Fuzzy Twistors and Emergent Gravity

Jonathan J. Heckman

hep-th/1104.2605 w/ H. Verlinde

hep-th/????.???? w/ H. Verlinde

Goal

Aim: Regulate $\mathcal{N} = 4\ U(N_c)$ SYM on S^4

via *finite* size matrices

But: Retain continuum theory symmetries

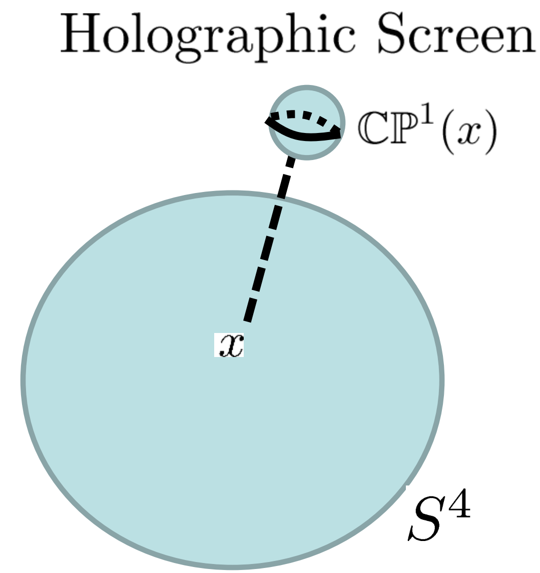
Eventual Applications: Holography with $\mathcal{R} > 0$

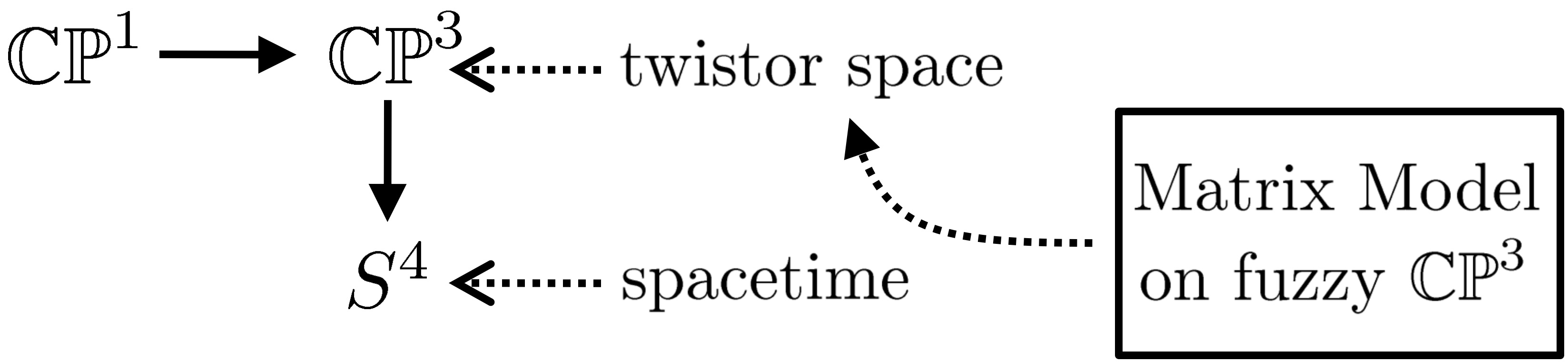
Geometric Picture

Spacetime $= S^4$

Over each $x \in S^4$ a fuzzy $\mathbb{CP}^1(x)$:

Functions = Finite Matrices





Twistors and $\mathcal{N} = 4$ SYM

We propose a Matrix Model for $\mathcal{N} = 4$ SYM

It relies on the deep connection between:

Self-dual $\mathcal{N} = 4$ SYM, e.g. $F = *F$

and: Ward '77, Chalmers Siegel '96, Witten '03

Holomorphic Chern-Simons on $\mathbb{CP}^{3|4}$

$$S_{hCS} = \int_{\mathbb{CP}^{3|4}} Tr\Omega \wedge (A\overline{\partial}A + \frac{2}{3}A^3)$$

All of $\mathcal{N} = 4$?

Getting all of $\mathcal{N} = 4$? Not so clear...

D-instantons / Twistor Strings:

Witten '03, Berkovits '04
Witten Berkovits '04

- $S_{eff} \sim hCS + \int d^{4|8}x \det \overline{\partial}_A^{(x)} +$ conformal gravity
- Main Issue: $\langle h^{(+2)}(k) h^{(-2)}(-k) \rangle \sim \frac{1}{k^4}$

"Phenomenological" Twistor Lagrangian

Boels Mason
Skinner '06

- $S_{eff} \sim hCS + \int d^{4|8}x \log \det \overline{\partial}_A^{(x)}$
- Why this action? Contact terms? Is gravity really decoupled?

The Proposal

JJH Verlinde '11
JJH Verlinde to appear

$$\mathcal{N} = 4 \; U(N_c) \text{ on } S^4 \;\; = \;\; \lim_{N \to \infty} (\text{Matrix Model})$$

$$g_{YM}^2 = g_{MM}^2 \times N$$

Note: N_c has nothing to do with N

$1/N$ Corrections $\Rightarrow$ Einstein Gravity

$$G_{Newton} \sim \frac{l_{fuzz}^2}{N}$$

Matrix Model Definition

hCS + Flux $\Rightarrow$ States in Landau Levels

($U(N) = U(n-1)$ Yang Monopole)

Theory of LLL = Matrix Model

Ground State Hilbert Spaces:

$$\mathcal{H}_{\mathbb{PT}}, \dim \mathcal{H}_{\mathbb{PT}} = k \sim n^3/6$$

$$\mathcal{H}_{\mathbb{CP}^1}, \dim \mathcal{H}_{\mathbb{CP}^1} = n$$

Matrix Model Fields

Fields = linear maps between fuzzy spaces

$\mathcal{H}_{\mathbb{PT}}$: Fuzzy Twistor Space

$\mathcal{H}_{\mathbb{CP}^1}$: Fuzzy Twistor Line

Bulk Modes: $A_{k\times k} : \mathcal{H}_{\mathbb{PT}} \to \mathcal{H}_{\mathbb{PT}}$
(D5/D5 strings)

Defect Modes:
(D1/D5 + D5/D1 strings)

$$\begin{cases} B_{n\times k} : \mathcal{H}_{\mathbb{PT}} \to \mathcal{H}_{\mathbb{CP}^1} \\ C_{k\times n} : \mathcal{H}_{\mathbb{CP}^1} \to \mathcal{H}_{\mathbb{PT}} \end{cases}$$

Matrix Model Action

Fields: $A_{k\times k}$, $B_{n\times k}$, $C_{k\times n}$ $\quad D_\alpha \equiv \overline{\partial}_\alpha + A_\alpha$

$$S_{total} = S_{bulk} + S_{defect}$$

This term breaks conformal symmetry

$$S_{bulk} = \frac{1}{g_{MM}^2}\mathrm{Tr}_{\mathbb{PT}\times U(N_c)}\psi^{\dagger 4}\varepsilon^{\alpha\beta\gamma\delta}p_\alpha D_\beta D_\gamma D_\delta$$

reference point

$$S_{defect} = \mathrm{Tr}_{\mathbb{CP}^1\times U(N_c)} B_{n\times k}p_\alpha I^{\alpha\beta}D_\beta C_{k\times n}$$

Continuum Limit

$N \to \infty$ and $l_{fuzz} \to 0$ and $g_{MM} \to 0$

But keep $l_{fuzz}^2 N$ fixed:

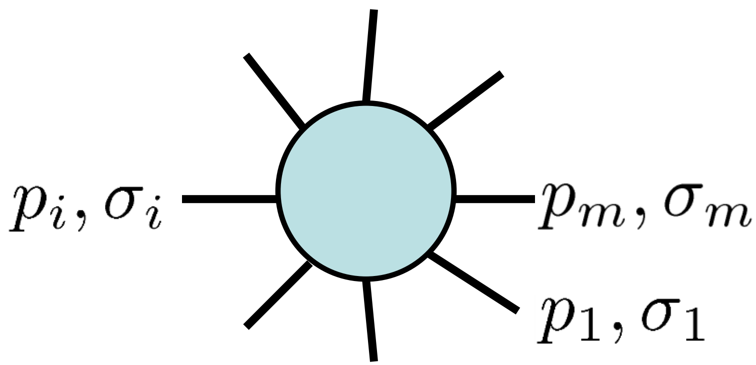
- $\mathrm{Vol}(\mathbb{PT}) \sim l_{fuzz}^6 N^3$
- $\mathrm{Vol}(S^4) \sim l_{fuzz}^4 N^2$
- $\mathrm{Vol}(\mathbb{CP}^1) \sim l_{fuzz}^2 N$

Pert theory in $g_{MM}^2 n N_c = g_{YM}^2 N_c = \lambda_{\text{'t Hooft}}$

Checking The Proposal

Check via scattering amplitudes

Formulate on $\mathbb{R}^4 = S^4 -$ North Pole $\rightarrow \mathbb{R}^{3,1}$



Matrix Model Correlators $\rightarrow$ Amplitudes

$$\langle\langle V_1 \cdots V_m \rangle\rangle_{MM} \rightarrow \mathcal{A}(p_1, \sigma_1; ...; p_m, \sigma_m)$$

Vertex Operators

Two Algebras:

$u(N_c)$: e.g. color, generators T_A

$u(k)$: e.g. fuzzy diffeos, generators $\mathcal{T}_{\mathfrak{A}}$

$$V_A^{gluon}(z,p) = Tr_{\mathbb{PT}\times U(N_c)}(B|z\rangle T_A e^{\langle p_\omega,\omega\rangle}\langle z|C)$$

$$V_{\mathfrak{A}}^{graviton}(z,p) = Tr_{\mathbb{PT}\times U(N_c)}(B|z\rangle \mathcal{T}_{\mathfrak{A}} e^{\langle p_\omega,\omega\rangle}\langle z|C)$$

MHV Amplitudes

Similar combinatorics for gauge theory *and* gravity

$$\langle\langle V_1 \cdots V_m \rangle\rangle_{MM}$$

$u(N_c)$ current algebra $\Rightarrow$ $\mathcal{A}^{gluons}_{MHV}$

Gluons: c.f. Nair '88, Witten '03
JJH Verlinde '11

$u(k)$ current algebra $\Rightarrow$ $\mathcal{A}^{gravitons}_{MHV}$

Gravitons: c.f. Nair '05
Mason Skinner '08,
JJH Verlinde to appear

Gravity !?

Leading order in $1/N \Rightarrow$ Einstein Gravity, not Conformal Gravity

One should expect corrections to amplitudes, as in string theory

Matrix Model can deviate from effective field theory:

- *High Energy* Regime via $1/N$ effects
- *Low Energy* Regime when # states $\sim O(N)$

Conclusions

- Fuzzy Twistors: Novel regulator for QFTs
- Continuum Limit $\rightarrow \mathcal{N} = 4$ SYM
- $1/N$ and Gravity
- Loop Amplitudes?