Antigravity Between Crunch and Bang in a Geodesically Complete Universe

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This talk is about solving cosmological equations analytically, no approximations. Found **all** the solutions for a specific model, and then discovered **model independent phenomena** that could not be noticed with approximate solutions. Among them is the notion of geodesic completeness, from which it follows there is a period of antigravity in the history of the universe. Also new general lessons for cosmology.

1) I.B. and S.H. Chen, **1004.0752**
2) I.B., and S.H. Chen and Neil Turok, **1105.3606**
3) I.B. + Chen + Turok + Steinhardt, to appear (several papers)

Cosmology with a scalar coupled to gravity

$$S=\int d^4x\sqrt{-g}\left\{\begin{array}{c}\frac{1}{2\kappa^2}R\left(g\right)-\frac{1}{2}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma-V\left(\sigma\right)\\ +radiation+matter\end{array}\right\}$$

Including relativistic matter and curvature

FRW

$$ds_E^2=-dt^2+a_E^2\left(t\right)ds_3^2=a^2\left(\tau\right)\left(-d\tau^2+ds_3^2\right),\qquad dt=a\left(\tau\right)d\tau$$

$$ds_3^2=\frac{dr^2}{1-kr^2/r_0^2}+r^2\left(d\theta^2+\sin^2\theta d\phi^2\right);\;k=0,\pm1.\qquad K=\frac{k}{r_0^2}$$

Friedmann equations

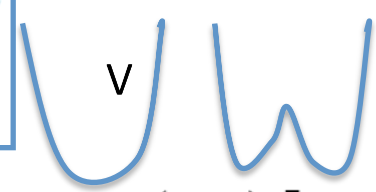
$$\frac{\dot a_E^2}{a_E^4}=\frac{\kappa^2}{3}\left[\frac{\dot\sigma^2}{2a_E^2}+V\left(\sigma\right)\right]-\frac{K}{a_E^2}+\frac{\rho_0}{a_E^4}$$

$$\frac{\ddot a_E}{a_E^3}-\frac{\dot a_E^2}{a_E^4}=-\frac{\kappa^2}{3}\left[\frac{\dot\sigma^2}{a_E^2}-V\left(\sigma\right)\right]-\frac{\rho_0}{3a_E^4}$$

$$\frac{\ddot\sigma}{a_E^2}+2\frac{\dot a_E\dot\sigma}{a_E^3}+V'\left(\sigma\right)=0.$$

6 parameters
4=b,c,K,ρ_0
2=(4-2) initial values

Also anisotropic metrics :
Kasner, Bianchi IX.
Two more fields in metric important only near Big Bang



Analytically solved with this V:
found ALL solutions

$$V\left(\sigma\right)=\left(\frac{\sqrt{6}}{\kappa}\right)^4\left[b\cosh^4\left(\frac{\kappa\sigma}{\sqrt{6}}\right)+c\sinh^4\left(\frac{\kappa\sigma}{\sqrt{6}}\right)\right]$$

Generic solution is geodesically incomplete in Einstein gravity.

There is a subset of geodesically complete solutions only with conditions on initial values and parameters of the model.

For geodesic compleness: a slight extension of Einstein gravity (with gauge degrees of freedom)

Local scaling symmetry (Weyl): allows only conformally coupled scalars (generalization possible)

(Plus gauge bosons, fermions , more conformal scalars , in complete Weyl invariant theory.)

$(\phi,s)\rightarrow(\phi,s)e^{\lambda(x)}, \quad g_{\mu\nu}\rightarrow g_{\mu\nu}e^{-2\lambda(x)}$

$$S=\int d^4x\sqrt{-g}\left(\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi-\frac{1}{2}g^{\mu\nu}\partial_\mu s\partial_\nu s+\frac{1}{12}\left(\phi^2-s^2\right)R\left(g\right)-\phi^4 f\left(\frac{s}{\phi}\right)\right)$$

no ghosts — gravitational parameter

A prediction of 2T-gravity in 4+2 dims.

Fundamental approach: Gauge symmetry in phase space
I.B. 0804.1585, I.B.+Chen 0811.2510

Also motivated by colliding branes scenario.

Khury + Seiberg +Steinhardt + Turok
McFadden + Turok 0409122

Gauge symmetry leftover from general coordinate transformations in extra 1+1 dims.

Weyl symmetry can be gauge fixed in several forms.

Einstein gauge $\frac{1}{12}\left(\phi_E^2-s_E^2\right)=\frac{1}{2\kappa^2}$ $\qquad \phi_E\left(x\right), s_E\left(x\right), g_E^{\mu\nu}\left(x\right)$

$$\phi_E\left(x\right)=\pm\frac{\sqrt{6}}{\kappa}\cosh\left(\frac{\kappa\sigma\left(x\right)}{\sqrt{6}}\right), \quad s_E\left(x\right)=\frac{\sqrt{6}}{\kappa}\sinh\left(\frac{\kappa\sigma\left(x\right)}{\sqrt{6}}\right)$$

$$S=\int d^4x\sqrt{-g}\left\{\frac{1}{2\kappa^2}R\left(g_E\right)-\frac{1}{2}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma-V\left(\sigma\right)\right\}$$

This is not the whole story: Einstein gauge is valid only in a patch of spacetime, when the *gauge invariant* quantity $\left[1-s^2\left(x^\mu\right)/\phi^2\left(x^\mu\right)\right]$ is positive

Can dynamics push this factor to negative values? ANTIGRAVITY in some regions of spacetime?

γ-gauge

$$\phi_\gamma, s_\gamma, g_\gamma^{\mu\nu}$$

Conformal factor of metric =1 for any metric. →

$$a_\gamma = 1$$

For all t,x dependence.

case of only time dependent fields

$$ds_\gamma^2 = -d\tau^2 + \frac{dr^2}{1 - kr^2/r_0^2} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right) \qquad \text{FRW}_\gamma$$

$$R(g_\gamma) = 6K, \text{ with } K \equiv \frac{k}{r_0^2}, \; k = 0, \pm 1.$$

$$L = \frac{1}{2}\left(-\dot{\phi}_\gamma^2 + \dot{s}_\gamma^2\right) - \frac{K}{2}\left(-\phi_\gamma^2 + s_\gamma^2\right) - \phi^4 f\left(\frac{s}{\phi}\right)$$

Nothing singular in γ-gauge

Plus the energy constraint: H=0 which compensates for the ghost.

This is equivalent to the 00 Einstein eq. $G_{00}=T_{00}$

connection between the γ-gauge and the Einstein gauge

BCRT transform
Bars Chen
Steinhardt Turok

$$a_E^2 = \frac{\kappa^2}{6}\left(\phi_\gamma^2 - s_\gamma^2\right) \qquad \sigma = \frac{\sqrt{6}}{\kappa}\frac{1}{2}\ln\left(\left|\frac{\phi_\gamma + s_\gamma}{\phi_\gamma - s_\gamma}\right|\right)$$

Positive region

$$\phi^2\left(1 - s^2/\phi^2\right)$$

BB singularity at a_E=0 in E-gauge : gauge invariant factor vanishes in γ-gauge, or any gauge!!

Analytic solutions – all of them!!

$$L = \frac{1}{2}\left(-\dot{\phi}_\gamma^2 + \dot{s}_\gamma^2\right) - \frac{K}{2}\left(-\phi_\gamma^2 + s_\gamma^2\right) - \phi^4 f\left(\frac{s}{\phi}\right)$$

Special case: $\phi^4 f(s/\phi) = b\phi^4 + cs^4$

BCST transform Friedmann equations become :

$$0 = \ddot{\phi}_\gamma - 4b\phi_\gamma^3 + K\phi_\gamma,$$

$$0 = \ddot{s}_\gamma + 4cs_\gamma^3 + Ks_\gamma,$$

$$0 = -\left(\frac{1}{2}\dot{\phi}_\gamma^2 - b\phi_\gamma^4 + \frac{1}{2}K\phi_\gamma^2\right) + \left(\frac{1}{2}\dot{s}_\gamma^2 + cs_\gamma^4 + \frac{1}{2}Ks_\gamma^2\right) + \rho_0$$

Completely decoupled equations, except for the zero energy condition. Solutions are **Jacobi elliptic functions,** with various boundary conditions.

First integral

$$\frac{1}{2}\dot{\phi}_\gamma^2 - b\phi_\gamma^4 + \frac{K}{2}\phi_\gamma^2 = E_\phi; \quad \frac{1}{2}\dot{s}_\gamma^2 + cs_\gamma^4 + \frac{K}{2}s_\gamma^2 = E_s \quad E_s \equiv E, \; E_\phi = E + \rho_0$$

Particle in a potential problem, intuitively solved by looking at the plot of the potential.

$$H(\phi) = \tfrac{1}{2}\dot{\phi}^2 + V(\phi) \qquad V(\phi) = \tfrac{1}{2}K\phi^2 - b\phi^4;$$

$$H(s) = \tfrac{1}{2}\dot{s}^2 + V(s) \qquad V(s) = \tfrac{1}{2}K\phi^2 + cs^4$$

Generic solution has 6 parameters $(b, c, K, \rho_0, E, \phi(\tau_0))$ with $s_\gamma(\tau_0) = 0$

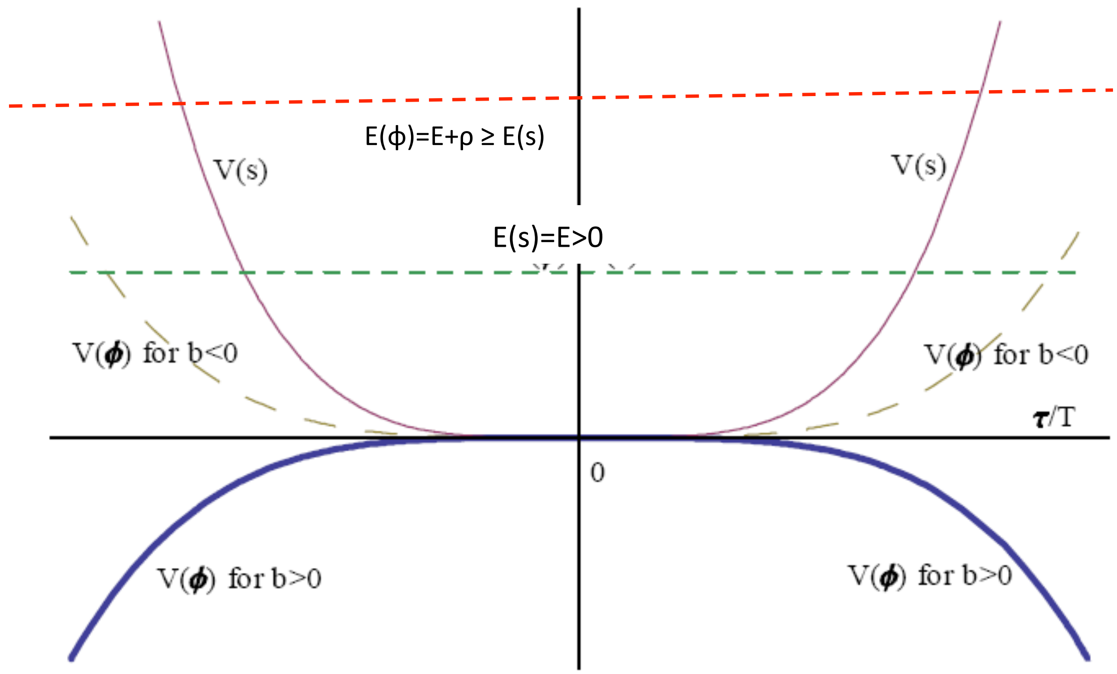
$$V(s) = \frac{1}{2}K\phi^2 + cs^4 \qquad V(\phi) = \frac{1}{2}K\phi^2 - b\phi^4.$$

K=0 case quartic potentials

$\phi(\tau)$, $s(\tau)$
=
A x F[sn(z|m), cn(z|m), dn(z|m)]

$z=(\tau-\tau_0)/T$

A,m,T depend on b,c,K,ρ_0,E



$\phi(\tau)$,$s(\tau)$ perform independent oscilations
For generic initial conditions, the sign of $(\phi^2\text{-}s^2)(\tau)$ changes over time.
Generic solution is geodesically incomplete in the Einstein gauge.
Geodesically complete with the natural extension in ϕ,s space.

There are special solutions that are geodesically complete in the restricted Einstein frame, but must constrain parameter space.

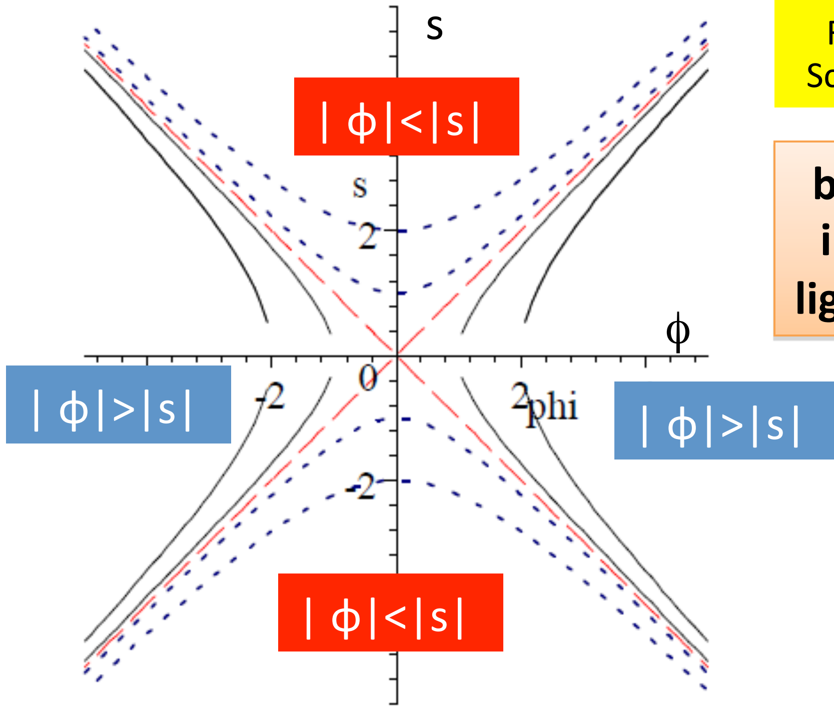
Geodesically complete larger space: ϕ_γ, s_γ plane

Generic solution:
$\phi(\tau), s(\tau)$ periodic

parametric plot using Mathematica

A smooth curve that spans the various quadrants (not shown)

Closed curve if periods relatively quantized.



Recall Kruskal-Szekeres versus Schwarzchild; now in field space.

big bangs or big crunches in spacetime ←→ at the lightcone in ϕ,s field space.

Generic solution is a cyclic universe with antigravity stuck between crunch and bang! Probably true for all V(σ).

No signature change

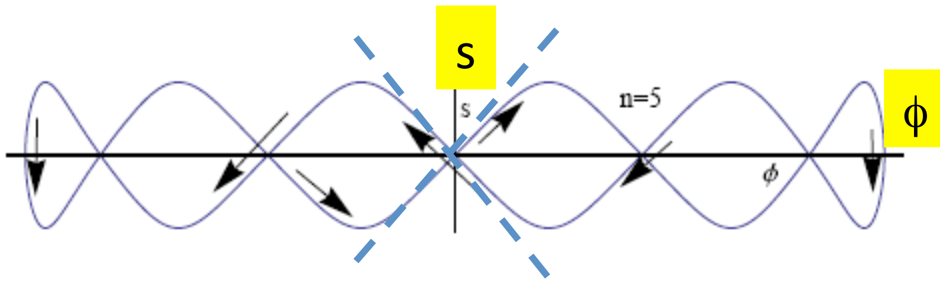
$$a_E^2 = |z| \quad , \; z \equiv \frac{\kappa^2}{6}\left(\phi_\gamma^2 - s_\gamma^2\right), \quad \sigma = \frac{\sqrt{6}}{\kappa}\frac{1}{2}\ln\left(\left|\frac{\phi_\gamma + s_\gamma}{\phi_\gamma - s_\gamma}\right|\right)$$

BCST Transform E-gauge to γ-gauge

$$\phi_\gamma = \pm\frac{\sqrt{6}}{\kappa}\sqrt{|z|}\cosh\left(\frac{\kappa\sigma}{\sqrt{6}}\right), \quad s_\gamma = \pm\frac{\sqrt{6}}{\kappa}\sqrt{|z|}\sinh\left(\frac{\kappa\sigma}{\sqrt{6}}\right).$$

Geodesically complete solutions in the Einstein gauge, without antigravity

b<0



Example n=5 → 5 nodes in picture

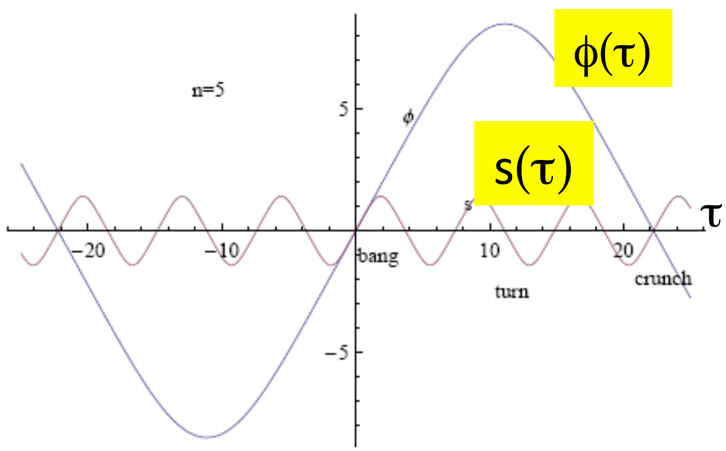
Conditions on 6 parameter space:

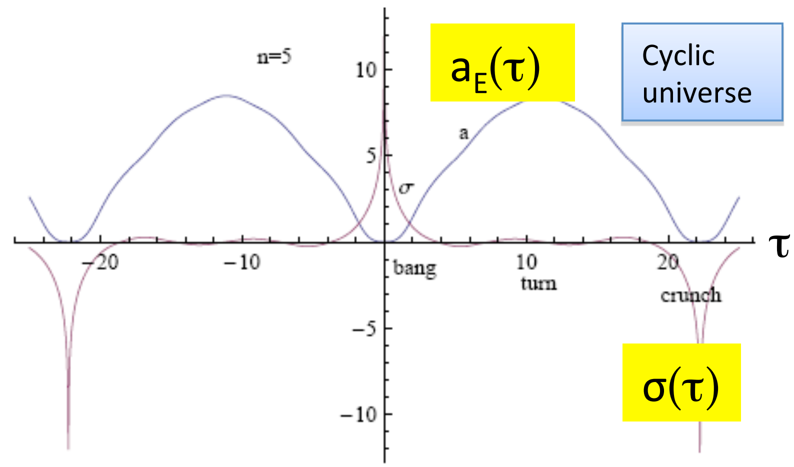
(1) Synchronized initial values

$\phi(0)=s(0)=0$

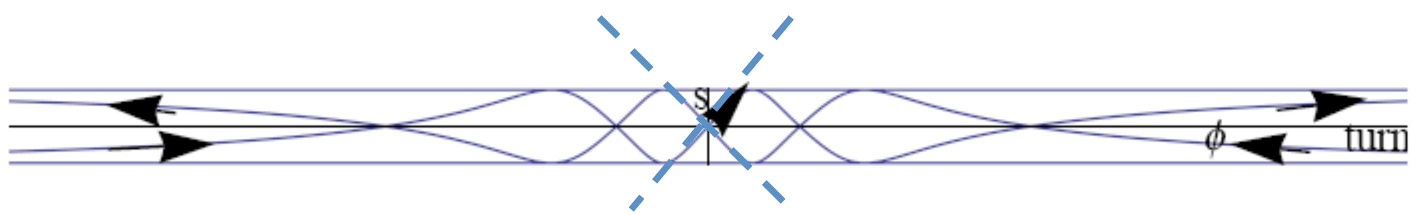
(2) Relative quantization of periods

$P_{\phi}(\text{5 parameters})=nP_{s}(\text{5 parameters})$





b>0



Universe expands to infinite size, turnaround at infinite size

Anisotropy

$$ds^2 = a^2(\tau)\left(-d\tau^2 + ds_3^2\right)$$ If K≠0, ds_3=Bianchi IX (Misner)

K→0 $$\left(ds_3^2\right)_{Kasner} = e^{2\alpha_1(\tau)+2\sqrt{3}\alpha_2(\tau)}dx^2 + e^{2\alpha_1(\tau)-2\sqrt{3}\alpha_2(\tau)}dy^2 + e^{-4\alpha_1(\tau)}dz^2$$

In Friedmann equations, 2 more fields $\alpha_1(\tau), \alpha_2(\tau)$, just like the $\sigma(\tau)$

Friedman Eqs: kinetic terms for α_1, α_2 just like σ, plus anisotropy potential if K≠0

$$V(\alpha_1, \alpha_2) = \frac{K}{\kappa^2 a^2}\left(e^{-8\alpha_1} + 4e^{\alpha_1}\sinh^2\left(2\sqrt{3}\alpha_2\right) - 4e^{-2\alpha_1}\cosh\left(2\sqrt{3}\alpha_2\right) + 3\right)$$

$p_1=(a_E)^2\partial_\tau\alpha_1$ etc.

Free scalars if K=0, then canonical conjugate momenta p_1,p_2 are constants of motion.
Near singularity, kinetic terms dominate, so all potentials, including V(σ) negligible.
Then σ momentum **q** is also conserved near the singularity.
For a range of q,p_1,p_2 mixmaster universe is avoided when σ is present (agree with BKL, etc.)

Without potentials can find all solutions analytically
for any (initial) anisotropy momenta $\mathbf{p_1}$,$\mathbf{p_2}$; or σ momentum **q**
including the parameters K,ρ_0.

Antigravity Loop (K=0 case)

q= σ momentum, $q=(a_E)^2\partial_\tau\sigma$
p= anisotropy momentum
$p_1=(a_E)^2\partial_\tau\alpha_1$ etc.

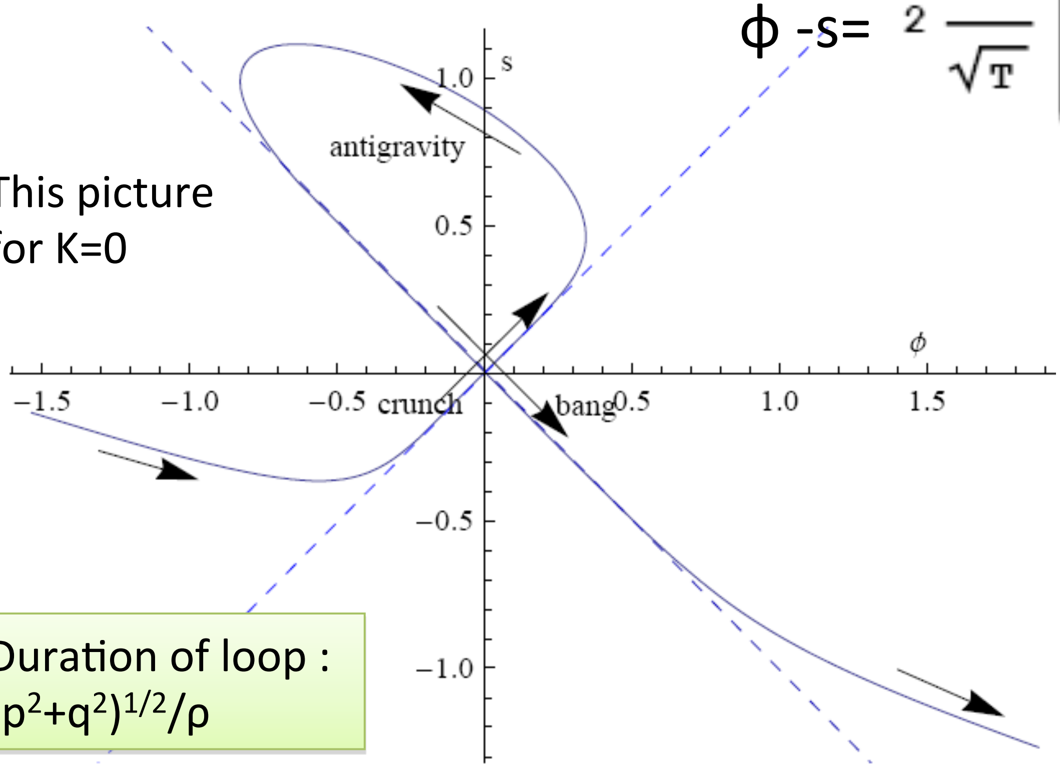
K=0: p_1,p_2 are conserved throughout motion.

q changes during the loop because of $V(\sigma)$.
If small loop, ≈no change.

$$\phi+s=\sqrt{T}\left(\sqrt{p^2+q^2}+\rho\,\tau\right)\left(\frac{\left(\frac{\tau}{T}\right)^2}{\left(\rho*\tau+\sqrt{p^2+q^2}\right)^2}\right)^{\frac{1}{4}\left(1+\frac{q}{\sqrt{p^2+q^2}}\right)}$$

$$\phi-s=2\,\frac{\tau}{\sqrt{T}}\left(\frac{\left(\frac{\tau}{T}\right)^2}{\left(\rho*\tau+\sqrt{p^2+q^2}\right)^2}\right)^{-\frac{1}{4}\left(1+\frac{q}{\sqrt{p^2+q^2}}\right)}$$

This picture for K=0



Duration of loop : $(p^2+q^2)^{1/2}/\rho$

ATTRACTOR MECHANISM !!
if p_1 or p_2 is not 0 :
both ϕ,s →0 at the big bang or crunch singularity,
FOR ALL INITIAL CONDITIONS.

ALWAYS
a period of antigravity sandwiched between crunch and bang

What have we learned?

- Found new techniques to solve cosmological equations **analytically**.
  Found **all** solutions for several special potentials V(σ).
  Several model independent general results: geodesic completeness,
  and an **attractor mechanism to the origin, ϕ,s→0, for any initial values.**

- Antigravity is very hard to avoid. Anisotropy + radiation + KE requires it.

- Studied Wheeler-deWitt equation (quantum) for the same system, can solve some cases exactly, others semi-classically. Same conclusions.

- Will this new insight survive the effects of a full quantum theory (very likely yes). Should be studied in string theory.

- These phenomena are direct predictions of 2T-physics in 4+2 dimensions.

- Open: What are the observational effects today of a past antigravity period? This is an important project. Study of small fluctuations and fitting to current observations of the CMB (under investigation).