

Deconfinement and chiral transition in finite temperature lattice QCD

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Deconfinement and chiral symmetry restoration are expected to happen at some temperature

At what temperatures these transition(s) take place ?

Does the chiral and deconfinement transition happen at the same temperatures ?

Is there is an interplay between these two transitions?

How wide are these transitions if they are not true phase transitions ?

Goal : answer these questions using 1st principle LQCD with controlled discretization errors

Continuum limit : $a \rightarrow 0$, $N_\tau \rightarrow \infty$ $a = 1/(TN_\tau)$ Computational costs: $\sim a^{-8}$, $\sim N_\tau^8$

reduce discretization effects by using improved action => **Highly Improved Staggered Quark (HISQ) action**

Bazavov and P.P., arXiv:1005.1131, arXiv:1009.4914, arXiv:1012.1257,

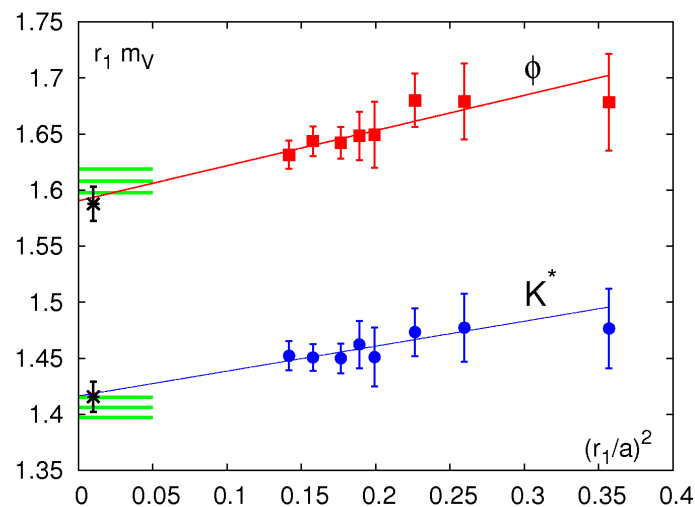
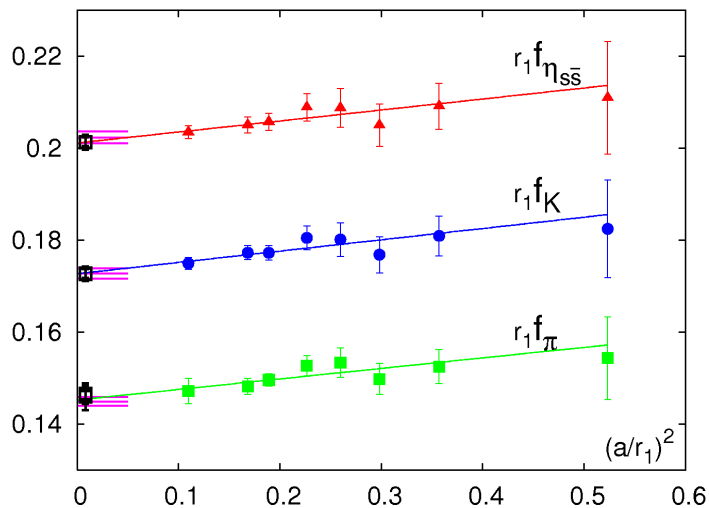
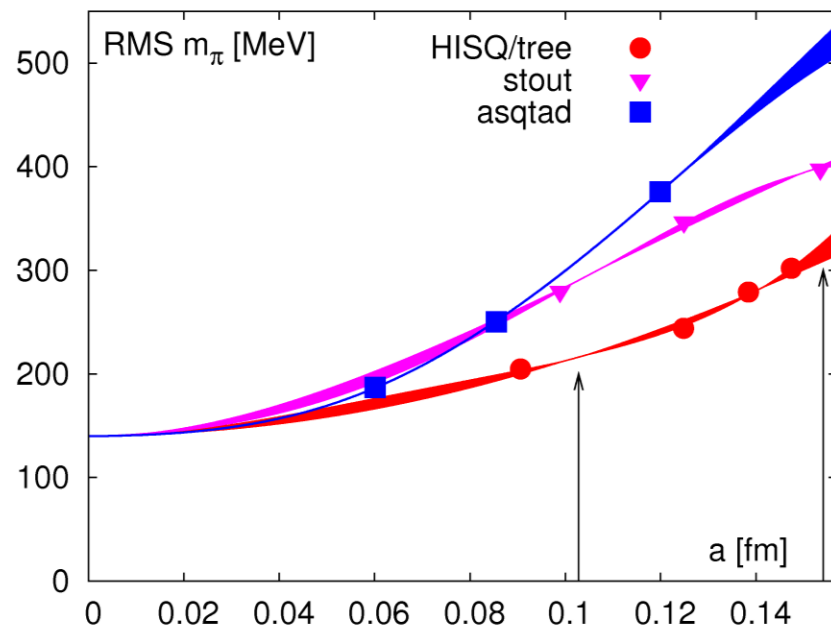
Improved staggered fermion actions

Calculations with **Highly Improved Staggered Quark (HISQ) action** using $N_\tau=6, 8$ and 12 lattices for, $m_l=m_s/20 \Rightarrow m_\pi \approx 160$ MeV + tree level improved gauge action \Rightarrow **HISQ/tree**

HISQ/tree action removes a^2 discretization effects as well as suppresses lattice artifact related to breaking of the flavor symmetry of the staggered fermions causing non-degenerate pion spectrum.

calculation with **asqtad** : $N_\tau=6, 8$ and 12 , $m_\pi \approx 160$ MeV

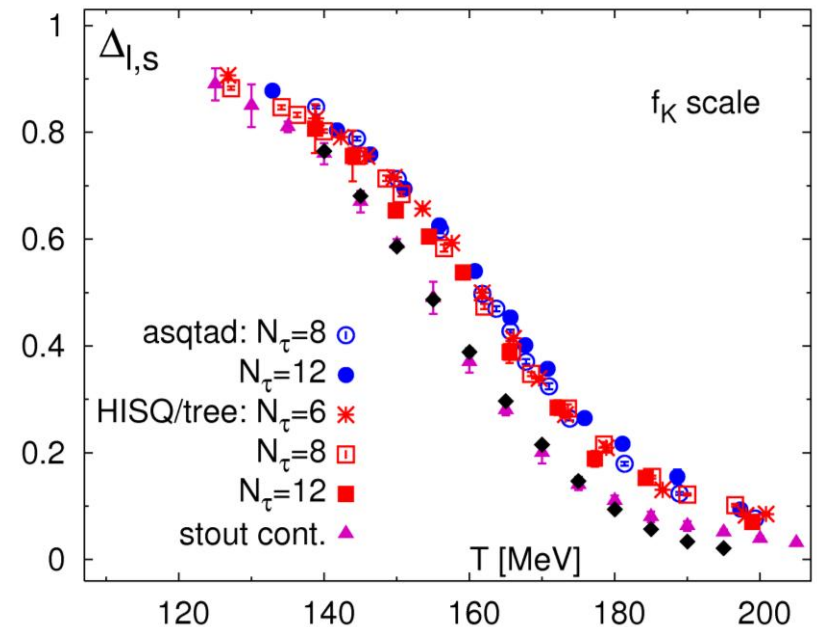
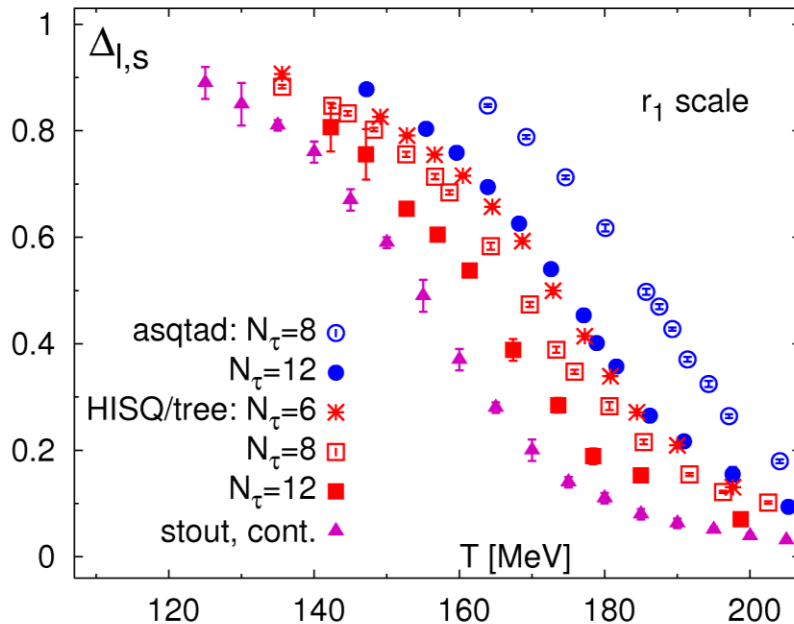
lattice spacing from $\left(\frac{dV_{q\bar{q}}(r)}{dr}\right)_{r=r_1} = 1.0$
 $r_1=0.3106(20)$ fm



The temperature dependence of chiral condensate

Chiral condensate needs multiplicative and additive renormalization for non-zero quark mass

$$\langle \bar{\psi}\psi \rangle_l \Rightarrow \Delta_{s,l}(T) = \frac{\langle \bar{\psi}\psi \rangle_{l,T} - \frac{m_l}{m_s} \langle \bar{\psi}\psi \rangle_{s,T}}{\langle \bar{\psi}\psi \rangle_{l,T=0} - \frac{m_l}{m_s} \langle \bar{\psi}\psi \rangle_{s,T=0}}$$



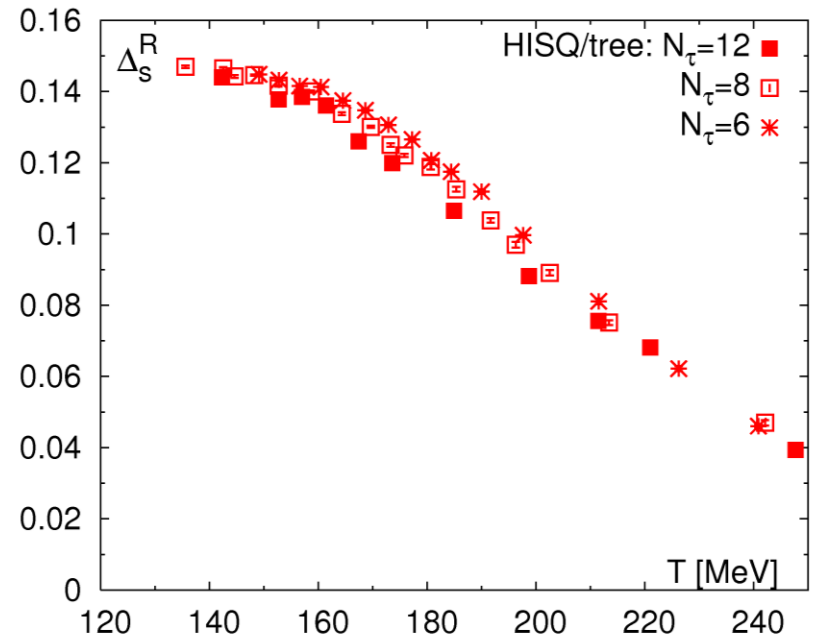
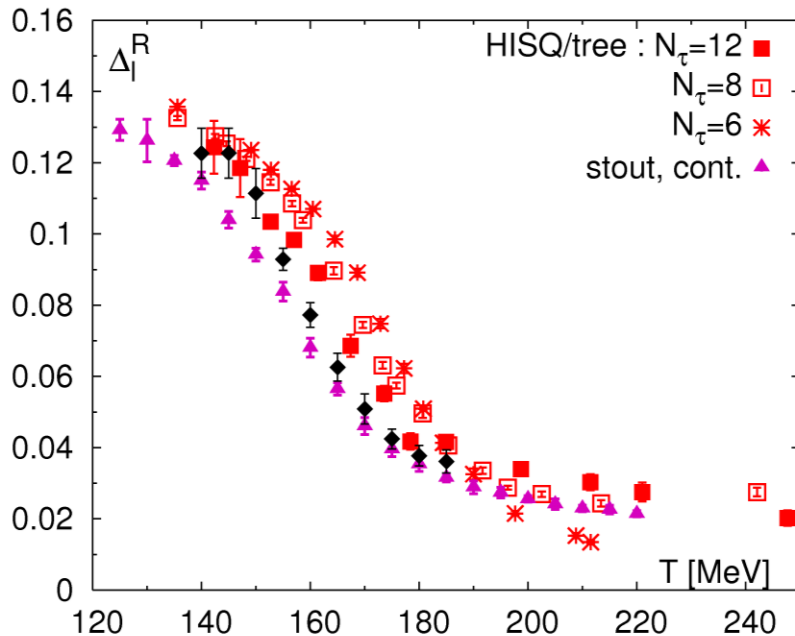
- Cut-off effects are significantly reduced when f_K is used to set the scale
- After quark mass interpolation based on $O(N)$ scaling the HISQ/tree results agree with the stout continuum result !

The temperature dependence of chiral condensate (cont'd)

Renormalized chiral condensate introduced by Budapest-Wuppertal collaboration

$$\langle \bar{\psi}\psi \rangle_q \Rightarrow \Delta_q^R(T) = m_{sr}^4 \left(\langle \bar{\psi}\psi \rangle_{q,T} - \langle \bar{\psi}\psi \rangle_{q,T=0} \right) + d, \quad q = l, s$$

with our choice $d=0.15$:



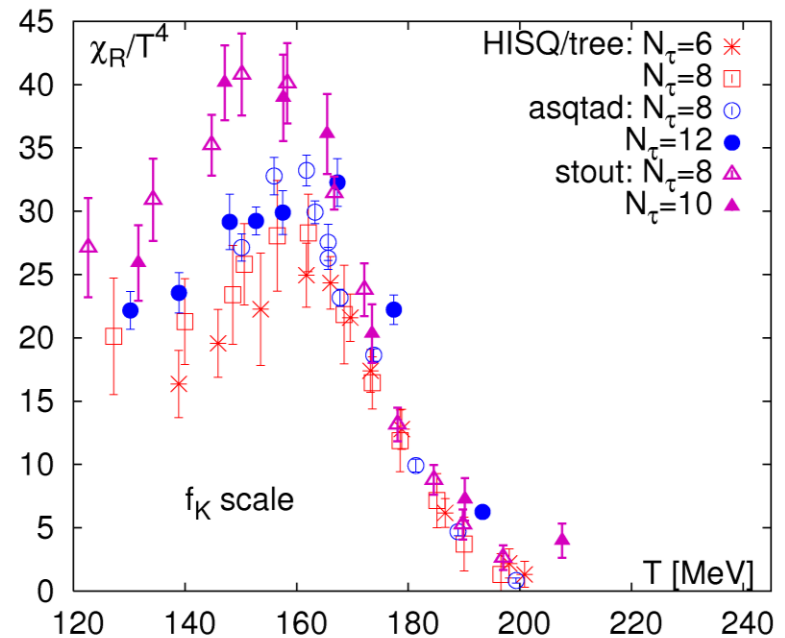
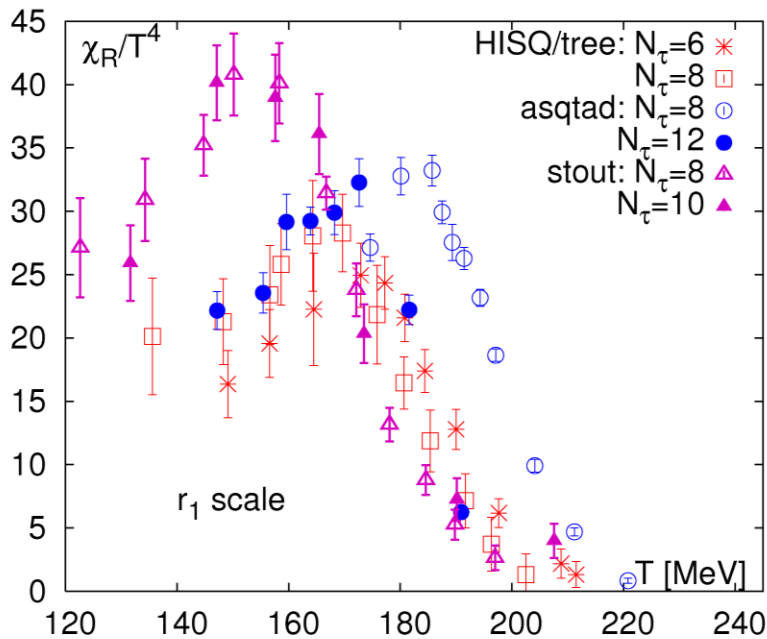
- after extrapolation to the continuum limit and physical quark mass HISQ/tree calculation agree with stout results !
- strange quark condensate does not show a rapid change at the chiral crossover => strange quark do not play a role in the chiral transition

The temperature dependence of chiral susceptibility

The renormalized 2 light flavor chiral susceptibility:

$$\chi_R(T) = \frac{m_l^2}{T^4} \left(\chi_{m,l}^{(n_f=2)}(T) - \chi_{m,l}^{(n_f=2)}(T=0) \right)$$

$$\chi_{m,l}^{(n_f=2)} = \frac{T}{V} \frac{\partial^2 \ln Z}{\partial m_l^2}$$



- Cut-off effects are significantly reduced when f_K is used to set the scale
- Differences in the peak region and at low temperatures between HISQ/tree and stout results (Aoki et al, [arXiv:hep-lat/0609068v2](https://arxiv.org/abs/hep-lat/0609068v2)) are due to difference in the quark mass

O(N) scaling and the chiral transition temperature

For sufficiently small m_l and in the vicinity of the transition temperature:

$$f(T, m_l) = -\frac{T}{V} \ln Z = f_{reg}(T, m_l) + f_s(t, h), \quad t = \frac{1}{t_0} \left(\frac{T - T_c^0}{T_c^0} + \kappa \frac{\mu_q^2}{T^2} \right), \quad H = \frac{m_l}{m_s}, \quad h = \frac{H}{h_0}$$

governed by universal $O(4)$ scaling
$$M = -\frac{\partial f_s(t, h)}{\partial H} = h^{1/\delta} f_G(z), \quad z = t/h^{1/\beta\delta}$$

T_c^0 is critical temperature in the mass-less limit, h_0 and t_0 are scale parameters

Pseudo-critical temperatures for non-zero quark mass are defined as peaks in the response functions (susceptibilities):

$$\chi_{m,l} = \frac{T}{V} \frac{\partial^2 \ln Z}{\partial m_l^2} \sim m_l^{1/\delta-1} \quad \chi_{t,l} = \frac{T}{V} \frac{\partial^2 \ln Z}{\partial m_l \partial t} \sim m_l^{\frac{\beta-1}{\beta\delta}} \quad \chi_{t,t} = \frac{T}{V} \frac{\partial^2 \ln Z}{\partial t^2} \sim |t|^{-\alpha}$$

\downarrow \downarrow \downarrow
 $T_{m,l}$ $T_{t,l}$ $T_{t,t} = T_c^0$

in the zero quark mass limit

$$\frac{\chi_{l,m}}{T^2} = \frac{T^2}{m_s^2} \left(\frac{1}{h_0} h^{1/\delta-1} f_\chi(z) + reg. \right)$$

universal scaling function has a peak at $z=z_p$



$$T_c(H) = T_{m,l} = T_c^0 + T_c^0 \frac{z_p}{z_0} H^{1/(\beta\delta)} + \dots$$

Caveat: staggered fermions O(2)

$m_l \rightarrow 0, a > 0,$

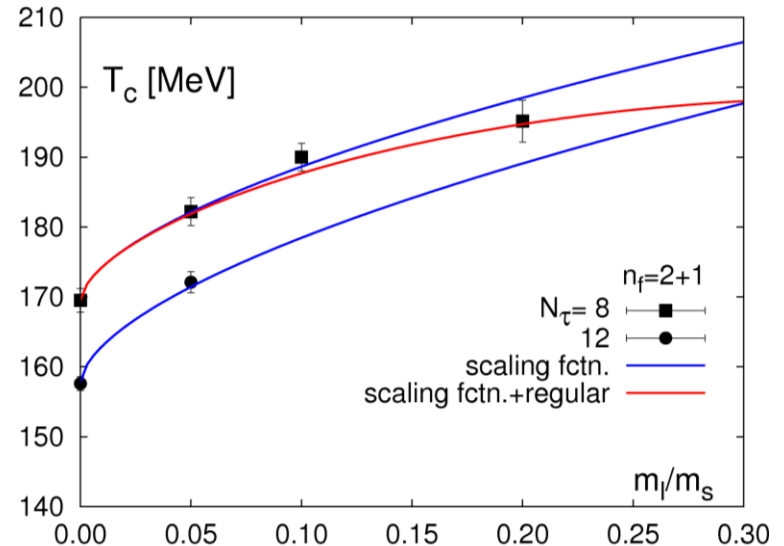
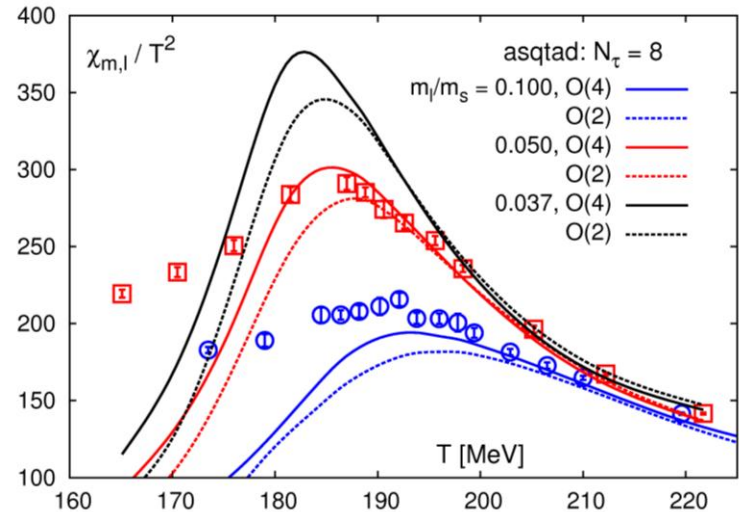
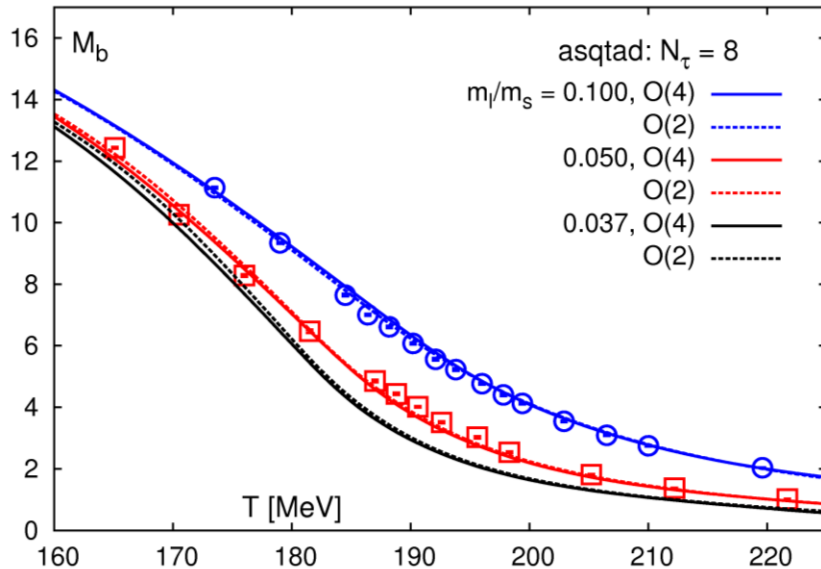
proper limit $a \rightarrow 0,$ before $m_l \rightarrow 0$

O(N) scaling and the lattice results : asqtad action

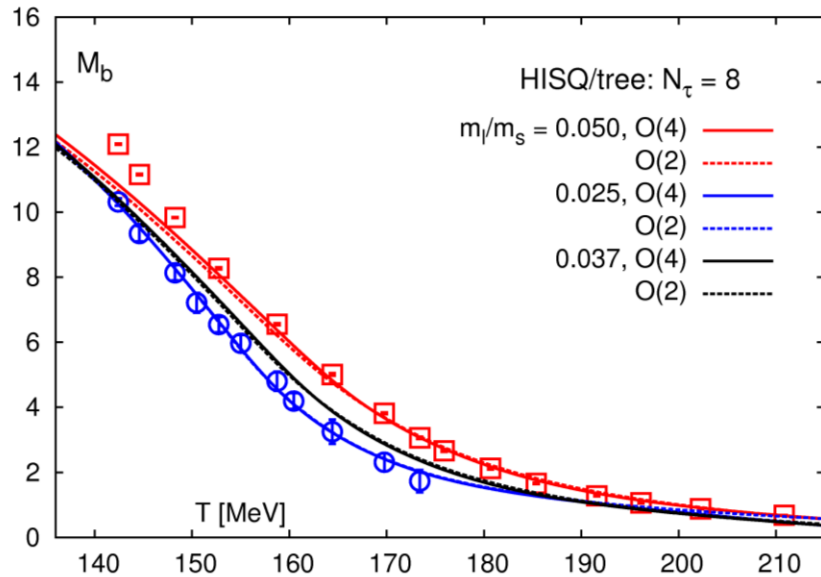
The notion of the transition temperature is only useful if it can be related to the critical temperature in the chiral limit : fit the lattice data on the chiral condensate with scaling form + simple Ansatz for the regular part

$$f_{reg}(T, H) = (a_t(T - T_c^0) + b_1)H$$

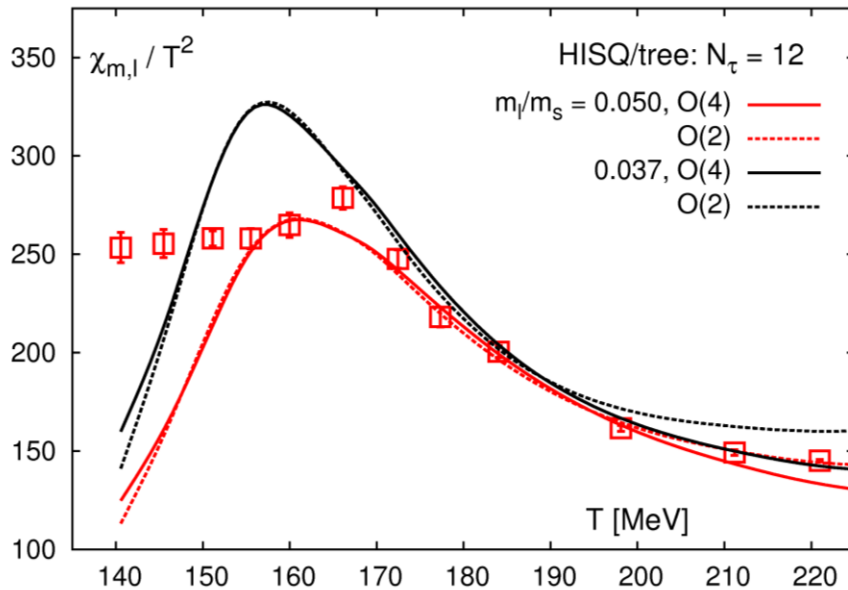
5 parameter fit : T_c^0 , t_0 , h_0 , a_t , b_1



O(N) scaling and the lattice results : HISQ/tree action

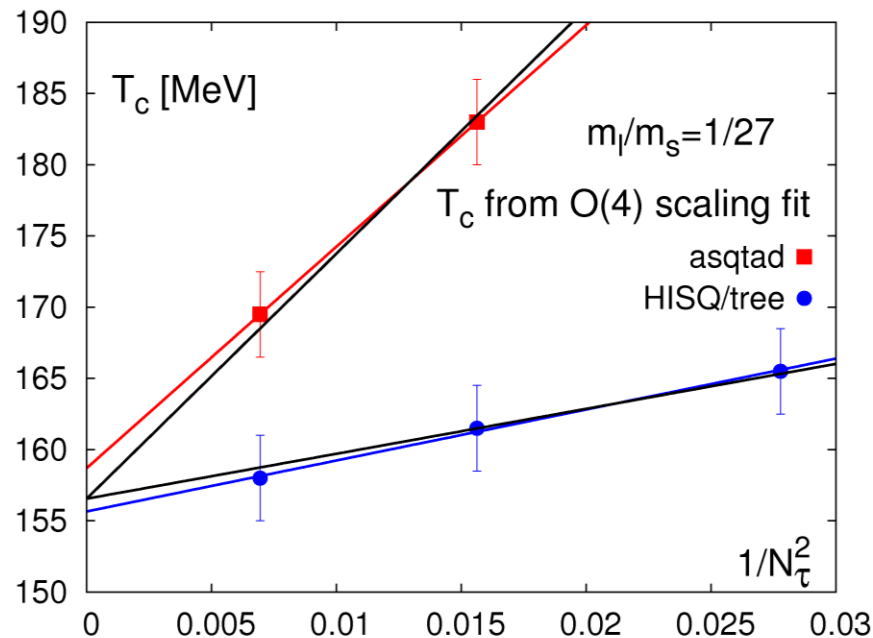
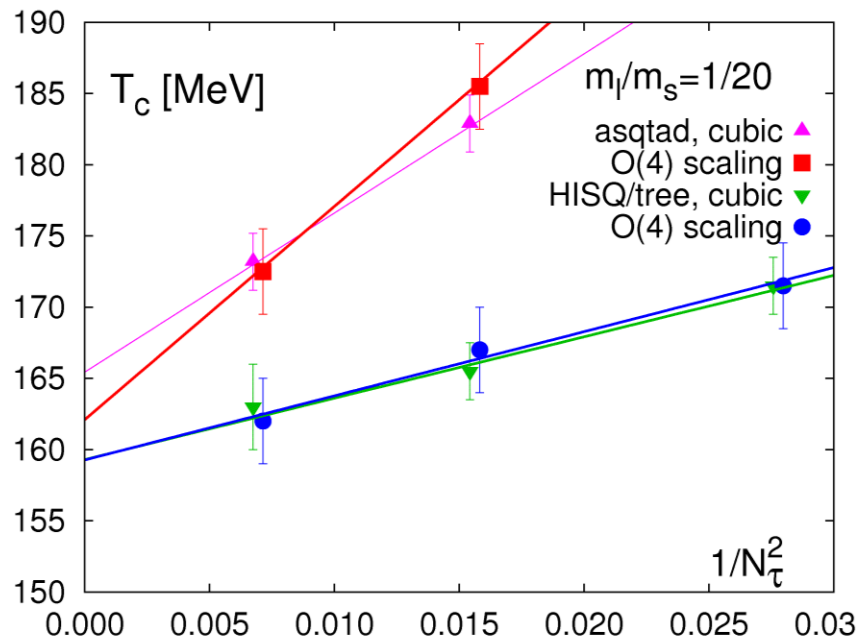


the analysis uses preliminary
RBC-Bielefeld data at $m_l = m_s/40$ for
 $N_\tau = 6$ and 8 lattices



The chiral transition temperature in the continuum limit

Separate and joint $1/N_\tau^2$ extrapolations for asqtad and HISQ/tree have been performed



$$T_c = (157 \pm 4(stat) \pm 3(ext) \pm 1(scale)) \text{MeV}$$

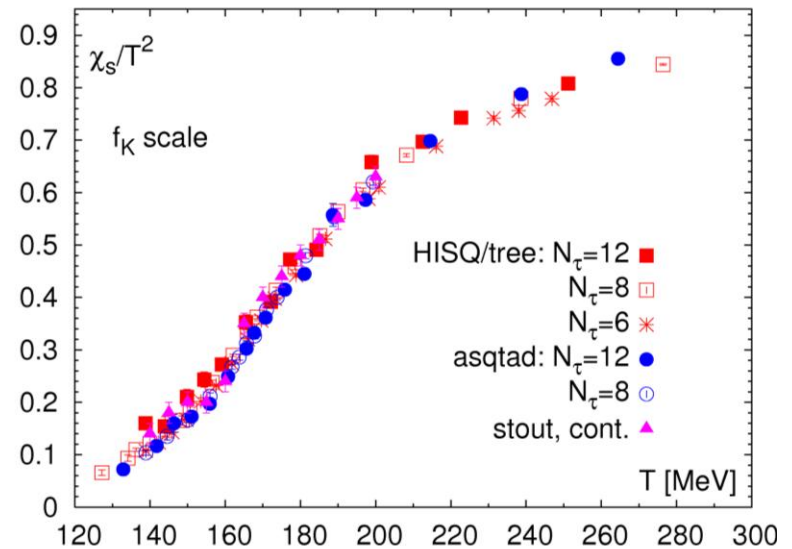
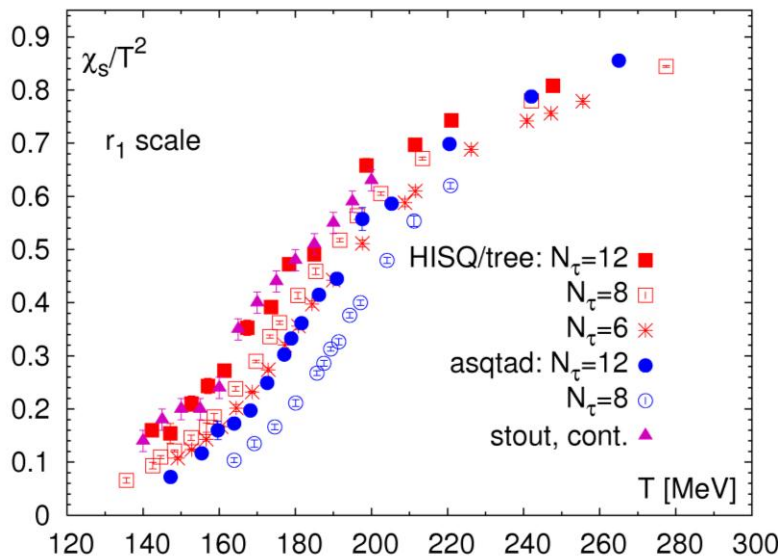
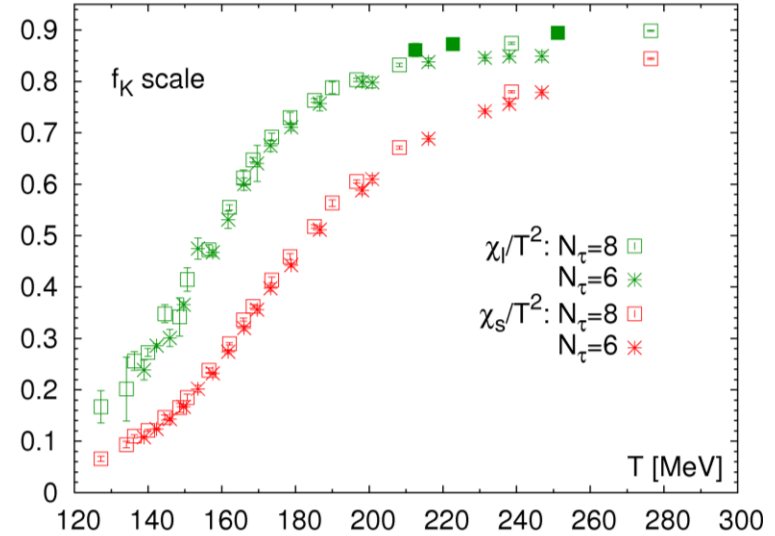
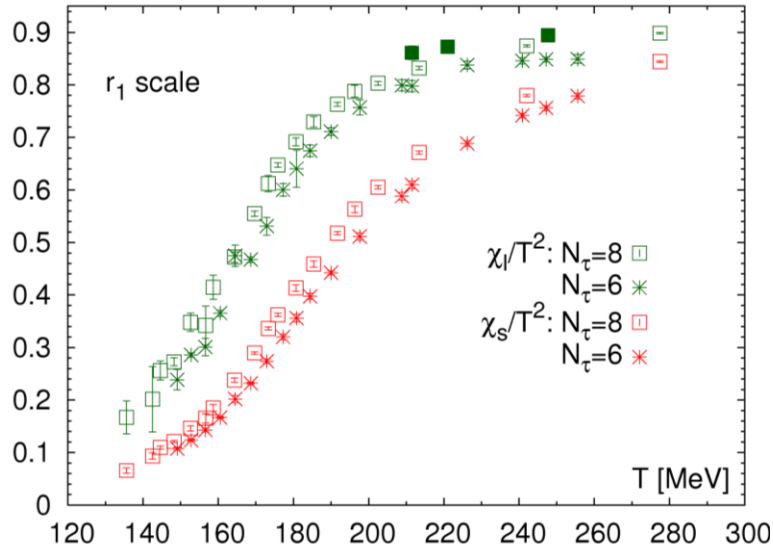
the error budget includes difference between asqtad and HISQ/tree, $O(2)$ vs. $O(4)$, possible a^4 corrections

in good agreement with the values obtained with stout action from chiral observables: $T_c = 147(2)(3)$ MeV, $157(3)(3)$ MeV, $155(3)(3)$ MeV, [Borsányi et al, arXiv:1005.3508](#)

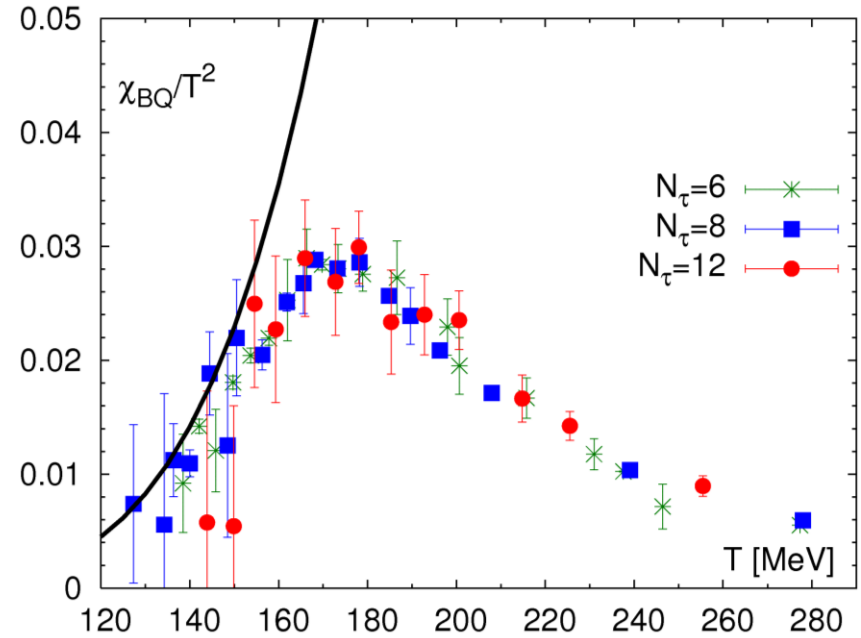
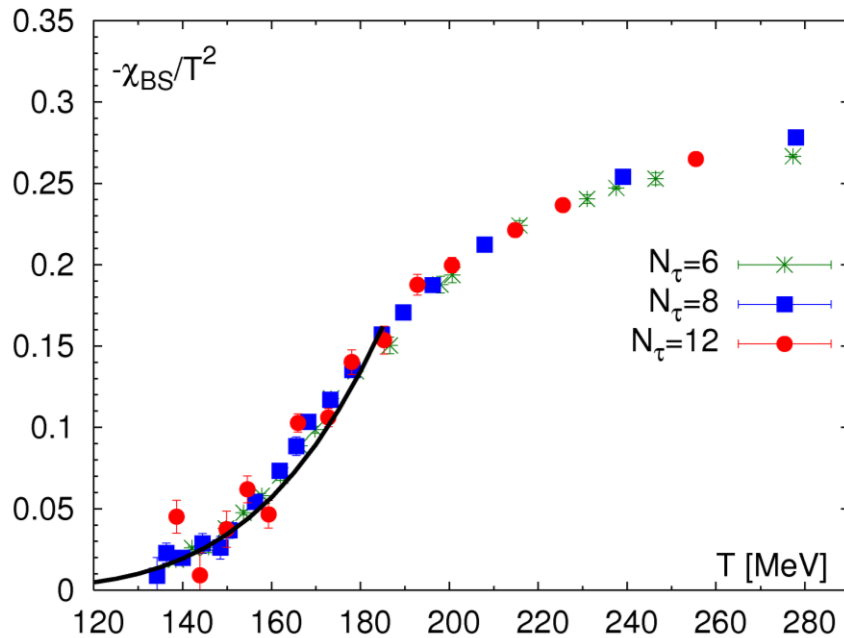
RBC-Bielefeld result $T_c = 192(7)(4)$ MeV ([Cheng et al, arXiv:hep-lat/0608013](#)) is overestimated due to large a^4 corrections that have been neglected in the extrapolations

Quark number susceptibility and deconfinement

$$\chi_q(T) = \frac{T}{V} \frac{\partial^2 \ln Z}{\partial \mu_q^2}, q = l, s \quad t = \frac{1}{t_0} \left(\frac{T - T_c}{T_c} + \kappa \left(\frac{\mu_q}{T} \right)^2 \right), \frac{\chi_q}{T^2} \sim A_{\pm} |t|^{1-\alpha} + \text{reg.}$$



Correlation of conserved charges and deconfinement



- At low temperatures correlations are well described by hadron resonance gas (HRG)
- At high T (>280 MeV) strangeness is carried by s-quarks \Rightarrow strangeness-baryon number correlation approaches $-1/3$
- All quarks (u,d and s) contribute to baryon-electric charge correlations at high T (>280 MeV) but the net charge is zero $\Rightarrow \chi_{BQ}=0$
- What are the carriers the quantum numbers for 190 MeV $< T < 280$ MeV

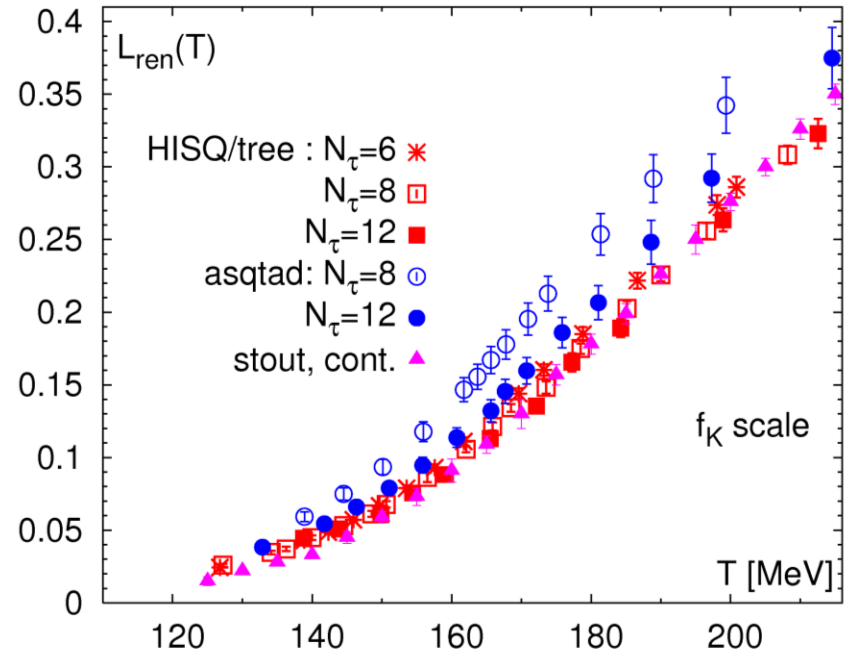
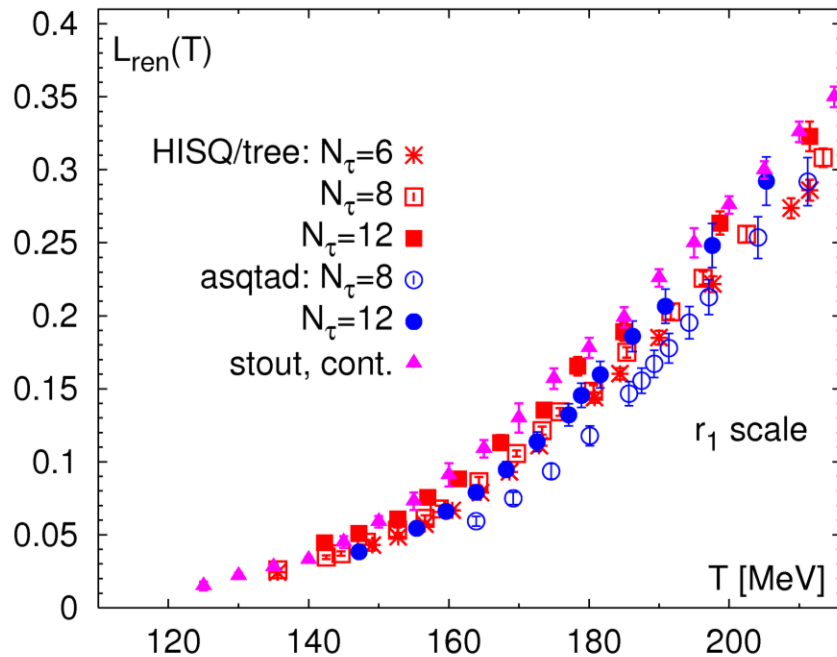
Polyakov loop and deconfinement

The Polyakov loop describes screening properties of the medium but is not related to critical behaviour in the chiral limit

$$L_{ren} = \exp(-F_Q(T)/T)$$

$$F_Q(T) \simeq \Lambda_{QCD} \quad \text{low } T$$

$$F_Q(T) \simeq -C_F \alpha_s m_D \quad \text{high } T$$



cutoff effects are smaller for HISQ/tree than for asqtad and further reduced when f_K is use to set the scale for HISQ/tree (but not asqtad)

and agree with stout continuum result [Borsányi et al, arXiv:1005.3508](https://arxiv.org/abs/1005.3508)

Lattice results on the trace of energy momentum tensor

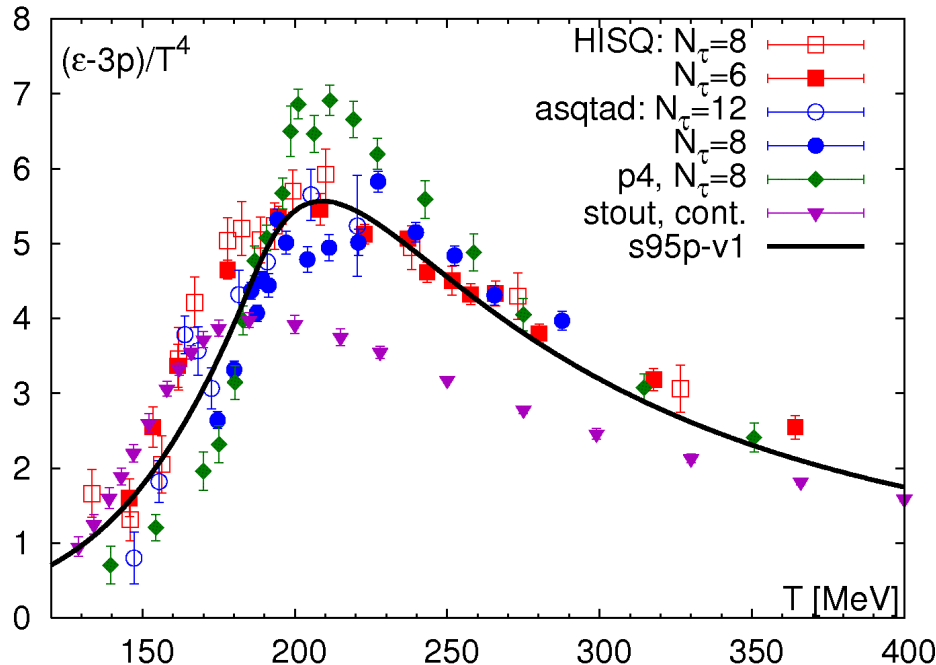
EoS is calculated from the trace anomaly $\Theta^{\mu\mu}(T) = \epsilon - 3p$ (integral method)

$$\frac{p(T)}{T^4} - \frac{p(T_0)}{T_0^4} = \int_{T_0}^T dT' \frac{\Theta^{\mu\mu}(T')}{T'^5},$$

For weakly interacting quarks and gluons:

$$\epsilon - 3p \sim \alpha_s^2(T) T^4 \ll T^4$$

Bazavov, P.P., arXiv:1005.1131
 Bazavov, P.P., arXiv:1012.1257



- HISQ results on the trace anomaly agree with previous HotQCD results for $T > 250 \text{ MeV}$

- A better agreement is achieved with HRG in the low T region with the HISQ action

- The HISQ results are compatible with asqtad calculations in the peak region

- HISQ results agree quite well with s95p-v1 parametrization of EoS that is based on HRG+LQCD and used in hydro models

Huovinen, P.P., NPA 837 (10) 26

Summary

- Lattice calculations with HISQ/tree action largely reduce cutoff effects in thermodynamic quantities and make possible to get close to the continuum limit with currently available computational resources.

- The chiral transition can be described in terms of universal $O(N)$ scaling as scaling violations are small

- The chiral transition defined through the chiral susceptibility is

$$T_c = (157 \pm 4(stat) \pm 3(ext) \pm 1(scale))MeV$$

- It is necessary to define the pseudo-critical temperature in terms mixed susceptibility and specific heat to establish the width of the chiral crossover

- Possible problems with T_c determination are due to lack of full chiral symmetry, e.g. $O(2)$ vs. $O(4)$ scaling, calculations chiral (e.g. DWF) will be necessary to crosscheck the above result.

- Deconfinement appears to be a rather smooth process with no well defined transition temperature. Depending on the observable it may happen at the chiral transition temperature or at higher temperatures