_3)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},u_{123},1;1\right)H(0;u_3)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},u_{123},\frac{1}{1-u_1};1\right)H(0;u_3)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},u_{123},\frac{1}{u_3};1\right)H(0;u_3)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{123},0;1\right)H(0;u_3)-$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},v_{123},\frac{1}{1-u_1};1\right)H(0;u_3)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{132},0;1\right)H(0;u_3)+$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},v_{132},\frac{1}{1-u_1};1\right)H(0;u_3)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},0,v_{213};1\right)H(0;u_3)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},0,v_{231};1\right)H(0;u_3)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},\frac{1}{1-u_2},v_{213};1\right)H(0;u_3)+$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},\frac{1}{1-u_2},v_{231};1\right)H(0;u_3)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},u_{231},0;1\right)H(0;u_3)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},u_{231},\frac{1}{1-u_2};1\right)H(0;u_3)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},u_{231},\frac{u_3-1}{u_2+u_3-1};1\right)H(0;u_3)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{213},0;1\right)H(0;u_3)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},v_{213},\frac{1}{1-u_2};1\right)H(0;u_3)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{231},0;1\right)H(0;u_3)+\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},v_{231},\frac{1}{1-u_2};1\right)H(0;u_3)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},0,v_{312};1\right)H(0;u_3)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},0,v_{321};1\right)H(0;u_3)+$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},\frac{1}{1-u_3},v_{312};1\right)H(0;u_3)+\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},\frac{1}{1-u_3},v_{321};1\right)H(0;u_3)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{312},1;1\right)H(0;u_3)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{312},\frac{1}{1-u_3};1\right)H(0;u_3)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{321},1;1\right)H(0;u_3)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{321},\frac{1}{1-u_3};1\right)H(0;u_3)-$$
$$\frac{3}{4}\mathcal{G}\left(v_{123},1,\frac{1}{1-u_1};1\right)H(0;u_3)-\frac{3}{4}\mathcal{G}\left(v_{123},\frac{1}{1-u_1},1;1\right)H(0;u_3)+$$
$$\frac{1}{4}\mathcal{G}\left(v_{132},1,\frac{1}{1-u_1};1\right)H(0;u_3)+\frac{1}{4}\mathcal{G}\left(v_{132},\frac{1}{1-u_1},1;1\right)H(0;u_3)-$$
$$\frac{1}{4}\mathcal{G}\left(v_{213},1,\frac{1}{1-u_2};1\right)H(0;u_3)-\frac{1}{4}\mathcal{G}\left(v_{213},\frac{1}{1-u_2},1;1\right)H(0;u_3)+$$

$$\frac{3}{4}\mathcal{G}\left(v_{231},1,\frac{1}{1-u_2};1\right)H(0;u_3)+\frac{3}{4}\mathcal{G}\left(v_{231},\frac{1}{1-u_2},1;1\right)H(0;u_3)+$$
$$\frac{1}{4}\mathcal{G}\left(v_{312},1,\frac{1}{1-u_3};1\right)H(0;u_3)+\frac{1}{4}\mathcal{G}\left(v_{312},\frac{1}{1-u_3},1;1\right)H(0;u_3)+$$
$$\frac{1}{4}\mathcal{G}\left(v_{321},1,\frac{1}{1-u_3};1\right)H(0;u_3)+\frac{1}{4}\mathcal{G}\left(v_{321},\frac{1}{1-u_3},1;1\right)H(0;u_3)+$$
$$\frac{1}{4}G\left(\frac{1}{u_1},\frac{1}{u_1+u_3};1\right)H(0;u_1)H(0;u_3)+$$
$$\frac{1}{4}G\left(\frac{1}{1-u_2},\frac{u_3-1}{u_2+u_3-1};1\right)H(0;u_1)H(0;u_3)+$$
$$\frac{1}{4}G\left(\frac{1}{u_3},\frac{1}{u_1+u_3};1\right)H(0;u_1)H(0;u_3)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},u_{231};1\right)H(0;u_1)H(0;u_3)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{213};1\right)H(0;u_1)H(0;u_3)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{231};1\right)H(0;u_1)H(0;u_3)+$$
$$\frac{5}{24}\pi^2H(0;u_1)H(0;u_3)+\frac{1}{4}G\left(\frac{1}{1-u_1},\frac{u_2-1}{u_1+u_2-1};1\right)H(0;u_2)H(0;u_3)+$$
$$\frac{1}{4}G\left(\frac{1}{u_2},\frac{1}{u_2+u_3};1\right)H(0;u_2)H(0;u_3)+\frac{1}{4}G\left(\frac{1}{u_3},\frac{1}{u_2+u_3};1\right)H(0;u_2)H(0;u_3)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},u_{123};1\right)H(0;u_2)H(0;u_3)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{123};1\right)H(0;u_2)H(0;u_3)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{132};1\right)H(0;u_2)H(0;u_3)+\frac{5}{24}\pi^2H(0;u_2)H(0;u_3)+$$
$$3H(0;u_2)H(0,0;u_1)H(0;u_3)+3H(0;u_1)H(0,0;u_2)H(0;u_3)+$$
$$\frac{1}{4}H(0;u_2)H\left(0,1;\frac{u_1+u_2-1}{u_2-1}\right)H(0;u_3)+\frac{1}{2}H(0;u_1)H(0,1;(u_1+u_3))H(0;u_3)+$$
$$\frac{1}{4}H(0;u_1)H\left(0,1;\frac{u_2+u_3-1}{u_3-1}\right)H(0;u_3)+\frac{1}{2}H(0;u_2)H(0,1;(u_2+u_3))H(0;u_3)+$$
$$\frac{3}{4}H(0;u_2)H(1,0;u_1)H(0;u_3)+\frac{3}{4}H(0;u_1)H(1,0;u_2)H(0;u_3)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{213};1\right)H(0,0;u_1)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{231};1\right)H(0,0;u_1)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{312};1\right)H(0,0;u_1)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{321};1\right)H(0,0;u_1)-\frac{23}{24}\pi^2H(0,0;u_1)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{123};1\right)H(0,0;u_2)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{132};1\right)H(0,0;u_2)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{312};1\right)H(0,0;u_2)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{321};1\right)H(0,0;u_2)-$$
$$\frac{25}{4}H(0,0;u_1)H(0,0;u_2)-\frac{23}{24}\pi^2H(0,0;u_2)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{123};1\right)H(0,0;u_3)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{132};1\right)H(0,0;u_3)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{213};1\right)H(0,0;u_3)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{231};1\right)H(0,0;u_3)+3H(0;u_1)H(0;u_2)H(0,0;u_3)-$$
$$\frac{25}{4}H(0,0;u_1)H(0,0;u_3)-\frac{25}{4}H(0,0;u_2)H(0,0;u_3)-\frac{23}{24}\pi^2H(0,0;u_3)+\frac{1}{12}\pi^2H(0,1;u_1)+$$
$$\frac{1}{12}\pi^2H(0,1;u_2)-\frac{1}{24}\pi^2H\left(0,1;\frac{u_1+u_2-1}{u_2-1}\right)+\frac{1}{2}H(0;u_1)H(0;u_2)H(0,1;(u_1+u_2))+$$

$$\frac{1}{12}\pi^2H(0,1;(u_1+u_2))+\frac{1}{12}\pi^2H(0,1;u_3)+\frac{1}{4}H(0;u_1)H(0;u_2)H\left(0,1;\frac{u_1+u_3-1}{u_1-1}\right)-$$
$$\frac{1}{24}\pi^2H\left(0,1;\frac{u_1+u_3-1}{u_1-1}\right)+\frac{1}{12}\pi^2H(0,1;(u_1+u_3))-\frac{1}{24}\pi^2H\left(0,1;\frac{u_2+u_3-1}{u_3-1}\right)+$$
$$\frac{1}{12}\pi^2H(0,1;(u_2+u_3))-\frac{1}{2}G\left(0,\frac{1}{u_1+u_2};1\right)H(1,0;u_1)-$$
$$\frac{1}{2}G\left(0,\frac{1}{u_1+u_3};1\right)H(1,0;u_1)+\frac{1}{4}G\left(\frac{1}{u_1},\frac{1}{u_1+u_2};1\right)H(1,0;u_1)+$$
$$\frac{1}{4}G\left(\frac{1}{u_1},\frac{1}{u_1+u_3};1\right)H(1,0;u_1)+\frac{1}{4}G\left(\frac{1}{u_2},\frac{1}{u_1+u_2};1\right)H(1,0;u_1)+$$
$$\frac{1}{4}G\left(\frac{1}{1-u_3},\frac{u_1-1}{u_1+u_3-1};1\right)H(1,0;u_1)+\frac{1}{4}G\left(\frac{1}{u_3},\frac{1}{u_1+u_3};1\right)H(1,0;u_1)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},u_{312};1\right)H(1,0;u_1)-\frac{3}{4}H(0,0;u_2)H(1,0;u_1)-\frac{3}{4}H(0,0;u_3)H(1,0;u_1)+$$
$$\frac{1}{4}H\left(0,1;\frac{u_1+u_3-1}{u_1-1}\right)H(1,0;u_1)-\frac{1}{3}\pi^2H(1,0;u_1)-\frac{1}{2}G\left(0,\frac{1}{u_1+u_2};1\right)H(1,0;u_2)-$$
$$\frac{1}{2}G\left(0,\frac{1}{u_2+u_3};1\right)H(1,0;u_2)+\frac{1}{4}G\left(\frac{1}{1-u_1},\frac{u_2-1}{u_1+u_2-1};1\right)H(1,0;u_2)+$$
$$\frac{1}{4}G\left(\frac{1}{u_1},\frac{1}{u_1+u_2};1\right)H(1,0;u_2)+\frac{1}{4}G\left(\frac{1}{u_2},\frac{1}{u_1+u_2};1\right)H(1,0;u_2)+$$
$$\frac{1}{4}G\left(\frac{1}{u_2},\frac{1}{u_2+u_3};1\right)H(1,0;u_2)+\frac{1}{4}G\left(\frac{1}{u_3},\frac{1}{u_2+u_3};1\right)H(1,0;u_2)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},u_{123};1\right)H(1,0;u_2)-\frac{3}{4}H(0,0;u_1)H(1,0;u_2)-\frac{3}{4}H(0,0;u_3)H(1,0;u_2)+$$
$$\frac{1}{4}H\left(0,1;\frac{u_1+u_2-1}{u_2-1}\right)H(1,0;u_2)-\frac{1}{4}H(1,0;u_1)H(1,0;u_2)-\frac{1}{3}\pi^2H(1,0;u_2)-$$
$$\frac{1}{2}G\left(0,\frac{1}{u_1+u_3};1\right)H(1,0;u_3)-\frac{1}{2}G\left(0,\frac{1}{u_2+u_3};1\right)H(1,0;u_3)+$$
$$\frac{1}{4}G\left(\frac{1}{u_1},\frac{1}{u_1+u_3};1\right)H(1,0;u_3)+\frac{1}{4}G\left(\frac{1}{1-u_2},\frac{u_3-1}{u_2+u_3-1};1\right)H(1,0;u_3)+$$
$$\frac{1}{4}G\left(\frac{1}{u_2},\frac{1}{u_2+u_3};1\right)H(1,0;u_3)-\frac{1}{3}\pi^2H(1,0;u_3)+\frac{1}{4}G\left(\frac{1}{u_3},\frac{1}{u_1+u_3};1\right)H(1,0;u_3)+$$
$$\frac{1}{4}G\left(\frac{1}{u_3},\frac{1}{u_2+u_3};1\right)H(1,0;u_3)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},u_{231};1\right)H(1,0;u_3)+$$
$$\frac{3}{4}H(0;u_1)H(0;u_2)H(1,0;u_3)-\frac{3}{4}H(0,0;u_1)H(1,0;u_3)-\frac{3}{4}H(0,0;u_2)H(1,0;u_3)+$$
$$\frac{1}{4}H\left(0,1;\frac{u_2+u_3-1}{u_3-1}\right)H(1,0;u_3)-\frac{1}{4}H(1,0;u_1)H(1,0;u_3)-\frac{1}{4}H(1,0;u_2)H(1,0;u_3)+$$
$$\frac{1}{24}\pi^2H(1,1;u_1)+\frac{1}{24}\pi^2H(1,1;u_2)+\frac{1}{24}\pi^2H(1,1;u_3)+\frac{1}{2}H(0;u_2)H(0,0,0;u_1)+$$
$$\frac{1}{2}H(0;u_3)H(0,0,0;u_2)+\frac{1}{2}H(0;u_1)H(0,0,0;u_3)-\frac{1}{2}H(0;u_2)H\left(0,0,1;\frac{u_1+u_2-1}{u_2-1}\right)-$$
$$\frac{1}{2}H(0;u_3)H\left(0,0,1;\frac{u_1+u_2-1}{u_2-1}\right)-H(0;u_1)H(0,0,1;(u_1+u_2))-$$
$$H(0;u_2)H(0,0,1;(u_1+u_2))-\frac{1}{2}H(0;u_1)H\left(0,0,1;\frac{u_1+u_3-1}{u_1-1}\right)-$$

$$\frac{1}{2}H(0;u_2)\,H\left(0,0,1;\frac{u_1+u_3-1}{u_1-1}\right)-H(0;u_1)\,H(0,0,1;(u_1+u_3))-$$
$$H(0;u_3)\,H(0,0,1;(u_1+u_3))-\frac{1}{2}H(0;u_1)\,H\left(0,0,1;\frac{u_2+u_3-1}{u_3-1}\right)-$$
$$\frac{1}{2}H(0;u_3)\,H\left(0,0,1;\frac{u_2+u_3-1}{u_3-1}\right)-H(0;u_2)\,H(0,0,1;(u_2+u_3))-$$
$$H(0;u_3)\,H(0,0,1;(u_2+u_3))-\frac{1}{2}H(0;u_2)\,H(0,1,0;u_1)-\frac{1}{2}H(0;u_3)\,H(0,1,0;u_2)-$$
$$\frac{1}{2}H(0;u_1)\,H(0,1,0;u_3)+\frac{1}{4}H(0;u_2)\,H\left(0,1,1;\frac{u_1+u_2-1}{u_2-1}\right)-$$
$$\frac{1}{4}H(0;u_3)\,H\left(0,1,1;\frac{u_1+u_2-1}{u_2-1}\right)+\frac{1}{4}H(0;u_1)\,H\left(0,1,1;\frac{u_1+u_3-1}{u_1-1}\right)-$$
$$\frac{1}{4}H(0;u_2)\,H\left(0,1,1;\frac{u_1+u_3-1}{u_1-1}\right)-\frac{1}{4}H(0;u_1)\,H\left(0,1,1;\frac{u_2+u_3-1}{u_3-1}\right)+$$
$$\frac{1}{4}H(0;u_3)\,H\left(0,1,1;\frac{u_2+u_3-1}{u_3-1}\right)+\frac{1}{2}H(0;u_2)\,H(1,0,0;u_1)-\frac{1}{2}H(0;u_3)\,H(1,0,0;u_1)-$$
$$\frac{1}{2}H(0;u_1)\,H(1,0,0;u_2)+\frac{1}{2}H(0;u_3)\,H(1,0,0;u_2)+\frac{1}{2}H(0;u_1)\,H(1,0,0;u_3)-$$
$$\frac{1}{2}H(0;u_2)\,H(1,0,0;u_3)-\frac{1}{4}H(0;u_3)\,H\left(1,0,1;\frac{u_1+u_2-1}{u_2-1}\right)-$$
$$\frac{1}{4}H(0;u_2)\,H\left(1,0,1;\frac{u_1+u_3-1}{u_1-1}\right)-\frac{1}{4}H(0;u_1)\,H\left(1,0,1;\frac{u_2+u_3-1}{u_3-1}\right)-$$
$$7H(0,0,0,0;u_1)-7H(0,0,0,0;u_2)-7H(0,0,0,0;u_3)+\frac{3}{2}H\left(0,0,0,1;\frac{u_1+u_2-1}{u_2-1}\right)+$$
$$3H(0,0,0,1;(u_1+u_2))+\frac{3}{2}H\left(0,0,0,1;\frac{u_1+u_3-1}{u_1-1}\right)+3H(0,0,0,1;(u_1+u_3))+$$
$$\frac{3}{2}H\left(0,0,0,1;\frac{u_2+u_3-1}{u_3-1}\right)+3H(0,0,0,1;(u_2+u_3))+\frac{9}{4}H(0,0,1,0;u_1)+$$
$$\frac{9}{4}H(0,0,1,0;u_2)+\frac{9}{4}H(0,0,1,0;u_3)-\frac{1}{2}H(0,1,0,0;u_1)-\frac{1}{2}H(0,1,0,0;u_2)-$$
$$\frac{1}{2}H(0,1,0,0;u_3)+\frac{1}{2}H\left(0,1,0,1;\frac{u_1+u_2-1}{u_2-1}\right)+\frac{1}{2}H\left(0,1,0,1;\frac{u_1+u_3-1}{u_1-1}\right)+$$
$$\frac{1}{2}H\left(0,1,0,1;\frac{u_2+u_3-1}{u_3-1}\right)+H(0,1,1,0;u_1)+H(0,1,1,0;u_2)+H(0,1,1,0;u_3)-$$
$$\frac{1}{4}H\left(0,1,1,1;\frac{u_1+u_2-1}{u_2-1}\right)-\frac{1}{4}H\left(0,1,1,1;\frac{u_1+u_3-1}{u_1-1}\right)-$$
$$\frac{1}{4}H\left(0,1,1,1;\frac{u_2+u_3-1}{u_3-1}\right)+H\left(1,0,0,1;\frac{u_1+u_2-1}{u_2-1}\right)+H\left(1,0,0,1;\frac{u_1+u_3-1}{u_1-1}\right)+$$
$$H\left(1,0,0,1;\frac{u_2+u_3-1}{u_3-1}\right)+2H(1,0,1,0;u_1)+2H(1,0,1,0;u_2)+2H(1,0,1,0;u_3)+$$
$$\frac{1}{4}H\left(1,1,0,1;\frac{u_1+u_2-1}{u_2-1}\right)+\frac{1}{4}H\left(1,1,0,1;\frac{u_1+u_3-1}{u_1-1}\right)+$$
$$\frac{1}{4}H\left(1,1,0,1;\frac{u_2+u_3-1}{u_3-1}\right)+\frac{1}{2}H(1,1,1,0;u_1)+\frac{1}{2}H(1,1,1,0;u_2)+\frac{1}{2}H(1,1,1,0;u_3)-$$
$$\frac{1}{24}\pi^2H(0;u_3)\,\mathcal{H}\left(1;\frac{1}{u_{123}}\right)-\frac{1}{24}\pi^2H(0;u_1)\,\mathcal{H}\left(1;\frac{1}{u_{231}}\right)-\frac{1}{24}\pi^2H(0;u_2)\,\mathcal{H}\left(1;\frac{1}{u_{312}}\right)+$$
$$\frac{1}{8}\pi^2H(0;u_2)\,\mathcal{H}\left(1;\frac{1}{v_{123}}\right)-\frac{1}{8}\pi^2H(0;u_3)\,\mathcal{H}\left(1;\frac{1}{v_{123}}\right)+\frac{1}{24}\pi^2H(0;u_2)\,\mathcal{H}\left(1;\frac{1}{v_{132}}\right)-$$

$$\frac{1}{24}\pi^2H(0;u_3)\,\mathcal{H}\left(1;\frac{1}{v_{132}}\right)-\frac{1}{24}\pi^2H(0;u_1)\,\mathcal{H}\left(1;\frac{1}{v_{213}}\right)+\frac{1}{24}\pi^2H(0;u_3)\,\mathcal{H}\left(1;\frac{1}{v_{213}}\right)-$$
$$\frac{1}{8}\pi^2H(0;u_1)\,\mathcal{H}\left(1;\frac{1}{v_{231}}\right)+\frac{1}{8}\pi^2H(0;u_3)\,\mathcal{H}\left(1;\frac{1}{v_{231}}\right)+\frac{1}{8}\pi^2H(0;u_1)\,\mathcal{H}\left(1;\frac{1}{v_{312}}\right)-$$
$$\frac{1}{8}\pi^2H(0;u_2)\,\mathcal{H}\left(1;\frac{1}{v_{312}}\right)+\frac{1}{24}\pi^2H(0;u_1)\,\mathcal{H}\left(1;\frac{1}{v_{321}}\right)-\frac{1}{24}\pi^2H(0;u_2)\,\mathcal{H}\left(1;\frac{1}{v_{321}}\right)-$$
$$\frac{1}{4}H(0;u_2)\,H(0;u_3)\,\mathcal{H}\left(0,1;\frac{1}{u_{123}}\right)-\frac{1}{4}H(1,0;u_2)\,\mathcal{H}\left(0,1;\frac{1}{u_{123}}\right)+\frac{1}{24}\pi^2\mathcal{H}\left(0,1;\frac{1}{u_{123}}\right)+$$
$$\frac{1}{24}\pi^2\mathcal{H}\left(0,1;\frac{1}{u_{231}}\right)-\frac{1}{4}H(0;u_1)\,H(0;u_3)\,\mathcal{H}\left(0,1;\frac{1}{u_{231}}\right)-\frac{1}{4}H(1,0;u_3)\,\mathcal{H}\left(0,1;\frac{1}{u_{231}}\right)-$$
$$\frac{1}{4}H(0;u_1)\,H(0;u_2)\,\mathcal{H}\left(0,1;\frac{1}{u_{312}}\right)-\frac{1}{4}H(1,0;u_1)\,\mathcal{H}\left(0,1;\frac{1}{u_{312}}\right)+\frac{1}{24}\pi^2\mathcal{H}\left(0,1;\frac{1}{u_{312}}\right)-$$
$$\frac{1}{4}H(0;u_2)\,H(0;u_3)\,\mathcal{H}\left(0,1;\frac{1}{v_{123}}\right)+\frac{1}{4}H(0,0;u_2)\,\mathcal{H}\left(0,1;\frac{1}{v_{123}}\right)+$$
$$\frac{1}{4}H(0,0;u_3)\,\mathcal{H}\left(0,1;\frac{1}{v_{123}}\right)+\frac{1}{6}\pi^2\mathcal{H}\left(0,1;\frac{1}{v_{123}}\right)-\frac{1}{4}H(0;u_2)\,H(0;u_3)\,\mathcal{H}\left(0,1;\frac{1}{v_{132}}\right)+$$
$$\frac{1}{4}H(0,0;u_2)\,\mathcal{H}\left(0,1;\frac{1}{v_{132}}\right)+\frac{1}{4}H(0,0;u_3)\,\mathcal{H}\left(0,1;\frac{1}{v_{132}}\right)+\frac{1}{6}\pi^2\mathcal{H}\left(0,1;\frac{1}{v_{132}}\right)-$$
$$\frac{1}{4}H(0;u_1)\,H(0;u_3)\,\mathcal{H}\left(0,1;\frac{1}{v_{213}}\right)+\frac{1}{4}H(0,0;u_1)\,\mathcal{H}\left(0,1;\frac{1}{v_{213}}\right)+$$
$$\frac{1}{4}H(0,0;u_3)\,\mathcal{H}\left(0,1;\frac{1}{v_{213}}\right)+\frac{1}{6}\pi^2\mathcal{H}\left(0,1;\frac{1}{v_{213}}\right)-\frac{1}{4}H(0;u_1)\,H(0;u_3)\,\mathcal{H}\left(0,1;\frac{1}{v_{231}}\right)+$$
$$\frac{1}{4}H(0,0;u_1)\,\mathcal{H}\left(0,1;\frac{1}{v_{231}}\right)+\frac{1}{4}H(0,0;u_3)\,\mathcal{H}\left(0,1;\frac{1}{v_{231}}\right)+\frac{1}{6}\pi^2\mathcal{H}\left(0,1;\frac{1}{v_{231}}\right)-$$
$$\frac{1}{4}H(0;u_1)\,H(0;u_2)\,\mathcal{H}\left(0,1;\frac{1}{v_{312}}\right)+\frac{1}{4}H(0,0;u_1)\,\mathcal{H}\left(0,1;\frac{1}{v_{312}}\right)+$$
$$\frac{1}{4}H(0,0;u_2)\,\mathcal{H}\left(0,1;\frac{1}{v_{312}}\right)+\frac{1}{6}\pi^2\mathcal{H}\left(0,1;\frac{1}{v_{312}}\right)-\frac{1}{4}H(0;u_1)\,H(0;u_2)\,\mathcal{H}\left(0,1;\frac{1}{v_{321}}\right)+$$
$$\frac{1}{4}H(0,0;u_1)\,\mathcal{H}\left(0,1;\frac{1}{v_{321}}\right)+\frac{1}{4}H(0,0;u_2)\,\mathcal{H}\left(0,1;\frac{1}{v_{321}}\right)+\frac{1}{6}\pi^2\mathcal{H}\left(0,1;\frac{1}{v_{321}}\right)-$$
$$\frac{1}{2}H(0;u_2)\,H(0;u_3)\,\mathcal{H}\left(1,1;\frac{1}{v_{123}}\right)+\frac{1}{2}H(0,0;u_2)\,\mathcal{H}\left(1,1;\frac{1}{v_{123}}\right)+$$
$$\frac{1}{2}H(0,0;u_3)\,\mathcal{H}\left(1,1;\frac{1}{v_{123}}\right)+\frac{11}{24}\pi^2\mathcal{H}\left(1,1;\frac{1}{v_{123}}\right)-\frac{1}{24}\pi^2\mathcal{H}\left(1,1;\frac{1}{v_{132}}\right)-$$
$$\frac{1}{24}\pi^2\mathcal{H}\left(1,1;\frac{1}{v_{213}}\right)-\frac{1}{2}H(0;u_1)\,H(0;u_3)\,\mathcal{H}\left(1,1;\frac{1}{v_{231}}\right)+\frac{1}{2}H(0,0;u_1)\,\mathcal{H}\left(1,1;\frac{1}{v_{231}}\right)+$$
$$\frac{1}{2}H(0,0;u_3)\,\mathcal{H}\left(1,1;\frac{1}{v_{231}}\right)+\frac{11}{24}\pi^2\mathcal{H}\left(1,1;\frac{1}{v_{231}}\right)-\frac{1}{2}H(0;u_1)\,H(0;u_2)\,\mathcal{H}\left(1,1;\frac{1}{v_{312}}\right)+$$
$$\frac{1}{2}H(0,0;u_1)\,\mathcal{H}\left(1,1;\frac{1}{v_{312}}\right)+\frac{1}{2}H(0,0;u_2)\,\mathcal{H}\left(1,1;\frac{1}{v_{312}}\right)+\frac{11}{24}\pi^2\mathcal{H}\left(1,1;\frac{1}{v_{312}}\right)-$$
$$\frac{1}{24}\pi^2\mathcal{H}\left(1,1;\frac{1}{v_{321}}\right)+\frac{1}{2}H(0;u_2)\,\mathcal{H}\left(0,0,1;\frac{1}{u_{123}}\right)+\frac{1}{2}H(0;u_3)\,\mathcal{H}\left(0,0,1;\frac{1}{u_{123}}\right)+$$
$$\frac{1}{2}H(0;u_1)\,\mathcal{H}\left(0,0,1;\frac{1}{u_{231}}\right)+\frac{1}{2}H(0;u_3)\,\mathcal{H}\left(0,0,1;\frac{1}{u_{231}}\right)+\frac{1}{2}H(0;u_1)\,\mathcal{H}\left(0,0,1;\frac{1}{u_{312}}\right)+$$
$$\frac{1}{2}H(0;u_2)\,\mathcal{H}\left(0,0,1;\frac{1}{u_{312}}\right)+\frac{1}{4}H(0;u_3)\,\mathcal{H}\left(0,1,1;\frac{1}{u_{123}}\right)+\frac{1}{4}H(0;u_1)\,\mathcal{H}\left(0,1,1;\frac{1}{u_{231}}\right)+$$

$$\frac{1}{4}H\left(0;u_2\right)\mathcal{H}\left(0,1,1;\frac{1}{u_{312}}\right)+\frac{1}{4}H\left(0;u_2\right)\mathcal{H}\left(0,1,1;\frac{1}{v_{123}}\right)-\frac{1}{4}H\left(0;u_3\right)\mathcal{H}\left(0,1,1;\frac{1}{v_{123}}\right)-$$
$$\frac{1}{4}H\left(0;u_2\right)\mathcal{H}\left(0,1,1;\frac{1}{v_{132}}\right)+\frac{1}{4}H\left(0;u_3\right)\mathcal{H}\left(0,1,1;\frac{1}{v_{132}}\right)+\frac{1}{4}H\left(0;u_1\right)\mathcal{H}\left(0,1,1;\frac{1}{v_{213}}\right)-$$
$$\frac{1}{4}H\left(0;u_3\right)\mathcal{H}\left(0,1,1;\frac{1}{v_{213}}\right)-\frac{1}{4}H\left(0;u_1\right)\mathcal{H}\left(0,1,1;\frac{1}{v_{231}}\right)+\frac{1}{4}H\left(0;u_3\right)\mathcal{H}\left(0,1,1;\frac{1}{v_{231}}\right)+$$
$$\frac{1}{4}H\left(0;u_1\right)\mathcal{H}\left(0,1,1;\frac{1}{v_{312}}\right)-\frac{1}{4}H\left(0;u_2\right)\mathcal{H}\left(0,1,1;\frac{1}{v_{312}}\right)-\frac{1}{4}H\left(0;u_1\right)\mathcal{H}\left(0,1,1;\frac{1}{v_{321}}\right)+$$
$$\frac{1}{4}H\left(0;u_2\right)\mathcal{H}\left(0,1,1;\frac{1}{v_{321}}\right)+\frac{1}{4}H\left(0;u_3\right)\mathcal{H}\left(1,0,1;\frac{1}{u_{123}}\right)+\frac{1}{4}H\left(0;u_1\right)\mathcal{H}\left(1,0,1;\frac{1}{u_{231}}\right)+$$
$$\frac{1}{4}H\left(0;u_2\right)\mathcal{H}\left(1,0,1;\frac{1}{u_{312}}\right)+\frac{1}{4}H\left(0;u_2\right)\mathcal{H}\left(1,0,1;\frac{1}{v_{123}}\right)-\frac{1}{4}H\left(0;u_3\right)\mathcal{H}\left(1,0,1;\frac{1}{v_{123}}\right)-$$
$$\frac{1}{4}H\left(0;u_2\right)\mathcal{H}\left(1,0,1;\frac{1}{v_{132}}\right)+\frac{1}{4}H\left(0;u_3\right)\mathcal{H}\left(1,0,1;\frac{1}{v_{132}}\right)+\frac{1}{4}H\left(0;u_1\right)\mathcal{H}\left(1,0,1;\frac{1}{v_{213}}\right)-$$
$$\frac{1}{4}H\left(0;u_3\right)\mathcal{H}\left(1,0,1;\frac{1}{v_{213}}\right)-\frac{1}{4}H\left(0;u_1\right)\mathcal{H}\left(1,0,1;\frac{1}{v_{231}}\right)+\frac{1}{4}H\left(0;u_3\right)\mathcal{H}\left(1,0,1;\frac{1}{v_{231}}\right)+$$
$$\frac{1}{4}H\left(0;u_1\right)\mathcal{H}\left(1,0,1;\frac{1}{v_{312}}\right)-\frac{1}{4}H\left(0;u_2\right)\mathcal{H}\left(1,0,1;\frac{1}{v_{312}}\right)-\frac{1}{4}H\left(0;u_1\right)\mathcal{H}\left(1,0,1;\frac{1}{v_{321}}\right)+$$
$$\frac{1}{4}H\left(0;u_2\right)\mathcal{H}\left(1,0,1;\frac{1}{v_{321}}\right)+H\left(0;u_2\right)\mathcal{H}\left(1,1,1;\frac{1}{v_{123}}\right)-H\left(0;u_3\right)\mathcal{H}\left(1,1,1;\frac{1}{v_{123}}\right)-$$
$$H\left(0;u_1\right)\mathcal{H}\left(1,1,1;\frac{1}{v_{231}}\right)+H\left(0;u_3\right)\mathcal{H}\left(1,1,1;\frac{1}{v_{231}}\right)+H\left(0;u_1\right)\mathcal{H}\left(1,1,1;\frac{1}{v_{312}}\right)-$$
$$H\left(0;u_2\right)\mathcal{H}\left(1,1,1;\frac{1}{v_{312}}\right)-\frac{3}{2}\mathcal{H}\left(0,0,0,1;\frac{1}{u_{123}}\right)-\frac{3}{2}\mathcal{H}\left(0,0,0,1;\frac{1}{u_{231}}\right)-$$
$$\frac{3}{2}\mathcal{H}\left(0,0,0,1;\frac{1}{u_{312}}\right)-3\mathcal{H}\left(0,0,0,1;\frac{1}{v_{132}}\right)-3\mathcal{H}\left(0,0,0,1;\frac{1}{v_{213}}\right)-3\mathcal{H}\left(0,0,0,1;\frac{1}{v_{321}}\right)-$$
$$\frac{1}{2}\mathcal{H}\left(0,0,1,1;\frac{1}{u_{123}}\right)-\frac{1}{2}\mathcal{H}\left(0,0,1,1;\frac{1}{u_{231}}\right)-\frac{1}{2}\mathcal{H}\left(0,0,1,1;\frac{1}{u_{312}}\right)-$$
$$\frac{1}{2}\mathcal{H}\left(0,1,0,1;\frac{1}{u_{123}}\right)-\frac{1}{2}\mathcal{H}\left(0,1,0,1;\frac{1}{u_{231}}\right)-\frac{1}{2}\mathcal{H}\left(0,1,0,1;\frac{1}{u_{312}}\right)+$$
$$\frac{1}{4}\mathcal{H}\left(0,1,1,1;\frac{1}{v_{123}}\right)+\frac{1}{4}\mathcal{H}\left(0,1,1,1;\frac{1}{v_{132}}\right)+\zeta_3H\left(0;u_1\right)+\zeta_3H\left(0;u_2\right)+\zeta_3H\left(0;u_3\right)+$$
$$\frac{5}{2}\zeta_3H\left(1;u_1\right)+\frac{5}{2}\zeta_3H\left(1;u_2\right)+\frac{5}{2}\zeta_3H\left(1;u_3\right)+\frac{1}{2}\zeta_3\mathcal{H}\left(1;\frac{1}{u_{123}}\right)+\frac{1}{2}\zeta_3\mathcal{H}\left(1;\frac{1}{u_{231}}\right)+$$
$$\frac{1}{2}\zeta_3\mathcal{H}\left(1;\frac{1}{u_{312}}\right)-\frac{1}{2}\mathcal{H}\left(1,0,0,1;\frac{1}{u_{123}}\right)-\frac{1}{2}\mathcal{H}\left(1,0,0,1;\frac{1}{u_{231}}\right)-\frac{1}{2}\mathcal{H}\left(1,0,0,1;\frac{1}{u_{312}}\right)+$$
$$\frac{1}{4}\zeta_3\mathcal{H}\left(1;\frac{1}{v_{123}}\right)+\frac{1}{4}\zeta_3\mathcal{H}\left(1;\frac{1}{v_{132}}\right)+\frac{1}{4}\zeta_3\mathcal{H}\left(1;\frac{1}{v_{213}}\right)+\frac{1}{4}\zeta_3\mathcal{H}\left(1;\frac{1}{v_{231}}\right)+\frac{1}{4}\zeta_3\mathcal{H}\left(1;\frac{1}{v_{312}}\right)+$$
$$\frac{1}{4}\zeta_3\mathcal{H}\left(1;\frac{1}{v_{321}}\right)+\frac{1}{4}\mathcal{H}\left(0,1,1,1;\frac{1}{v_{213}}\right)+\frac{1}{4}\mathcal{H}\left(0,1,1,1;\frac{1}{v_{231}}\right)+\frac{1}{4}\mathcal{H}\left(0,1,1,1;\frac{1}{v_{312}}\right)+$$
$$\frac{1}{4}\mathcal{H}\left(0,1,1,1;\frac{1}{v_{321}}\right)+\frac{1}{4}\mathcal{H}\left(1,0,1,1;\frac{1}{v_{123}}\right)+\frac{1}{4}\mathcal{H}\left(1,0,1,1;\frac{1}{v_{132}}\right)+\frac{1}{4}\mathcal{H}\left(1,0,1,1;\frac{1}{v_{213}}\right)+$$
$$\frac{1}{4}\mathcal{H}\left(1,0,1,1;\frac{1}{v_{231}}\right)+\frac{1}{4}\mathcal{H}\left(1,0,1,1;\frac{1}{v_{312}}\right)+\frac{1}{4}\mathcal{H}\left(1,0,1,1;\frac{1}{v_{321}}\right)+\frac{1}{4}\mathcal{H}\left(1,1,0,1;\frac{1}{v_{123}}\right)+$$
$$\frac{1}{4}\mathcal{H}\left(1,1,0,1;\frac{1}{v_{132}}\right)+\frac{1}{4}\mathcal{H}\left(1,1,0,1;\frac{1}{v_{213}}\right)+\frac{1}{4}\mathcal{H}\left(1,1,0,1;\frac{1}{v_{231}}\right)+\frac{1}{4}\mathcal{H}\left(1,1,0,1;\frac{1}{v_{312}}\right)+$$

If that were the best we could do, we all ought to quit now, there would be no hope...

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Goncharov, Vergu, Volovich and I used some mathematical magic called the symbol of a transcendental function to simplify this expression down to a single line, involving nothing more complicated than the "standard" Li_n polylogarithm functions.

The worth of our formula is NOT that it computes the value of the 2-loop 6-point MHV amplitude!

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Rather, it is important because it provides hope, where previously was none, that we might really unlock the secrets to all orders...

The Philosophy of Amplitudeology

1. Simplifications do not happen by accident.
2. This is an experimental science.
   (Get the answer first, by any means necessary, then analyze it.)
3. Simplicity has to be believed to be seen.