A New, Analytic, Non-Perturbative, Gauge Invariant Formulation of "Realistic" QCD

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Outline

A. Explaining the Title (and what is to come)

- "New" = Less than 2 years old;
- "Non-Perturbative" = Summation of all Feynman graphs corresponding to multi-gluon exchanges between quarks and/or anti-quarks; individual gluon exchanges are replaced by "Gluon Bundles" containing an infinite number of exchanged gluons; "Bundle Diagrams" replace Feynman diagrams.
- Distinguishes "Ideal" from "Realistic" QCD = After non-perturbative gluon sums defining "gluon bundles" are performed, fundamental "transverse imprecision" of Qs and $\bar{Q}$s is necessary. Why? Because quarks are asymptotically bound, their transverse coordinates cannot — in principle — be defined precisely; and absurdities appear which MUST be removed by relevant changes in the original Lagrangian.

A. Explaining the Title (and what is to come)

- “Gauge-Invariance” = All gluon bundle exchanges are strictly gauge-invariant, as all gauge-dependent, intermediary gluon propagators exactly cancel! Just as for the “dressed” photon propagator in QED, only the original “free” gluon propagator retains its original gauge dependence, but all its radiative corrections are strictly gauge-invariant. (No Fadeev-Popov ghosts needed here!)
- Displays the remarkable property of “Effective Locality” = A new feature of gauge-invariant, non-perturbative sums, wherein each gluon bundle appears to be emitted and absorbed at one space-time point on a quark line (modulo transverse imprecision). EL replaces functional integrals by (a few sets of) ordinary integrals. A fantastic simplification!

B. Results at this Stage

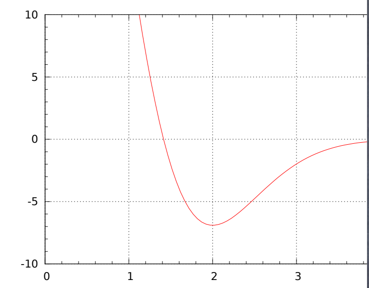
N.B.: For simplicity, we treat only one type of massive quark, along with zero-mass gluons, and neglect spin and angular momenta of quarks and bound states. Flavors, and electroweak effects, as well as all spin effects, can be inserted, as desired.

- Simple, analytic construction of a $Q, \bar{Q}$ Binding Potentials, with qualitative estimates of pion and nucleon ground states.

  $Q - \bar{Q}$ Potential has the form: $V(r) = \xi(\mu r)^{(1+\xi)}$, where μ is a mass scale (very close to the pion mass), and ξ a small, real parameter (on the order of .1), appearing in the definition of transverse imprecision. The numerical values of ξ and of m are fixed by comparison with the ground states corresponding to the model pion $Q - \bar{Q}$ and model nucleon QQQ.

B. Results at this Stage

- Nucleon-nucleon Scattering and Binding Potential: An explicit construction of a model n-p binding to form a deuteron.



$V(x) = V_0(2 - x^2)\exp[-(1/2)x^2]$

The potential MUST go negative for large r, with parameters appropriate to form a deuteron of Binding Energy = 2.2 MeV. (Closely resembles the average of Jastrow's 1951 singlet and triplet potentials.) To our knowledge, this is the first example of Nuclear Physics from basic QCD.

C. Functional Methodology

1. We employ an exact Schwinger solution for QCD GF, given in terms of a gauge-dependent gluon propagator, $D_c(\rho)$,

$$\tilde{D}^{ab\,(\rho)}_{c,\,\mu\nu}(k) = \delta_{ab}(1/k^2)\left[\delta_{\mu\nu} - \rho\, k_\mu k_\nu/k^2\right],\ \rho = \lambda/(1-\lambda)\,;$$

a quark propagator (for motion in a fictious gluon field A^a_μ) $G_c[A]$;

$$G_c(x, y\,|A) = \langle x|[m + \gamma_\mu\,(\partial_\mu + i\,g\,A^a_\mu\,\lambda_a)]^{-1}|y\rangle\,;$$

and the closed-fermion functional,

$$L[A] = \mathrm{tr}\ln[1 - i\,g\,(\gamma A\,\lambda)\,S_c]\,,\ S_c = G_c[0]\,.$$

C. Functional Methodology

1. Re-arrange that Schwinger solution in the unique way which preserves GI. (This manipulation overlooked for many decades! It can be done in QCD, but not in QED. Why? Because of the existence of cubic and quartic gluon interactions!)

2. Employ an exact Fradkin functional representation for $G_c[A]$ and $L[A]$.

C. Functional Methodology

NB: $L[A]$ is MGI, by construction, and so preserved in its Fradkin representation.

- NB': $G_c[A]$ and $L[A]$ are Potential-Theory constructs, each containing A-dependence not more complicated than Gaussian. Therefore, ALL functional operations can be computed exactly — corresponding to the sum over all relevant Feynman graphs — and results expressed in terms of the Fradkin representations for G_c and L, which are subject to simple approximations, especially at high energies.

- We suggest this formulation as a simple and elegant approach to analytic, non-perturbative, gauge-invariant QCD calculations.

D. References

1. EPJC (2010) 65,395-411. Explains rearrangements of Schwinger generating functional to give gauge-invariance in a quenched, eikonal model.
2. Proof of EL in QCD, without approximation and without exception: arXiv:hep-th/1003.2936
3. “Ideal” vs. “Realistic" QCD: Transverse Imprecision: arXiv:hep-th/1103.4179
4. Quark Binding Potentials,for pion and nucleon: arXiv:hep-th/1104.4663
5. Nucleon-Nucleon Scattering and Binding Potentials: arXiv:(soon)