

Testing Inflation with Dark Matter Halos

Primordial non-Gaussianity in large-scale structure

Marilena Loverde (Institute for Advanced Study)

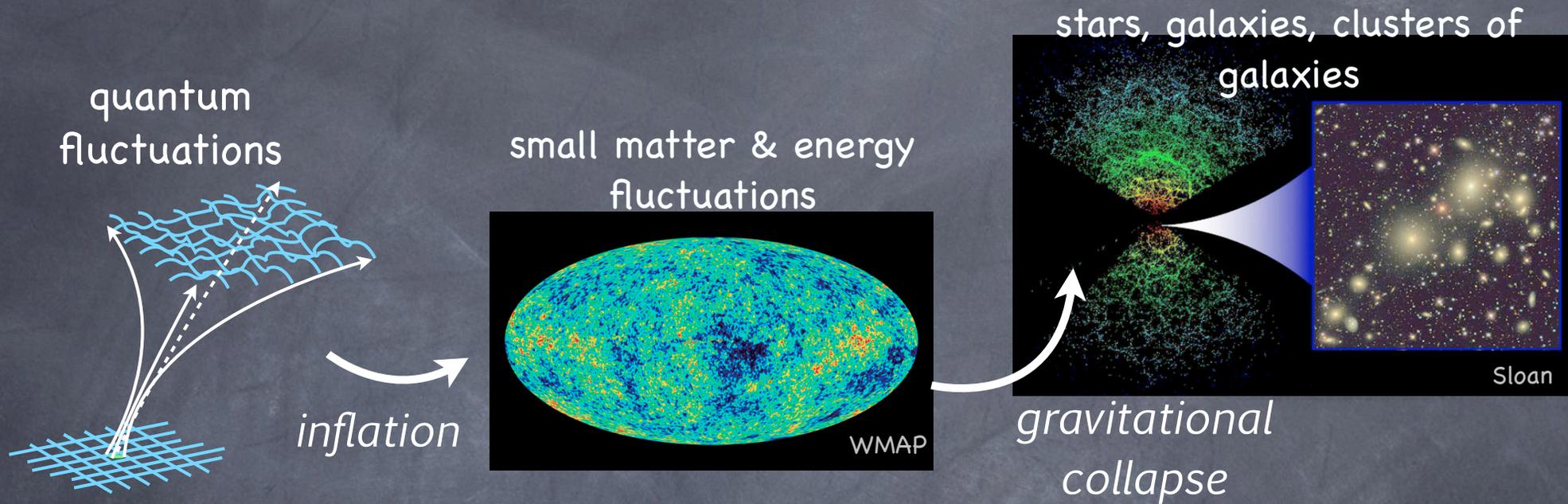
K. M. Smith & ML arXiv: 1010.0055, ML & K.M. Smith arXiv:1102.1439
K. M. Smith, S. Ferraro, & ML arXiv:1106.0503

ML is grateful for support from the Friends of the Institute for Advanced Study

Outline

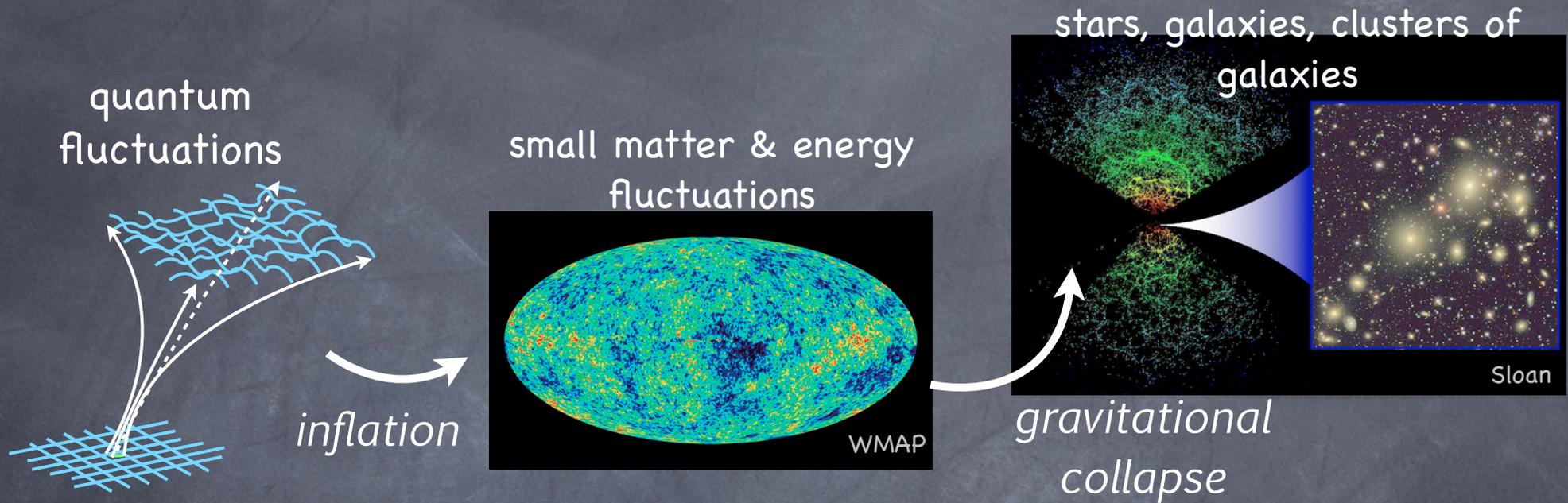
- Inflation as the origin of structure
- Statistics of primordial perturbations
- Impact of (primordial) non-Gaussian statistics on large-scale structure: Analytics & N-body simulations
- Conclusion & Outlook

Inflation as the origin of structure



$$\delta\varphi_{\text{inflaton}} \longrightarrow \Phi_{\text{curvature}} \rightsquigarrow \begin{matrix} \delta T_{\text{CMB}} \\ \delta\rho_{\text{matter}} \\ \delta n_{\text{galaxies}} \end{matrix}$$

Inflation as the origin of structure



probe of this era

$\delta\varphi_{\text{inflaton}}$



$\Phi_{\text{curvature}}$



statistics of these

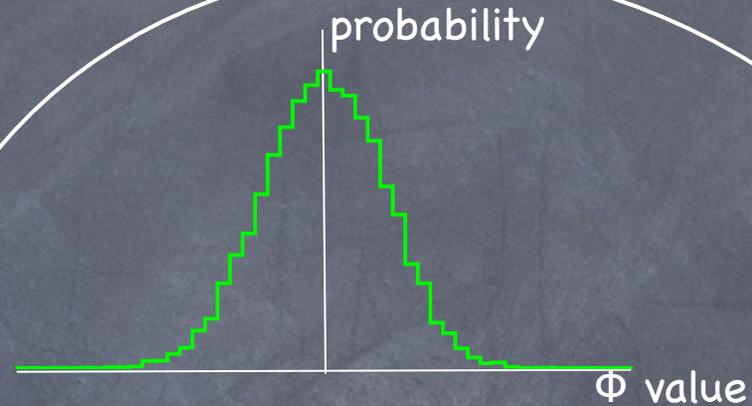
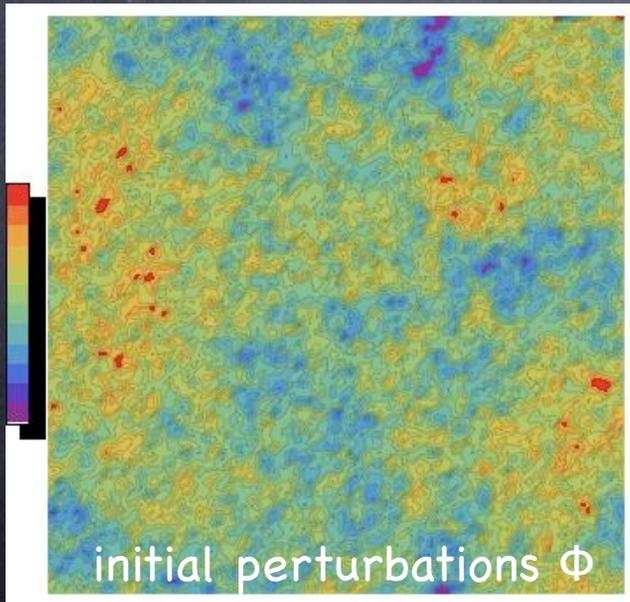
δT_{CMB}

$\delta\rho_{\text{matter}}$

$\delta n_{\text{galaxies}}$



What can we learn from the statistics of perturbations?



$$\langle \Phi(x)\Phi(y) \rangle \leftrightarrow P_{\Phi}(k)$$

$$\langle \Phi(x)\Phi(y)\Phi(z) \rangle \leftrightarrow B_{\Phi}(k_1, k_2, k_3)$$

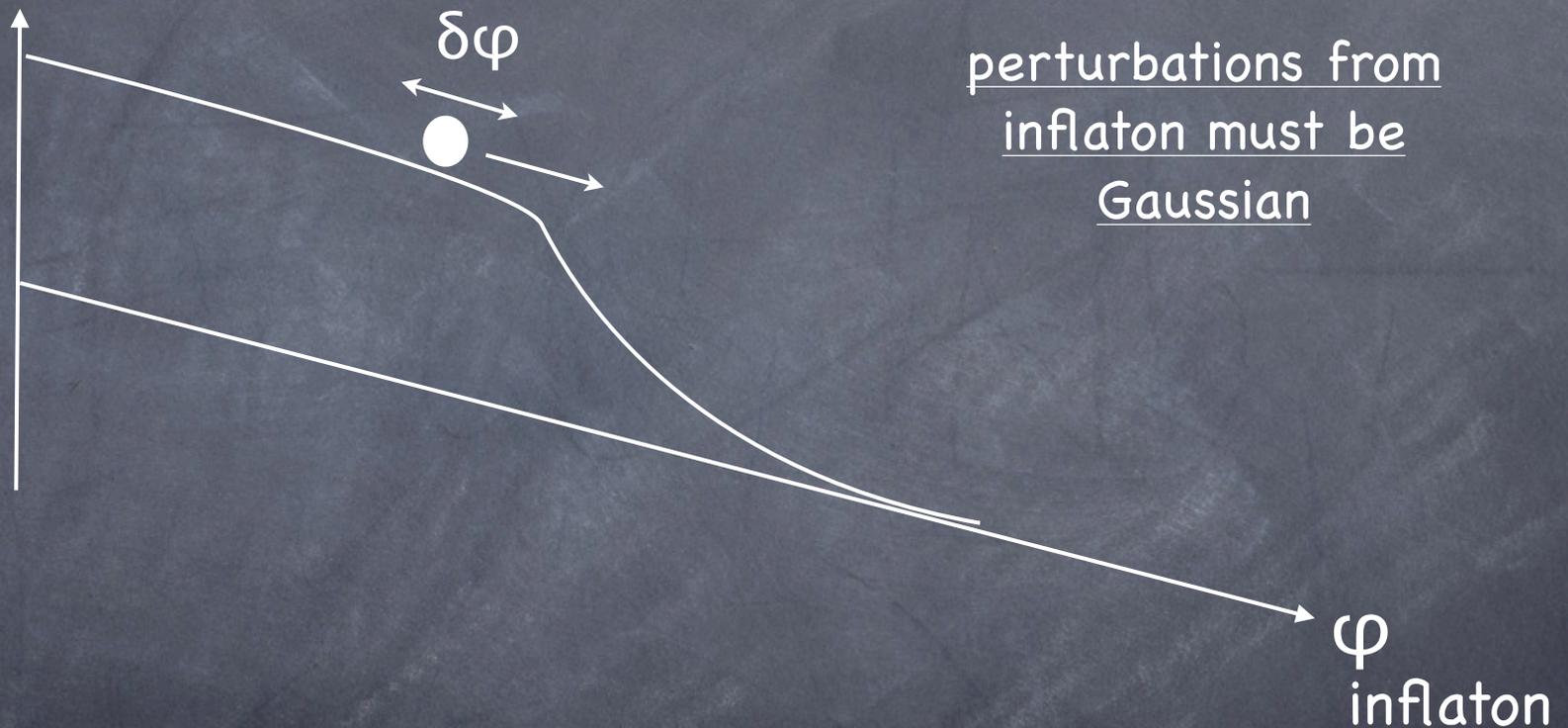
...

For instance

total energy dominated by inflaton:

$$H^2 = 8\pi G/3 V(\varphi, \sigma)$$

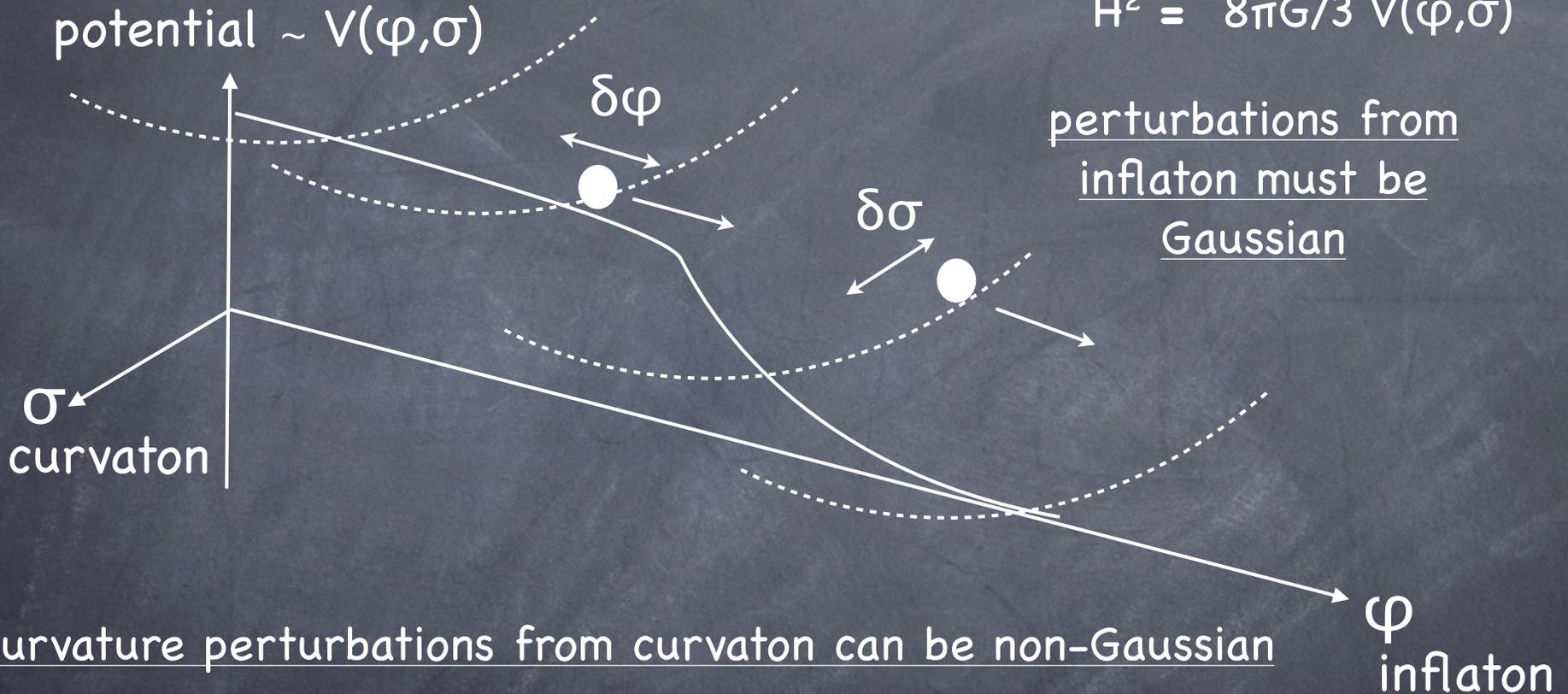
potential $\sim V(\varphi)$



For instance

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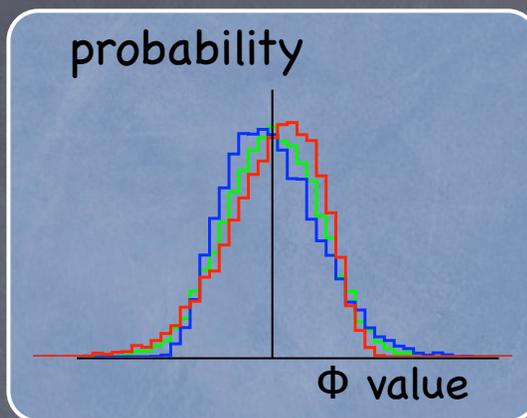


$$\Phi \sim \delta\sigma + \delta\sigma^2$$

$$\Phi \sim \delta\sigma + \delta\sigma^3 + \dots$$

$$\Phi \sim \delta\varphi + \delta\sigma + \delta\sigma^2 + \dots$$

" f_{NL} " $\Phi \sim \delta\sigma + f_{NL} \delta\sigma^2$



skewness $\sim f_{NL}$

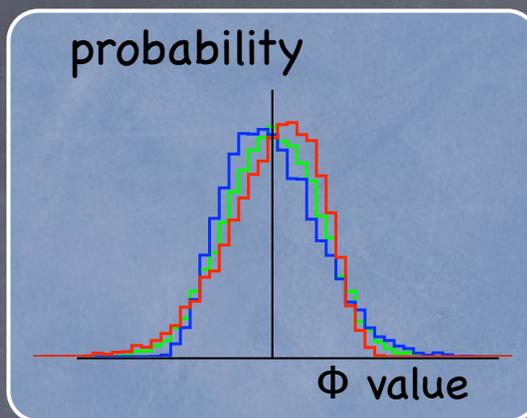
kurtosis $\sim f_{NL}^2$

...

(Φ =primordial gravitational potential)

Φ value

" f_{NL} " $\Phi \sim \delta\sigma + f_{NL} \delta\sigma^2$

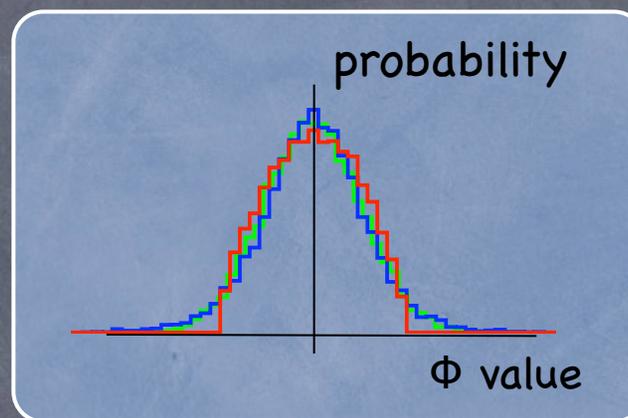


skewness $\sim f_{NL}$

kurtosis $\sim f_{NL}^2$

...

" g_{NL} " $\Phi \sim \delta\sigma + g_{NL} \delta\sigma^3 + \dots$



skewness ~ 0

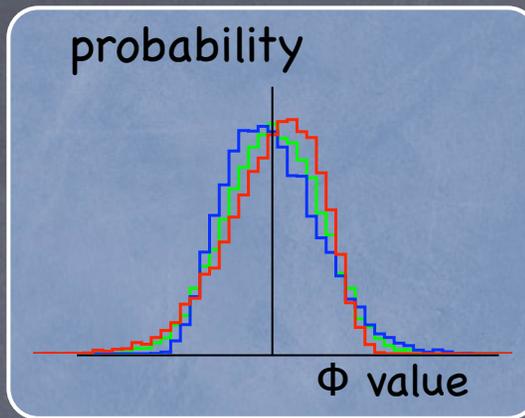
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_____ Φ value

(Φ =primordial gravitational potential)

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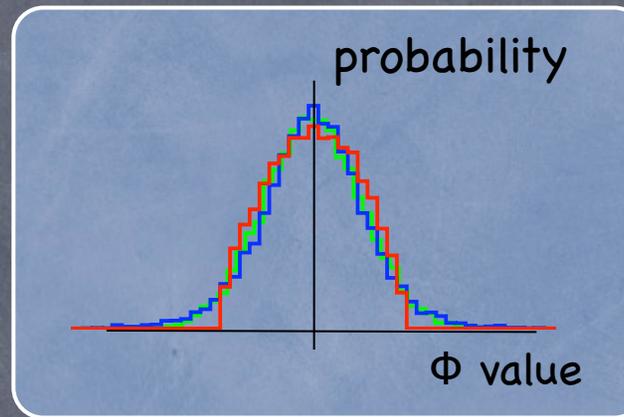


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" g_{NL} " $\Phi \sim \delta\sigma + g_{NL} \delta\sigma^3 + \dots$



skewness ~ 0

kurtosis $\sim g_{NL}$

...

" T_{NL} " $\Phi \sim \delta\varphi + \delta\sigma + \tilde{f}_{NL} \delta\sigma^2 + \dots$

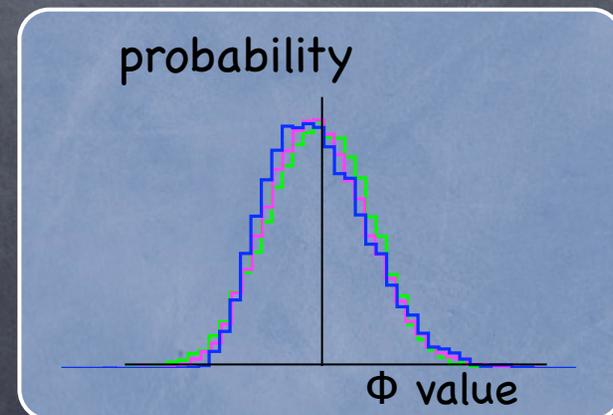
$\tilde{f}_{NL} = f_{NL}(1 + P_{\varphi\varphi}/P_{\sigma\sigma})^2$

$T_{NL} = f_{NL}^2(1 + P_{\varphi\varphi}/P_{\sigma\sigma})$

and $P_{\varphi\sigma} = 0$

skewness $\sim f_{NL}$

kurtosis $\sim T_{NL}$



(Φ =primordial gravitational potential)

$$\text{"}f_{\text{NL}}\text{" } \Phi \sim \delta\sigma + f_{\text{NL}} \delta\sigma^2$$

$$\langle \Phi_{\text{short}}^2 \rangle = \langle \sigma_{\text{G,short}}^2 \rangle (1 + 4 f_{\text{NL}} \sigma_{\text{G,long}}(\mathbf{x}))$$



small-scale power depends on large-scale fluctuations!

(Φ =primordial gravitational potential)

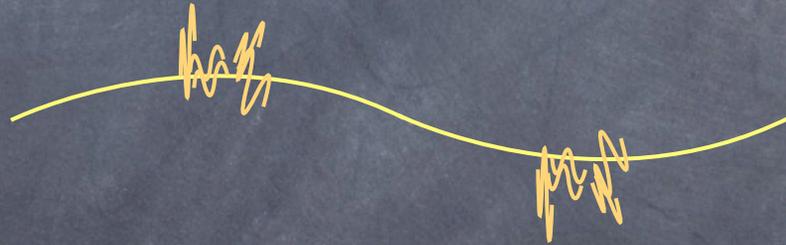
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" g_{NL} " $\Phi \sim \delta\sigma + g_{\text{NL}} \delta\sigma^3 + \dots$

$$\langle \Phi_{\text{short}}^3 \rangle = 18 g_{\text{NL}} \langle \sigma_{\text{G,short}}^2 \rangle^2 \sigma_{\text{G,long}}(\mathbf{x}) \equiv f_{\text{NL}}^{\text{eff}}(\mathbf{x}) \langle \sigma_{\text{G,short}}^2 \rangle^2$$



(Φ =primordial gravitational potential)

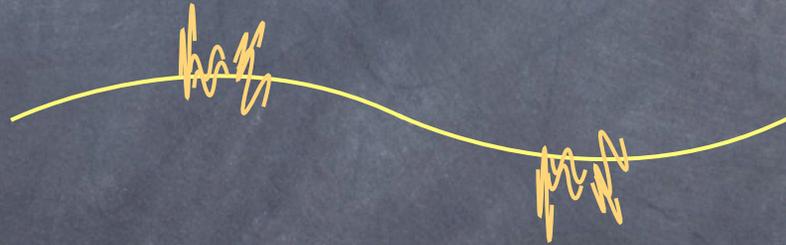
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" T_{NL} " $\Phi \sim \delta\varphi + \delta\sigma + \tilde{f}_{NL} \delta\sigma^2 + \dots$

$$\langle \Phi_s^2 \rangle = \langle \Phi_{G,\text{short}}^2 \rangle (1 + 4 \tilde{f}_{NL} \sigma_{G,\text{long}}(\mathbf{x}))$$



(Φ =primordial gravitational potential)

These “local” models (i.e. $\Phi_{\text{NG}}(\mathbf{x})=F(\sigma_{\text{G}}(\mathbf{x}))$) of non-Gaussianity are just some examples

Can also get non-Gaussianity from self interactions of inflaton (e.g. k-inflation, DBI, . . .)

but not with such extreme couplings of perturbations on short and long length scales

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BUT, single-field inflation requires

$$\langle \Phi(\mathbf{k})\Phi(\mathbf{k}')\Phi(\mathbf{k}'' \rightarrow 0) \rangle \approx f_{\text{NL}}(2\pi)^3 \delta(\mathbf{k}+\mathbf{k}') P_{\Phi}(\mathbf{k}) P_{\Phi}(\mathbf{k}'') \approx 0$$

so $f_{\text{NL}} \gtrsim$ few rules out single field inflation

Acquaviva, Bartolo, Matarrese, Riotto 2003; Maldacena 2003; Creminelli & Zaldarriaga 2004

(similar arguments hold for g_{NL} , T_{NL})

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Acquaviva, Bartolo, Matarrese, Riotto 2003; Maldacena 2003; Creminelli & Zaldarriaga 2004
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$$-10 < f_{\text{NL}} < 74$$

WMAP, Komatsu et al 2010

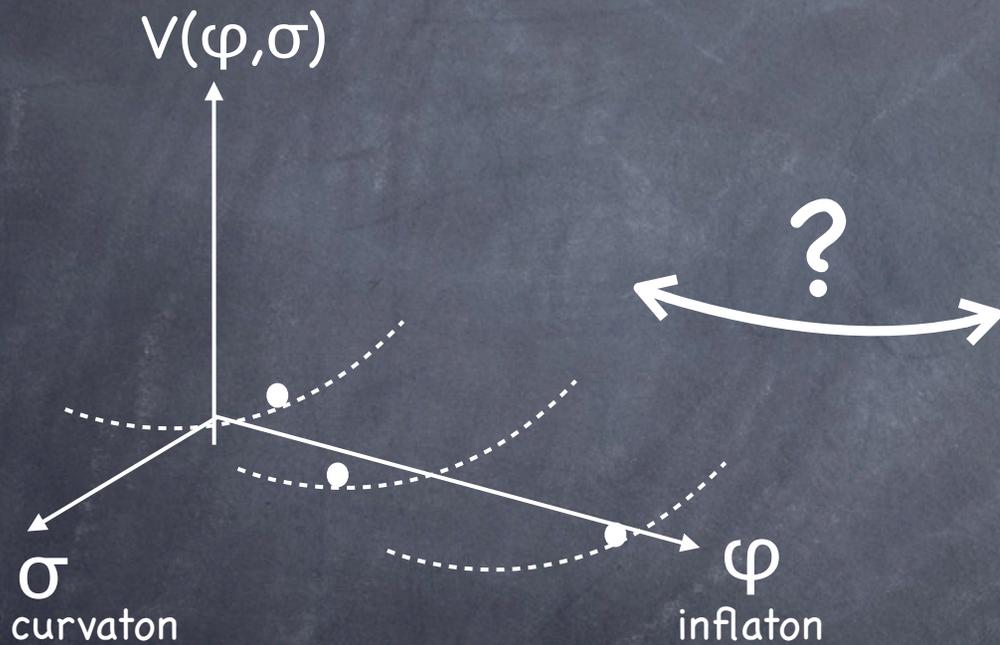
$$-12 < g_{\text{NL}} / 10^5 < 16$$

(WMAP, Fergusson et al 2010)

$$-6000 < \tau_{\text{NL}} < 33,000$$

(WMAP, Smidt et al 2010)

How does the large-scale halo distribution see primordial non-Gaussianity?



Signatures in LSS I: more/fewer massive halos

dark matter halos form in peaks of the density field



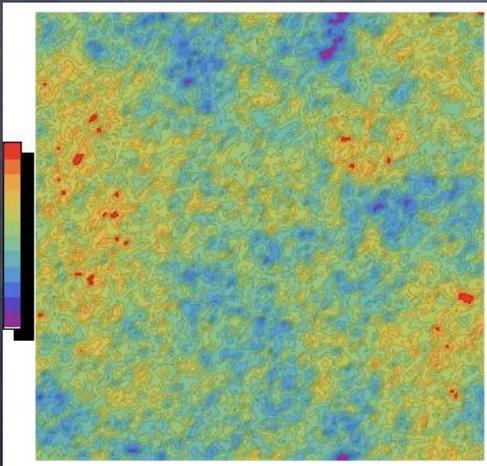
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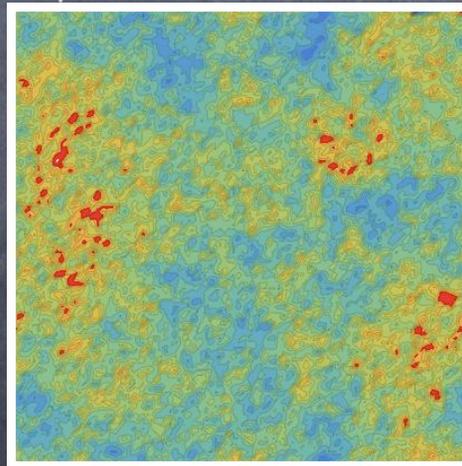


non-Gaussianity changes the number density of **peaks**

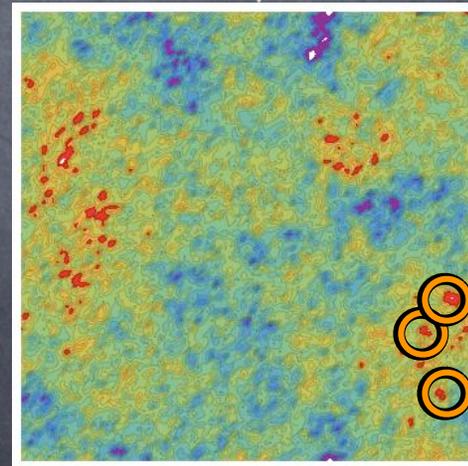
Gaussian



positive skewness



no skewness, positive kurtosis



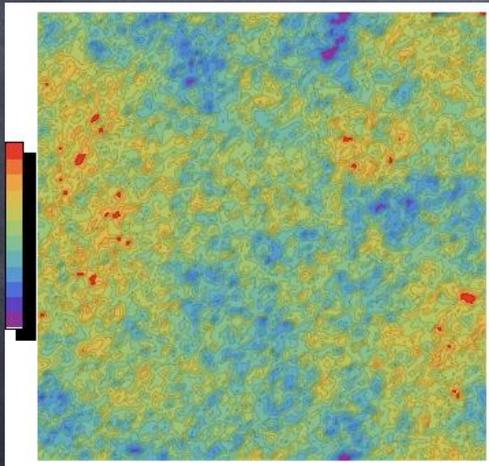
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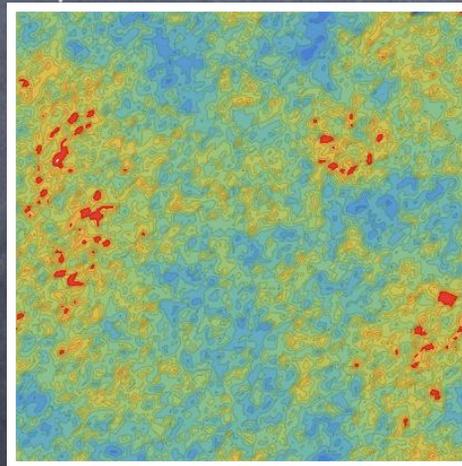


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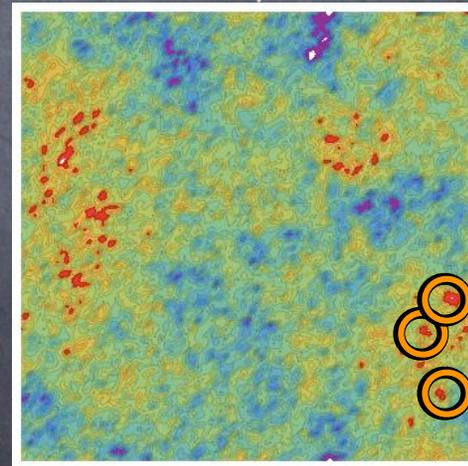
Gaussian



positive skewness



no skewness, positive kurtosis



number of peaks \Leftrightarrow number of dark matter halos

Lucchin & Matarrese 1988; Chiu, Ostriker, Strauss 1998; Robinson, Gawiser, Silk 2000

Signatures in LSS II: scale-dependent halo bias

a dark matter halo forms when $\delta\rho/\rho$ is larger than the collapse threshold

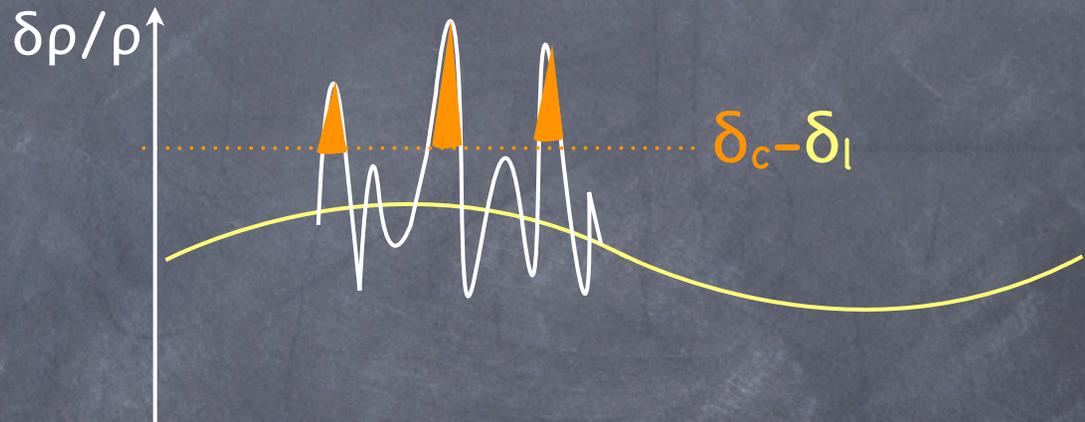


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which is easier to reach on top of a **long wavelength** density perturbation

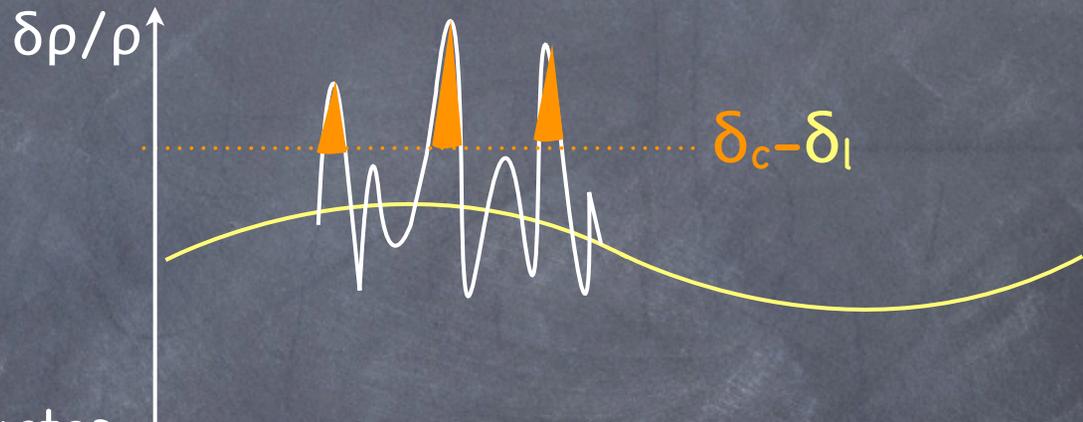


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so the number of halos fluctuates depending on δ_l

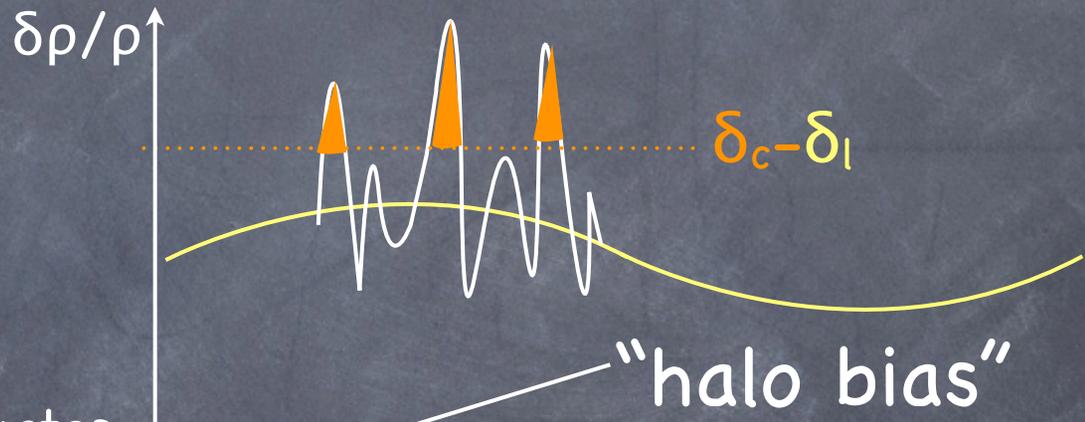
$$\delta n = \frac{\partial n}{\partial \delta} \delta_l \dots$$

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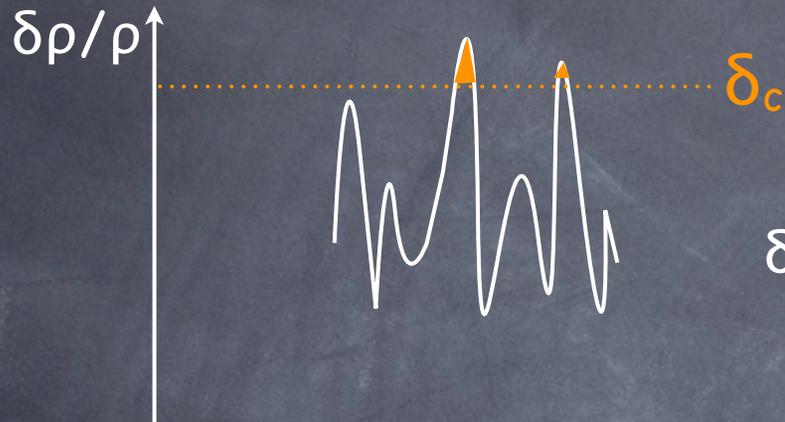
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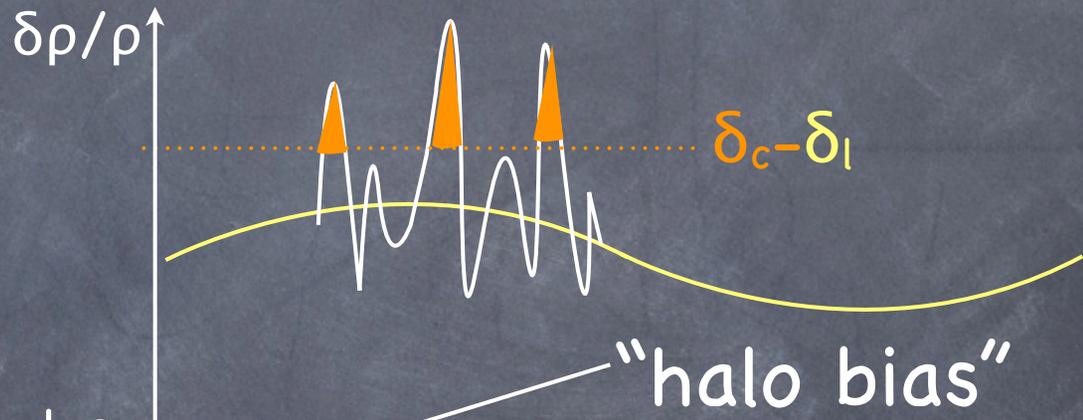
"halo bias"

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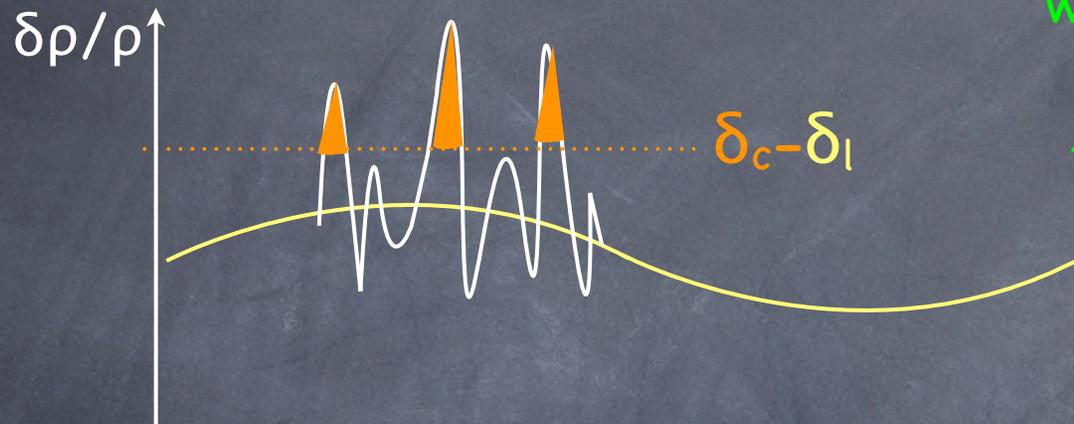
so the number of halos fluctuates depending on δ_l

$$\delta n = \frac{\partial n}{\partial \delta} \delta_l \dots$$

but with f_{NL} non-Gaussianity, the small-scale power also fluctuates depending on Φ_l

Signatures in LSS II: scale-dependent halo bias

a dark matter halo forms when $\delta\rho/\rho$ is larger than the collapse threshold



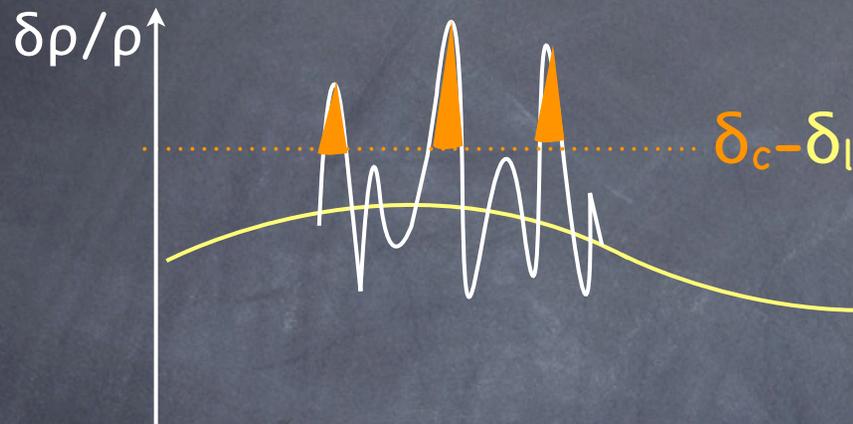
with f_{NL} non-Gaussianity, the small-scale power P_s also fluctuates depending on Φ_l

so the number of halos fluctuates depending on δ_l and Φ

$$\delta n = \frac{\partial n}{\partial \delta} \delta_l + 4f_{NL} \frac{\partial n}{\partial P_s} \Phi_l \dots$$

Signatures in LSS II: scale-dependent halo bias

a dark matter halo forms when $\delta\rho/\rho$ is larger than the collapse threshold



with f_{NL} non-Gaussianity, the small-scale power P_s also fluctuates depending on Φ_l

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$$\nabla^2\Phi_l \sim 4\pi G \delta_l$$

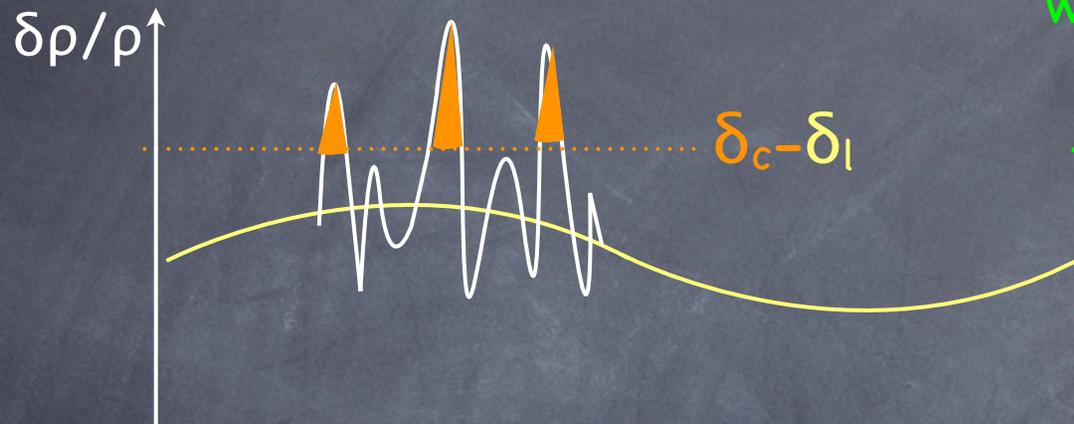
$$\delta n = \frac{\partial n}{\partial \delta} \delta_l + 4f_{NL} \frac{\partial n}{\partial P_s} \Phi_l \dots$$

$$\approx \left(\frac{\partial n}{\partial \delta} + 4f_{NL} \frac{\partial n}{\partial P_s} / k^2 \right) \delta_l(k) \dots$$

bias depends on Fourier scale k

Signatures in LSS II: scale-dependent halo bias

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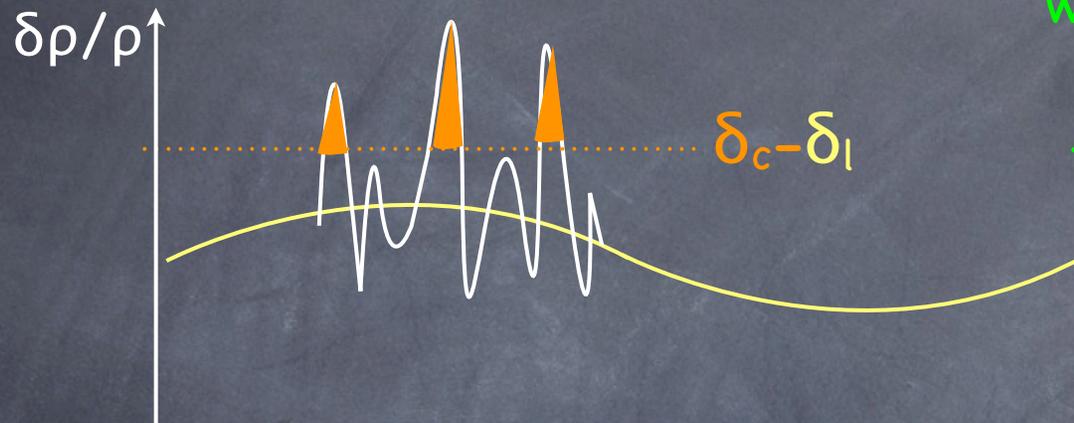
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this $1/k^2$ scaling is hard to generate with local (post-inflationary) processes

Signatures in LSS II: scale-dependent halo bias

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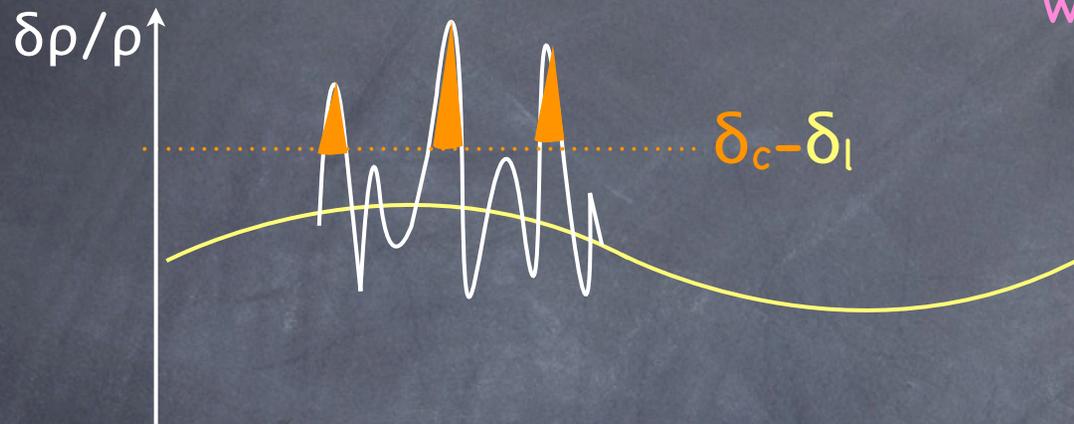
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this $1/k^2$ scaling is hard to generate with local (post-inflationary) processes \longrightarrow powerful test!

Signatures in LSS II: scale-dependent halo bias

a dark matter halo forms when $\delta\rho/\rho$ is larger than the collapse threshold



with g_{NL} non-Gaussianity, the small-scale skewness fluctuates with Φ_l

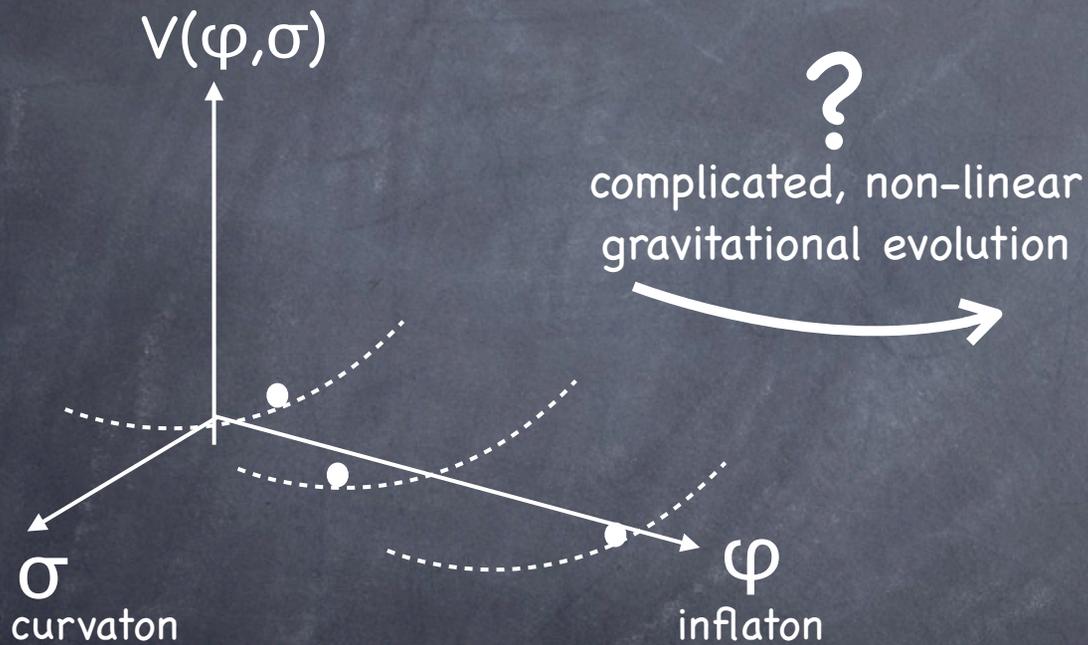
so the number of halos fluctuates depending on δ_l and Φ

$$\delta n = \frac{\partial n}{\partial \delta} \delta_l + 18g_{NL} \frac{\partial n}{\partial S_3} \Phi_l \dots$$

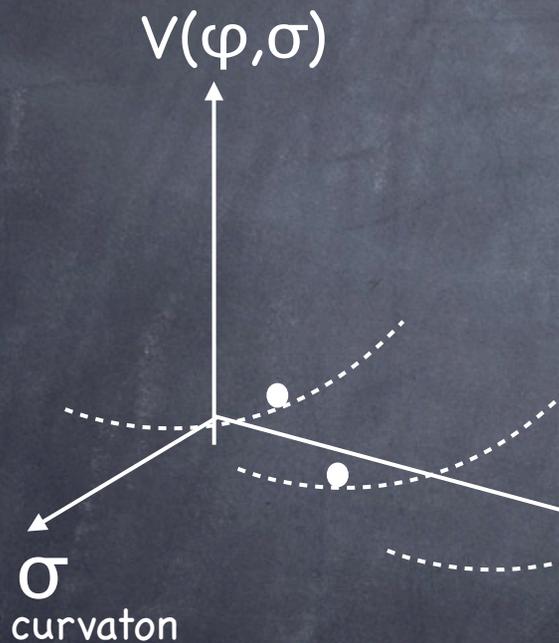
$$\nabla^2 \Phi_l \sim 4\pi G \delta_l \quad \longrightarrow \quad \approx \left(\frac{\partial n}{\partial \delta} + 18g_{NL} \frac{\partial n}{\partial S_3} / k^2 \right) \delta_l(k) \dots$$

bias depends on Fourier scale k

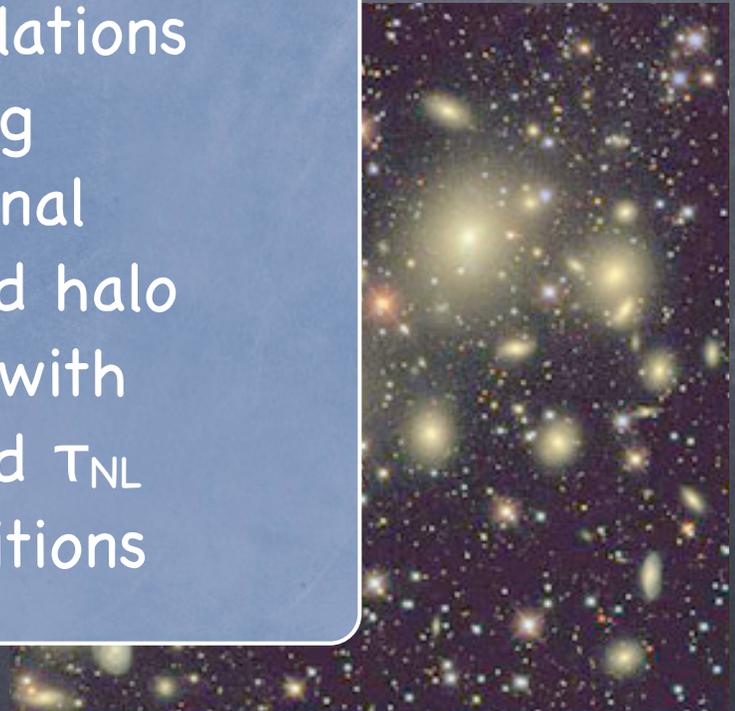
Can we model the non-Gaussian effects on halos reliably?



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N-body simulations modeling gravitational evolution and halo formation with f_{NL} , g_{NL} , and τ_{NL} initial conditions

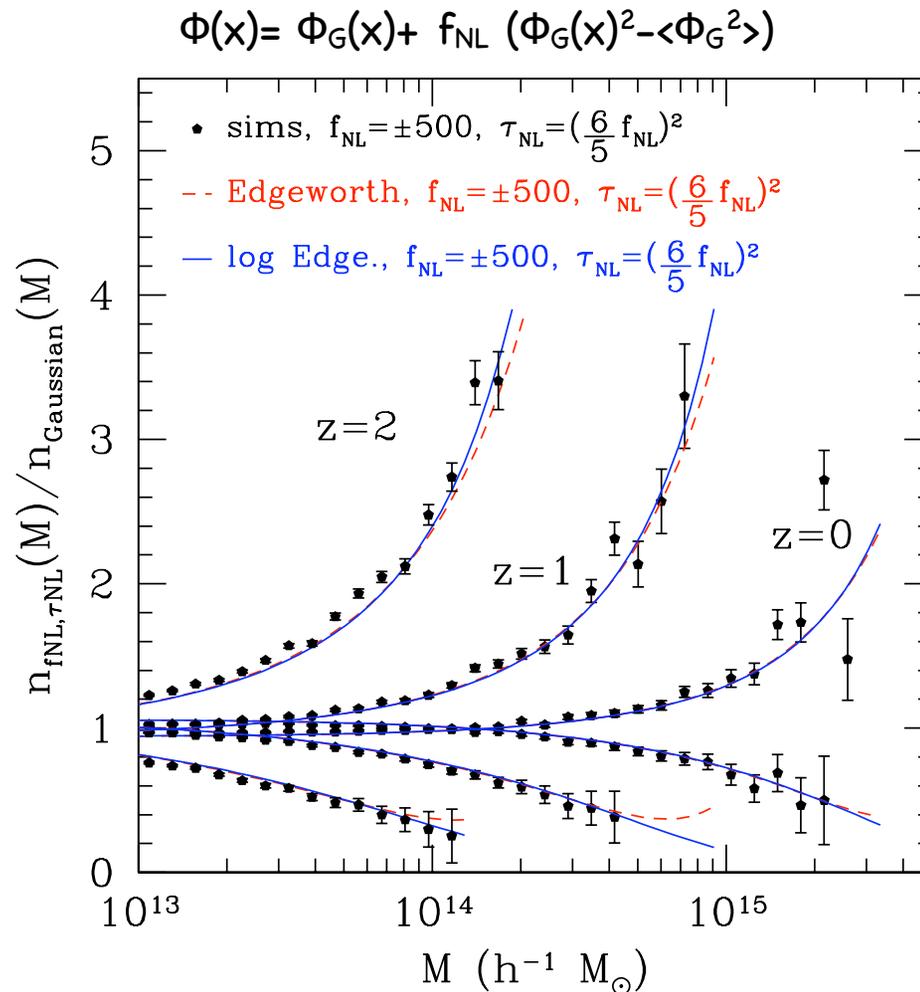


Signatures in LSS I: more/fewer massive halos

N-body simulations with f_{NL} , g_{NL} , and τ_{NL}

f_{NL}

of halos relative to Gaussian cosmology



curves are simple models of halo abundance (Press-Schechter + spherical collapse + approximate expressions for P.D.F. of δ)

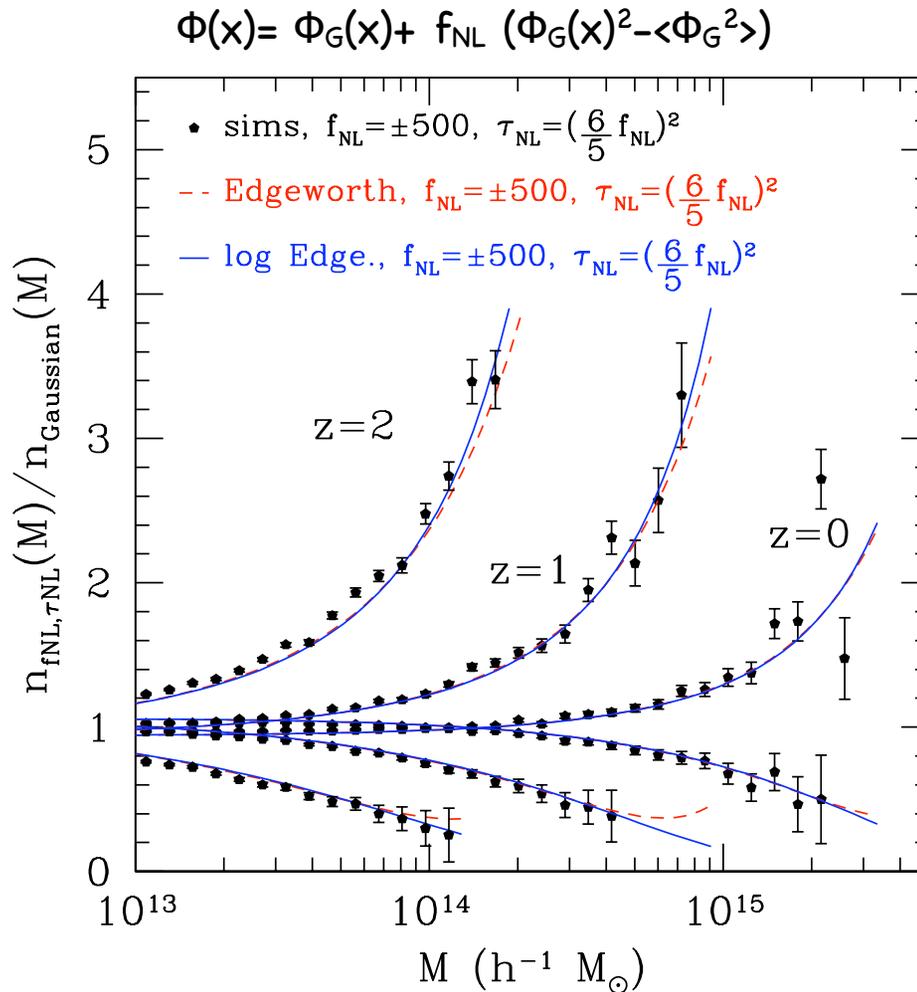
see also Dalal, Dore, Huterer, Shirokov 2007; Grossi et al 2009; Kang, Norberg, Silk 2009; Pillepich, Porciani, Hahn 2009 ; many others

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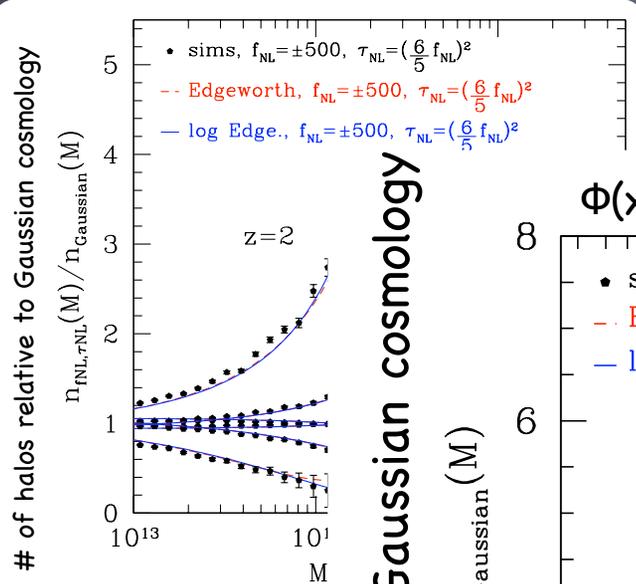
curves are simple models of halo abundance (Press-Schechter + spherical collapse + approximate expressions for P.D.F. of δ)

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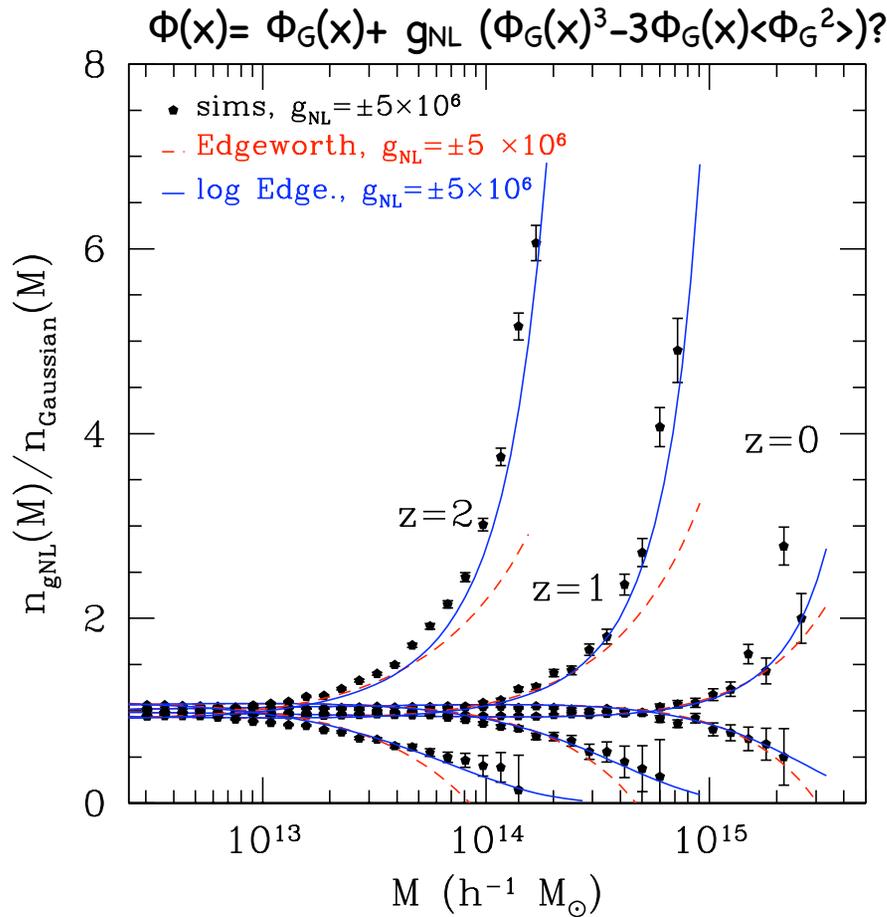
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Signatures in LSS I: more/fewer massive halos

N-body simulations with f_{NL} , g_{NL} , and τ_{NL}

$f_{\text{NL}}, \tau_{\text{NL}}$ independent

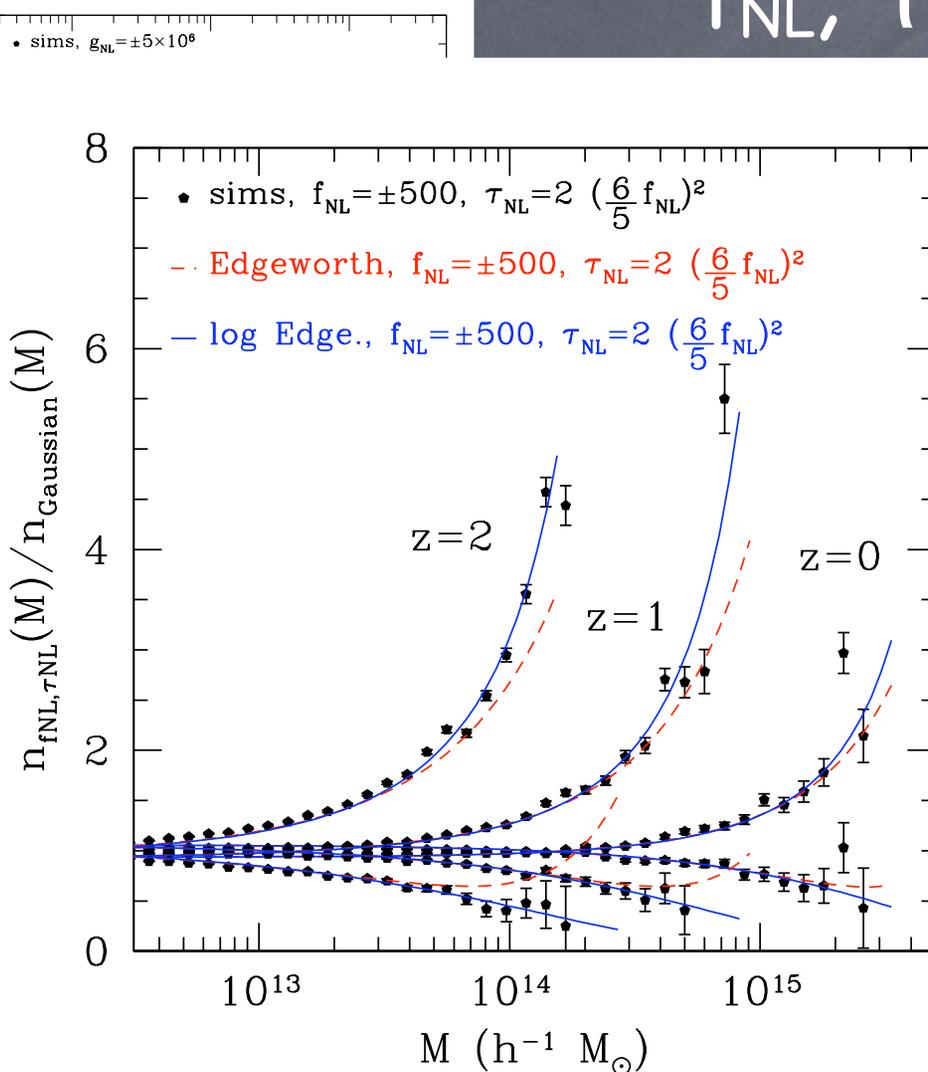
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of halos relative to Gaussian cosmology

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Signatures in LSS I: more/fewer massive halos

N-body simulations with f_{NL} , g_{NL} , and τ_{NL}

$f_{\text{NL}}, \tau_{\text{NL}}$ independent

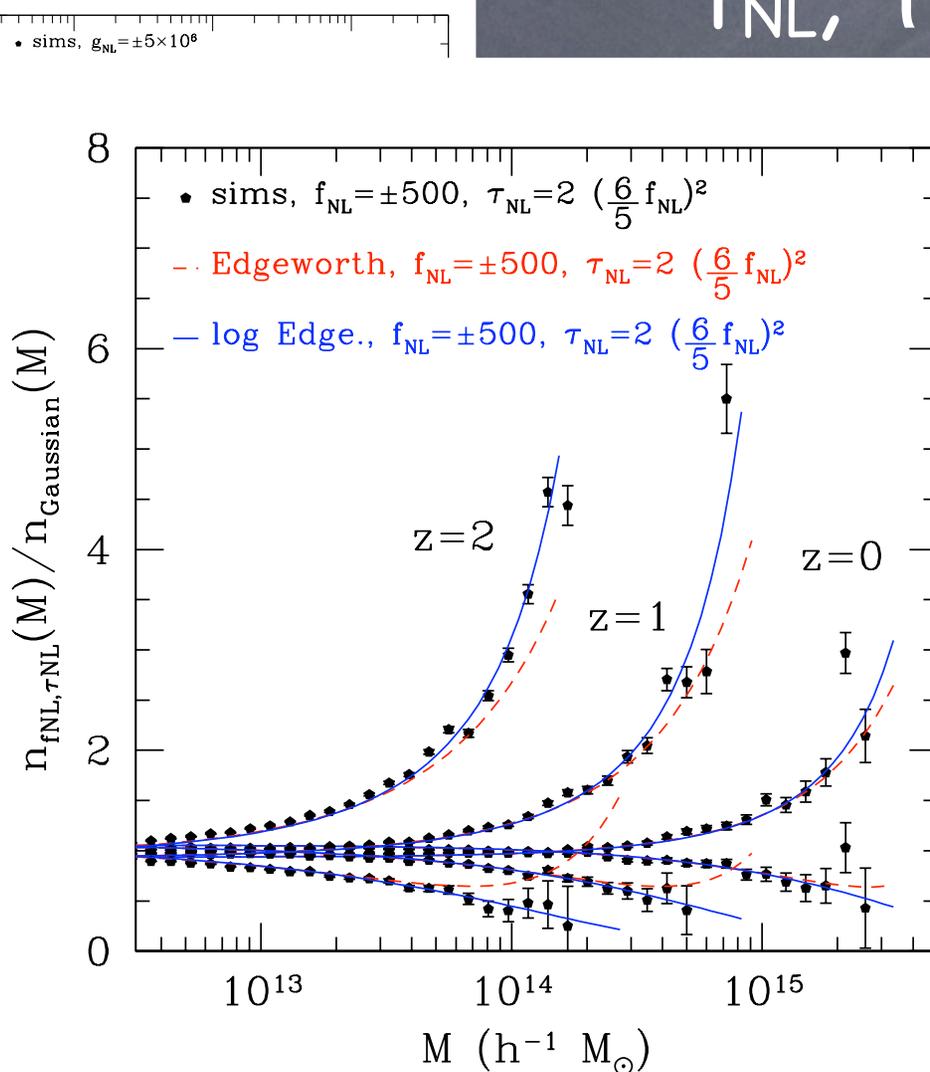
the old mass function is ok for f_{NL}

the new mass function works for $f_{\text{NL}}, g_{\text{NL}}$, and τ_{NL} types of non-Gaussianity

of halos relative to Gaussian cosmology

of halos relative to Gaussian cosmology

of halos relative to Gaussian cosmology



Signatures in LSS II: scale-dependent halo bias

$$\text{Recall, } \nabla^2 \Phi_l \sim 4\pi G \delta_l \text{ or } k^2 \Phi_l(k) \sim 4\pi G \delta_l(k)$$

$$\text{so on large scales } \delta n \approx \left(\frac{\partial n}{\partial \delta} + 4f_{\text{NL}} \frac{\partial n}{\partial P_s} / k^2 \right) \delta_l(k) \dots$$

OR

$$P_{n\delta}(k) \approx \left(\frac{\partial n}{\partial \delta} + 4f_{\text{NL}} \frac{\partial n}{\partial P_s} / k^2 \right) P_{\delta\delta}(k) \dots$$

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$$\text{so on large scales } \delta n \approx \left(\frac{\partial n}{\partial \delta} + 4f_{\text{NL}} \frac{\partial n}{\partial P_s} / k^2 \right) \delta_l(k) \dots$$

OR

$$P_{n\delta}(k) \approx \left(\frac{\partial n}{\partial \delta} + 4f_{\text{NL}} \frac{\partial n}{\partial P_s} / k^2 \right) P_{\delta\delta}(k) \dots$$

given a Gaussian mass function, we can calculate these derivatives and predict the coefficients

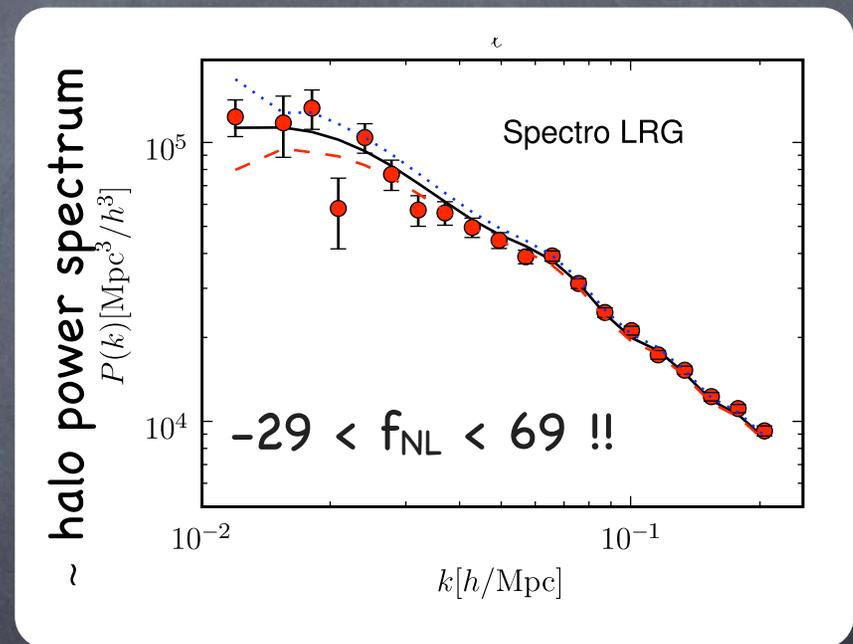
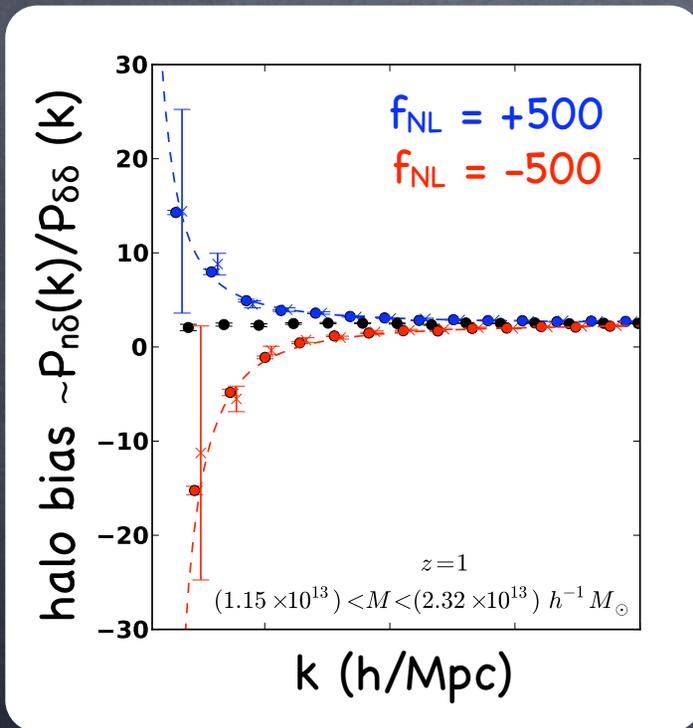
Signatures in LSS II: scale-dependent halo bias

Recall, $\nabla^2 \Phi_l \sim 4\pi G \delta_l$ or $k^2 \Phi_l(k) \sim 4\pi G \delta_l(k)$

so on large scales $\delta n \approx \left(\frac{\partial n}{\partial \delta} + 4f_{NL} \frac{\partial n}{\partial P_s} / k^2 \right) \delta_l(k) \dots$

AND IT WORKS!

example data



Slosar, Hirata, Seljak, Ho,
Padmanabhan 2008

Dalal, Doré, Huterer, Shirokov 2007

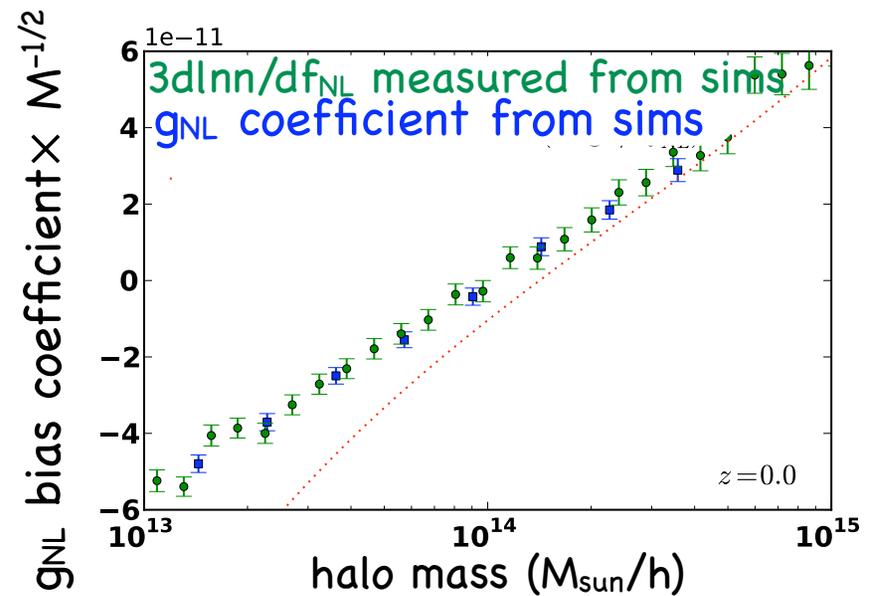
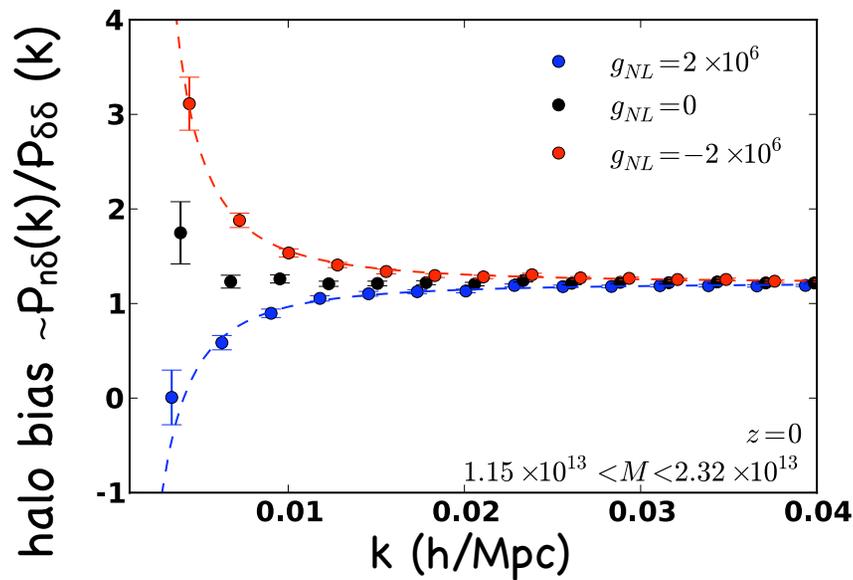
Pillepich, Porciani, Hahn 2008; Desjacques, Seljak, Iliev 2008; Grossi et al 2009

Signatures in LSS II: scale-dependent halo bias

And for g_{NL} :
$$\delta n \approx \left(\frac{\partial n}{\partial \delta} + 18g_{NL} \frac{\partial n}{\partial S_3} / k^2 \right) \delta_l(k) \dots$$

$$= \left(\frac{\partial n}{\partial \delta} + 3g_{NL} \frac{\partial n}{\partial f_{NL}} / k^2 \right) \delta_l(k) \dots$$

AND IT WORKS AGAIN!

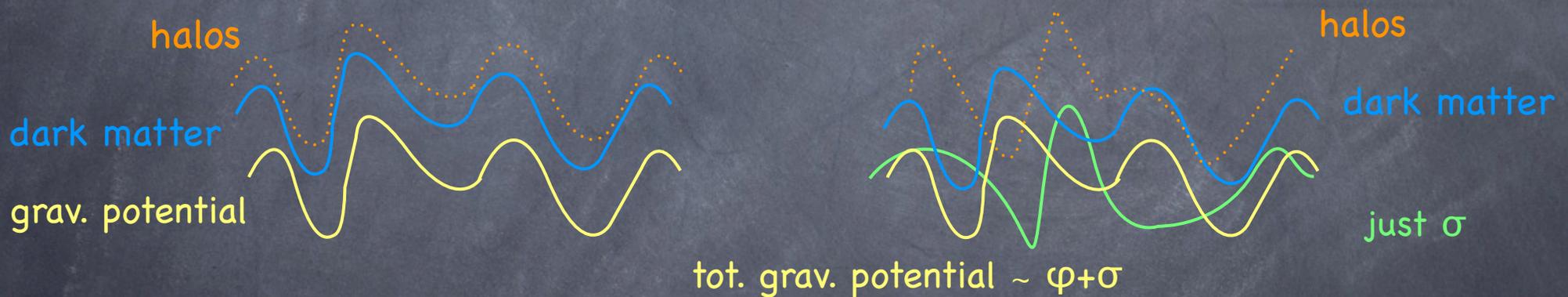


Signatures in LSS III: stochastic halo bias

f_{NL} , τ_{NL} non-Gaussianity gives both scale dependent bias and makes halo # fluctuations stochastic w.r.t. dark matter fluctuations

non-stochastic

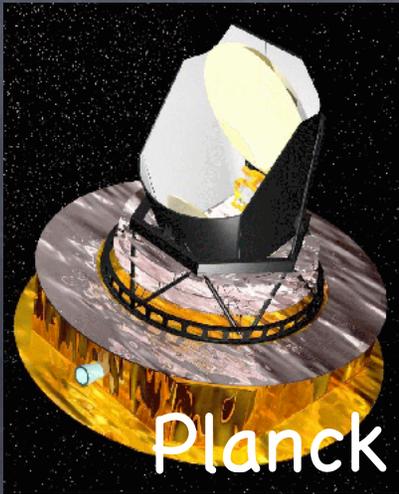
stochastic



Tseliakhovich, Hirata, Slosar 2010

τ_{NL} dependent stochasticity is present in N-body sims! But the predicted amplitude is not as accurate as f_{NL} and g_{NL} biases . . .

Smith & ML 2010



Planck



DES



HETDEX

lots of large-scale
structure data!



SPT



SDSS



ACT

. . . and more

see e.g. Shandera, Dalal, Huterer 2010
Oguri and Takada 2010
Carbone, Verde, Matarrese 2009 many more . . .

Summary

- Non-Gaussian initial conditions significantly change the abundance and clustering of dark matter halos
- We have an analytic description for the halo mass function that compares well to N-body for f_{NL} , g_{NL} and τ_{NL} -- perhaps it works for more general forms of NG?
- Analytic descriptions of halo bias agree well with sims, for f_{NL} and g_{NL} too!
- *Large-scale structure is a promising probe of the early universe*

Signatures in LSS III: stochastic halo bias

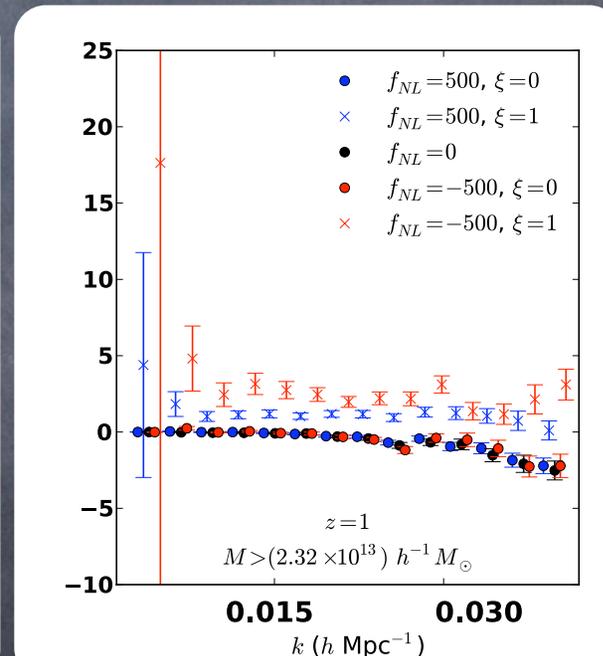
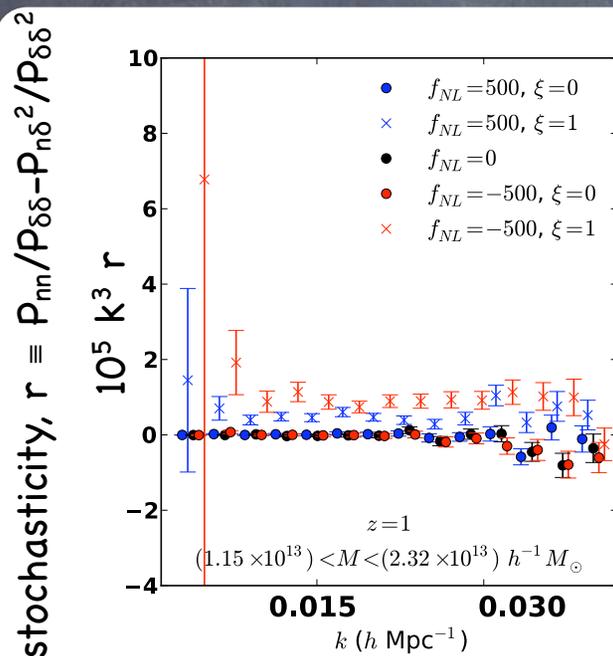
$$P_{n\delta}(k) \sim \left(\frac{\partial n}{\partial \delta} + 4f_{NL} \frac{\partial n}{\partial P_s} / k^2 \right) P_{\delta\delta}$$

$$P_{nn}(k) \sim \left(\frac{\partial n}{\partial \delta} + 4f_{NL} \frac{\partial n}{\partial P_s} / k^2 \right)^2 P_{\delta\delta} + \left(4f_{NL} \frac{\partial n}{\partial P_s} \right)^2 \xi^2 P_{\delta\delta}$$

stochasticity

$$\xi^2 = P_{\varphi\varphi}(k) / P_{\sigma\sigma}(k)$$

$$\tau_{NL} = (1 + \xi^2) f_{NL}^2$$



N.B. the bias factor in $P_{n\delta}$ is unchanged from f_{NL} -only model

Signatures in LSS III: stochastic halo bias

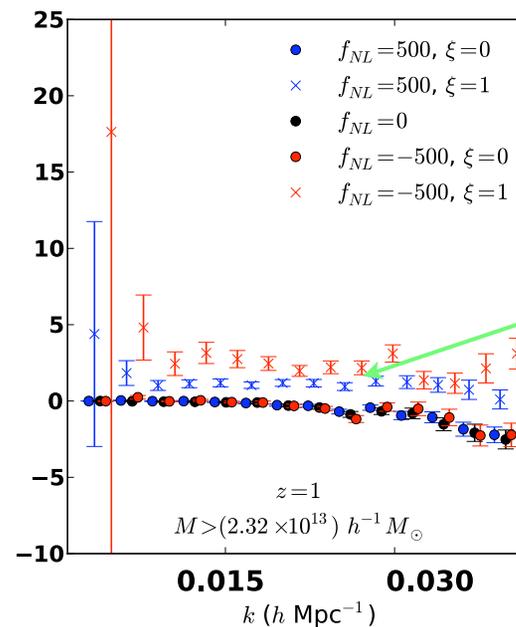
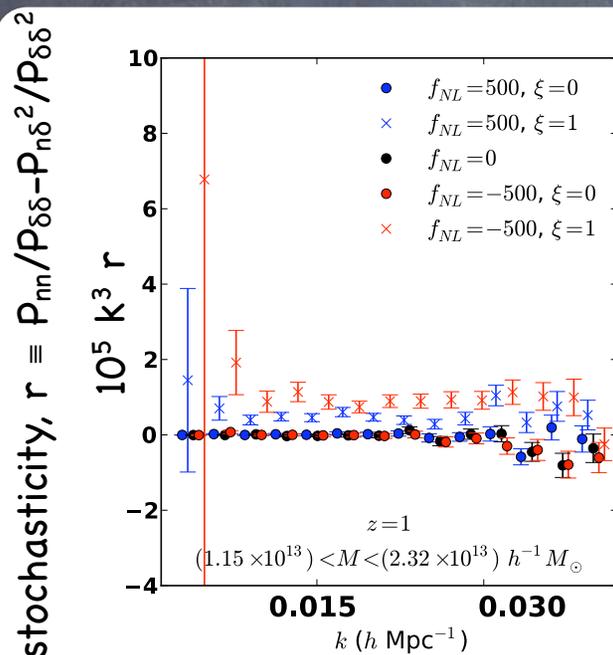
$$P_{n\delta}(k) \sim \left(\frac{\partial n}{\partial \delta} + 4f_{NL} \frac{\partial n}{\partial P_s} / k^2 \right) P_{\delta\delta}$$

$$P_{nn}(k) \sim \left(\frac{\partial n}{\partial \delta} + 4f_{NL} \frac{\partial n}{\partial P_s} / k^2 \right)^2 P_{\delta\delta} + \left(4f_{NL} \frac{\partial n}{\partial P_s} \right)^2 \xi^2 P_{\delta\delta}$$

stochasticity

$$\xi^2 = P_{\varphi\varphi}(k) / P_{\sigma\sigma}(k)$$

$$\tau_{NL} = (1 + \xi^2) f_{NL}^2$$



models with $\xi \neq 0$
indeed stochastic

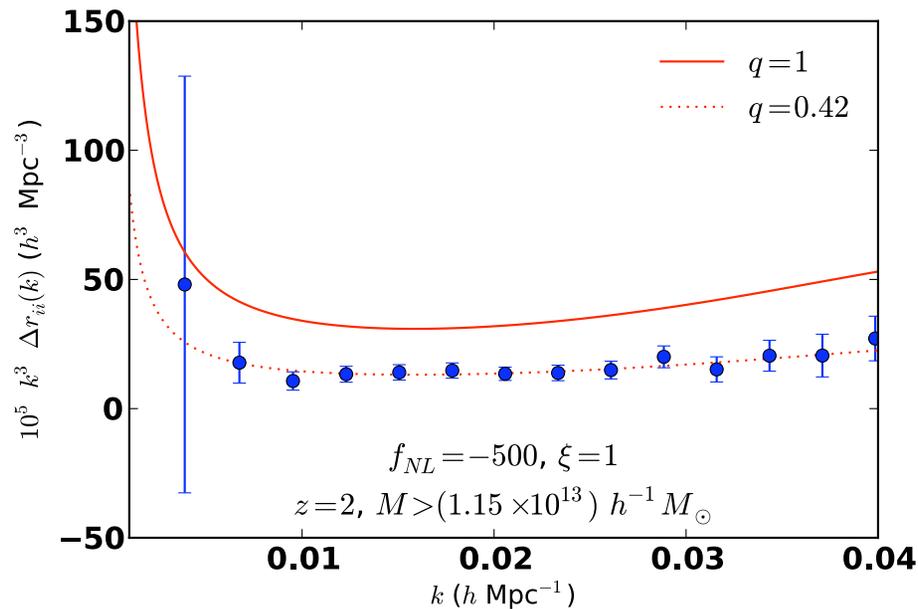
N.B. the bias factor in $P_{n\delta}$ is unchanged from f_{NL} -only model

Signatures in LSS III: stochastic halo bias

does stochasticity agree with predictions?

$$P_{nn}(k) \sim \left(\frac{\partial n}{\partial \delta} + 4f_{NL} \frac{\partial n}{\partial P_s} / k^2 \right)^2 P_{\delta\delta} + \left(4f_{NL} \frac{\partial n}{\partial P_s} \right)^2 \xi^2 P_{\delta\delta}$$

excess stochasticity above Gaussian

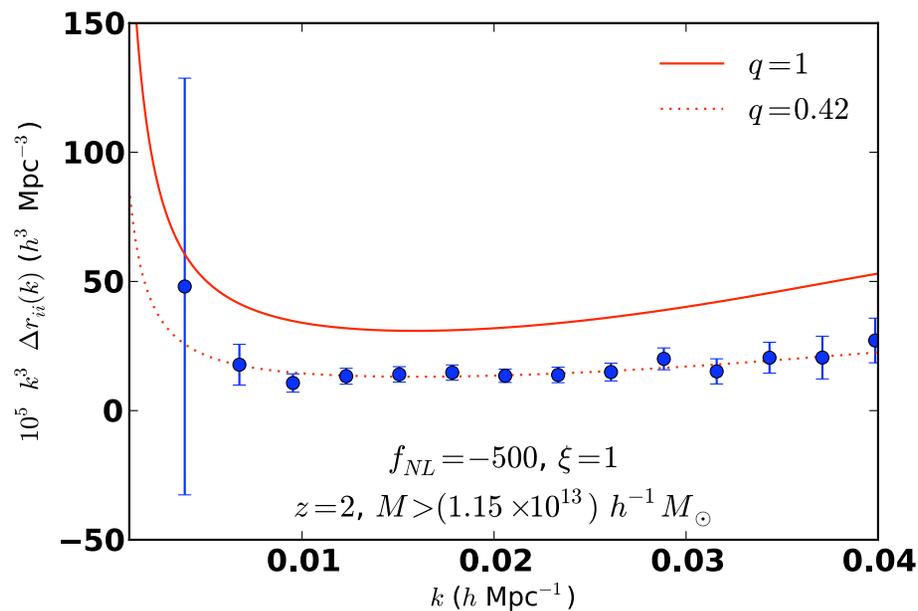


Signatures in LSS III: stochastic halo bias

does stochasticity agree with predictions?

$$P_{nn}(k) \sim \left(\frac{\partial n}{\partial \delta} + 4f_{NL} \frac{\partial n}{\partial P_s} / k^2 \right)^2 P_{\delta\delta} + \left(4f_{NL} \frac{\partial n}{\partial P_s} \right)^2 \xi^2 P_{\delta\delta}$$

excess stochasticity above Gaussian



um, shape looks good but not amplitude

tends to look better at low masses, low f_{NL}