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# Momentum broadening in weakly coupled quark-gluon plasma


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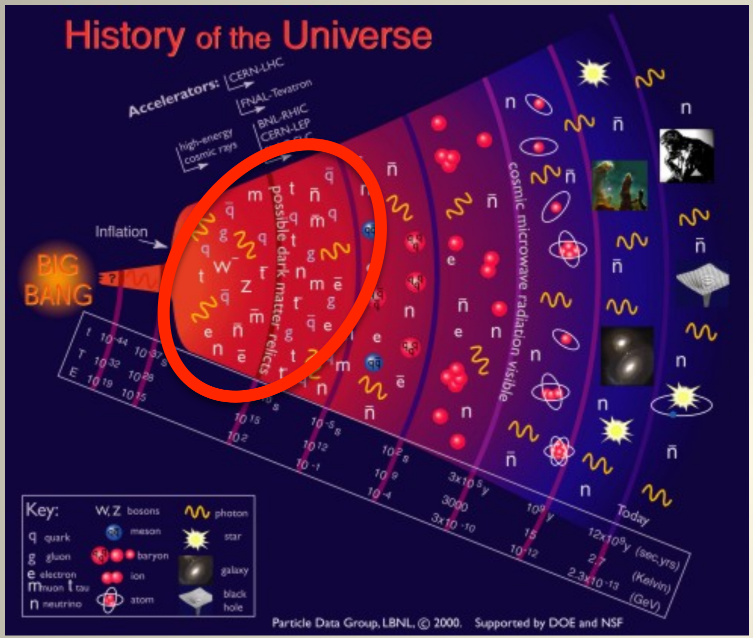
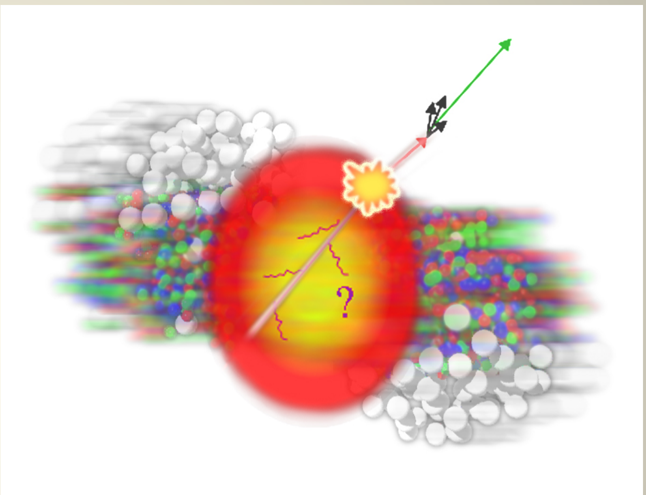
# Introduction




Jet energy loss mechanism



- Heavy Ion Collisions - a way to investigate quark-gluon plasma (QGP)
- One of the biggest puzzles in HIC is the energy loss mechanism of a probe quark/gluon that shoots through the medium with high velocity
- One of the quantities to look at – jet quenching parameter

# Outline

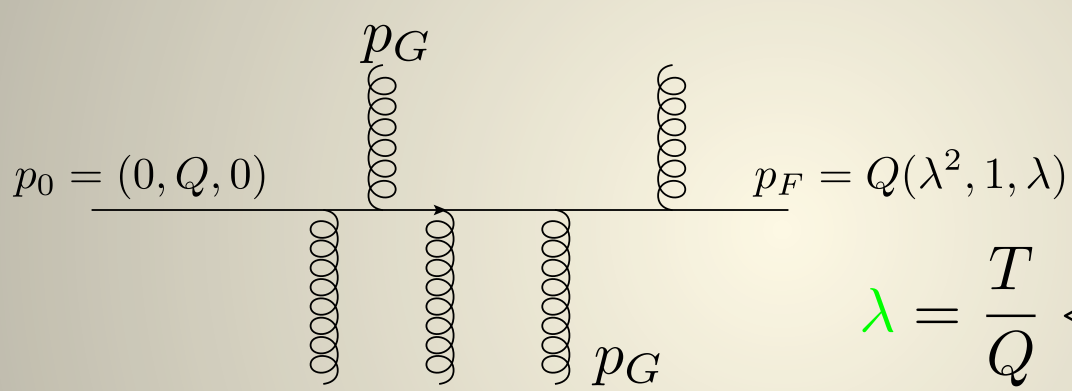
- Definition of momentum broadening distribution function (jet quenching parameter) in terms of Wilson lines

- Application of the result to the weakly coupled equilibrium quark-gluon plasma

# Hard probe

Parton traveling through the medium experiences:

- Energy loss
- Transverse momentum broadening:




$$\lambda = \frac{T}{Q} \ll 1$$

Glauber gluon momentum scaling:

$$p_G = (\lambda^2, \lambda^2, \lambda)$$

$p^+$ $p^-$ $p_\perp$

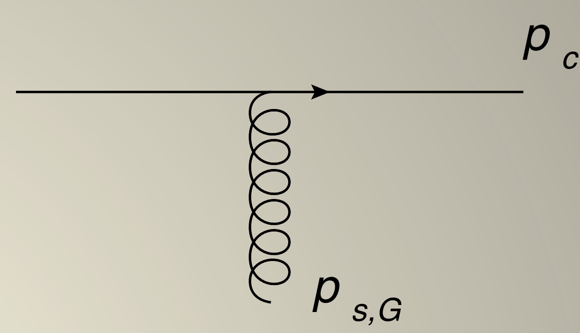
Soft Collinear Effective Theory (SCET) is well suited for problems involving separated scales.

Parton stays on-shell after interaction: Glauber gluons do not induce radiation.

Other applications of SCET for finite T medium:

Idilbi, Majumder (2009); Ovanesyan, Vitev (2011)

# Modes of SCET




Off-shell modes with $P^2 \gg Q^2\lambda^2$ are integrated out.

$$\lambda = \frac{T}{Q} \ll 1$$

- Collinear modes: $p_c = Q(\lambda^2, 1, \lambda)$

  The mode of energetic parton

- Soft modes: $p_s = Q(\lambda, \lambda, \lambda)$

  After interaction puts collinear mode off-shell and induces radiation, thus not relevant for momentum broadening or $\hat{q}$

- Glauber modes: $p_G = Q(\lambda^2, \lambda^2, \lambda)$

  Keep collinear mode on-shell, induce momentum broadening only

# Transverse momentum broadening

- Momentum broadening of quark (gluon) traveling through medium is calculated using

$$P(k_\perp) = \int d^2x_\perp \, e^{-ik_\perp \cdot x_\perp} \, \mathcal{W}_\mathcal{R}(x_\perp)$$

Casalderrey-Solana and Salgado (2007)
Liang, Wang and Zhou (2008)
D'Eramo, Liu and Rajagopal (2010)

$$\mathcal{W}_\mathcal{R}(x_\perp) = \frac{1}{d(\mathcal{R})} \left\langle \mathrm{Tr} \left[ \mathrm{W}^\dagger_\mathcal{R}[\mathrm{x}^+ = 0, \mathrm{x}_\perp] \, \mathrm{W}_\mathcal{R}[\mathrm{x}^+ = 0, 0] \right] \right\rangle$$

where $\mathcal{R}$ is the $SU(N)$ representation to which the collinear particle belongs and $d(\mathcal{R})$ is the dimension of this representation.

- Normalization condition $\int \frac{d^2k_\perp}{(2\pi)^2} P(k_\perp) = 1$
- Valid for both weak and strong coupling, general medium
- $\mathcal{N} = 4$ SYM case was considered

Liu, Rajagopal, Wiedemann (2006)

# Wilson lines in weakly coupled equilibrium quark-gluon plasma

$$\mathcal{W}_{\mathcal{R}}(x_\perp) = \frac{1}{d(\mathcal{R})} \left\langle \mathrm{Tr}\left[ \mathrm{W}^\dagger_{\mathcal{R}}[\mathrm{x}^+ = 0, \mathrm{x}_\perp]\, \mathrm{W}_{\mathcal{R}}[\mathrm{x}^+ = 0, 0] \right] \right\rangle$$

Average is taken over the specific medium, which in our case is weakly coupled equilibrium quark-gluon plasma.

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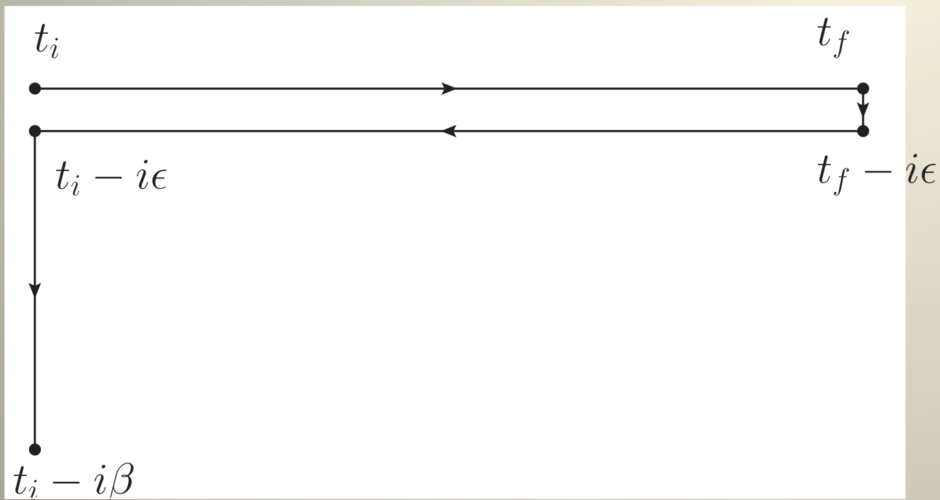
Wilson line takes the form

$$W_{\mathcal{R}}\left[y^+, y_\perp\right] \equiv P\left\{ \exp\left[ ig \int_0^{L^-} dy^-\, A^+_{\mathcal{R}}(y^+, y^-, y_\perp) \right] \right\}$$

$P$ stands for path ordering

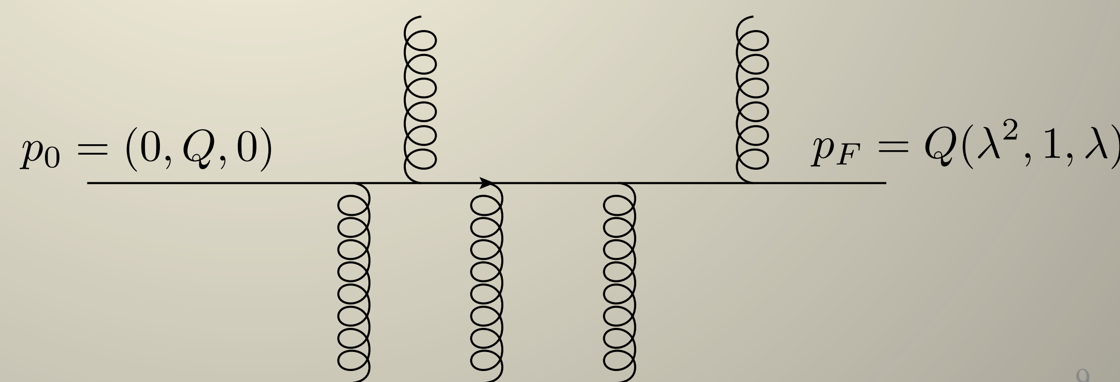


Schwinger-Keldysh countour

Since gluon operators are path ordered, Wilson lines are separated by $i\epsilon$ on Schwinger-Keldysh countour.

# Jet quenching parameter

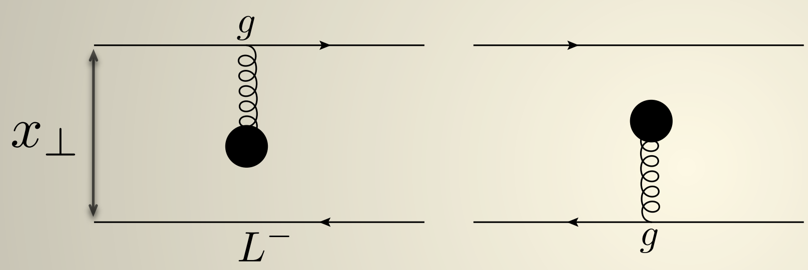
- Defined by $\hat{q} = \frac{1}{L}\int \frac{d^2k_\perp}{(2\pi)^2} k_\perp^2 P(k_\perp)$
- Estimates transverse momentum picked-up by particle per distance traveled.
- Second moment of probability distribution $P(k_\perp)$ (higher moments might be of interest too).
- $\hat{q}$ plays a central role in jet energy loss calculations.
- "Clean" field theoretical definition, experimentally definition is more elaborate.



# Counting powers of $g$

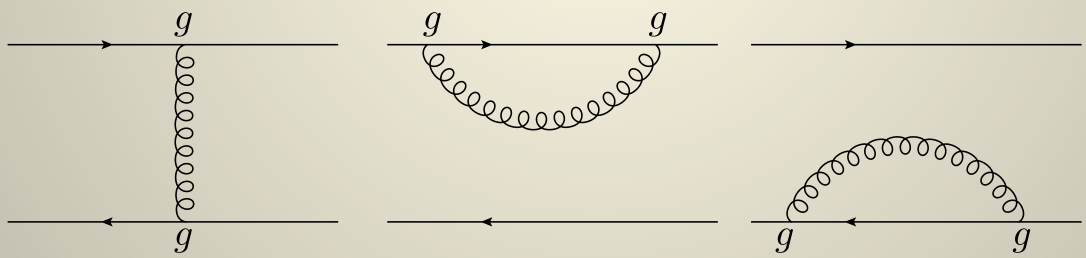
- Let's find the LO contributions by counting powers of $g$ explicitly

$g$ :




Vanish due to $\mathrm{tr}[A] = 0$

$g^2$ :

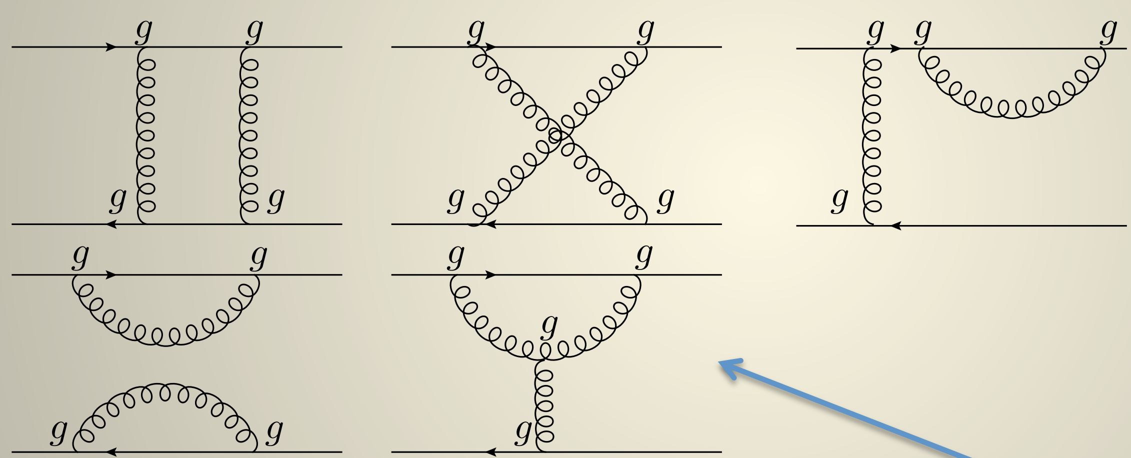



Vanish due to $g_{++} = 0$

$g^3$ : No diagrams contributing

# Counting powers of $g$

$g^4$ tree level diagrams:

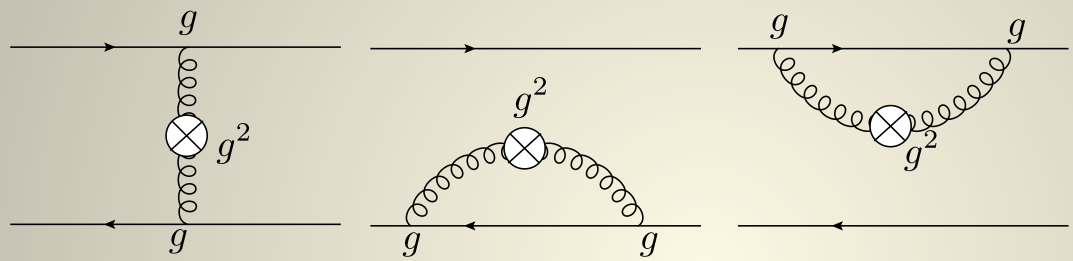


\+ permutations

Vanish also due to $g_{++} = 0$ and due to $\Gamma^{abc}_{+++} = 0$

# Counting powers of $g$

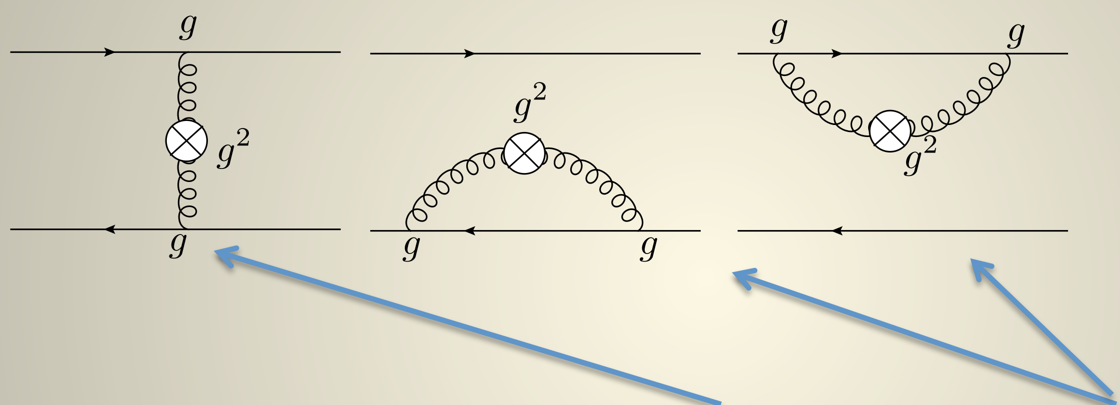
$g^4$ diagrams involving an effective propagator:




Non-vanishing!

# Counting powers of $g$

$g^4$ diagrams involving an effective propagator:




Non-vanishing!

$$P(k_\perp) = (2\pi)^2\delta^2(k_\perp) + P_>(k_\perp) + \delta^2(k_\perp)P_{11}$$

Leading order is $g^4$

# Probability distribution

- One can express probability distribution

$$P(k_\perp) = (2\pi)^2\delta^2(k_\perp) + P_>(k_\perp) - \delta^2(k_\perp)\int d^2q_\perp P_>(q_\perp)$$

- $P_>(k_\perp)$ is IR divergent, which is

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1. Irrelevant for an evaluation of jet quenching parameter (second moment of distribution)
2. Important if we care about $P(k_\perp)$ itself
3. Solution: use plus distribution function to extract delta function part from the second term and show that the divergent part cancels the divergent part of the third term

$$P(k_\perp) = \delta^2(k_\perp)g(k_{\perp 0}) + [P_>(k_\perp)]_+$$

# IR limit (HTL)

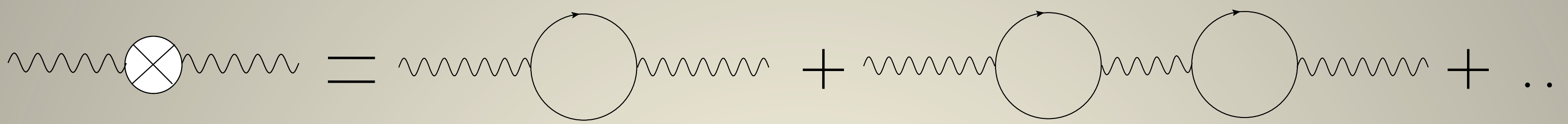
- For soft external momentum, need to use resummed effective theory – Hard Thermal Loops.
- Soft momentum: $q_0, |\vec{q}| \approx gT$
- Hard momentum: $q_0 \approx T$ or $|\vec{q}| \approx T$

Braaten and Pisarski (1990)
Frenkel and Taylor (1990)
Le Bellac (1996) for a pedagogical review

- Loop corrections are of order $(gT)^2/q^2$. For soft external momentum: corrections comparable to tree level propagator.
- In such case hard loop momentum gives the main contribution, self-energies simplify.
- Satisfies Ward identities.
- Valid only for soft external momentum!
- Need to use HTL resummed propagator and vertices to have valid perturbative expansion in powers of $g$ in IR limit.

# Resummation

- Resummation for QED and analogous for QCD:



- General form for Retarded propagator in covariant gauge at finite temperature:

$$(-i)\, D^R_{\mu\nu}(Q) = \frac{P^L_{\mu\nu}}{Q^2 - F} + \frac{P^T_{\mu\nu}}{Q^2 - G} - \zeta\frac{Q_\mu Q_\nu}{Q^4} + c(q) C_{\mu\nu} \qquad F = \frac{Q^2}{q^2}\Pi^L_R \qquad G = \Pi^T_R$$

- F and G are longitudinal and transverse self energies (gauge independent) which in static limit correspond to electric and magnetic masses
- On light-cone:

$Q_+ = 0$ : no explicit dependence on $\zeta$

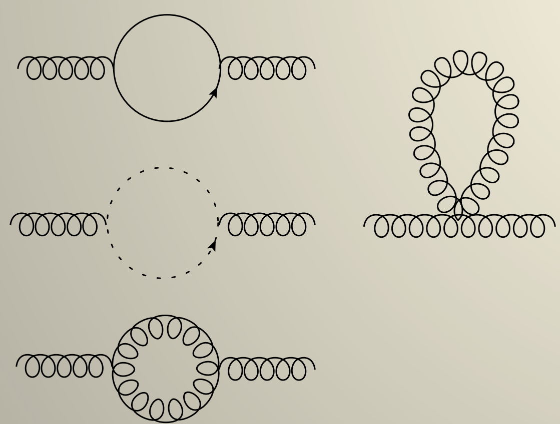
$C_{++} = 0$ : possibly gauge dependent term $c(q)$ vanishes

- Resummed propagator $D^R_{++}$ has no explicit gauge dependendence!

# IR and UV limits

**IR limit**

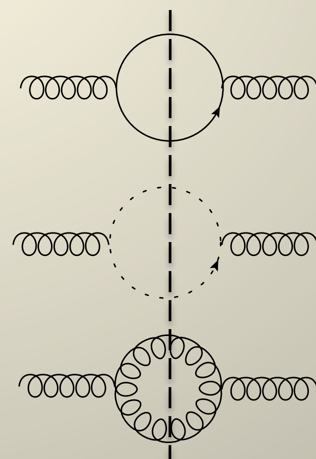
- Resummed propagator. Due to HTL approximation, real and imaginary parts of $F_{HTL}$ and $G_{HTL}$ are known analytically.



$$D_> = -\frac{1}{1-e^{-\beta k_0}} 2\mathrm{Re} D_R$$

**UV limit**

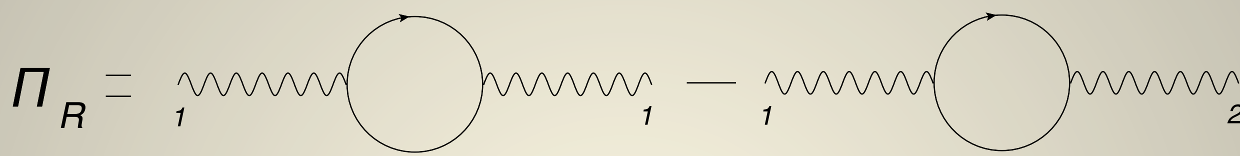
- In UV limit it is enough to calculate non-amputated propagator, resummation is not necessary, propagator is proportional to Im part of self energies.



$$D_> = -\frac{1}{k^2 k_\perp^2} \frac{1}{1-e^{-\beta k_0}} \left( \mathrm{Im}\Pi_R^T + \frac{k_\perp^2}{k^2} \mathrm{Im}\Pi_R^L \right)$$

# Transverse momentum broadening

$$P_>(k_\perp) = g^2 C_R L^- \sqrt{2} \int_{-\infty}^{\infty} \frac{dk_0}{2\pi} D_>^{++}(k^+ = 0, k_0, k_\perp)$$

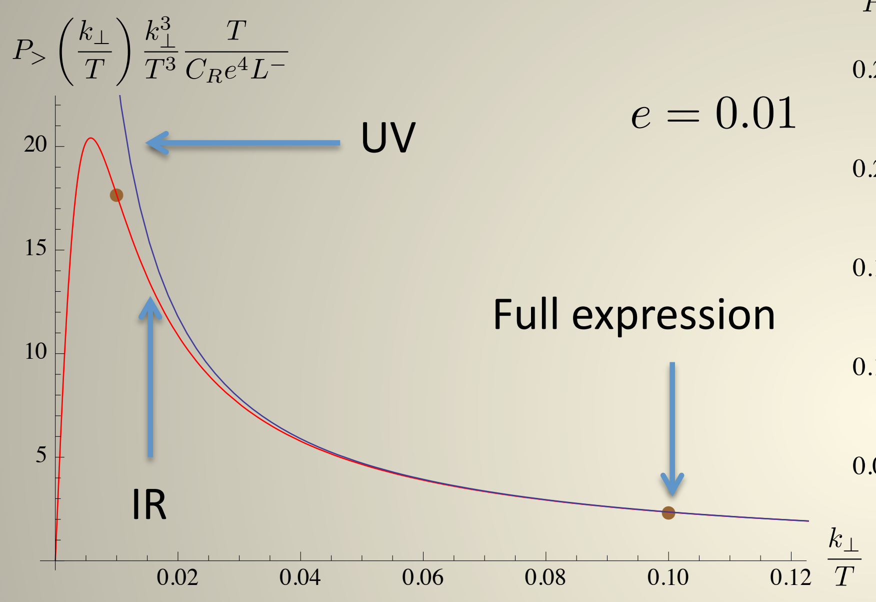


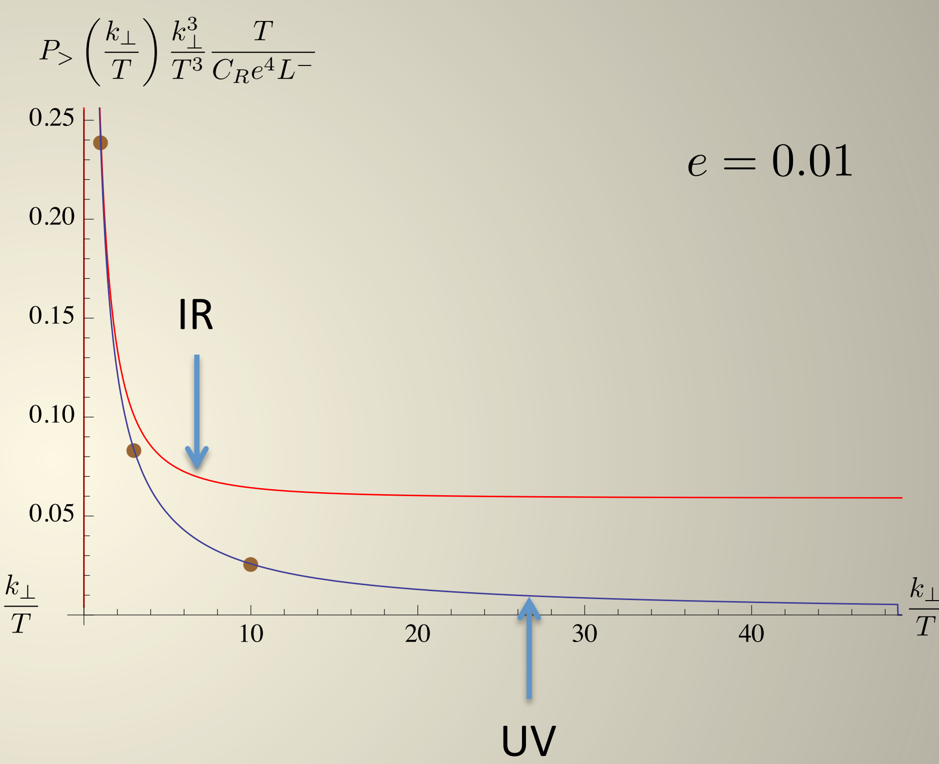

- We calculated full self-energies in the kinematical region of $k^+ = 0$ :

  Im$\Pi_{\mathrm{R}}$ – analytically

  Re$\Pi_{\mathrm{R}}$ – numerically
- In the regime of soft momentum ($k_0, k_\perp \sim gT$) reproduce HTL result as expected, but $\Pi_{\mathrm{R}}$ is <u>valid for all momentum space</u> and not restricted to soft momentum region.
- Can calculate momentum broadening in UV limit immediately, and obtain that $P_>(k_\perp) \propto 1/k_\perp^4$ which must be the case according to general arguments.

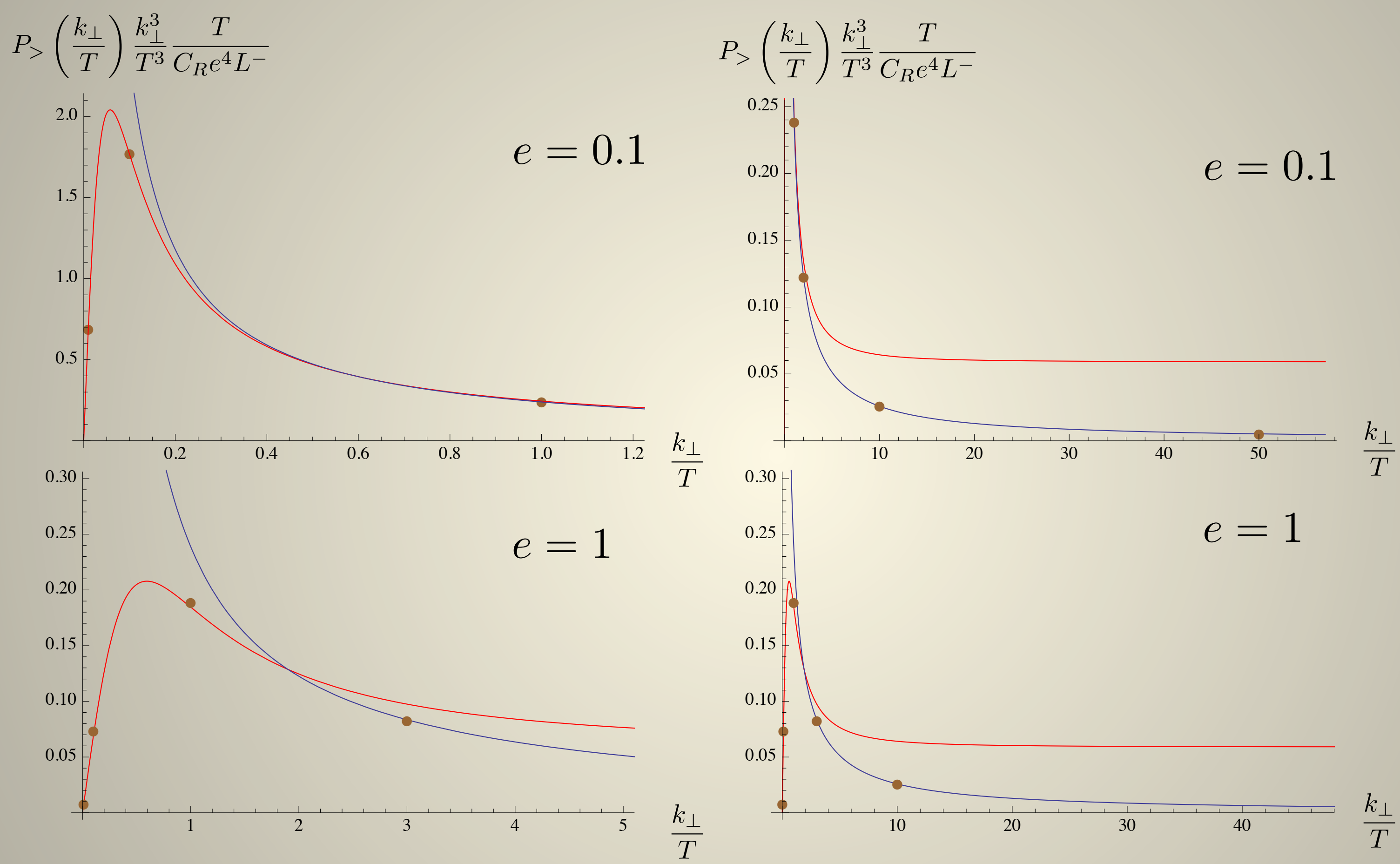
# Transverse momentum broadening



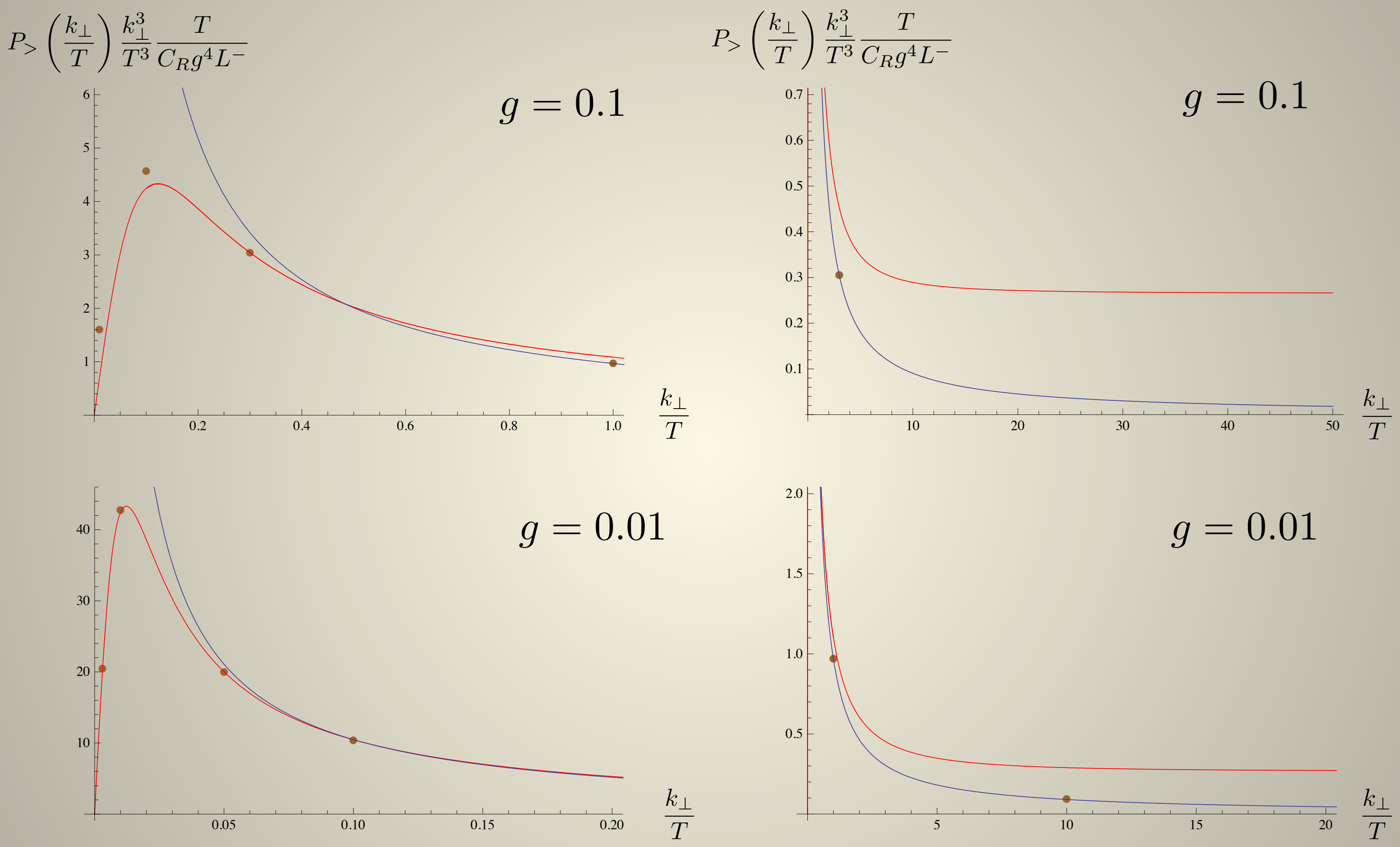

- Full expression is obtained with no approximations on $\Pi_R$
- UV and IR limits smoothly overlap
- In IR region, HTL is a good approximation as expected
- Can integrate to obtain jet quenching parameter $\hat{q}$

# Transverse momentum broadening (QED)




IR (HTL) and UV limits agree well with full expression (brown dots).

# Transverse momentum broadening (QCD)




Open question: is HTL valid in IR limit in QCD case (magnetic mass effects)?

# Comparison to literature

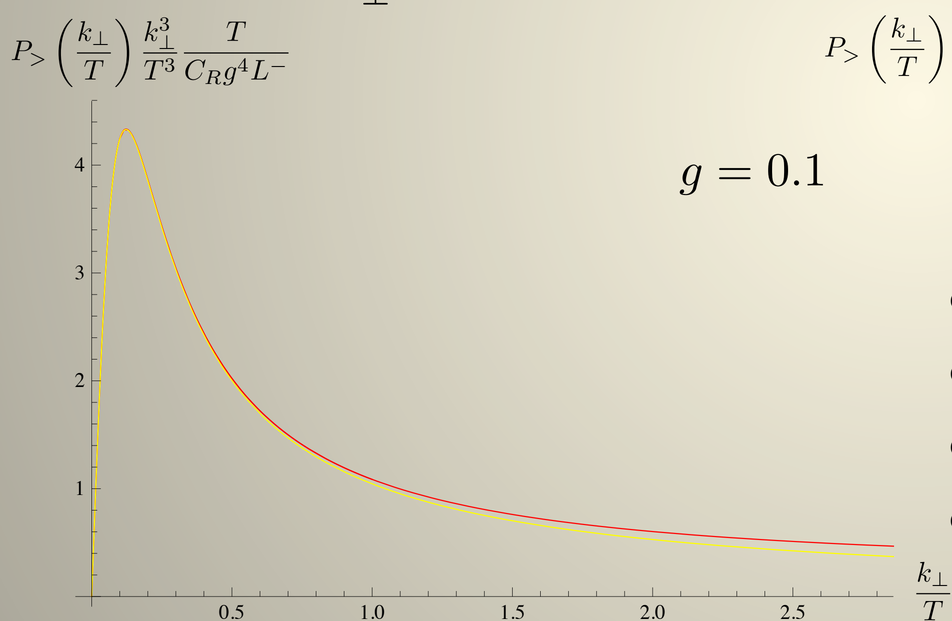
- In IR an UV limits we agree with the established limits in the literature

$$P(k_\perp) = C_R \frac{g^2 T m_D^2}{k_\perp^2 (k_\perp^2 + m_D^2)}, \quad k_\perp \ll T$$ Aurenche, *et al.* (2002)

$$P(k_\perp) = C_R \frac{g^4 \mathcal{N}}{k_\perp^4}, \quad k_\perp \gg T$$ Arnold, Dogan (2008)

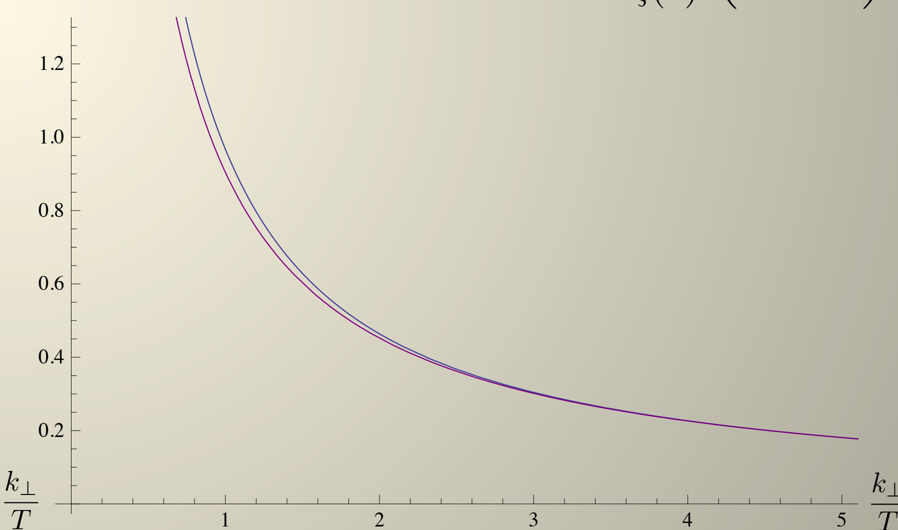
$$m_D^2 = \frac{1}{6}\left(1 + \frac{N_f}{2}\right) g^2 T^2$$

$$\mathcal{N} = \frac{\zeta(3)}{\zeta(2)}\left(1 + \frac{N_f}{4}\right) T^3$$




$g = 0.1$

$P_>\left(\frac{k_\perp}{T}\right)\frac{k_\perp^3}{T^3}\frac{T}{C_R g^4 L^-}$




- We have the automatic interpolation of $P(k_\perp)$ for all values of $k_\perp$

# Summing up

- Wilson line expression for momentum broadening obtained using SCET is taken one step further by evaluating leading order contribution in weakly coupled equilibrium plasma
- Full field theoretical calculation beyond HTL approximation performed
- General result, first attempt to calculate jet quenching parameter using SCET
- In IR and UV limits, $P_{>}(k_{\perp})$ agrees with results in the literature, automatic connection of UV and IR limits

# Summing up

- Wilson line expression for momentum broadening obtained using SCET is taken one step further by evaluating leading order contribution in weakly coupled equilibrium plasma
- Full field theoretical calculation beyond HTL approximation performed
- General result, first attempt to calculate jet quenching parameter using SCET
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Thank you for the attention!

# Back-up slides

# Explicit expressions

- Probability distribution function

$$P(k_\perp) = (2\pi)^2\delta^2(k_\perp) - (2\pi)^2\delta^2(k_\perp)2g^2C_R\int_0^{L^-}\mathrm{d}y_1^-\int_0^{y_1^-}\mathrm{d}y_2^-\mathrm{Re}\left[D_{11}^{++}(0,y_1^- - y_2^-,0)_{g^2}\right] +$$
$$g^2C_R\int_0^{\infty}\mathrm{d}^2x_\perp\int_0^{L^-}\mathrm{d}y_1^-\int_0^{L^-}\mathrm{d}y_2^- e^{-ik_\perp\cdot x_\perp}D_>^{++}(0,y_1^- - y_2^-,x_\perp)_{g^2}$$
$$\equiv (2\pi)^2\delta^2(k_\perp) + (2\pi)^2P_{11}(k_\perp) + P_>(k_\perp)$$

$$\text{Quadratic Casimir } C_R = \begin{cases} N_c & \text{for fundamental representation (quarks)} \\ \frac{N_c^2-1}{2N_c} & \text{for adjoint representation (gluons)} \end{cases}$$

- Taking the limit $L^- \to \infty$

$$P(k_\perp) \equiv (2\pi)^2\delta^2(k_\perp) + P_>(k_\perp) + (2\pi)^2P_{11}(k_\perp)$$
$$= (2\pi)^2\delta^2(k_\perp) + g^2C_RL^-\int\frac{dq_-}{2\pi}D_>^{++}(0,q_-,k_\perp)_{g^2} -$$
$$\delta^2(k_\perp)g^2C_RL^-\int\frac{dq_-d^2q_\perp}{2\pi}\mathrm{Re}D_{11}^{++}(0,q_-,q_\perp)_{g^2}$$

# Hard Thermal Loops

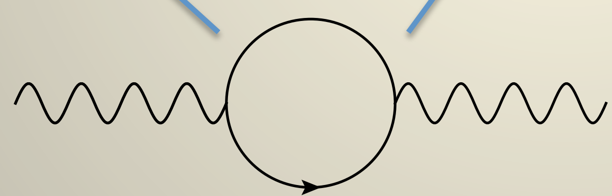
- Retarded propagator for photon

$$D^R_{\mu\nu}(q_\mu) = \frac{i}{Q^2 - G} P^T_{\mu\nu} + \frac{i}{Q^2 - F} P^L_{\mu\nu} - i\zeta \frac{q_\mu q_\nu}{Q^2}$$

$\zeta$ - gauge fixing parameter

- HTL "self energies"

$$F = \frac{iQ^2}{q_0 q} \Pi_{tz} \qquad G = \Pi_{xx} = m^2 - \frac{F}{2} \qquad q \equiv |\vec{q}|$$



- It turns out that for gluon field $D_R^{\mu\nu} \to \delta^{ab} D_R^{\mu\nu}$ with thermal mass

$$m^2 \to m^2_{QCD} = \frac{3}{4}(gT)^2$$

# IR behavior, Plus Distribution Func.

- The Plus Distribution Function for some function $g(x)$ is defined by

$$[\theta(x)g(x)]_+ = \lim_{\beta\to 0}\frac{d}{dx}[\theta(x-\beta)G(x)] \quad \text{with} \quad G(x) = \int_{x_0}^{x} dx' g(x')$$

Ligeti, Stewart, Tackman (2008)
Stewart, Tackman, Waalewijn (2010)

$$P(k_\perp) = (2\pi)^2\delta^2(k_\perp) + P_>(k_\perp) - \delta^2(k_\perp)\int d^2q_\perp P_>(q_\perp)$$

- Extract $\delta^2(k_\perp)$ contribution from the second term to see cancelation.
- Introducing the scale $k_{\perp 0}$, we see the finite IR behavior:

$$P(k_\perp) = \delta^2(k_\perp)\left((2\pi)^2 - \int_{k_{\perp 0}}^{\infty} dq_\perp 2\pi q_\perp P_>(q_\perp)\right) + [P_>(k_\perp)]_+$$

Satisfy

$$\int_0^{k_{\perp 0}} dk_\perp [P_>(k_\perp)]_+ = 0$$

- Can interpret $[P_>(k_\perp)]_+$ as $P(k_\perp)$.