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Probing scalar mesons below and above 1 GeV

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Main refs.:

A.F., R. Jora, J. Schechter, Phys. Rev. D 79, 074014 (2009)
A.F., R. Jora, J. Schechter, M.N. Shahid, arXiv:1106.4835v1[hep-ph]

Related refs.:

A.F., R. Jora, J. Schechter, PRD 72, 034001 (2005); PRD 76, 014011 (2007); PRD 76, 114001 (2007);
PRD 77, 034006 (2008); PRD 77, 094004 (2008)

D. Black, A.F., J. Shechter, PRD 61, 074001 (2000).
D. Black, A.F., S. Moussa, S. Nasri, J. Shechter, PRD 64, 014031 (2001).

Outline:

1) Introduction

2) Generalized linear sigma model framework

3) Results

4) Summary & future works

Introduction:

Scalar mesons play important roles in low - energy QCD :

- Induce spontaneous chiral symmetry breaking

- Are probes of QCD vacuum

- Are important intermediate states in low energy processes (such as $\pi\pi$, πK, $\pi\eta$ scatterings, and decays such as $\eta^{/} \to \eta\pi\pi$, semileptonic decays of D_s,...)

Understanding the properties of scalar mesons is known to be nontrivial :

- Some of these states are broad and therefore interfere with nearby states

- Their mass spectrum doesn't follow the conventional SU(3) multiplets

Mass

$I = 0 \quad f_0(1500)\ \&\ f_0(1710)$

$I = 1 \quad a_0(1450)$

$I = 1/2 \quad K_0^*(1430)$

$I = 0 \quad f_0(1370)$

$f_0(1500)\ \&\ f_0(1710):$ could contain high glue comp.

PDG : likely candidates for a $q\bar{q}$ nonet

$$m[a_0(1450)] = 1474 \pm 19 \text{ MeV} > m[K_0^*(1430)] = 1425 \pm 50 \text{ MeV}$$

$$\frac{\Gamma[a_0{}^{tot.}]}{\Gamma[K_0^* \to \pi K]} = \begin{cases} 1.0 \pm 0.5 \text{ (PDG)} \\ 1.5 \quad \text{SU(3)} \end{cases}$$

$$\frac{\Gamma[a_0 \to K\bar{K}]}{\Gamma[a_0 \to \pi\eta]} = \begin{cases} 0.88 \pm 0.23 \text{ (PDG)} \\ 0.55 \quad \text{SU(3)} \end{cases}$$

$$\frac{\Gamma[a_0 \to \pi\eta']}{\Gamma[a_0 \to \pi\eta]} = \begin{cases} 0.35 \pm 0.16 \text{ (PDG)} \\ 0.16 \quad \text{SU(3)} \end{cases}$$

1 GeV

$I = 0\&1 \quad f_0(980) \& a_0(980)$

$I = 1/2 \quad K^*(800) \leftrightarrow k$

$I = 0 \quad f_0(600) \leftrightarrow \sigma$

Ideally mixed $q\bar{q}$ nonet

$I = 0 \quad \leftrightarrow \quad s\bar{s}$

$I = 1/2 \quad \leftrightarrow \quad n\bar{s}$

$I = 0\&1 \quad \leftrightarrow \quad n\bar{n}$

Vector meson nonet

$\phi(1020)$

$K^*(892)$

$\omega(776)\ \&\ \rho(783)$

Ideally mixed $qq\bar{q}\bar{q}$ nonet
(MIT bag model, Jaffe, 1977)

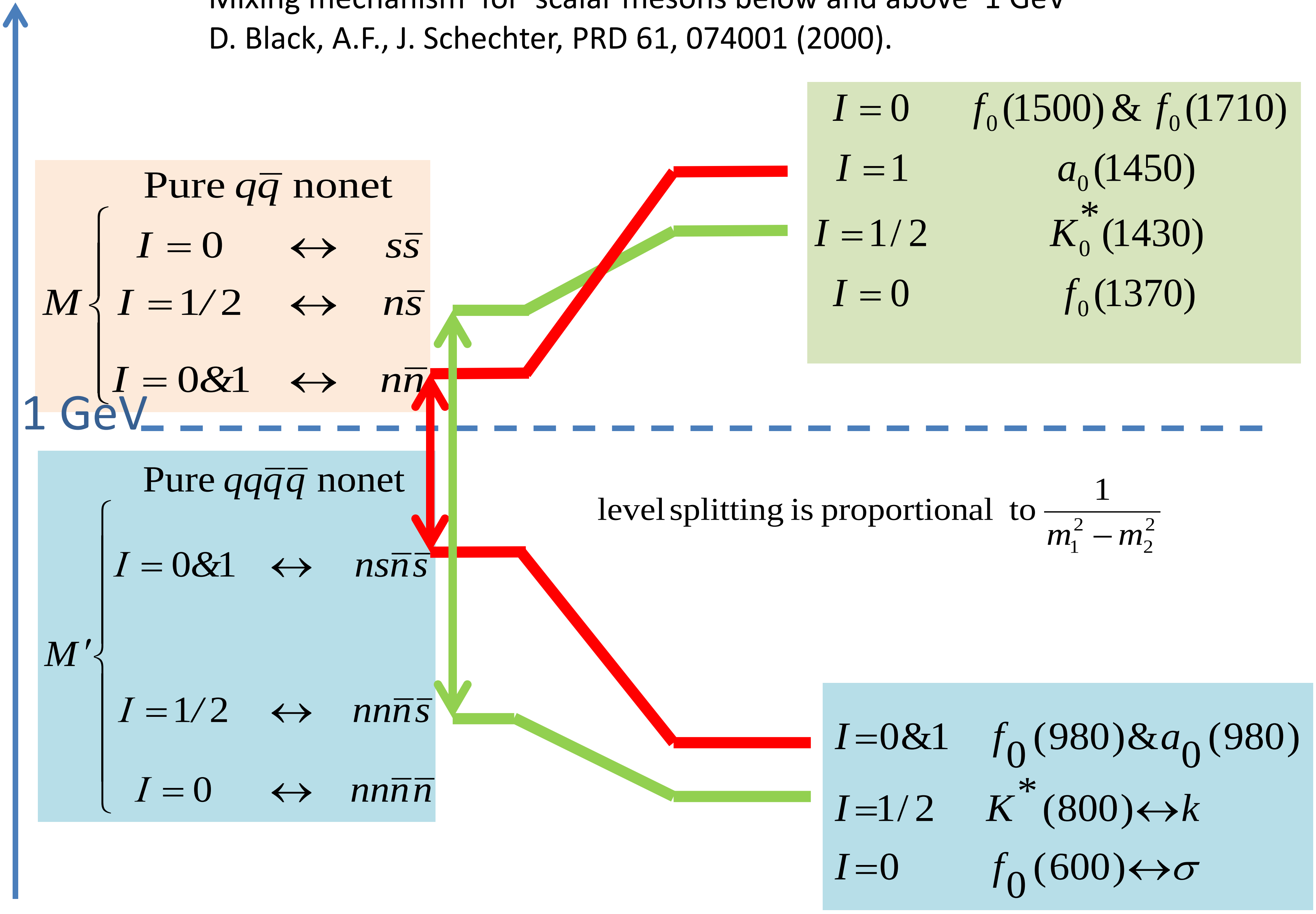
$I = 0\&1 \quad \leftrightarrow \quad ns\bar{n}\bar{s}$

$I = 1/2 \quad \leftrightarrow \quad nn\bar{n}\bar{s}$

$I = 0 \quad \leftrightarrow \quad nn\bar{n}\bar{n}$

Mass

Mixing mechanism for scalar mesons below and above 1 GeV
D. Black, A.F., J. Schechter, PRD 61, 074001 (2000).



Pure $q\bar{q}$ nonet

$$M\begin{cases} I=0 & \leftrightarrow & s\bar{s} \\ I=1/2 & \leftrightarrow & n\bar{s} \\ I=0\&1 & \leftrightarrow & n\bar{n} \end{cases}$$

1 GeV

Pure $qq\bar{q}\bar{q}$ nonet

$$M'\begin{cases} I=0\&1 & \leftrightarrow & ns\bar{n}\bar{s} \\ I=1/2 & \leftrightarrow & nn\bar{n}\bar{s} \\ I=0 & \leftrightarrow & nn\bar{n}\bar{n} \end{cases}$$

$I=0$	$f_0(1500)\,\&\,f_0(1710)$
$I=1$	$a_0(1450)$
$I=1/2$	$K_0^*(1430)$
$I=0$	$f_0(1370)$

level splitting is proportional to $\dfrac{1}{m_1^2-m_2^2}$

$I=0\&1$	$f_0(980)\&a_0(980)$
$I=1/2$	$K^*(800)\leftrightarrow k$
$I=0$	$f_0(600)\leftrightarrow\sigma$

Generalized Linear Sigma Model Framework:

The two chiral nonets are defined in terms of scalar and pseudoscalar nonets:

$$M = S + i\phi$$

$$M' = S' + i\phi'$$

Under $SU(3)_L \times SU(3)_R \times U(1)_A$:

$$M \rightarrow e^{2i\nu}\, U_{\mathrm{L}} M U_{\mathrm{R}}^{\dagger} \qquad\qquad M' \rightarrow e^{-4i\nu}\, U_{\mathrm{L}} M' U_{\mathrm{R}}^{\dagger}$$

At the quark level:

$$M_a^{\dot b} = (q_{bA})^\dagger \gamma_4 \frac{1+\gamma_5}{2} q_{aA}$$

$$M' \left\{ \begin{array}{ll} \epsilon_{acd}\epsilon^{\dot b \dot e \dot f}(M^\dagger)^c_{\dot e}(M^\dagger)^d_{\dot f} & \\ (L^{gA})^\dagger R^{fA} & \left\{ \begin{array}{l} L^{gE} = \epsilon^{gab}\epsilon^{EAB} q^T_{aA} C^{-1} \frac{1+\gamma_5}{2} q_{bB} \\ R^{\dot g E} = \epsilon^{\dot g \dot a \dot b}\epsilon^{EAB} q^T_{\dot a A} C^{-1} \frac{1-\gamma_5}{2} q_{\dot b B} \end{array} \right. \\ \left(L^g_{\mu\nu,AB}\right)^\dagger R^f_{\mu\nu,AB} & \left\{ \begin{array}{l} L^g_{\mu\nu,BA} = \epsilon^{gab} q^T_{aA} C^{-1} \sigma_{\mu\nu} \frac{1+\gamma_5}{2} q_{bB} \\ R^{\dot g}_{\mu\nu,BA} = \epsilon^{\dot g \dot a \dot b} q^T_{\dot a A} C^{-1} \sigma_{\mu\nu} \frac{1-\gamma_5}{2} q_{\dot b B} \end{array} \right. \end{array} \right.$$

Lagrangian:

$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}\left(\partial_\mu M \partial_\mu M^\dagger\right) - \frac{1}{2}\mathrm{Tr}\left(\partial_\mu M' \partial_\mu M'^\dagger\right) - V_0\left(M, M'\right) - V_{SB}$$

V_0 is invariant under $SU(3)_L \times SU(3)_R$, but not $U(1)_A$
V_{SB} is the symmetry breaking term

Approach 1:

Without making a specific choice for chiral invariant part of V_0 and with modeling:

$$\begin{cases} \text{i)} & \text{axial anomaly} \\ \text{ii)} & V_{SB} \\ \text{iii)} & \text{condensates} \end{cases} \Rightarrow \begin{cases} \text{i)} & \pi, \mathrm{K}, \eta \text{ contain small four - quak components} \\ \text{ii)} & \kappa \text{ contains large four - quark component} \\ \text{iii)} & \text{no predictions for: } a_0 \text{ and } f_0 \end{cases}$$

[A.F., R. Jora, J. Schechter, PRD 72, 034001 (2005)]

Approach 2:

Making a specific choice for chiral invariant part of V_0 and modeling:

$$\begin{cases} \text{i)} & \text{axial anomaly} \\ \text{ii)} & V_{SB} \\ \text{iii)} & \text{condensates} \end{cases} \Rightarrow \begin{cases} \text{i)} & \text{Pseudoscalars contain small four - quak components} \\ \text{ii)} & \text{Scalars contain large four - quark components} \\ \text{iii)} & \text{Predictions for low - energy processes such as } \pi\pi, \pi K \text{ scatterings} \end{cases}$$

[A.F., R. Jora, J. Schechter, PRD 76, 014011 (2007); PRD 76, 114001 (2007);
PRD 77, 034006 (2008); PRD 77, 094004 (2008); PRD 79, 074014 (2009);...]

In principle, V_0 contains infinite terms. Terms with dimension ≤ 4 are:

Break $U(1)_A$

N=4 N=6 N=8 N=8

$$V_0 = \; - \; c_2 \, \mathrm{Tr}(MM^\dagger) + \underline{\tilde{c}_3 \, (\det M + \mathrm{h.c.})} + c_4^a \, \mathrm{Tr}(MM^\dagger MM^\dagger) + c_4^b \, \left(\mathrm{Tr}(MM^\dagger)\right)^2$$

N=8 N=12 N=16

$$+ \; d_2 \, \mathrm{Tr}(M'M'^\dagger) + \underline{d_3 \, (\det M' + \mathrm{h.c.})} + d_4^a \, \mathrm{Tr}(M'M'^\dagger M'M'^\dagger)$$

N=16 N=6 N=8

$$+ \; d_4^b \, \left(\mathrm{Tr}(M'M'^\dagger)\right)^2 \quad + \; \underline{e_2 \, (\mathrm{Tr}(MM'^\dagger) + \mathrm{h.c.})} + e_3^a(\epsilon_{abc}\epsilon^{def} M_d^a M_e^b M_f'^c + \mathrm{h.c.})$$

N=10 N=12 N=12

$$+ \; \underline{e_3^b(\epsilon_{abc}\epsilon^{def} M_d^a M_e'^b M_f'^c + \mathrm{h.c.})} + \; e_4^a \, \mathrm{Tr}(MM^\dagger M'M'^\dagger) + e_4^b \, \mathrm{Tr}(MM'^\dagger M'M^\dagger)$$

N=12 N=10

$$+ \; \underline{e_4^c \, [\mathrm{Tr}(MM'^\dagger MM'^\dagger) + \mathrm{h.c.}]} + \underline{e_4^d \, [\mathrm{Tr}(MM^\dagger MM'^\dagger) + \mathrm{h.c.}]}$$

N=14 N=12

$$+ \; \underline{e_4^e \, [\mathrm{Tr}(M'M'^\dagger M'M^\dagger) + \mathrm{h.c.}]} \; + \; e_4^f \, \mathrm{Tr}(MM^\dagger)\mathrm{Tr}(M'M'^\dagger)$$

N=12 N=16

$$+ \; e_4^g \, \mathrm{Tr}(MM'^\dagger)\mathrm{Tr}(M'M^\dagger) \; + e_4^h \, [(\mathrm{Tr}(M'M'^\dagger))^2 + \mathrm{h.c.}]$$

N=10 N=14

$$+ \; \underline{e_4^i \, [\mathrm{Tr}(MM^\dagger)\mathrm{Tr}(MM'^\dagger) + \mathrm{h.c.}]} + \underline{e_4^j \, [\mathrm{Tr}(M'M'^\dagger)\mathrm{Tr}(M'M^\dagger) + h.c.]}$$

N = number of quark or antiquark lines at each vertex

= 2 × number of M or M^+ + 4 × number of M' or M'^+

As our first step, we consider terms up to N = 8 with single Tr :

Break $U(1)_A$

N=4 N=6 N=8 N=8

$$V_0 = -\ c_2\, \mathrm{Tr}(MM^\dagger) + \tilde{c}_3\, (\det M + \mathrm{h.c.}) + c_4^a\, \mathrm{Tr}(MM^\dagger MM^\dagger) + c_4^b\, \left(\mathrm{Tr}(MM^\dagger)\right)^2$$

N=8

$$+\ d_2\, \mathrm{Tr}(M'M'^\dagger) + e_2\, (\mathrm{Tr}(MM'^\dagger) + \mathrm{h.c.}) + e_3^a(\epsilon_{abc}\epsilon^{def} M_d^a M_e^b M_f'^c + \mathrm{h.c.})$$

N=6 N=8

To exactly mock up $U(1)_A$:

$$\boxed{c_3 \left[\gamma_1 \ln\left(\frac{\det M}{\det M^\dagger}\right) + (1-\gamma_1)\ln\frac{\mathrm{Tr}(MM'^\dagger)}{\mathrm{Tr}(M'M^\dagger)}\right]^2}$$

C. Rosenzweig, J. Schechter and G. Trahern, Phys. Rev. **D21**, 3388 (1980)

J. Schechter, Phys. Rev. **D21**, 3393 (1980)

Terms in V_{SB} that are linear in flvor symmetry breaking matrix $A = diag.(A_1, A_2, A_3)$ and dim. ≤ 4 are :

$$
\begin{aligned}
V_{SB} = \; & + \underline{k_1[\mathrm{Tr}(AM) + \mathrm{h.c.}]} + k_2[\mathrm{Tr}(AM') + \mathrm{h.c.}] \\
& + k_3[\mathrm{Tr}(AMM^\dagger M) + \mathrm{h.c.}] + k_4[\mathrm{Tr}(AMM'^\dagger M') + \mathrm{h.c.}] \\
& + k_5[\mathrm{Tr}(AMM^\dagger M') + \mathrm{h.c.}] + k_6[\mathrm{Tr}(AMM'^\dagger M) + \mathrm{h.c.}] \\
& + k_7[\mathrm{Tr}(AM'M'^\dagger M') + \mathrm{h.c.}] + k_8[\mathrm{Tr}(AM'M^\dagger M) + \mathrm{h.c.}] \\
& + k_9[\mathrm{Tr}(AM'M'^\dagger M) + \mathrm{h.c.}] + k_{10}[\mathrm{Tr}(AM'M^\dagger M') + \mathrm{h.c.}] \\
& + k_{11}[\mathrm{Tr}(AM) + \mathrm{h.c.}]\mathrm{Tr}(MM^\dagger) \\
& + k_{12}[\mathrm{Tr}(AM) + \mathrm{h.c.}]\mathrm{Tr}(M'M'^\dagger) \\
& + k_{13}[\mathrm{Tr}(AM)\mathrm{Tr}(MM'^\dagger) + \mathrm{h.c.}] + k_{14}[\mathrm{Tr}(AM)\mathrm{Tr}(M'M^\dagger) + \mathrm{h.c.}] \\
& + k_{15}[\mathrm{Tr}(AM') + \mathrm{h.c.}]\mathrm{Tr}(MM^\dagger) \\
& + k_{16}[\mathrm{Tr}(AM') + \mathrm{h.c.}]\mathrm{Tr}(M'M'^\dagger) \\
& + k_{17}[\mathrm{Tr}(AM')\mathrm{Tr}(MM'^\dagger) + \mathrm{h.c.}] + k_{18}[\mathrm{Tr}(AM')\mathrm{Tr}(M'M^\dagger) + \mathrm{h.c.}] \\
& + k_{19} A_a^b \epsilon_{bcd} \epsilon^{aef} M_e^c M_f^d + \mathrm{h.c.} \\
& + k_{20} A_a^b \epsilon_{bcd} \epsilon^{aef} M'^c_e M'^d_f + \mathrm{h.c.} \\
& + k_{21} A_a^b \epsilon_{bcd} \epsilon^{aef} M_e^c M'^d_f + \mathrm{h.c.}
\end{aligned}
$$

$$
\begin{aligned}
V \;=\; & -\; c_2\,\mathrm{Tr}(MM^\dagger) + c_4^a\,\mathrm{Tr}(MM^\dagger MM^\dagger) + \; d_2\,\mathrm{Tr}(M'M'^\dagger) \\
& + e_3^a(\epsilon_{abc}\epsilon^{def} M_d^a M_e^b M'^c_f + h.c.) \\
& + \; c_3 \left[\gamma_1 \ln(\frac{\det M}{\det M^\dagger}) + (1-\gamma_1)\ln\frac{\mathrm{Tr}(MM'^\dagger)}{\mathrm{Tr}(M'M^\dagger)}\right]^2 \; -2\,\mathrm{Tr}(A\,S)
\end{aligned}
$$

$$
\text{Assuming isospin symmetry:} \begin{cases} A = diag.(A_1, A_2, A_3) & \text{with: } A_1 = A_2 \neq A_3 \\ \langle S_a^a \rangle = \alpha_a & \text{with: } \alpha_1 = \alpha_2 \neq \alpha_3 \\ \langle S_a'^a \rangle = \beta_a & \text{with: } \beta_1 = \beta_2 \neq \beta_3 \end{cases}
$$

All together there are 12 unknown parameters: $c_2, c_4^a, d_2, e_3^a, c_3, \gamma_1, \alpha_1, \alpha_3, \beta_1, \beta_3, A_1, A_3$

$$
\text{Exp. Inputs} \begin{cases}
m[a_0(980)] = 984.7 \pm 1.2\,\mathrm{MeV} \\
m[a_0(1450)] = 1474 \pm 19\,\mathrm{MeV} \\
m[\pi(1300)] = 1300 \pm 100\,\mathrm{MeV} \\
m_\pi = 137\,\mathrm{MeV} \\
F_\pi = 131\,\mathrm{MeV} \\
A_3 / A_1 \approx 20 \rightarrow 30 \\
\det M_\eta^2 = (\det M_\eta^2)_{EXP} \\
\mathrm{Tr}\, M_\eta^2 = (\mathrm{Tr}\, M_\eta^2)_{EXP}
\end{cases}
\quad \oplus \quad \text{Miminum conds.} \begin{cases}
\left\langle \frac{\partial V}{\partial S_1^1} \right\rangle = 0 \\
\left\langle \frac{\partial V}{\partial S_3^3} \right\rangle = 0 \\
\left\langle \frac{\partial V}{\partial S_1'^1} \right\rangle = 0 \\
\left\langle \frac{\partial V}{\partial S_3'^3} \right\rangle = 0
\end{cases}
$$

$$(M_\pi^2) = \begin{bmatrix} 4\,e_3^a\,\beta_3 - 2\,c_2 + 4\,c_4^a\,\alpha_1^2 & 4\,e_3^a\,\alpha_3 \\ 4\,e_3^a\,\alpha_3 & 2\,d_2 \end{bmatrix}$$

$$(M_K^2) = \begin{bmatrix} 4\,e_3^a\,\beta_1 - 2\,c_2 + 4\,c_4^a\,\alpha_1^2 - 4\,c_4^a\,\alpha_1\,\alpha_3 + 4\,c_4^a\,\alpha_3^2 & 4\,e_3^a\,\alpha_1 \\ 4\,e_3^a\,\alpha_1 & 2\,d_2 \end{bmatrix}$$

$$(X_a^2) = \begin{bmatrix} -4\,e_3^a\,\beta_3 - 2\,c_2 + 12\,c_4\,\alpha_1^2 & -4\,e_3^a\,\alpha_3 \\ -4\,e_3\,\alpha_3 & 2\,d_2 \end{bmatrix}$$

$$(X_\kappa^2) = \begin{bmatrix} -4\,e_3^a\,\beta_1 - 2\,c_2 + 4\,c_4\,\alpha_1^2 + 4\,c_4^a\,\alpha_1\,\alpha_3 + 4\,c_4\,\alpha_3^2 & -4\,e_3^a\,\alpha_1 \\ -4\,e_3^a\,\alpha_1 & 2\,d_2 \end{bmatrix}$$

$$(X_0^2) = \begin{bmatrix} 4\,e_3^a\,\beta_3 - 2\,c_2 + 12\,c_4^a\,\alpha_1^2 & 4\,\sqrt{2}\,e_3^a\,\beta_1 & 4\,e_3^a\,\alpha_3 & 4\,\sqrt{2}\,e_3^a\,\alpha_1 \\ 4\,\sqrt{2}\,e_3^a\,\beta_1 & -2\,c_2 + 12\,c_4^a\,\alpha_3^2 & 4\,\sqrt{2}\,e_3^a\,\alpha_1 & 0 \\ 4\,e_3^a\,\alpha_3 & 4\,\sqrt{2}\,e_3^a\,\alpha_1 & 2\,d_2 & 0 \\ 4\,\sqrt{2}\,e_3^a\,\alpha_1 & 0 & 0 & 2\,d_2 \end{bmatrix}$$

$$\begin{aligned} (M_\eta^2)_{11} \;=\; & (16\,c_4\,\alpha_1^6\beta_1{}^2 + 16\,c_4\,\alpha_1{}^5\alpha_3\,\beta_1\,\beta_3 + 4\,c_4\,\alpha_1{}^4\alpha_3{}^2\beta_3{}^2 - 16\,e_3\,\alpha_1{}^4\beta_1{}^2\beta_3 - 16\,e_3\,\alpha_1{}^3\alpha_3\,\beta_1\,\beta_3{}^2 \\ & -4\,e_3\,\alpha_1{}^2\alpha_3{}^2\beta_3{}^3 - 8\,c_2\,\alpha_1{}^4\beta_1{}^2 - 8\,c_2\,\alpha_1{}^3\alpha_3\,\beta_1\,\beta_3 - 2\,c_2\,\alpha_1{}^2\alpha_3{}^2\beta_3{}^2 - 16\,c_3\,\gamma_1{}^2\alpha_1{}^2\beta_1{}^2 \\ & -32\,c_3\,\gamma_1{}^2\alpha_1\,\alpha_3\,\beta_1\,\beta_3 - 16\,c_3\,\gamma_1{}^2\alpha_3{}^2\beta_3{}^2 - 32\,c_3\,\gamma_1\,\alpha_1{}^2\beta_1{}^2 - 32\,c_3\,\gamma_1\,\alpha_1\,\alpha_3\,\beta_1\,\beta_3 \\ & -16\,c_3\,\alpha_1{}^2\beta_1{}^2) \,/\, \left((2\,\alpha_1\,\beta_1 + \alpha_3\,\beta_3)^2\,\alpha_1{}^2\right) \end{aligned}$$

Results:

🟦 Inputs

🟥 Predictions

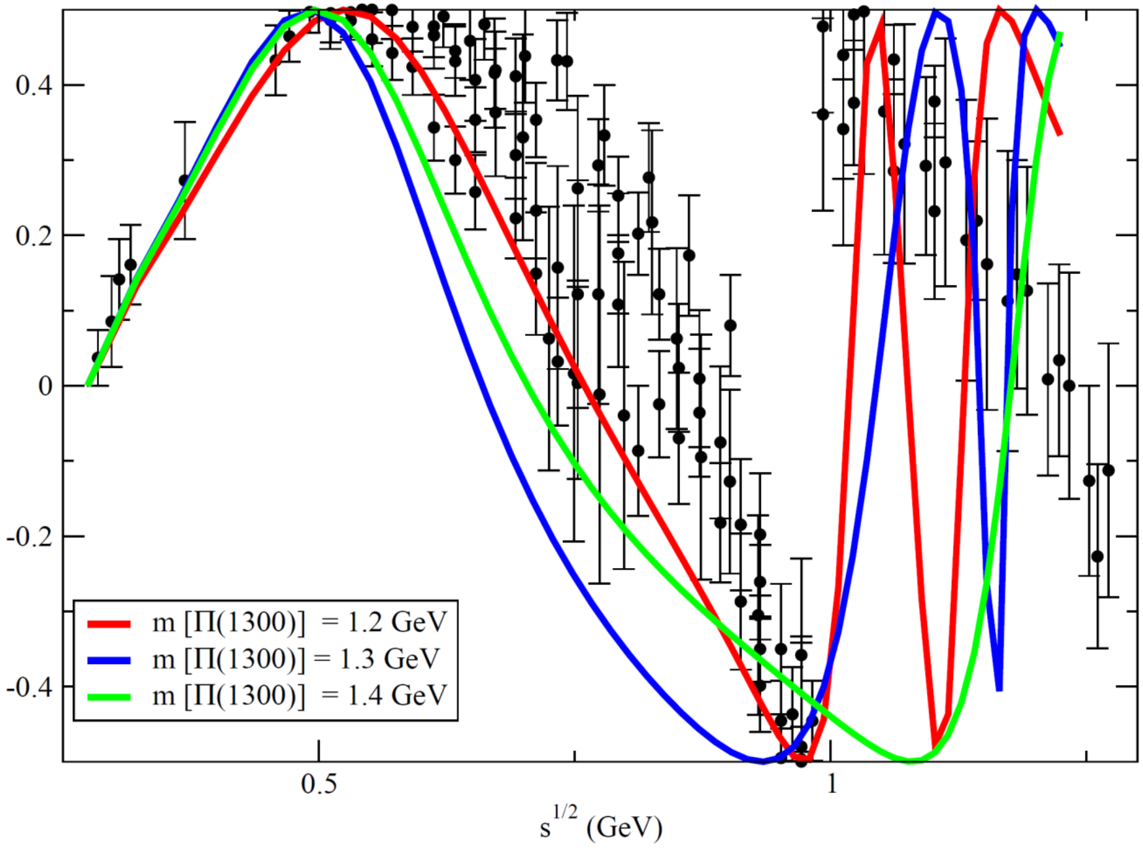
State	$\bar{q}q\%$	$\bar{q}\bar{q}qq\%$	m (GeV)	EXP.
π	85	15	0.137	
π'	15	85	1.215	
K	86	14	0.515	0.496
K'	14	86	1.195	1.460
η_1	89	11	0.553	0.547
η_2	78	22	0.982	0.958
η_3	32	68	1.225	? 1.294 1.410 1.476 1.617
η_4	1	99	1.794	

State	$\bar{q}q\%$	$\bar{q}\bar{q}qq\%$	m (GeV)	EXP.
a	24	76	0.984	
a'	76	24	1.474	
κ	8	92	1.067	0.676
κ'	92	8	1.624	1.425
f_1	40	60	0.742	$0.4-1.2$
f_2	5	95	1.085	0.980
f_3	63	37	1.493	? $1.2-1.5$ 1.505 1.720 •••
f_4	93	7	1.783	

Receive corrections due to the unitarization of ☐ ☐ scattering amplitude

Pole	Mass (MeV)	Width (MeV)
1	483	455
2	1012	154
3	1082	35
4	1663	2.1

K-matrix unitarization of prediction for real part of I=J=0 □ □ scattering amplitue



Summary:

The generalized linear sigma model seems to provide a consistent picture for the mixing of scalar and pseudoscalar mesons below 2 GeV

Scalar meson content is predicted to be reverse of the pseudoscalar meson Contant

Future works:

Pi K and pi eta scatterings

Eta prime decays

Inclusion of glueballs