Large Nc Gauge Theories on the lattice

Rajamani Narayanan
rajamani.narayanan@fiu.edu
Florida International University

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Large Nc Yang-Mills theories in 3D

- SU(3) $\to$ SU(N) with $\lambda = g^2 N$ fixed as $N \to \infty$ and $g \to 0$.
- $\lambda \to 0$ is the continuum limit.
- $Z = \int [dU] e^{S(U)}$.
- $S(U) = 2bN \sum_p \mathrm{Re}\ U_p$; $b = \frac{1}{\lambda}$.
- $U_p = \mathrm{Tr}\ U_\mu(\mathbf{n}) U_\nu(\mathbf{n}+\hat{\mu}) U_\mu^\dagger(\mathbf{n}+\hat{\nu}) U_\nu^\dagger(\mathbf{n})$.
- $\mathbf{n}$ is a point on a d-dimensional periodic lattice.
- Gauge fields obey periodic boundary conditions on all directions.
- Global Z_N^d ($\to U^d(1)$ as $N \to \infty$) symmetry:

$$U_\mu(n_1, \cdots, n_{\mu-1}, L_\mu, n_{\mu+1}, \cdots, n_d)$$

$$\to$$

$$e^{i\frac{2\pi k}{N}} U_\mu(n_1, \cdots, n_{\mu-1}, L_\mu, n_{\mu+1}, \cdots, n_d);$$

$k = 0, \cdots, N-1$; $\mu = 1, \cdots, d$.

Transition in the plaquette operator

- The transition in the plaquette operator is a unphysical transition in the lattice theory – The continuum theory is always in the phase with a gap.
- It facilitates the lattice realization of gauge field topology – The eigenvalue distribution of the interpolating field between two lattice gauge fields with two different topological charge, will not satisfy the condition of the gap for some value of the interpolating parameter.
- It is known as the Gross-Witten transition in $d = 2$ and occurs at $b = 0.5$. The analytical calculation shows that tt is a third order transition.
- Numerical calculations in $d = 3$ suggest that it is possibly a third order transition and it occurs aroung $b \approx 0.43$.
- It is first order transition in $d = 4$ and it occurs around $b \approx 0.36$.

Various continuum phases

- Set $b > 0.43$ to be in the continuum side of the bulk transition.
- The tadpole improved coupling $b_I = b\langle\mathrm{Plaquette}\rangle$ can be used to set the scale.
- Consider a symmetric three torus with the physical size $l = \frac{L}{b_I}$ kept fixed as L and b_I are taken to ∞.
- The continuum theory exists in many phases:
  - 0c: $l_1 < l$ – None of the three U(1) symmetries are broken – Confined phase.
  - 1c: $l_2 < l < l_1$ – One of the three U(1) symmetries are broken – Deconfined phase.
  - 2c: $l_3 < l < l_2$ – Two of the three U(1) symmetries are broken – QCD in a small box at low temperatures.
  - 3c: $l < l_3$ – All three U(1) symmetries are broken – QCD in a small box at high temperatures.

Physics is independent of the size of any unbroken direction.

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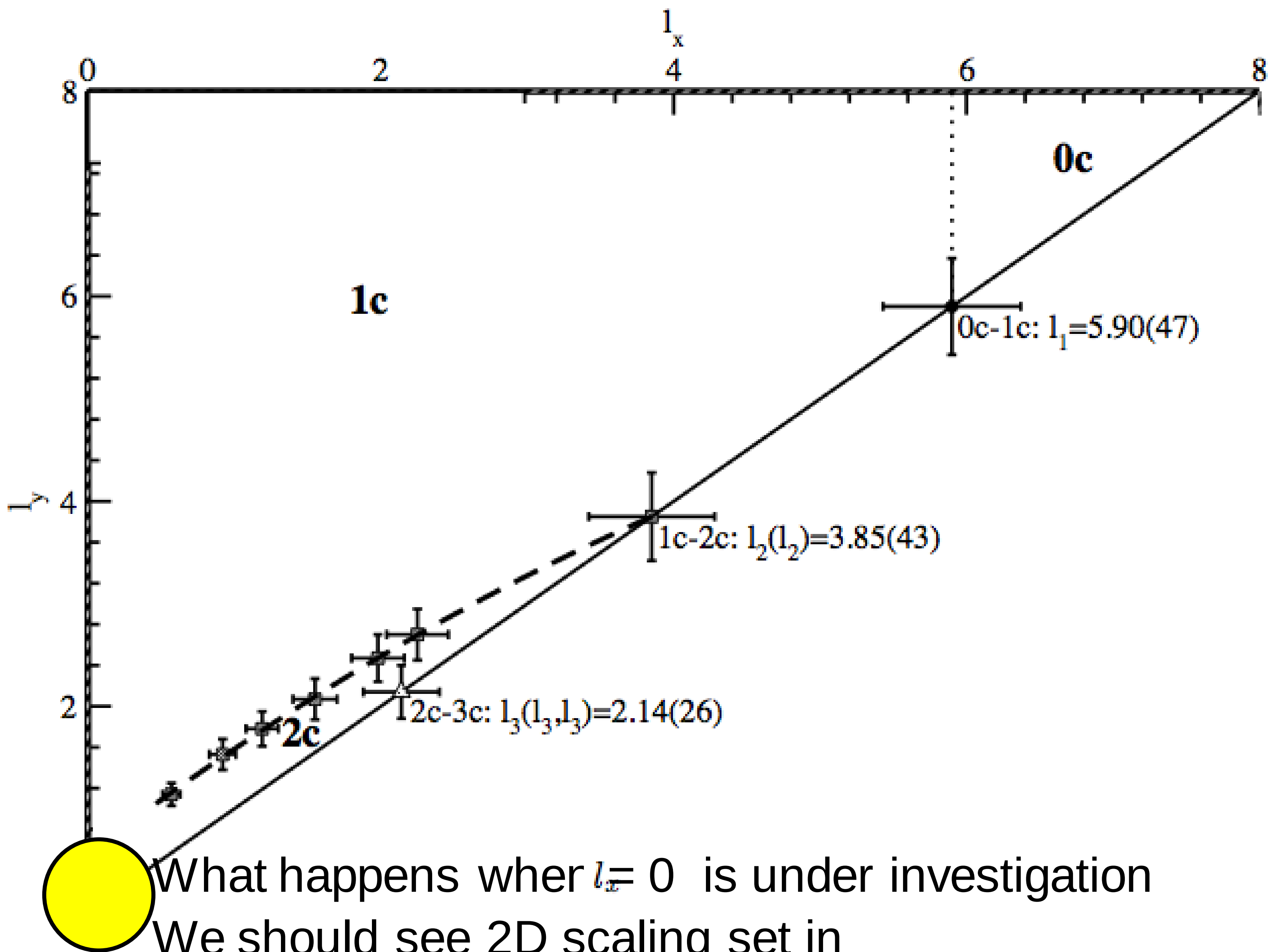
Phase diagram so far in 3D

$l_x \le l_y \le l_z$



What happens when $l_x = 0$ is under investigation

We should see 2D scaling set in

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Large Nc QCD with adjoint fermions

Eguchi-Kawai reduction is expected to hold and one can work on a single site lattice

Gauge action
$$S^g = -bN \sum_{\mu\neq\nu=1}^{4} \mathrm{Tr}\left[U_\mu U_\nu U_\mu^\dagger U_\nu^\dagger\right]$$

Fermion action
$$S_f = \mathrm{Tr}\left[\bar{\Psi}_k D_o(m_q)\Psi_k\right] \qquad D_o(m_q) = \frac{1}{2}\left[(1+m_q)+(1-m_q)V\right]$$

$$V = \gamma_5 \epsilon(H)$$

$$\epsilon(H) = \sum_{k=1}^{n} \frac{r_k H}{H^2 + p_k}; \quad 0 < p_1 < p_2 \cdots < p_n,$$

$$\begin{aligned} H &= \begin{pmatrix} 4-m-\frac{1}{2}\sum_\mu \left(V_\mu + V_\mu^t\right) & \frac{1}{2}\sum_\mu \sigma_\mu \left(V_\mu - V_\mu^t\right) \\ -\frac{1}{2}\sum_\mu \sigma_\mu^\dagger \left(V_\mu - V_\mu^t\right) & -4+m+\frac{1}{2}\sum_\mu \left(V_\mu + V_\mu^t\right) \end{pmatrix} \quad m \in [0,2] \\ &= (4-m)\gamma_5 - \sum_\mu \left(w_\mu V_\mu + w_\mu^\dagger V_\mu^t\right) \end{aligned}$$

$$w_\mu = \frac{1}{2}\begin{pmatrix} 1 & -\sigma_\mu \\ \sigma_\mu^\dagger & -1 \end{pmatrix}$$

$$V_\mu \Psi = U_\mu \Psi U_\mu^\dagger \qquad V_\mu^t \Psi = U_\mu^\dagger \Psi U_\mu$$

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Weak Coupling Perturbation Theory

Polyakov loop eigenvalues

$$D_\mu^{ij} = e^{i\theta_\mu^i}\delta_{ij}$$

Perturbation

$$U_\mu = e^{ia_\mu} D_\mu e^{-ia_\mu}$$

Leading order result

$$S_g = \sum_{i\neq j} \ln\left[\sum_\mu \sin^2 \frac{1}{2}\left(\theta_\mu^i - \theta_\mu^j\right)\right]$$

$$S_f = -2\sum_{i\neq j}\ln\left[\frac{1+m_q^2}{2} + \frac{1-m_q^2}{2}\,\frac{2\sum_\mu \sin^2\frac{\theta_\mu^i-\theta_\mu^j}{2} - m}{\sqrt{\left(2\sum_\mu \sin^2\frac{\theta_\mu^i-\theta_\mu^j}{2} - m\right)^2 + \sum_\mu \sin^2(\theta_\mu^i - \theta_\mu^j)}}\right]$$

If all $\theta_\mu^i = 0$ then $S_g \to -\infty$

But, if $m_q = 0$ then $S_f \to \infty$

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Single site Polyakov loop eigenvalues and momenta on the infinite lattice

At lowest order in weak coupling perturbation theory
adjoint fermions on a single site lattice see momentum modes

$$e^{i(\theta^i_\mu - \theta^j_\mu)} \quad 1 \le i,j \le N$$

The N^2-1 angles, $\theta^i_\mu - \theta^j_\mu$ approach a continuum of momenta, p_μ as N approaches infinity

We want the measure to be $\int \prod_\mu dp_\mu$ in order to reproduce correct infinite volume perturbation theory

<u>Naive fermions</u>

$$S_g = \sum_{i\neq j} \ln\left[\sum_\mu \sin^2 \frac{1}{2}\left(\theta^i_\mu - \theta^j_\mu\right)\right] \qquad S_f = \sum_{i\neq j} \ln\left[\sum_\mu \sin^2(\theta^i_\mu - \theta^j_\mu)\right]$$

Momenta, $p_\mu \in [\frac{\pi}{2}, \pi]$ will spoil the uniform measure in the large N limit.

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Massless overlap fermions

$$S_f = -2\sum_{i\neq j}\ln\left[\frac{1}{2}+\frac{1}{2}\frac{2\sum_\mu \sin^2\frac{\theta^i_\mu-\theta^j_\mu}{2}-m}{\sqrt{\left(2\sum_\mu \sin^2\frac{\theta^i_\mu-\theta^j_\mu}{2}-m\right)^2+\sum_\mu \sin^2(\theta^i_\mu-\theta^j_\mu)}}\right]$$

Unlike naive fermions, momenta $p_\mu \in [0,\frac{\pi}{2}]$ and $p_\mu \in [\frac{\pi}{2},\pi]$ are not identified

We need $m > 2\sum_\mu \sin^2\frac{\theta^i_\mu-\theta^j_\mu}{2}$ for the mode corresponding to that momentum to be massless

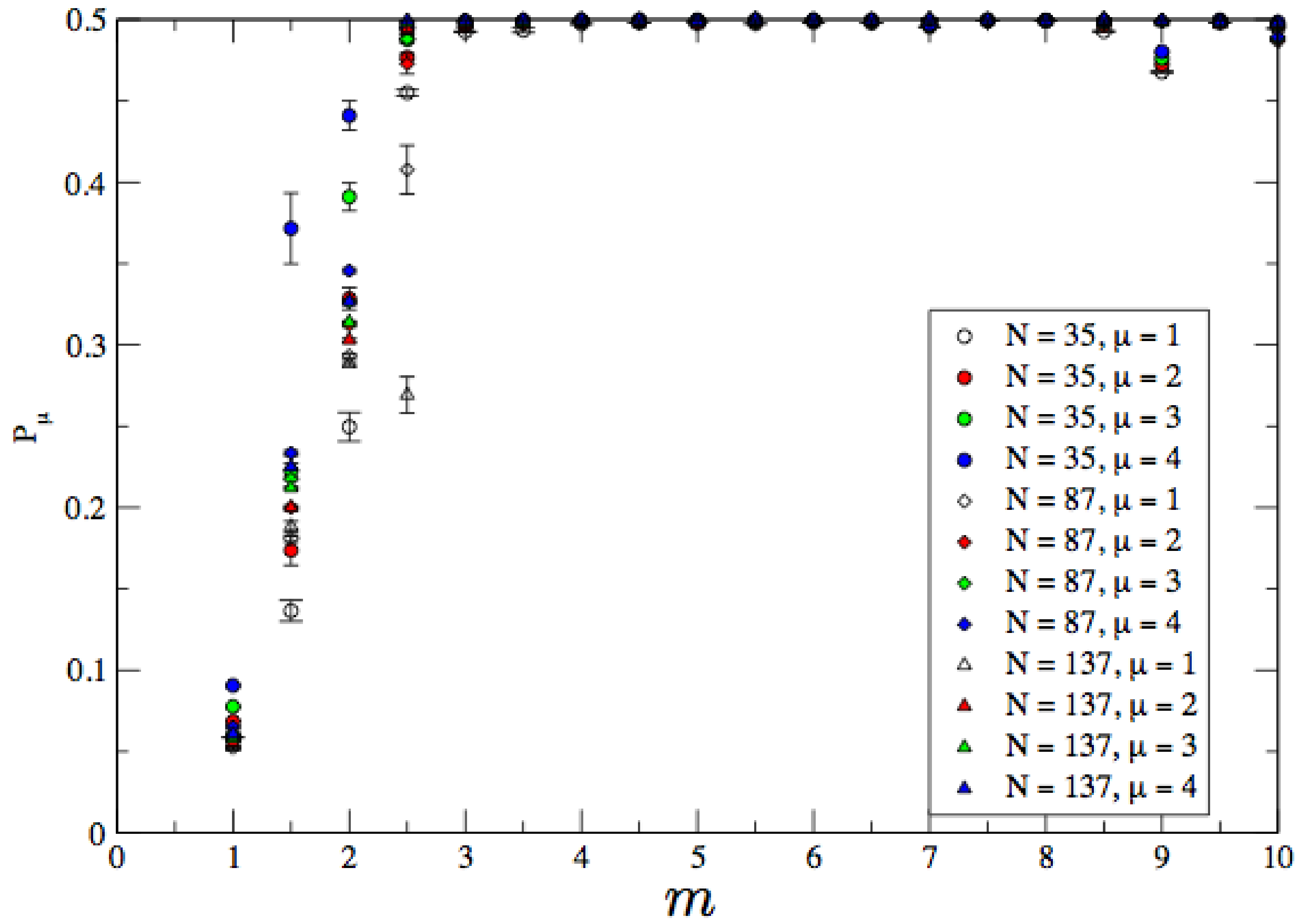
$m\rightarrow\infty$; the naive fermion limit and so we cannot make it too large

Making it too small will restrict the region inside the Brillouin zone, $p_\mu \in [-\pi,\pi]$

where we have massless fermions and therefore the correct momentum measure

Distribution of Polyakov loop eigenvalues

$$P_\mu = \frac{1}{2}\left(1 - \frac{1}{N^2}|\mathrm{Tr}U_\mu|^2\right) = \frac{1}{N^2}\sum_{i,j}\sin^2\frac{1}{2}\left(\theta^i_\mu - \theta^j_\mu\right)$$



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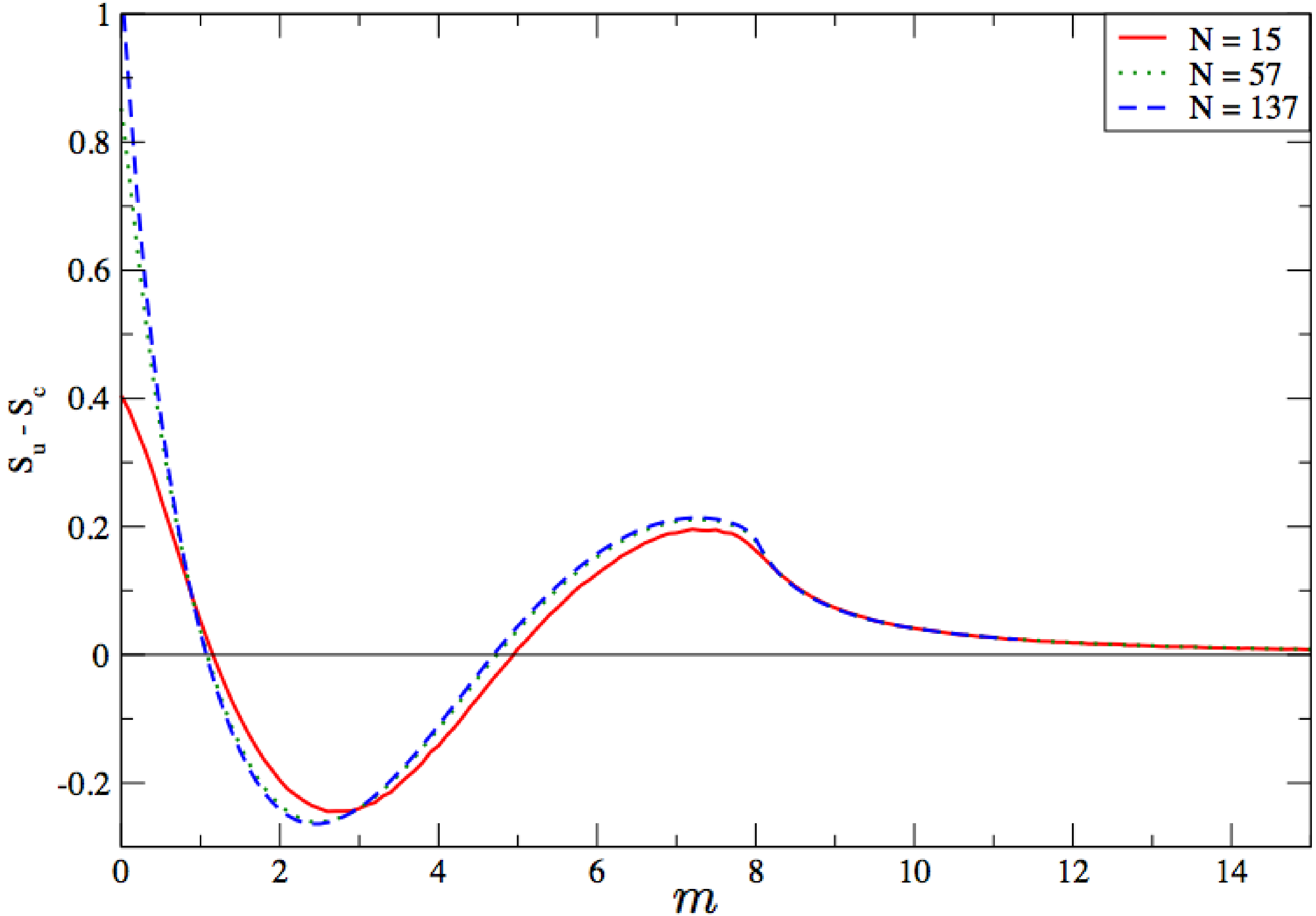
Correlated versus uncorrelated momenta

Correlated momenta $\theta_\mu^j = \frac{2\pi j}{N}$ $j = 1 \cdots, N$ in all four directions S_c: action

Uncorrelated momenta $\theta_\mu^j = \frac{2\pi \pi_j^\mu}{N}$ π^μ is a permutation of $j = 1 \cdots, N$ S_u: action



$m \in [3.5, 4.5]$ is a good choice

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A numerical proposal

- Use Hybrid Monte Carlo Algorithm with Pseudo-fermions.
- Works for integer number of Dirac flavors.
- A direct fermion HMC algorithm for non-integer Dirac flavors.
- Pick N and b such that we are in the large N limit for that b.
- We expect N to increase as b increases.
- Pick a quantity to set the scale.
- Lowest positive eigenvalue of the overlap Dirac operator.
- Strong to weak coupling transition.
- Find the region of b where we observe scaling.
- Measure physically interesting quantities.

We will report on progress toward this goal by showing some results with

$$f = 1$$

$$N = 18$$

$$b = 0.32, 0.35, 0.40$$

$$m_q = 0.05, 0.1$$

$$m = 4$$

Strong to weak coupling transition

Consider eigenvalues of Wilson loop operator

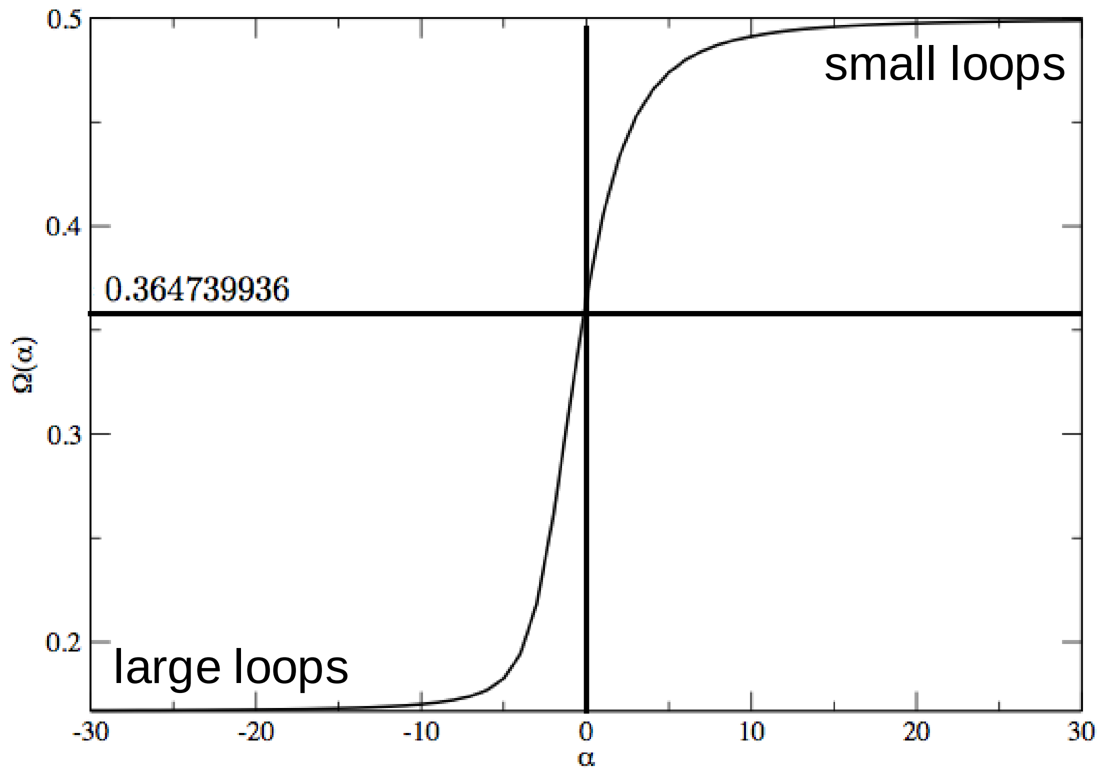
- Gauge invariant
- Distributed on a unit circle
- Sharply peaked for loops with small area (distribution has a gap at infinite N)
- Uniform for very large area (distribution has no gap at infinite N)
- Deviation from uniform distribution gives the string tension
- Phase transition from small area to large area in the limit of large N (Durhuus-Olesen transition)
- Universal behavior in the double scaling limit where the area becoming critical and the gap c

$$O_N(y,b) = \left\langle \det\left(e^{\frac{y}{2}} + e^{-\frac{y}{2}} W\right) \right\rangle$$

$$O_N(y,b) = C_0(b,N) + C_1(b,N)y^2 + C_2(b,N)y^4 + \cdots$$

$$\Omega(b,N) = \frac{C_0(b,N)C_2(b,N)}{C_1^2(b,N)}$$

$$b = b_c(L,N)\left[1 + \frac{\alpha}{\sqrt{3N}a_2(L,N)}\right]$$



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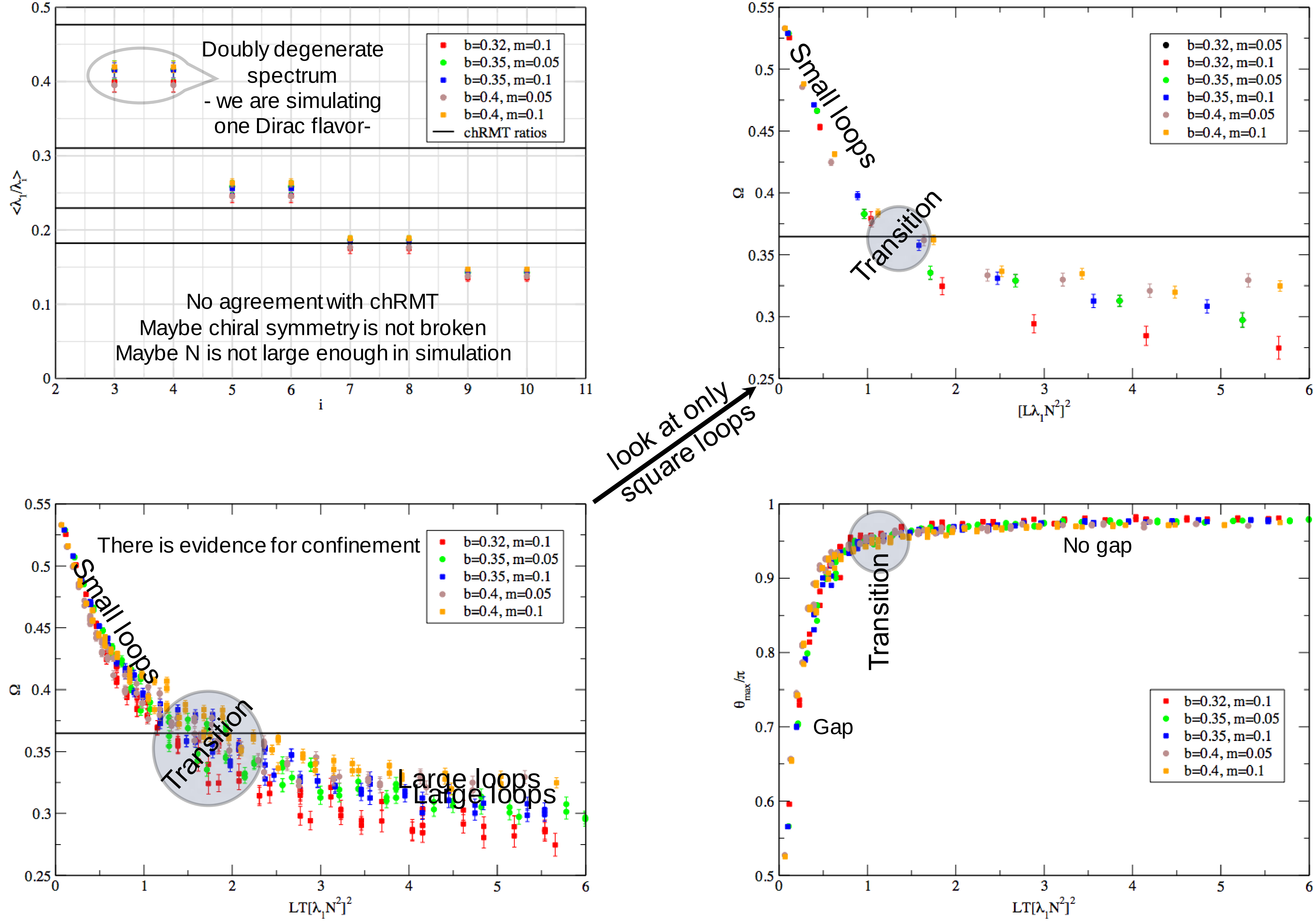
Numerical tests of scaling



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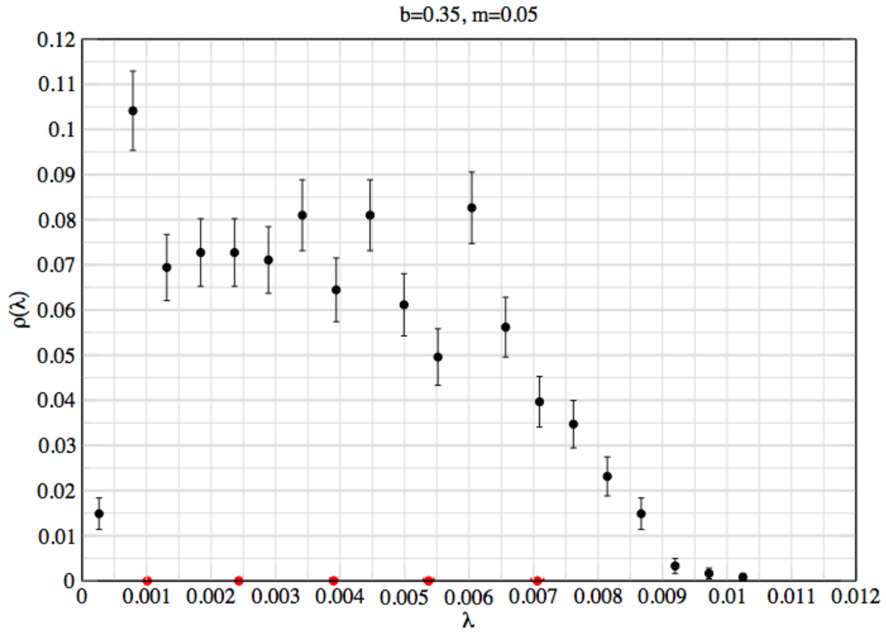
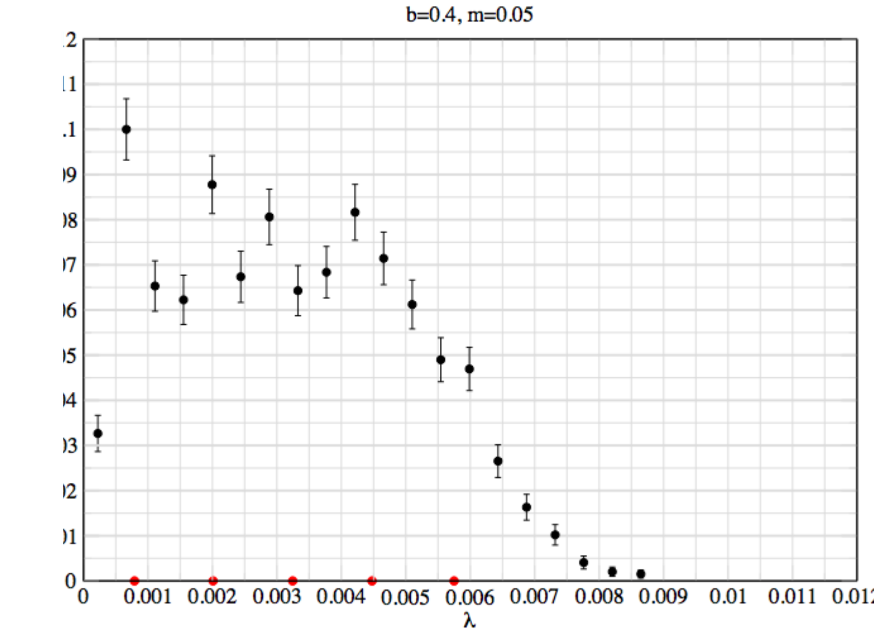
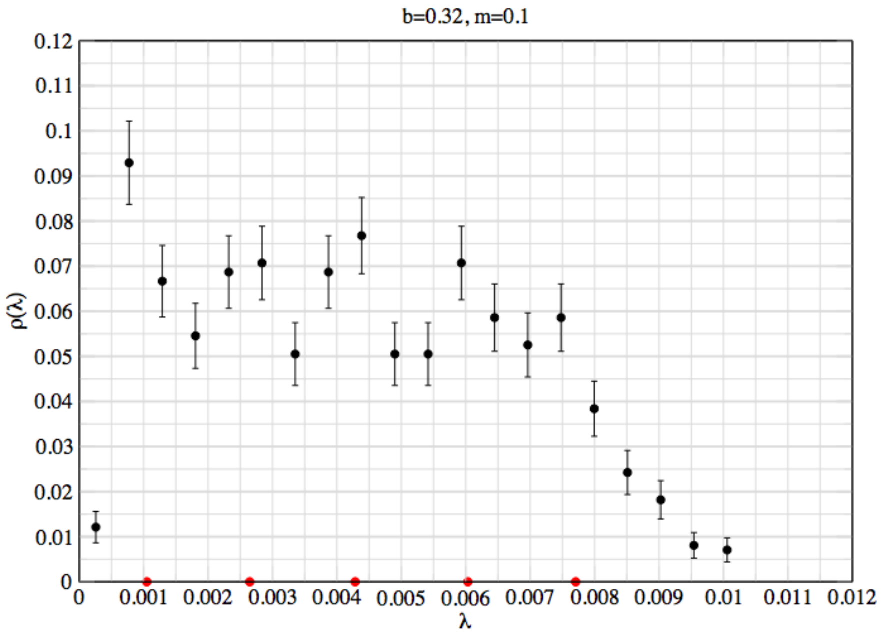
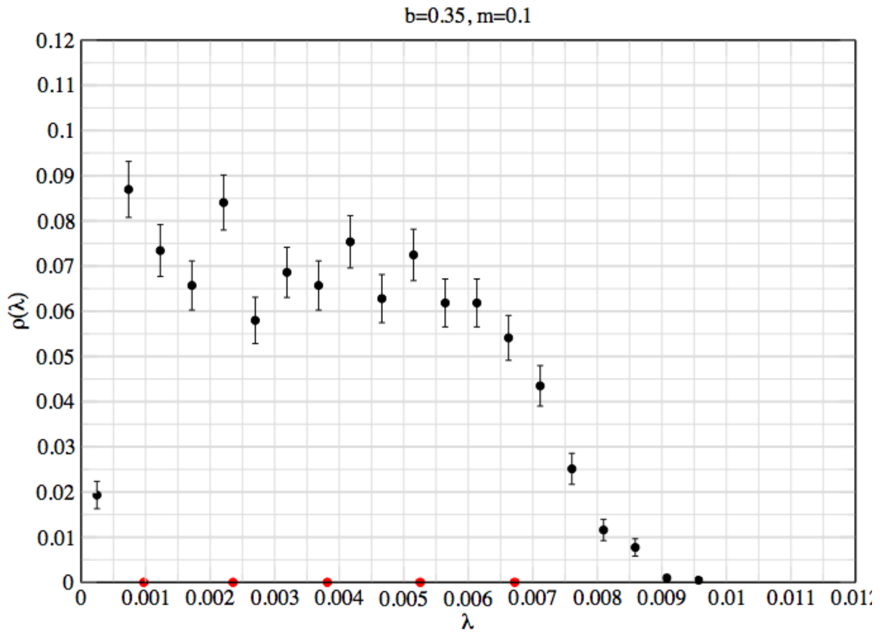
Is chiral symmetry broken?

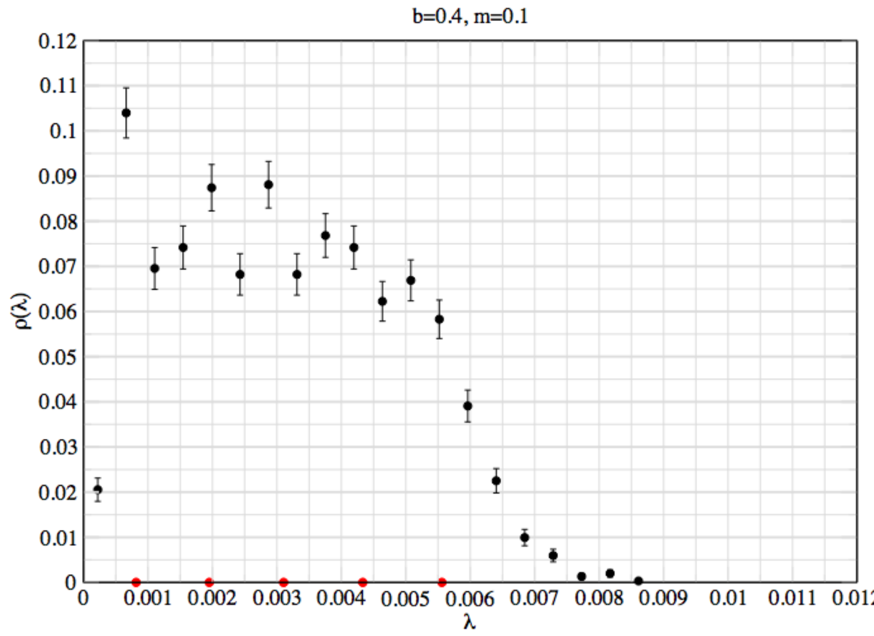
- Look at the distribution of lowest eigenvalues of the adjoint overlap Dirac operator
- Red dots are the averages of the five lowest distinct eigenvalues
- Not enough statistics to see clean peaks in the distribution associated with the five eigenvalues
- Is there a flattening of the distribution as one approaches zero eigenvalue? May be.
- But ratios of eigenvalues do not match predictions from chiral random matrix theory.











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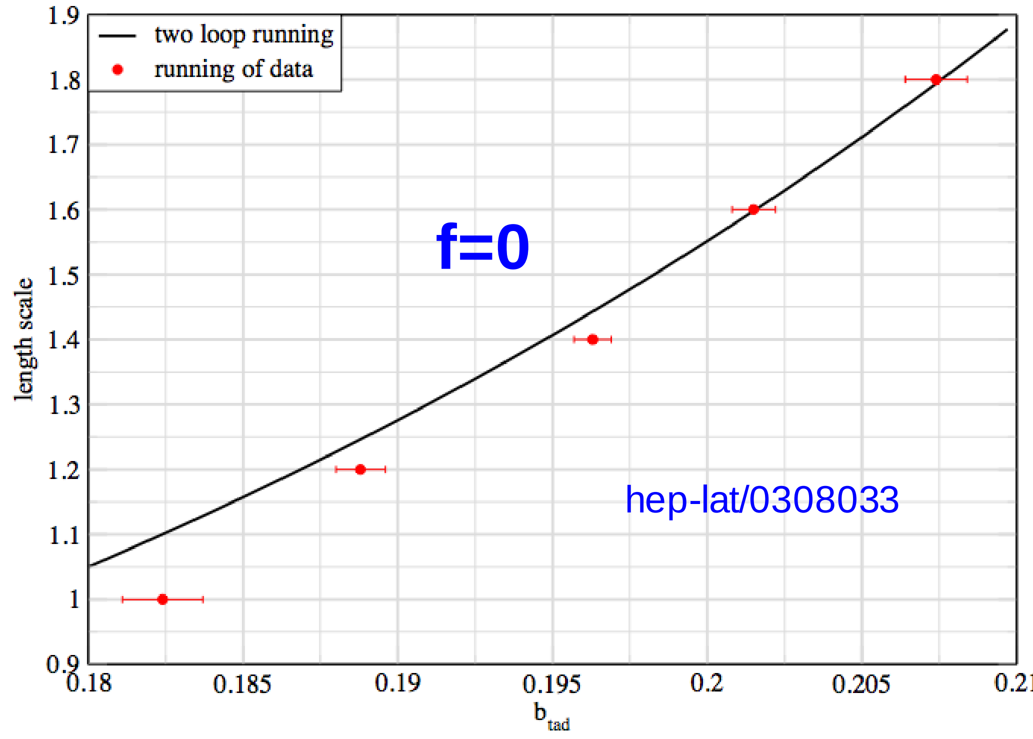
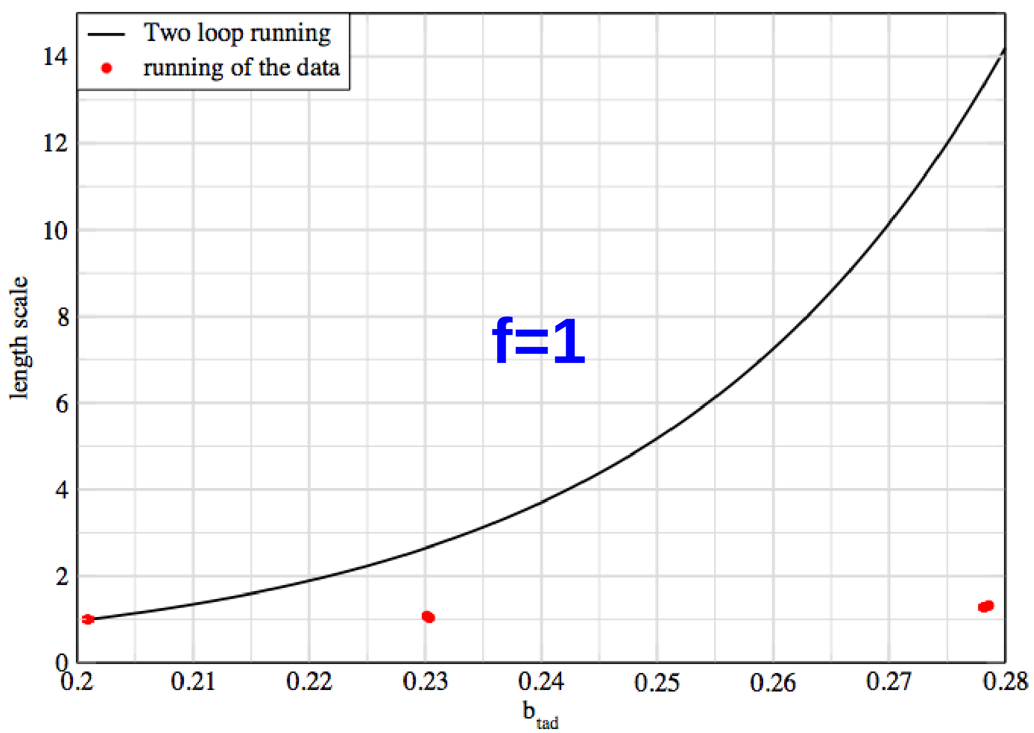
Does the theory with f=1 walk or run?

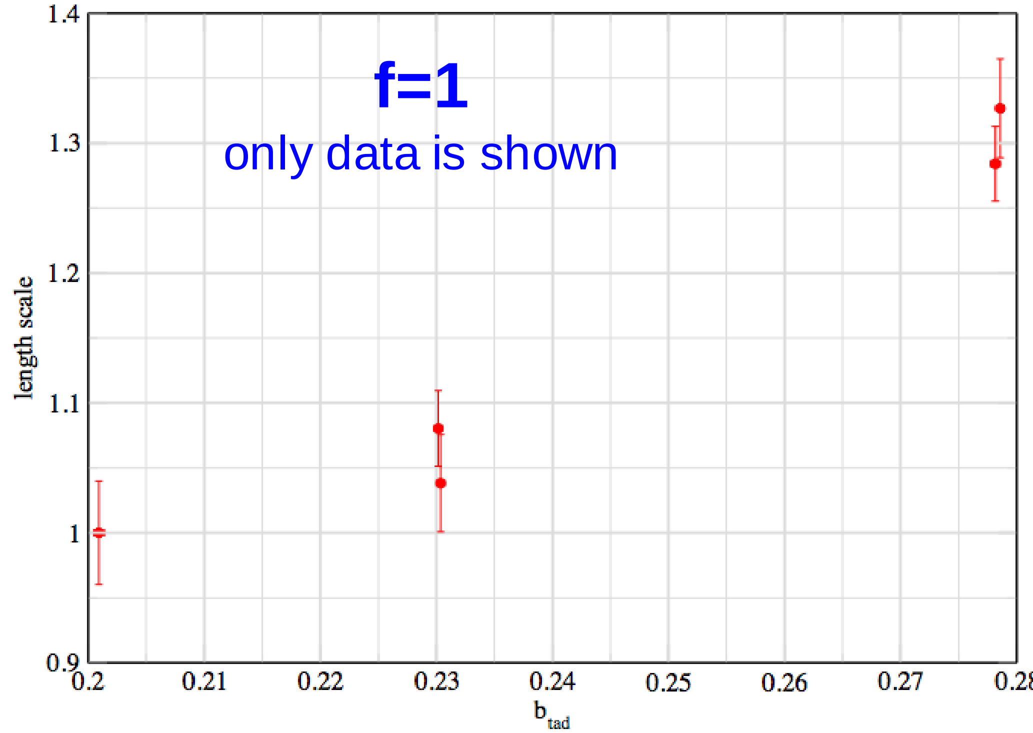
Two loop beta function

$$\alpha = \frac{1}{16\pi^2 b}$$

$$a\frac{d\alpha}{da} = 2\left(\frac{11}{3} - \frac{4}{3}f\right)\alpha^2 + 2\left(\frac{34}{3} - \frac{32}{3}f\right)\alpha^3 + \cdots$$







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