Conformal hydrodynamics in Minkowski and de-Sitter spacetimes

Amos Yarom

With S. Gubser

A boost invariant solution

$$T^{\mu\nu} = \frac{1}{3}\epsilon\left(4u^{\mu}u^{\nu} + \eta^{\mu\nu}\right)$$

$$\partial_{\mu}T^{\mu\nu} = 0$$

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$$\epsilon = \frac{\hat{\epsilon}_0}{\tau^{4/3}} \frac{(2q)^{8/3}}{\left(1 + 2q^2(\tau^2 + x_\perp^2) + q^4(\tau^2 - x_\perp^2)^2\right)^{4/3}}$$

$$-\frac{u_\perp}{u_\tau} = \frac{2q^2 \tau x_\perp}{1 + q^2\tau^2 + q^2 x_\perp^2}$$

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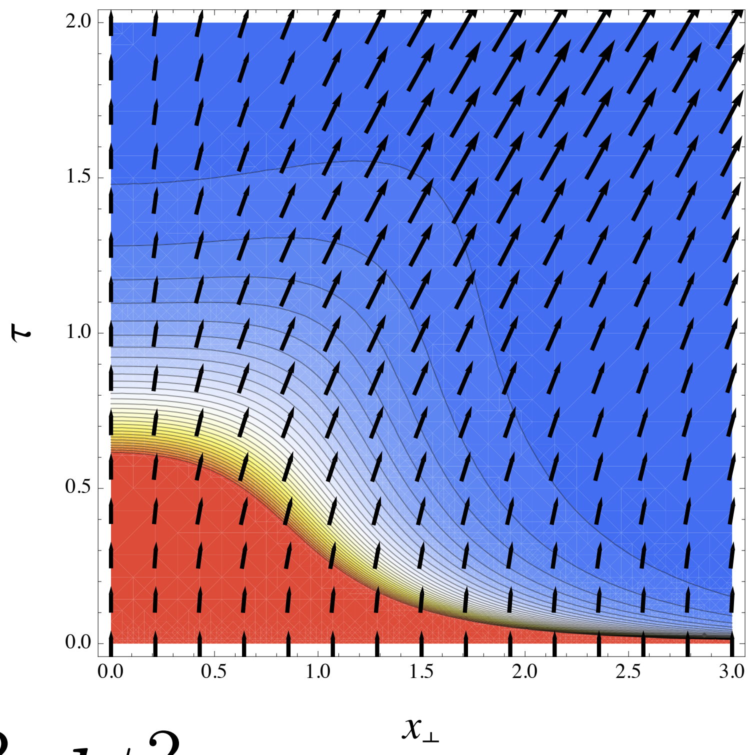

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Hydrodynamics

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u^μ

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$$u^\mu$$

$$(u^\mu u_\mu = -1)$$

Hydrodynamics

u^μ ϵ

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$u^\mu \qquad \epsilon(T)$

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$$T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu \;+P\eta^{\mu\nu} \;-\eta\sigma^{\mu\nu} \;-\zeta\Delta^{\mu\nu}\partial_\alpha u^\alpha$$

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Hydrodynamics

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$$\sigma_{\mu\nu} = \partial_{\langle\mu}u_{\nu\rangle}$$

Hydrodynamics

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$$\Delta^{\mu\nu} = \eta^{\mu\nu} + u^\mu u^\nu$$

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inviscid

Hydrodynamics

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$$T^{\mu\nu} = \underbrace{(\epsilon + P)u^\mu u^\nu + P\eta^{\mu\nu}}_{\text{inviscid}} \underbrace{- \eta\sigma^{\mu\nu} - \zeta\Delta^{\mu\nu}\partial_\alpha u^\alpha}_{\text{viscous}}$$

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$$T^\mu_\mu = 0 \quad \Rightarrow \quad P = \epsilon/3 \qquad \zeta = 0$$

Conformal hydrodynamics

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$$\partial_\mu T^{\mu\nu} = 0$$

$$T^{\mu\nu} = \frac{1}{3}\epsilon(4u^\mu u^\nu + \eta^{\mu\nu}) - \eta\sigma^{\mu\nu}$$

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Solve:

$$\epsilon = \epsilon_0$$

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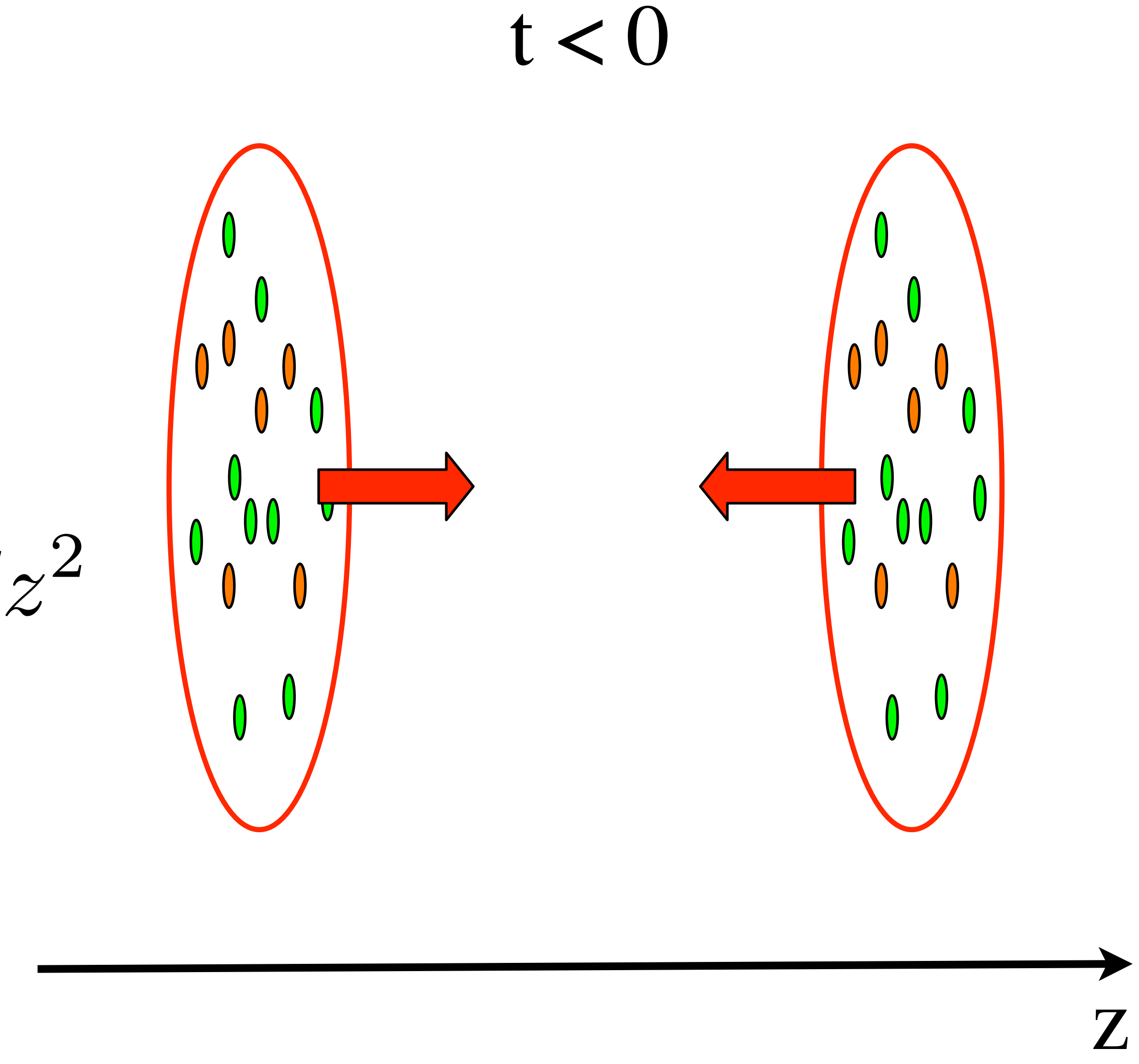

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$$\tau = \sqrt{z^2 - t^2}$$

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Solve:

$$\epsilon = \epsilon_0 \left(\frac{\tau_0}{\tau}\right)^{4/3}$$

Conformal hydrodynamics

$$D_\mu T^{\mu\nu} = 0$$

$$T^{\mu\nu} = \frac{1}{3}\epsilon(4u^\mu u^\nu + g^{\mu\nu})$$

Strategy:

- Find a useful isometry
- Use the right coordinate system
- Construct an ansatz respecting the symmetry
- Solve

Extending Bjorken flow

$$D_\mu T^{\mu\nu} = 0$$

$$T^{\mu\nu} = \frac{1}{3}\epsilon(4u^\mu u^\nu + g^{\mu\nu})$$

$$ds^2 = -d\tau^2 + dx_\perp^2 + x_\perp^2 d\phi^2 + \tau^2 d\eta^2$$

$$u^\mu = \gamma(1, \beta(x_\perp), 0, 0) \qquad \epsilon(\tau, x_\perp)$$

Extending Bjorken flow

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$$u^\mu \rightarrow \Omega^{-1} u^\mu = \hat{u}^\mu$$

$$\epsilon \rightarrow \Omega^{-4} \epsilon = \hat{\epsilon}$$

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de Sitter space

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de Sitter space

line

Extending Bjorken flow

$dS_3 \times \mathbf{R}$

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$$\begin{aligned}\widehat{ds^2} &= \frac{1}{\tau^2}\left(-d\tau^2 + dx_\perp^2 + x_\perp^2 d\phi^2\right) + d\eta^2 \\ &= -d\mathcal{T}^2 + \cosh^2\mathcal{T}(d\theta^2 + \sin^2\theta d\phi^2) + d\eta^2\end{aligned}$$

Extending Bjorken flow

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Choose a useful coordinate system:

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Use SO(1,1)×SO(3)×Z_2 ansatz:

$$\hat{u}^\mu = (1,0,0,0) \quad \hat{\epsilon} = \hat{\epsilon}(\mathcal{T})$$

Extending Bjorken flow

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Solve:

$$\hat{\epsilon} = \hat{\epsilon}_0(\cosh\mathcal{T})^{-8/3}$$

Extending Bjorken flow

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Use $SO(1,1)\times SO(3)\times Z_2$ ansatz:

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$$\tan\theta = \frac{2qx_\perp}{1 + q^2\tau^2 - q^2 x_\perp^2}$$

Choose a useful coordinate system:

$$\widehat{ds^2} = \frac{1}{\tau^2}\left(-d\tau^2 + dx_\perp^2 + x_\perp^2 d\phi^2\right) + d\eta^2$$

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Use SO(1,1)×SO(3)×Z_2 ansatz:

$$\hat{u}^\mu = (1,0,0,0) \quad \hat{\epsilon} = \hat{\epsilon}(\mathcal{T})$$

Solve:

$$\hat{\epsilon} = \hat{\epsilon}_0(\cosh\mathcal{T})^{-8/3}$$

Extending Bjorken flow

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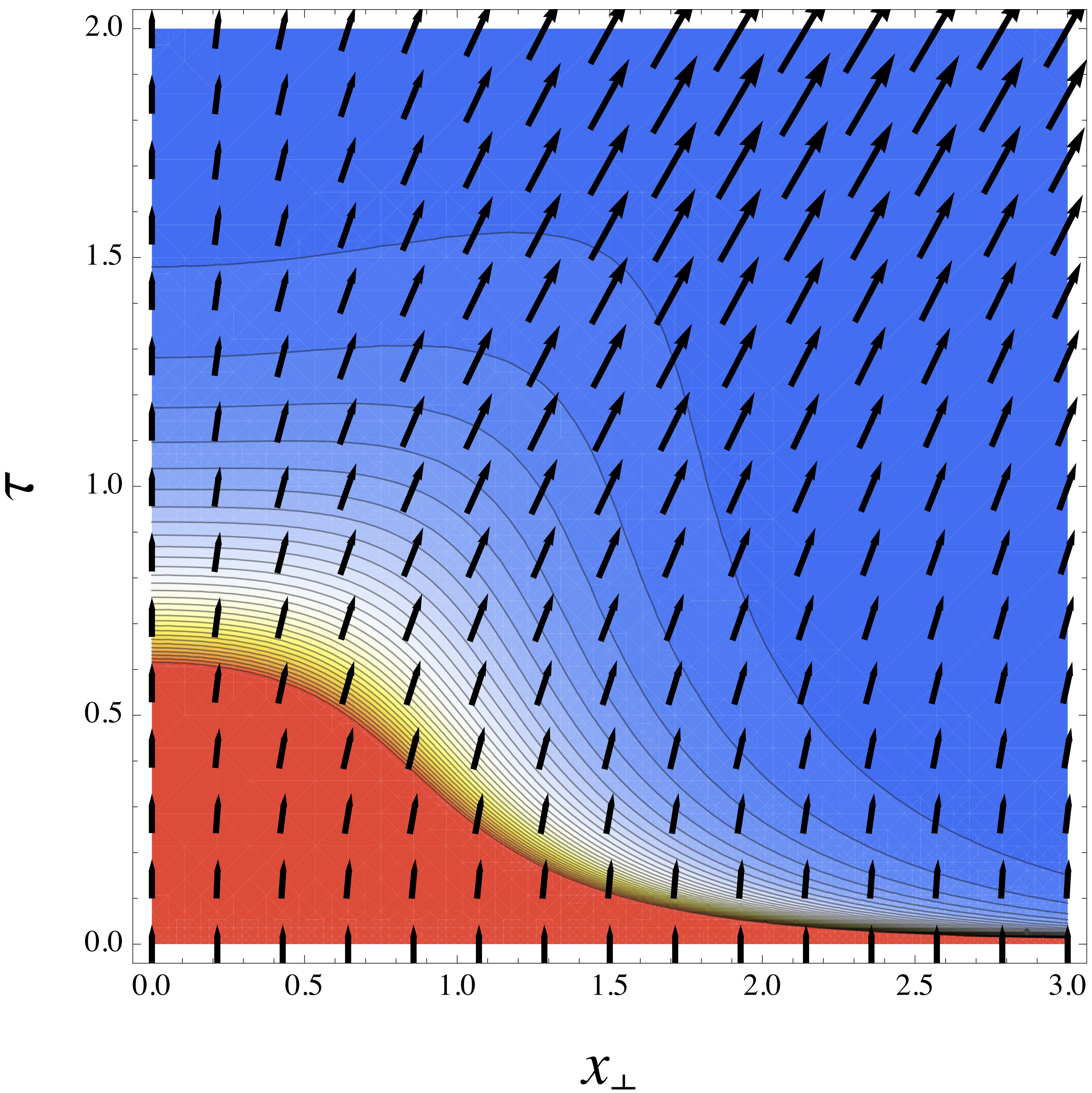

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Extensions

Extensions

- Include a chemical potential

Extensions

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$$\partial_\mu J^\mu = 0$$

Extensions

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- Include shear

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$$J^\mu = \rho u^\mu - \kappa\Delta^{\mu\nu}\partial_\nu \frac{\mu}{T}$$

Extensions

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- Generalize to higher dimensions

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$d \geq 4$

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$$\hat{\epsilon} = \frac{P_\theta \ell^{3/2}}{3\pi \ell_0^2} \frac{1 - \ell(\sqrt{2+\ell^2} - \ell)}{\sqrt{\sqrt{2+\ell^2} - \ell}}$$

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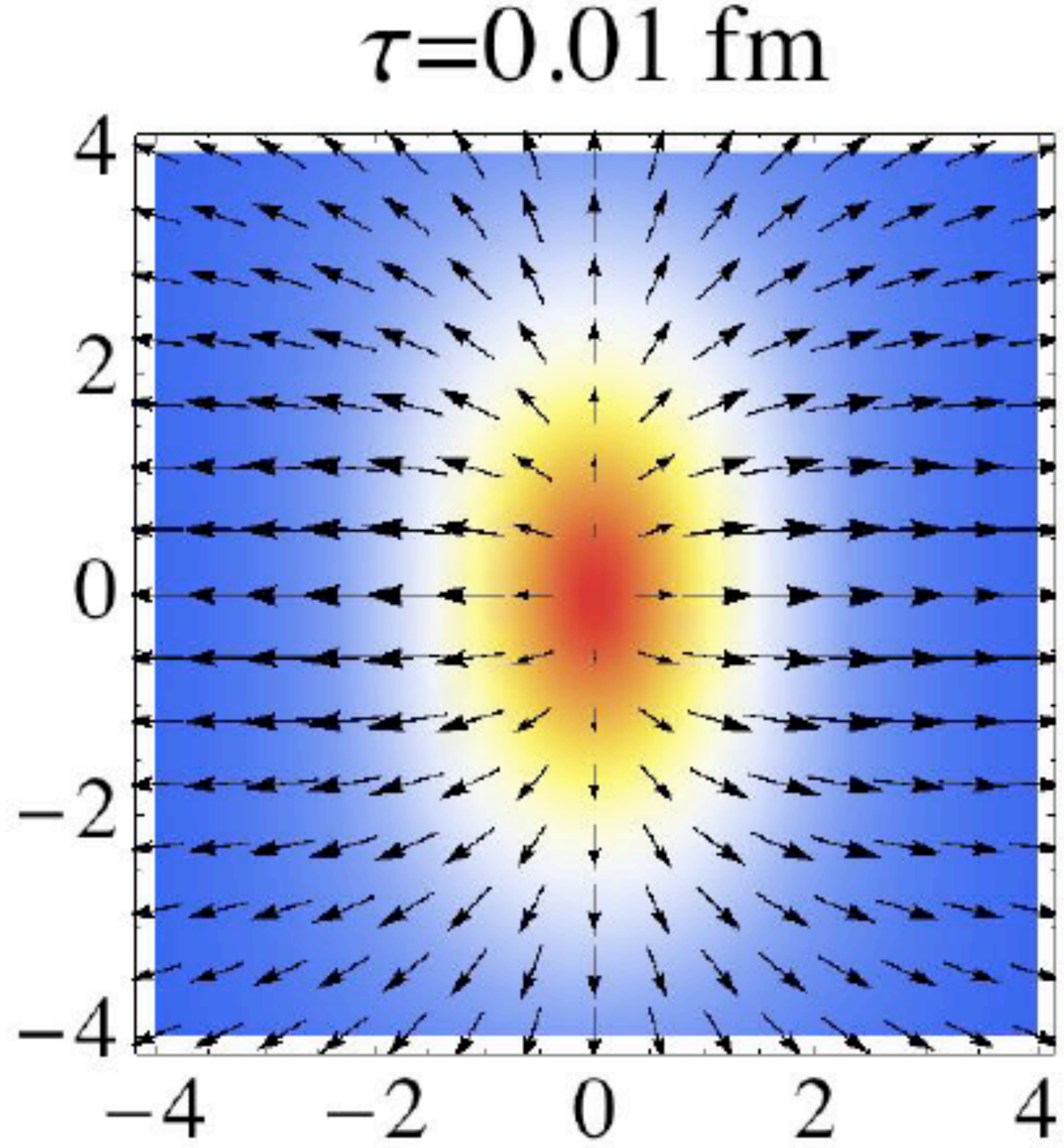

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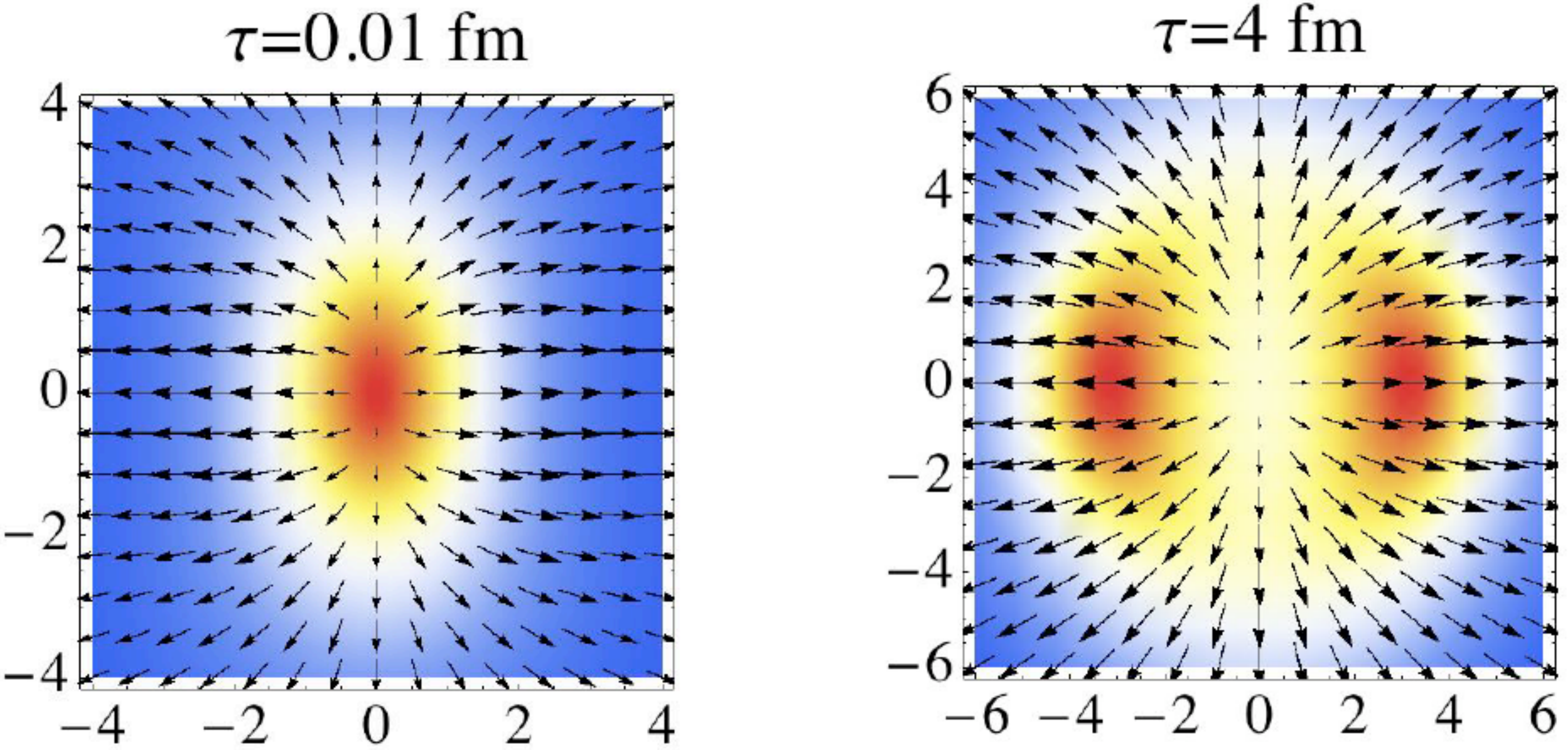

Extensions

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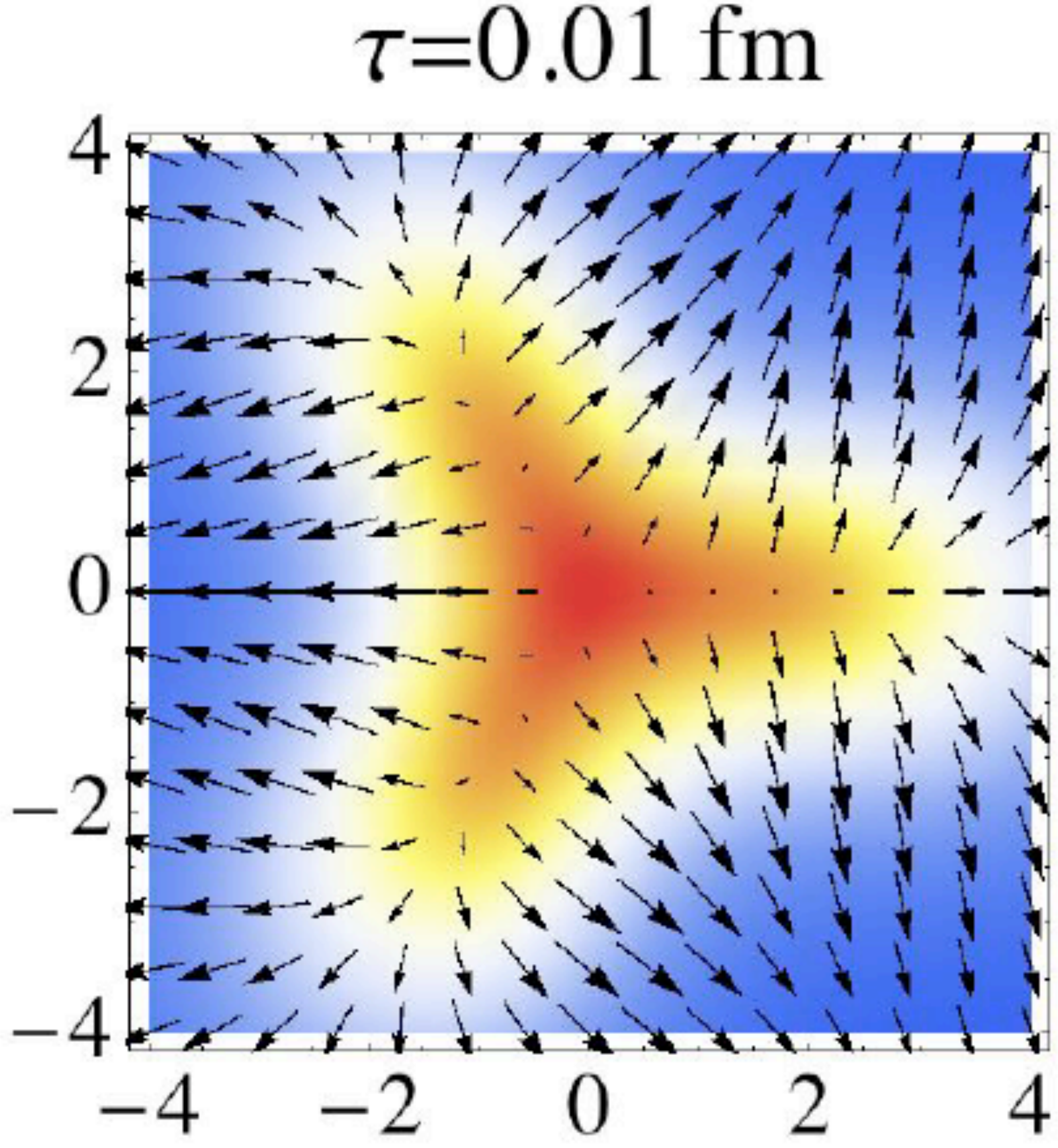

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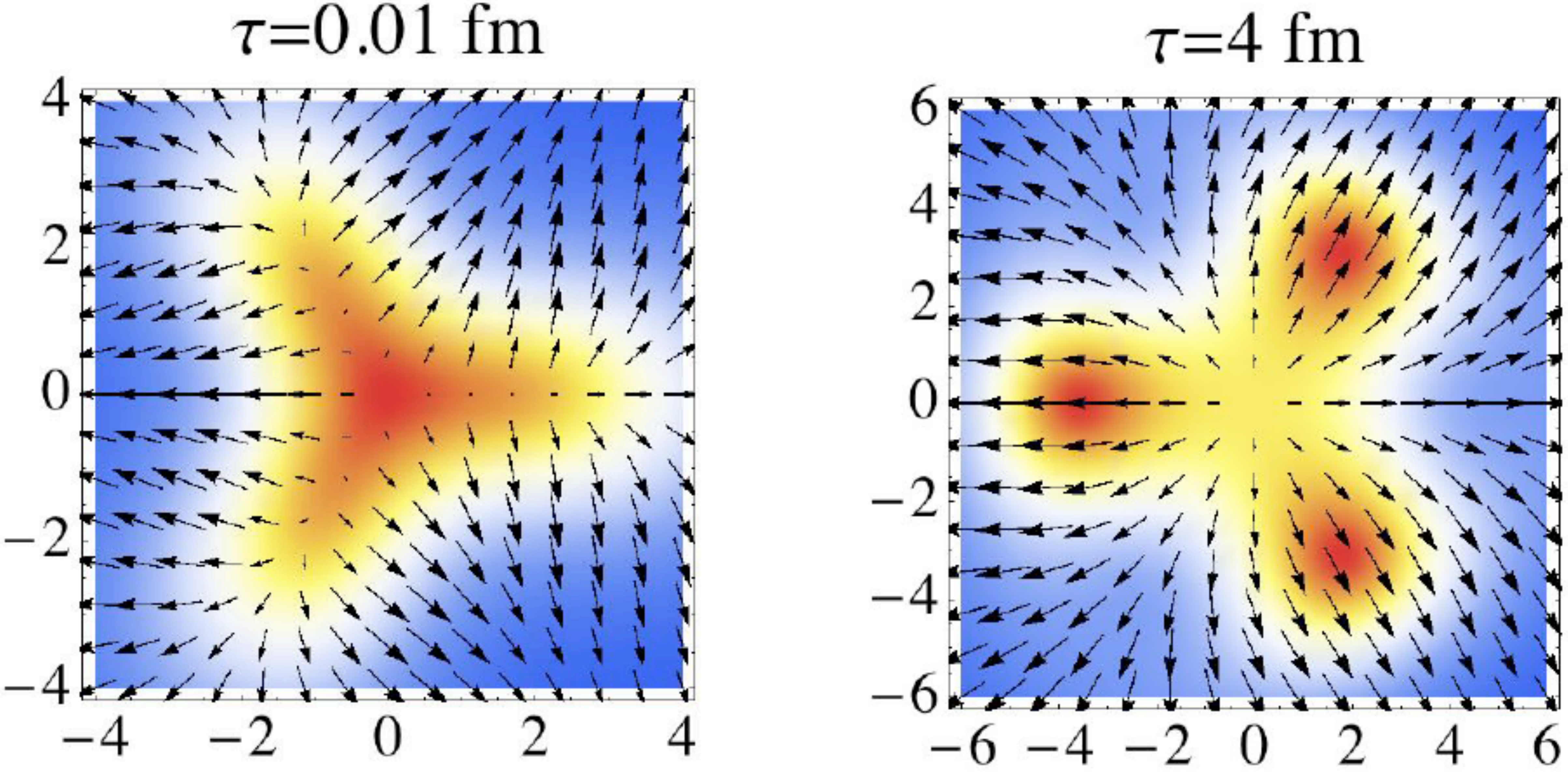

Extensions

- Include a chemical potential
- Include shear
- Generalize to higher dimensions
- Linearized perturbations
- Stability

Extensions

- Include a chemical potential
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- ...

Thank you