# A MODEL FOR SOFT INTERACTIONS MOTIVATED BY AdS/CFT AND QCD

**Errol Gotsman**
**Tel Aviv University**

(work done with Genya Levin and Uri Maor)

summarizing EPJC71 (2011)171 and arXiv:1103.4509

## Outline

- GLM model for high energy soft interactions incorporating multi eikonal scattering plus multi-Pomeron vertices.

- Comparison with competing models

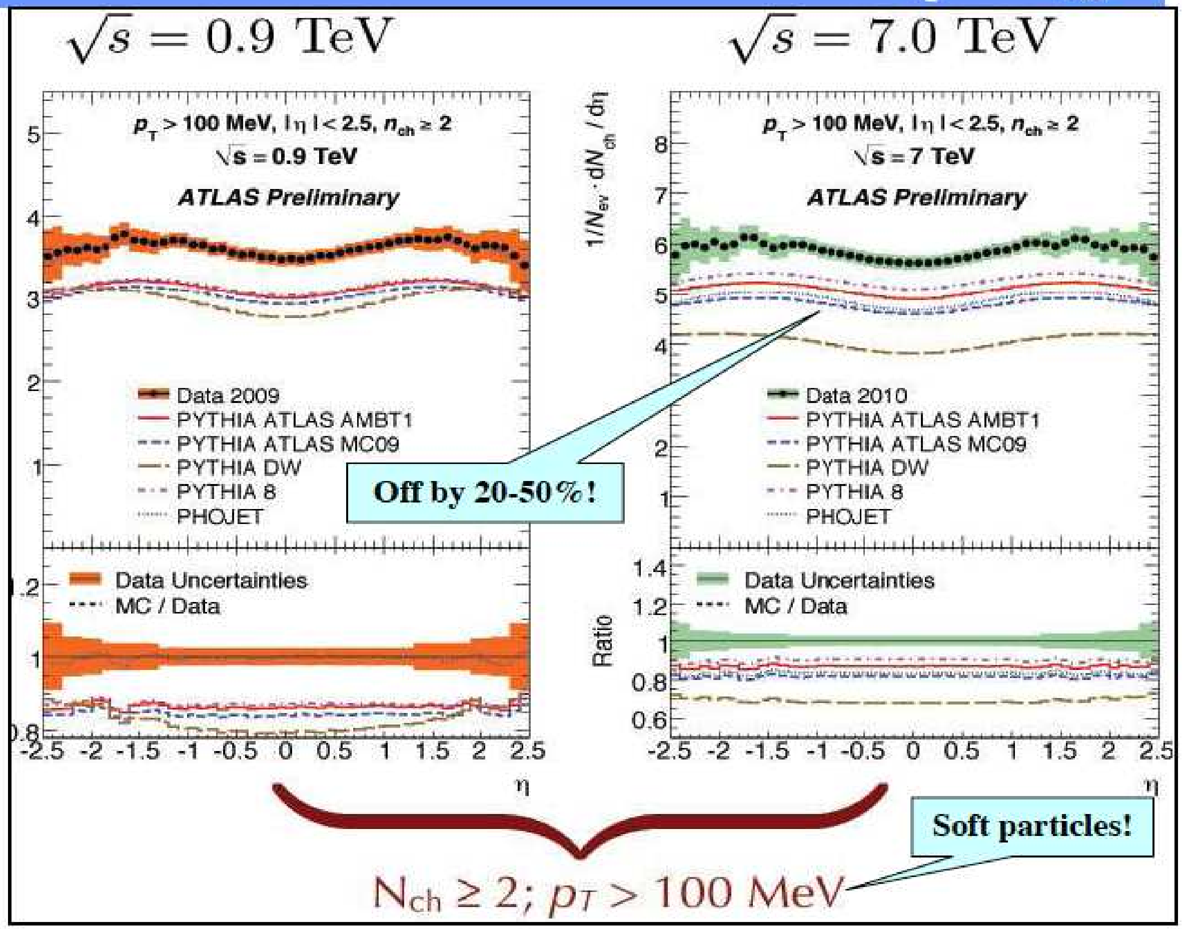
- Inclusive distributions - Model Predictions - Monte Carlo - Experimental Results

- Summary

# ATLAS INEL dN/dη

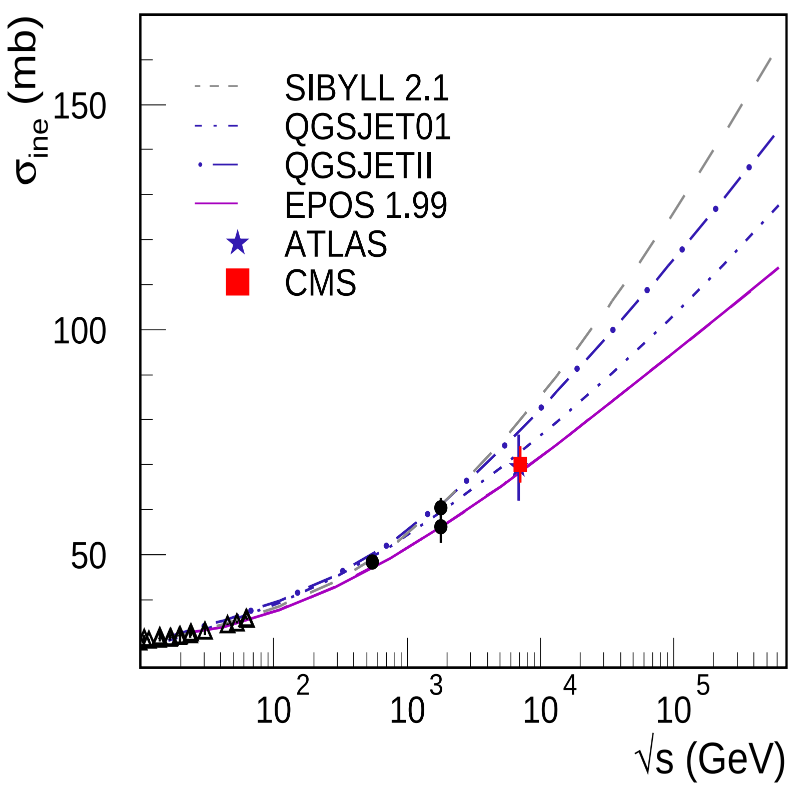
- None of the tunes fit the ATLAS INEL dN/dη data with PT > 100 MeV! They all predict too few particles.

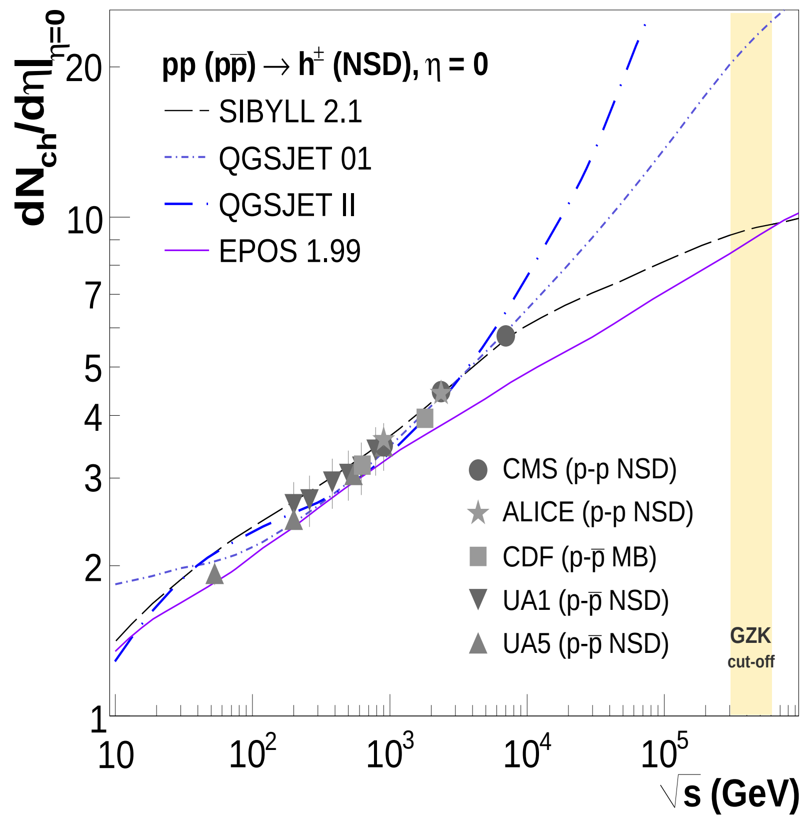
- The ATLAS Tune AMBT1 was designed to fit the inelastic data for Nchg ≥ 6 with $p_T$ > 0.5 GeV/c!



# Predictions of Monte Carlo Generators for High Energies

d'Enterria et al arXiv:1106.2453




Energy dependence of the inelastic $p - p(\bar{p}p)$ cross section (left) and of midrapidity charged hadron multiplicity density (right).

# Guiding criteria for GLM Model

- The model should be built using Pomerons and Reggeons.
- The intercept of the Pomeron should be relatively large. In AdS/CFT correspondence we expect $\Delta_{I\!P} = \alpha_{I\!P}(0) - 1 = 1 - 2/\sqrt{\lambda} \approx 0.11 \div 0.33$. The estimate for $\lambda$ from the cross section for multiparticle production as well as from DIS at HERA is $\lambda = 5 \div 9$;
- $\alpha'_{I\!P}(0) = 0$;
- A large Good-Walker component is expected, as in the AdS/CFT approach the main contribution to shadowing corrections comes from elastic scattering and diffractive production.
- The Pomeron self-interaction should be small (of the order of $2/\sqrt{\lambda}$ in AdS/CFT correspondence), and much smaller than the vertex of interaction of the Pomeron with a hadron, which is of the order of $\lambda$;
- The last requirement follows from the natural matching with perturbative QCD: where the only vertex that contributes is the triple Pomeron vertex.

## Good-Walker Formalism

Diffractively produced hadrons at a given vertex are considered as a single hadronic state described by the wave function $\Psi_D$, which is orthonormal to the wave function $\Psi_h$ of the incoming hadron (proton in the case of interest)

$$< \Psi_h | \Psi_D >= 0$$

We introduce two wave functions $\psi_1$ and $\psi_2$ that diagonalize the 2x2 interaction matrix $\mathbf{T}$

$$A_{i,k} =< \psi_i \, \psi_k | \mathbf{T} | \psi_{i'} \, \psi_{k'} >= A_{i,k} \, \delta_{i,i'} \, \delta_{k,k'}.$$

In this representation the observed states are written in the form

$$\psi_h = \alpha \, \psi_1 + \beta \, \psi_2 \, ,$$

$$\psi_D = -\beta \, \psi_1 + \alpha \, \psi_2$$

where, $\alpha^2 + \beta^2 = 1$

## Good-Walker Formalism-2

Unitarity constraints:

$$Im\, A_{i,k}\,(s,b) = |A_{i,k}\,(s,b)\,|^2 + G^{in}_{i,k}(s,b),$$

$G^{in}_{i,k}$ is the contribution of all non diffractive inelastic processes i.e. it is the summed probability for these final states to be produced in the scattering of particle $i$ off particle $k$.

A simple solution to the above equation is:

$$A_{i,k}(s,b) = i\left(1 - \exp\left(-\frac{\Omega_{i,k}(s,b)}{2}\right)\right),$$

$$G^{in}_{i,k}(s,b) = 1 - \exp\left(-\Omega_{i,k}(s,b)\right).$$

## Good-Walker Formalism-3

Note

$$P^S_{i,k} = \exp\left(-\Omega_{i,k}(s,b)\right)$$

is the probability that the initial projectiles $(i,k)$ reach the final state interaction unchanged, regardless of the initial state rescatterings, (i.e. no inelastic interactions).

Amplitudes in two channel formalism are:

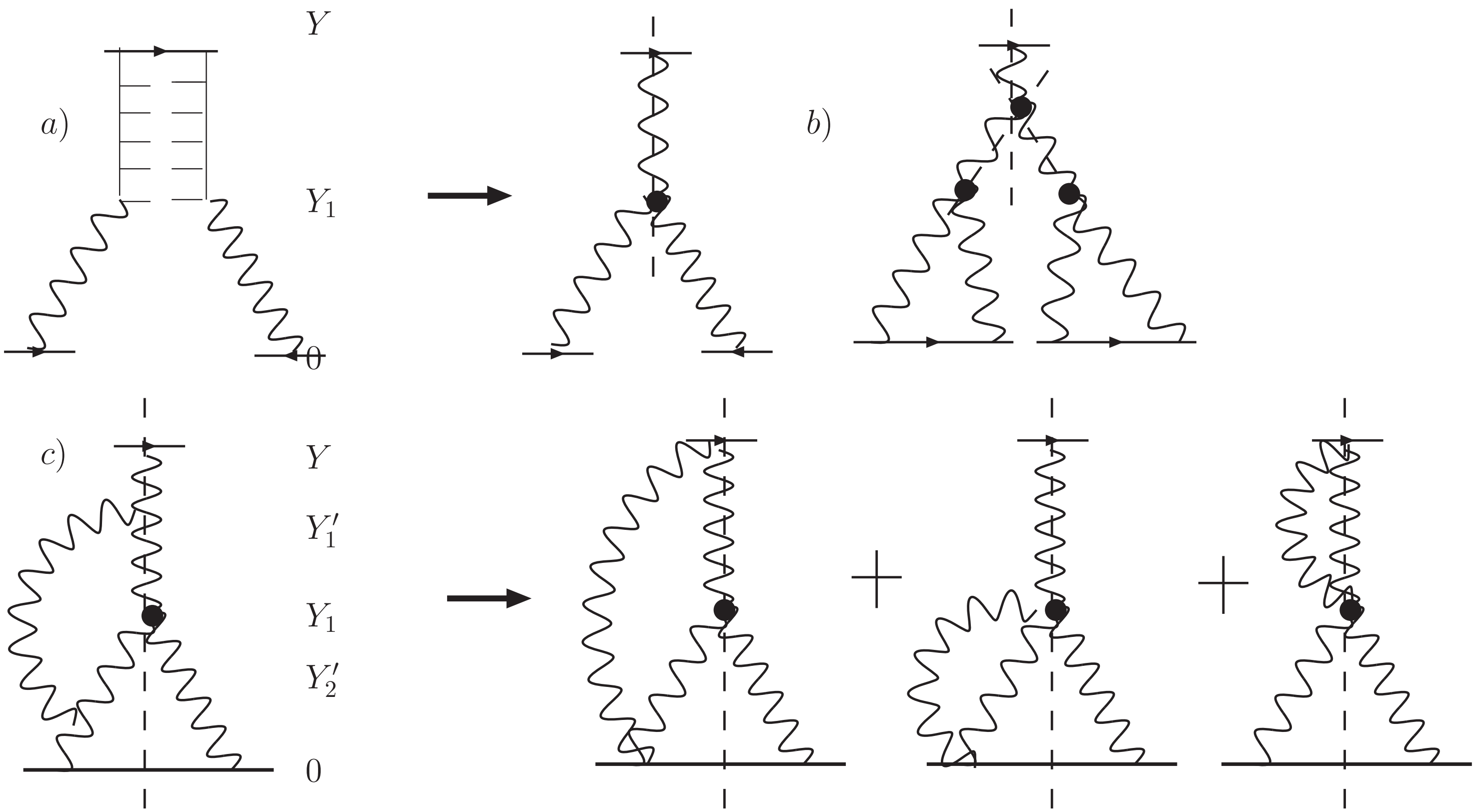
$$a_{el}(s,b) = i\{\alpha^4 A_{1,1} + 2\alpha^2\beta^2 A_{1,2} + \beta^4 A_{2,2}\},$$

$$a_{sd}(s,b) = i\alpha\beta\{-\alpha^2 A_{1,1} + (\alpha^2 - \beta^2)A_{1,2} + \beta^2 A_{2,2}\},$$
$$a_{dd}(s,b) = i\alpha^2\beta^2\{A_{1,1} - 2A_{1,2} + A_{2,2}\}.$$

With the G-W mechanism $\sigma_{el}$ , $\sigma_{sd}$ and $\sigma_{dd}$ occur due to elastic scattering of $\psi_1$ and $\psi_2$, the correct degrees of freedom.

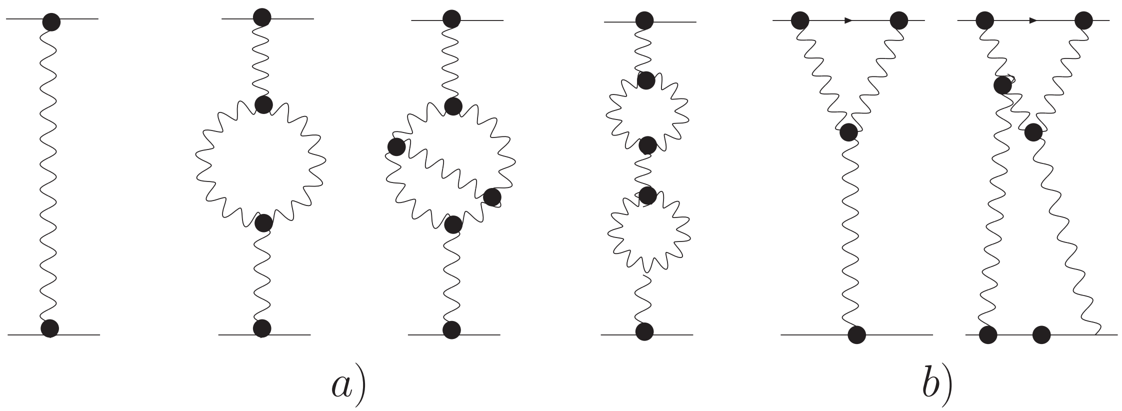
# Examples of Pomeron diagrams

# leading to diffraction NOT included in G-W mechanism




Examples of the Pomeron diagrams that lead to a different source of the diffractive dissociation that cannot be described in the framework of the G-W mechanism. (a) is the simplest diagram that describes the process of diffraction in the region of large mass $Y - Y_1 = \ln(M^2/s_0)$. (b) and (c) are examples of more complicated diagrams in the region of large mass. The dashed line shows the cut Pomeron, which describes the production of hadrons.

# Example of enhanced and semi-enhanced diagram




Different contributions to the Pomeron Green's function
a) examples of enhanced diagrams ; b) examples of semi-enhanced diagrams
Multi-Pomeron interactions are crucial for the production of LARGE MASS DIFFRACTION

## Tel Aviv approach for summing interacting Pomeron diagrams

In the spirit of LO pQCD we write a generating function

$$Z(y,\,u)\;=\;\sum_n\;P_n(y)\;u^n,$$

$P_n(y)$ is the probability to find $n$-Pomerons (dipoles) at rapidity $y$.

The solution, with boundry conditions, gives us the sum of enhanced diagrams.

For the function $Z\,(u)$ the following evolution equation can be written

$$-\frac{\partial\,Z(y,\,u)}{\partial\,y}\;=\;-\,\Gamma(1\rightarrow 2)\,u\,(1-u)\;\frac{\partial\,Z(y,\,u)}{\partial\,u}\;+\;\Gamma(2\rightarrow 1)\,u\,(1-u)\;\frac{\partial^2\,Z(y,\,u)}{\partial^2\,u},$$

This is no more than the Fokker-Planck diffusion equation

$\Gamma(1\rightarrow 2)$ describes the decay of one Pomeron (dipole) into two Pomerons (dipoles)

while $\Gamma(2\rightarrow 1)$ relates to the merging of two Pomerons (dipoles) into one Pomeron (dipole).

## Tel Aviv approach for summing interacting Pomeron diagrams contd.

Using the functional $Z$, we find the scattering amplitude, using the following formula:

$$N\,(Y) \;\equiv\; \mathrm{Im}A_{el}\,(Y) \;=\; \sum_{n=1}^{\infty} \frac{(-1)^n}{n!}\;\frac{\partial^n\,Z(y,\,u)}{\partial^n\,u}|_{u=1}\,\gamma_n(Y=Y_0,b),$$

$\gamma_n(Y=Y_0,b)$ is the scattering amplitude of $n$-partons (dipoles) at low energy.

Using the MPSI approximation (where only large $I\!P$ loops of rapidity size O($Y$) contribute) we obtain the exact Pomeron Green's function

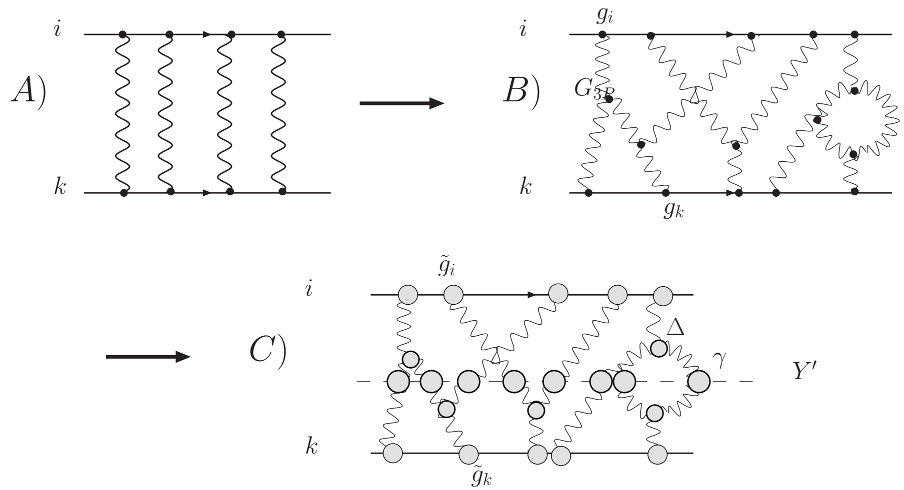
$$G_{I\!P}\,(Y) \;=\; 1\;-\;\exp\left(\frac{1}{T(Y)}\right)\frac{1}{T(Y)}\,\Gamma\left(0,\frac{1}{T(Y)}\right),$$

$\Gamma\,(0,x)$ is the incomplete gamma function and

$$T\,(Y) \;=\; \gamma\,e^{\Delta_{I\!P}\,Y}.$$

$\gamma$ is the amplitude of the parton (colorless dipole) interaction with the target at arbitrary Y. MPSI approximation is valid only for $Y \leq \frac{1}{\gamma}$.

## Full set of diagrams that need to be summed




A) shows the sum of enhanced diagrams in two channel approach,
B) shows the full set of the diagrams which in C) is pictured in the way that is most suitable to illustrate the MPSI approach.
The bold wave line stands for the exact Pomeron Green function that includes all enhanced diagrams.

The summed amplitude has the form:

$$A_{i,k}\left(Y;b\right) \;=\; 1 \;-\; \exp\left\{ -\,\frac{1}{2}\int d^2b' \;\frac{\left(\tilde{g}_i\left(\vec{b}'\right)\;\tilde{g}_k\left(\vec{b}-\vec{b}'\right)\;T(Y)\right)}{1\,+\,T(Y)\;\left[\tilde{g}_i\left(\vec{b}'\right)+\tilde{g}_k\left(\vec{b}-\vec{b}'\right)\right]}\right\}$$

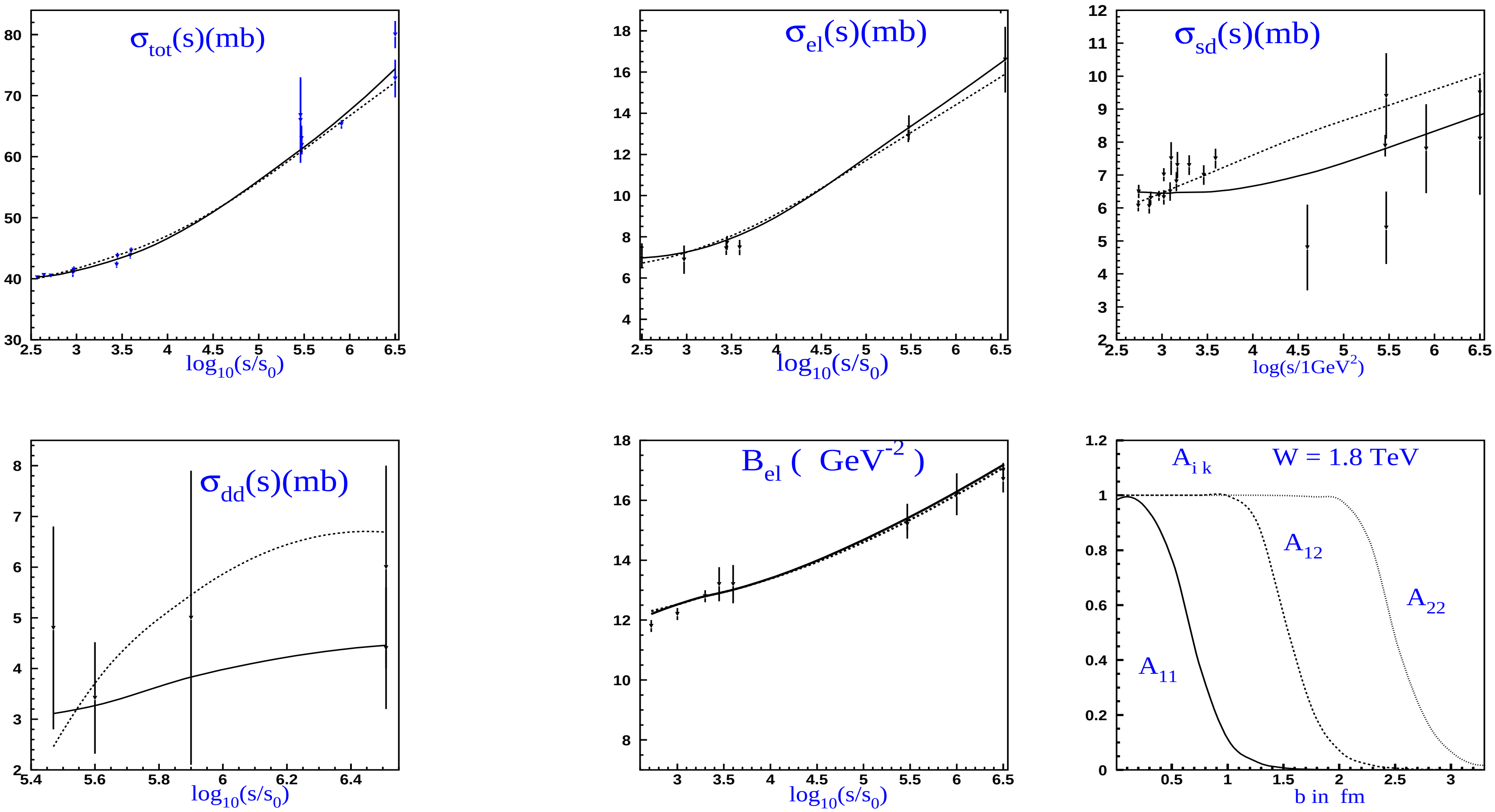
Parameters for our model fit includes G-W

+ enhanced + semi-enhanced Pomeron diagrams

| | | | | | | |
|---|---|---|---|---|---|---|
| $\Delta_{I\!P}$ | $\beta$ | $\alpha'_{I\!P}$ | $g_1$ | $g_2$ | $m_1$ | $m_2$ |
| 0.2 | 0.388 | 0.020 $GeV^{-2}$ | 2.53 $GeV^{-1}$ | 88.4 $GeV^{-1}$ | 2.648 $GeV$ | 1.37 $GeV$ |
| $\Delta_{I\!R}$ | $\gamma$ | $\alpha'_{I\!R}$ | $g_1^{I\!R}$ | $g_2^{I\!R}$ | $R^2_{0,1}$ | $G_{3I\!P}$ |
| - 0.466 | 0.0033 | 0.4 $GeV^{-2}$ | 14.5 $GeV^{-1}$ | 1343 $GeV^{-1}$ | 4.0 $GeV^{-2}$ | 0.0173$GeV^{-1}$ |

- $g_1(b)$ and $g_2(b)$ describe the vertices of interaction of the Pomeron with state 1 and state 2
- The Pomeron intercept is $\Delta_{I\!P}(0)$ - 1
- $\gamma$ denotes the low energy amplitude of the dipole-target interaction
- $\tilde{g}_i \;=\; g_i/\sqrt{\gamma}$
- For $\tilde{g}_i(b)$, we use the phenomenological assumption $\tilde{g}_i(b) \;=\; \tilde{g}_i\, S(b) \;=\; \frac{\tilde{g}_i}{4\pi}\, m_i^3\, b\, K_1\left(m_i\, b\right)$,
where $S(b)$ is the Fourier transform of the dipole formula for the form factor $1/(1+q^2/m_i^2)^2$.

# Energy dependence of GLM cross sections



## Comparison of cross sections obtained in GLM, Phythia6(MC09) and Phojet and Experimental Data

| $\sqrt{s}$ TeV | | Pythia6 (mb) | Phojet (mb) | GLM (mb) |
|---|---|---|---|---|
| 0.9 | ND | 34.4 | 40.0 | 39.2 |
| 0.9 | SD | 11.7 | 10.5 | 8.2 |
| 0.9 | DD | 6.4 | 3.5 | 3.8 |
| 0.9 | INEL | 52.5 | 54.0 | 52.1 |
| 7.0 | ND | 48.5 | 61.6 | 51.6 |
| 7.0 | SD | 13.7 | 10.7 | 10.2 |
| 7.0 | DD | 9.3 | 3.9 | 6.5 |
| 7.0 | INEL | 59.5 | 76.2 | 68.3 |
| | | ALICE | ATLAS | CMS |
| 7.0 | INEL | $72.7 \pm 1.1$ mb | $69.4 \pm 2.4$ mb | 63 to 70 mb |

## Comparison of results obtained in GLM, Ostapchenko and KMR models

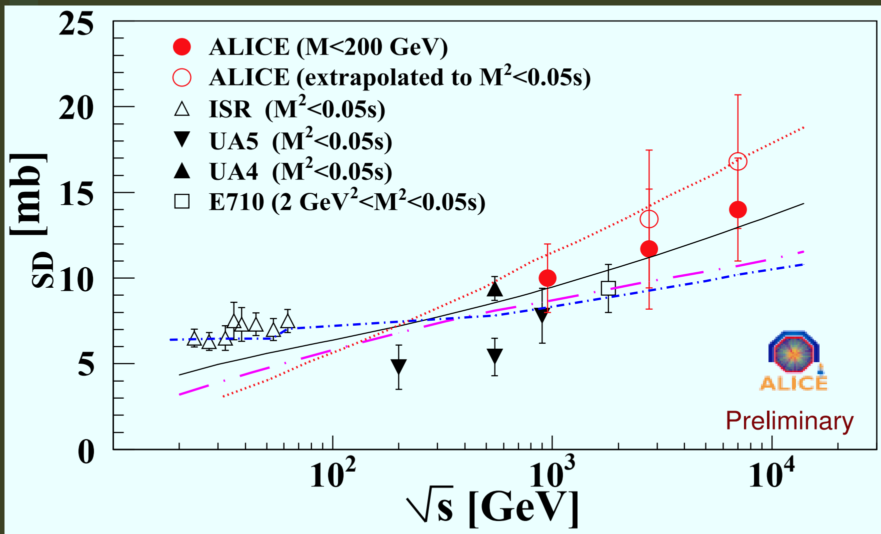
Ostapchenko (Phys.Rev.D81,114028(2010)) has made a comprehensive calculation in the framework of Reggeon Field Theory based on the resummation of both enhanced and semi-enhanced Pomeron diagrams.

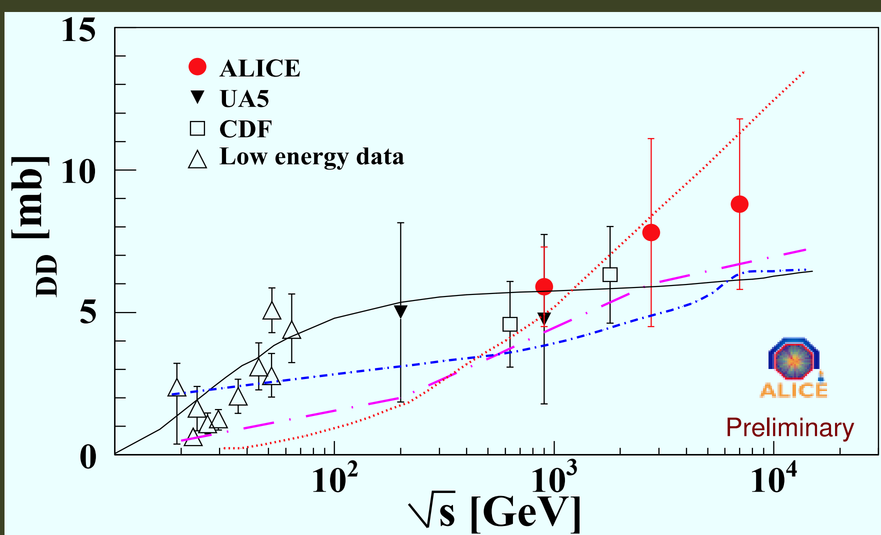
To fit the total and diffractive cross sections he assumes TWO POMERONS: "SOFT POMERON" $\alpha^{Soft} = 1.14 + 0.14t$ "HARD POMERON" $\alpha^{Hard} = 1.31 + 0.085t$
The above are the values for Set (C) of his fit including the E710 value of $\sigma_{tot} = 72.8 \pm 2.24$ mb at the Tevatron.

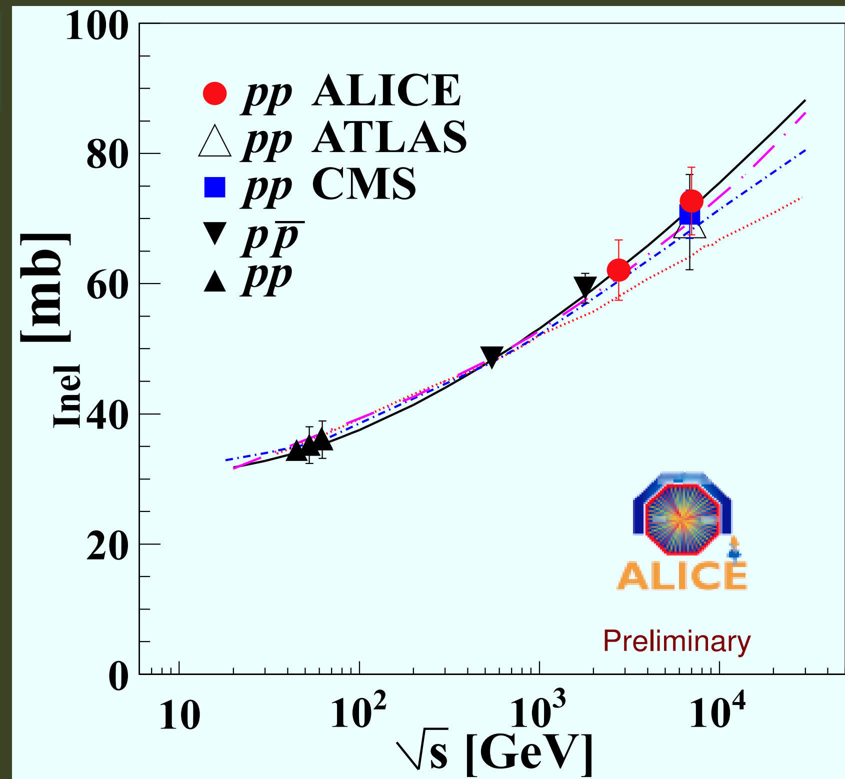
The Durham Group (Khoze, Martin and Ryskin), have a model which is similar to spirit to GLM, the main difference lies in the technique of summing the "Pomeron loop" diagrams. Over the years they have improved their model, the latest version (EPJ C71, 1617(2011) includes $k_t$ evolution.

| | Tevatron (1.8 TeV) | | | | | LHC (14 TeV) | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | GLMM | GLM | KMR(07) | KMR(11) | OS(C) | GLMM | GLM | KMR(07) | KMR(11) | OS(C) |
| $\sigma_{tot}$(mb) | 73.3 | 74.4 | 74.0 | 72.8/72.5 | 73.0 | 92.1 | 101.0 | 88.0 | 98.3/94.6 | 114.0 |
| $\sigma_{el}$(mb) | 16.3 | 17.5 | 16.3 | 16.3/16.8 | 16.8 | 20.9 | 26.1 | 20.1 | 25.1/24.2 | 33.0 |
| $\sigma_{sd}$(mb) | 9.8 | 8.9 | 10.9 | 11.4/13.0 | 9.6 | 11.8 | 10.8 | 13.3 | 17.6/18.8 | 11.0 |
| $\sigma_{dd}$(mb) | 5.4 | 3.5 | 7.2 | 7.0 | 3.9 | 6.1 | 6.5 | 13.4 | 13.5 | 4.8 |

# Comparison with other experiments and models




Gotsman et al., arXiv:1010.5323, EPJ. C74, 1553 (2011)
Kaidalov et al., arXiv:0909.5156, EPJ. C67, 397 (2010)
Ostapchenko, arXiv:1010.1869, PR D83 114018 (2011)
Khoze et al., EPJ. C60 249 (2009), C71 1617 (2011)

Model predictions:
SD $M^2 < 0.05s$
DD $\Delta\eta > 3$

Quark Matter 2011 M.Poghosyan 19

## Experimental Results for Inclusive Production

The three experimental groups ALICE, CMS and ATLAS have slight differences in the presentation of their results for for psuedo-rapidity distributions at the LHC.

$$\sigma_{tot} \ = \ \sigma_{ND} + \sigma_{el} + \sigma_{SD} + \sigma_{DD} = \sigma_{el} + \sigma_{inel}$$

ATLAS give results for $\sigma_{ND}$

CMS display

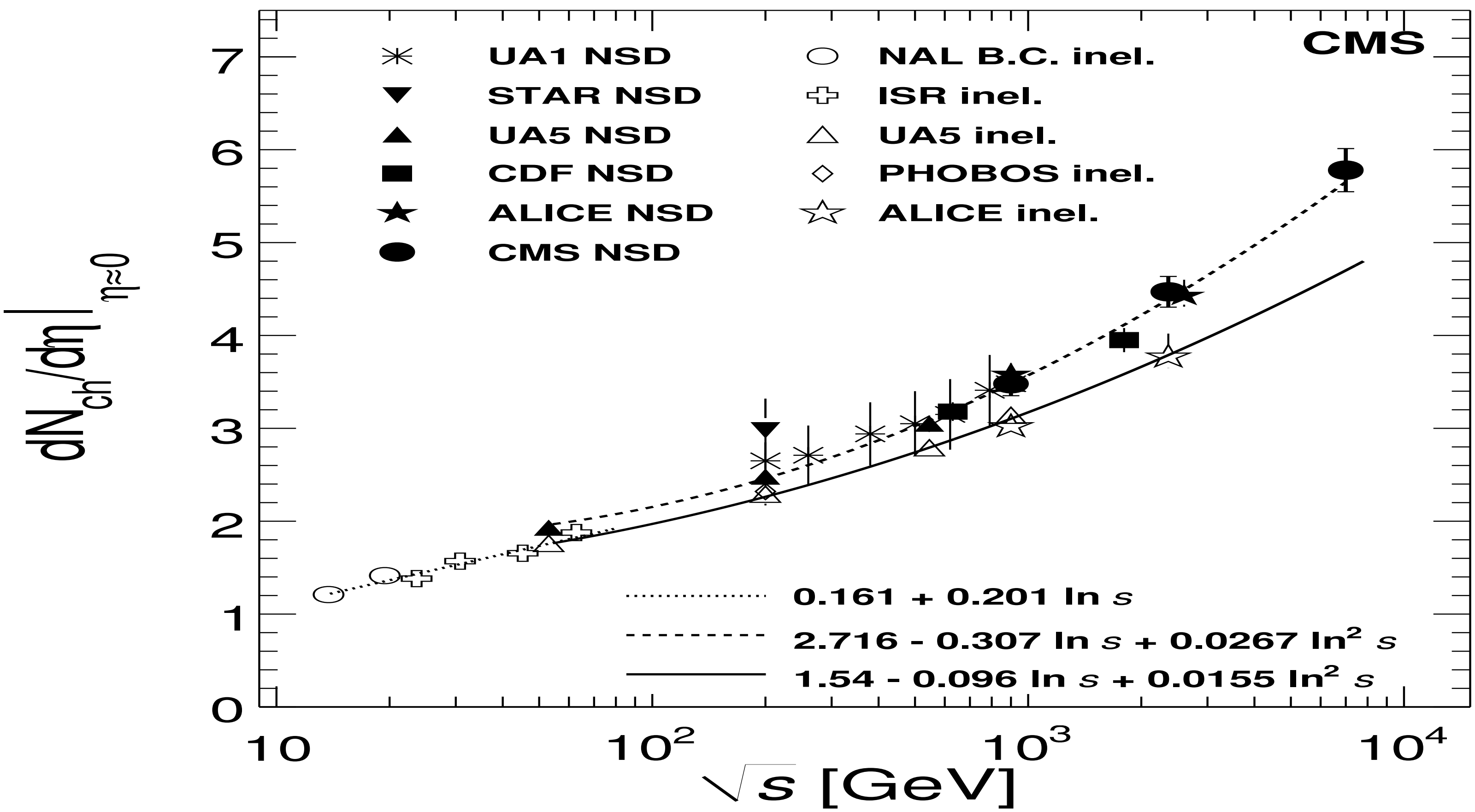
$$\sigma_{NSD} \ = \ \sigma_{ND} + \sigma_{DD} = \sigma_{tot} - \sigma_{el} - \sigma_{SD}$$

$$\sigma_{inel} = \sigma_{ND} + \sigma_{SD} + \sigma_{DD}$$

ALICE also present $\sigma_{NSD}$ for W $=$0.9 and 2.36 TeV, however, for W $=$ 7 TeV they impose an additional constraint requiring at least one charged particle in the interval $\mid \eta \mid < 1$ $(inel > 0_{|\eta|<1})$.

# RESULTS 3 CMS 2

CMS Collaboration JHEP (2010)041



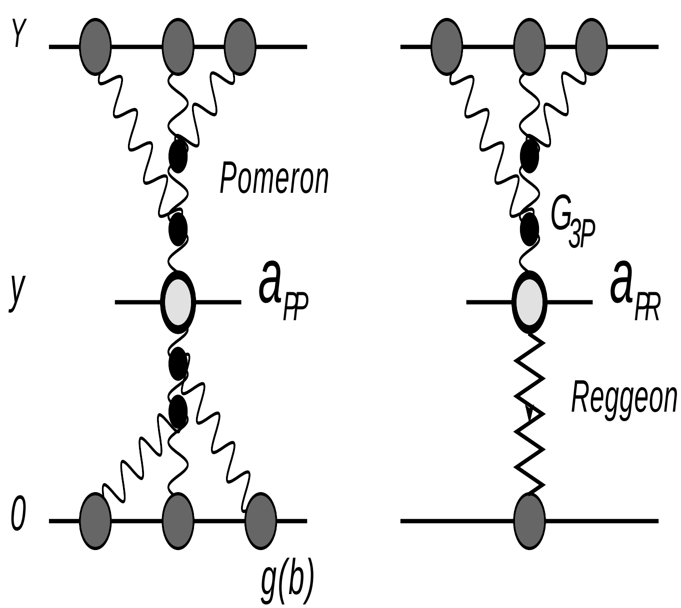

Average value of $dN_{ch}/d\eta$ in the central $\eta$ region as a function of the c.m. energies in pp and $\bar{p}$p collisions

# Single inclusive cross section 1

We expand our approach to describe rapidity distributions at high energies e.g. the single inclusive cross section.

**Assumptions**

- $\alpha'_{I\!P} = 0$.
- Only the triple Pomeron vertex is included to describe the interaction of the soft Pomerons.
- The single inclusive cross section in the framework of the Pomeron calculus can be calculated using Mueller diagrams shown.



## Single Inclusive cross section 2

They lead to the following expression for the single inclusive cross section

$$\frac{1}{\sigma_{NSD}}\frac{d\sigma}{dy} \;=\; \frac{1}{\sigma_{NSD}(Y)}\,\Big\{a_{I\!PI\!P}\Big(\int d^2b\Big(\alpha^2\,G_1(b,Y/2-y)+\beta^2 G_2(b,Y/2-y)\Big)$$

$$\times\int d^2b\Big(\alpha^2\,G_1(b,Y/2+y)+\beta^2 G_2(b,Y/2+y)\Big)$$

$$-\;a_{I\!PI\!R}\;(\alpha^2\,g_1^{I\!R}+\beta^2 g_2^{I\!R})\,\Big[\alpha^2\int d^2b\Big(\alpha^2\,G_1(b,Y/2-y)+\beta^2 G_2(b,Y/2-y)\Big)\,e^{\Delta_{I\!R}\,(Y/2+y)}$$

$$+\int d^2b\Big(\alpha^2\,G_1(b,Y/2+y)+\beta^2 G_2(b,Y/2+y)\Big)\,e^{\Delta_{I\!R}\,(Y/2-y)}\Big]\Big\}\,.$$

$G_i\,(b,Y)$ denotes the sum of 'fan' diagrams

$$G_i\,(b,Y)\;=\;(g_i\,(b)\,/\gamma)\;G_{enh}(y)/\Big(1+(G_{3I\!P}/\gamma)\;g_i\,(b)\;G_{enh}(y)\Big),$$

where the Green's function of the Pomeron, obtained by the summation of the enhanced diagrams, is equal to

$$G_{enh}\,(Y)\;=\;1\;-\;\exp\left(\frac{1}{T\,(Y)}\right)\,\frac{1}{T\,(Y)}\,\Gamma\left(0,\frac{1}{T\,(Y)}\right).\quad T\,(Y)\;=\;\gamma e^{\Delta_{I\!P}\,Y}$$

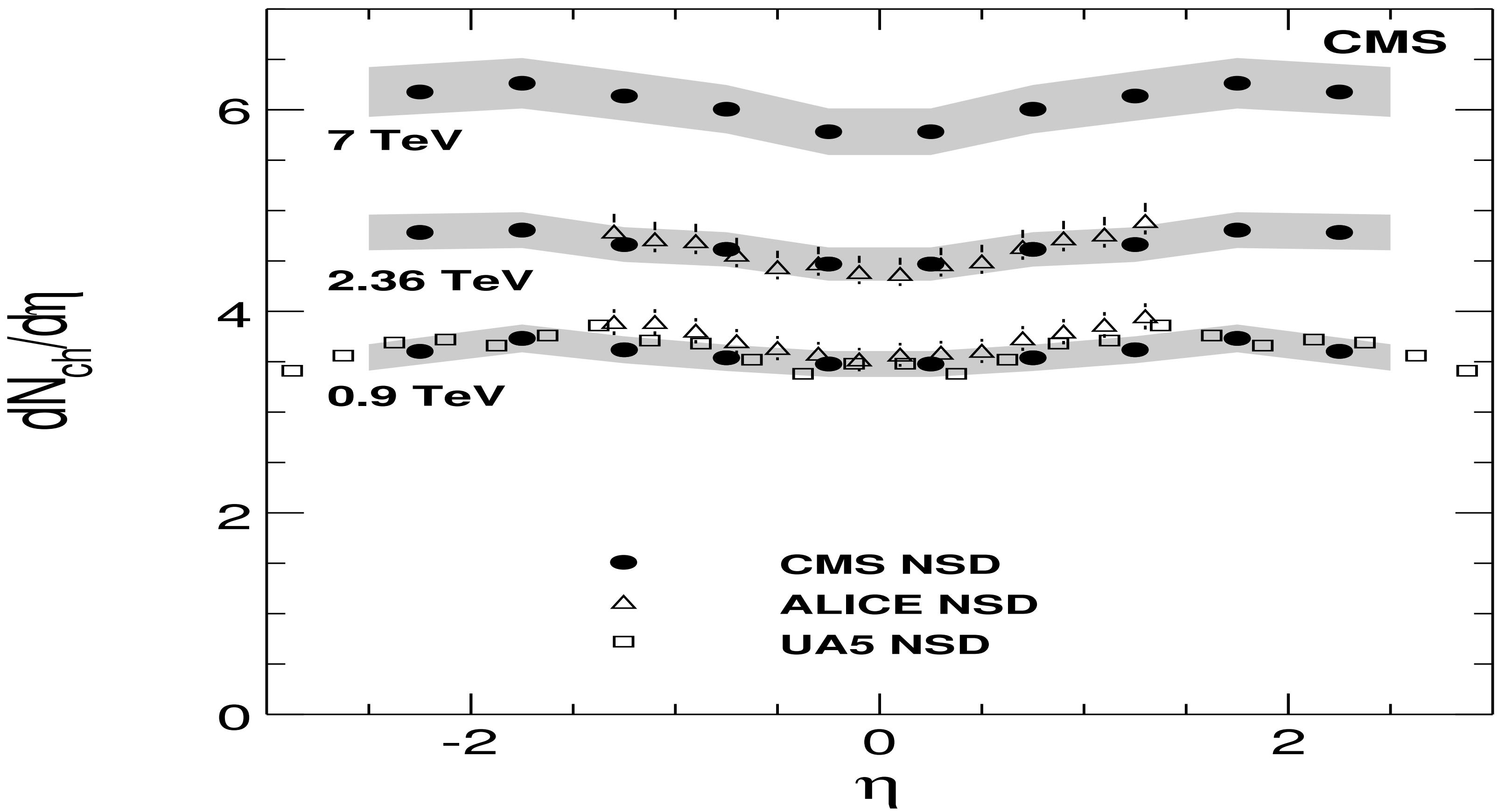
## Single Inclusive cross section 3

- We need to introduce two new phenomenological parameters: $a_{\mathbb{P}\mathbb{P}}$ and $a_{\mathbb{R}\mathbb{P}}$ to describe the emission of hadrons from the Pomeron and the Reggeon.
- As well as two dimensional parameters $Q$ and $Q_0$, $Q$ is the average transverse momentum of produced minijets, and $\frac{Q_0}{2}$ denotes the mass of the slowest hadron produced in the decay of the minijet.
- We extract the three new parameters: $a_{\mathbb{P}\mathbb{P}}, a_{\mathbb{P}\mathbb{R}}$ and $Q_0/Q$ from the experimental inclusive data.
- The ratio $Q_0/Q$ determines the shape of the inclusive spectra.
- We made two separate fits:
  (a) fitting only the CMS data at different LHC energies and
  (b) fitting all inclusive data for $W \geq 546\,GeV$.

| Data | $a_{\mathbb{P}\mathbb{P}}$ | $a_{\mathbb{P}\mathbb{R}}$ | $Q_0/Q$ | $\chi^2/d.f.$ |
|---|---|---|---|---|
| All | 0.396 | 0.186 | 0.427 | 0.9 |
| CMS | 0.413 | 0.194 | 0.356 | 0.2 |

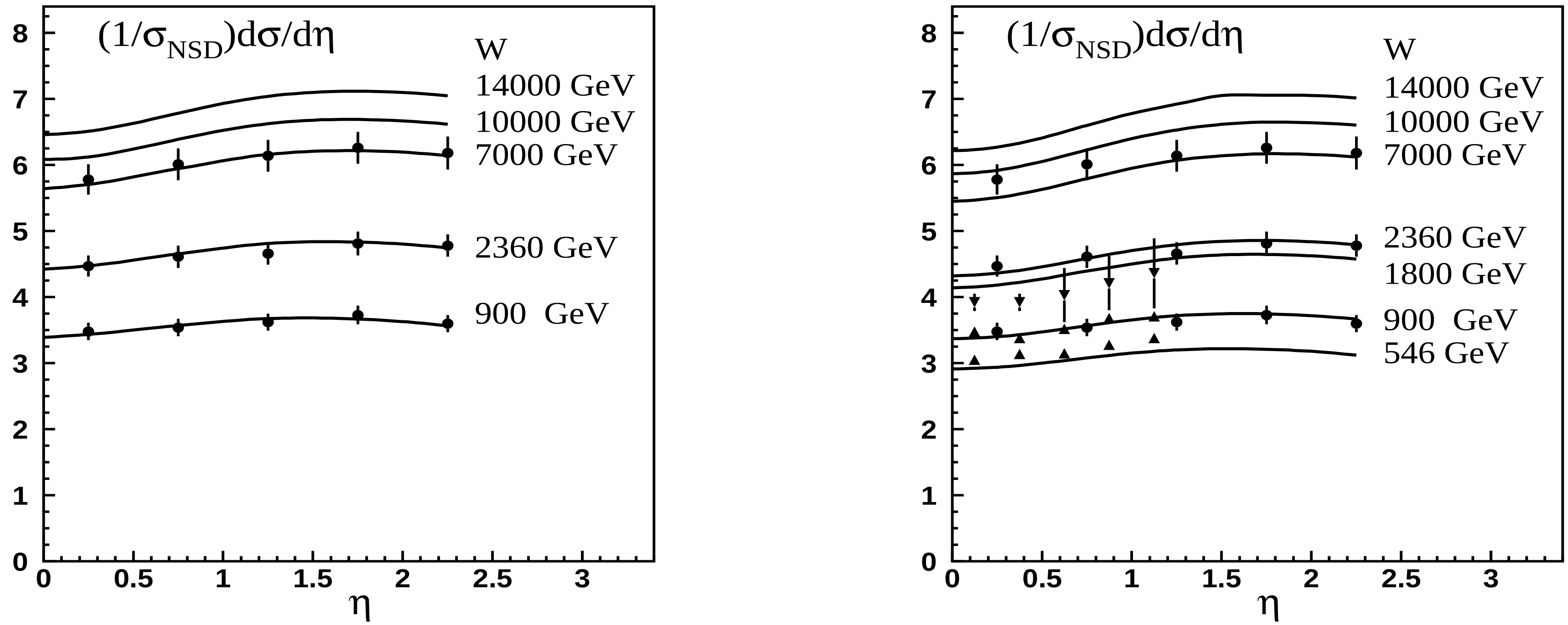
# RESULTS 2 CMS 1

CMS Collaboration JHEP (2010)041




Charged particle $\eta$ distributions in region $|\eta| < 2.4$ for $dN_{ch}/d\eta$. Comparison of UA5 , ALICE and CMS results for NSD events.

## Results of GLM for single inclusive cross section




The single inclusive density versus energy.
The data were taken from ALICE,CMS, and ATLAS Collaborations and from PDG.
The fit to the CMS data is plotted in (a), while (b) presents the description of all inclusive spectra with $W \geq 546\, GeV$.

# Summary

- We present a model for soft interactions having two components:
  (i) G-W mechanism for elastic and low mass diffractive scattering.
  (ii) Pomeron enhanced contributions for high mass diffractive production.
- Enhanced $I\!\!P$ diagrams, make important contributions to both $\sigma_{sd}$ and $\sigma_{dd}$.
- Most phenomenological models which successfully describe LHC data, are found lacking at lower energies.
- Monto Carlo generators which were successful in describing data for W $\leq$ 1.8 TeV need to be RETUNED to describe LHC data.
- GLM model (with parameters determined by data for W $\leq$ 1.8 TeV ) underestimates Inclusive rapidity distribution ($\sigma_{NSD}$) data for $\sqrt{s} = 7$ TeV.
  Fit made with LHC data (ONLY), is successful.
- THERE APPEARS TO BE AN INHERENT DIFFICULTY IN EXTRAPOLATING FROM ISR TO LHC ENERGIES.