Axion monodromy

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I. Introduction: inflation vs UV completeness

II. Monodromy from strongly coupled QFT
   arxiv:1105.3740 with Sergei Dubovsky (NYU), AL, and Matthew Roberts (NYU)

III. 4d model for large field inflation
   arxiv:1101.0026 with Nemanja Kaloper (UC Davis), AL, and Lorenzo Sorbo (U Mass Amherst)

IV. Conclusion
I. Introduction

“Slow roll” inflation matches CMB/LSS data well

\[ V(\varphi) \]
\[ \delta \varphi \]
\[ \varphi(t) \]
\[ \varphi \]

Slow roll + vacuum dominance:
\[ \epsilon = m_p^2 \left( \frac{V'}{V} \right)^2 \ll 1 \]
\[ \eta = m_p^2 \frac{V''}{V} \ll 1 \]

Spacetime approximately de Sitter:
\[ ds^2 = -dt^2 + e^{2 \int dt H \, dx_3^2} \]
\[ H^2 = \frac{V}{m_p^2} \]

Observed flatness of universe requires expansion factor \( \sim e^{60} \)

Need shallow potential

Quantum fluctuations generate structure:
\[ \frac{\delta \rho}{\rho} \sim \frac{V^{3/2}}{m_p^3 V^2} \sim \frac{\delta T_{CMB}}{T_{CMB}} \sim 5 \times 10^{-5} \]

Quantum fluctuations of graviton generate gravitational waves:
\[ \mathcal{P}_g \sim \frac{V}{m_p^4} \quad \text{Detectable via CMB polarization experiments} \]
Building sensible inflation models

The game is to get a potential which is shallow over a large range, consistent with naturalness

Impose softly broken shift symmetry? \( \varphi \rightarrow \varphi + a \)

- “Small field” models \( \Delta \varphi < m_{pl} \)
  
  Sub-Planckian physics sufficient: operators of form
  
  \[ \mathcal{O}_n = \frac{\varphi^n}{m_{pl}^{n-4}} \]

  remain small

- “Large field” models \( \Delta \varphi > m_{pl} \)
  
  We must control \( \mathcal{O}_n \) for all \( n \)

  Requires UV complete theory at Planck scale

NB we are using the “reduced Planck mass”

\[ m_{pl} = \frac{1}{\sqrt{8\pi G_N}} \]

\[ \sim 2 \times 10^{18} \text{ GeV} \]
Large field models?

- Gravity waves and “Lyth bound”

Observational upper bound on GW: \( V \lesssim 10^{16} \text{ GeV} \sim M_{GUT} \)

Close to “unification scale”. If GW observable, V must be close to this scale

Observable GW \( \Rightarrow \Delta \varphi \gg m_{pl} \) \hspace{1cm} Lyth, hep-ph/9606387

- PNGB-driven inflation

Scalar field \( \varphi \equiv \varphi + 2\pi f \) \hspace{1cm} Corrections of form \( \delta V(\varphi) \sim \varphi^n \) forbidden

“Natural inflation”: inflaton potential takes form

\[ V(\varphi) = \Lambda^4 \left( 1 - \cos \left( \frac{\varphi}{f} \right) \right) \]

\text{eg via instanton effects}

Shallow if \( \Lambda \ll f \) \hspace{1cm} can easily be done naturally

Fits CMB data if \( f \sim \text{few} \times m_{pl} \)
Large field models and UV completions

- Quantum gravity breaks global symmetries
- Instanton actions get small when $f > m_{pl}$

$\Rightarrow$ Symmetries cannot control operators like $\frac{\varphi^n}{m_{pl}^{n-4}} \cos \frac{n\varphi}{f}$ for $n \gg 1$

Spoils slow roll inflation
Candidate solution: monodromy in field space

Consider compact scalar field $\varphi \sim \varphi + f$ ; $f \ll m_{pl}$

Theory invariant under shift $\varphi \rightarrow \varphi + f$ physical state need not be

Let axion wind $N$ times such that $N f_{\phi} \gg m_{pl}$

Compactness of field space seems to control quantum corrections
• Known string realizations seem to give flat potentials, with relatively small powers

\[ V \sim M^{4-p} \varphi^{p<2} \]

Dong, Horn, Silverstein, and Westphal

• Scenario seems to be viable in string theory but: quantum corrections studied model by model: these are complicated, and physical reason for flat potentials is not completely transparent.

Strategies: study calculable 4d models (part II)
effective field theory analysis (part III)

Some lessons:

• Must take care that metastable states stay metastable, do not decay too quickly

• Quadratic inflation viable only if moduli coupling to inflaton are heavy
II. Monodromy from strongly coupled QFT

Goal: study axion monodromy in a calculable context, via gauge/gravity duality

N type IIA D4-branes wrapped on $S^1$ with radius $\beta$

Antiperiodic boundary conditions for fermions break SUSY

Bosons get mass from loops

Massless sector: U(N) gauge theory

$\theta$ angle from D-brane coupling to RR 1-form potential

$$S_{WZW} = \int_{S^1 \times R^4} C^{(1)} \wedge \text{Tr} F \wedge F$$

For constant RR field polarized along $S^1$ (Wilson line)

$$\theta = \frac{2\pi C_\beta}{\sqrt{\alpha'}}$$
Decoupling limit and gravitational dual

\[ \sqrt{\alpha'} \to 0, g_s \to \infty \quad \text{such that} \quad g_{5,YM}^2, g_{4,YM}^2 \quad \text{held fixed} \]

\[ N \to \infty, \lambda = g_{4,YM}^2 N \quad \text{fixed} \]

massless open strings decouple from closed strings, oscillator modes at low energies

\[ u \to 0 \quad \text{throat is locally} \quad R^4 \times S^1 \times R_u \times S^4 \]

Dual gravity solution for small \[ \theta \ll N/\lambda = g_{4,YM}^{-2} \]

found by Witten (1998)
Phases of theory

(1) “Throat” is infinite -- no mass gap. “Deconfined” phase.

Vacuum energy independent of $\theta$

(II) “Throat” ends at $u = u_0$

Mass gap at $\Lambda_{QCD} \sim u_0/\lambda$ \quad ($u_0 \sim \lambda/\beta$ for small $\theta$) \quad Less useful for studying 4d confinement (at small $x$)

$E(\theta) \sim \lambda N^2 V \left( x = \frac{\lambda \theta}{4\pi^2 N} \right)$ \quad Witten; DLR

This always has lower energy

Energy dependence implies monodromy potential for $\theta$

Think of $\theta$ as nondynamical axion $\theta = \phi/f_\phi$
Three Branches of Vacua
Large-\(x\) behavior

\[
\int_{S^1_{u=\infty}} d\chi C^{(1)}_\chi = \int dud\chi F_{u\chi} = \theta + 2\pi n
\]

For \(x \sim \frac{\lambda n}{2\pi N} \gg 1\) must take backreaction of 2-form flux into account

- \(\Lambda_{QCD} \sim \frac{u_0}{\lambda} \sim \frac{1}{\beta(1+x^2)}\)

Throat recedes into IR, glueballs become 4d objects

- \(\frac{E}{V_3} \left( x = \frac{\lambda \theta}{4\pi^2 N} \right) = \frac{2\lambda N^2}{3^7 \pi^2 \beta^4} \left( 1 - \frac{1}{(1+x^2)^3} \right) \rightarrow x \rightarrow \infty \frac{2\lambda N^2}{3^7 \pi^2 \beta^4} \left( 1 - \frac{1}{x^6} \right)\)

Potential flattens (response of \(E\) to \(\theta\) depends on \(\Lambda_{QCD}\) )

Viable model of inflation
Stability at large $x$

$R_u \times S^1 \uparrow$ becomes long, thin cylinder

- Winding modes about $\chi$ when $x = \frac{\lambda \theta}{4\pi^2 N} \gg \lambda^{1/3}$

- Casimir forces dominate over RR 2-form flux when $x^7 \gg N \lambda^{1/2}$

Result in both cases is to “pinch off” cylinder for $u > u_0(x)$

But we already know a solution; branch with lower energy.
Conjecture: a given branch with $x = 0$ at minimum ceases to exist at large $x$
Nonperturbative instabilities

D6-brane is a source for RR 2-form charge.

Two candidate domain wall solutions

• D6-brane wrapping $S^4$ sitting at $u = u_0$  
  Witten

• D6-brane wrapping $S^3(\varphi) \subset S^4$ filling $\mathbb{R}^4$
  
  $\varphi$ appears as QFT mode
  
  analogous to Kachru, Pearson, Verlinde

Domain wall when $\varphi$ varies in space

Nucleation of second domain wall has lower action at large $x$
\[ \phi = 0, \quad \phi = \pi \]

- Height of barrier \( \Delta E \sim \frac{\lambda^2 N}{\beta^4 x^{11}} \) at large \( x \)

- Scaling applied to DBI action of D6 \( S \sim \frac{\lambda^2 N}{x^{11}} \)

metastable branch beginning at \( x = 0 \) should end when \( x^{11} \gg \lambda^2 N \)
III. 4d models of axion monodromy

**Axion-four form model** Kaloper and Sorbo

\[
S_{\text{class}} = \int d^4 x \sqrt{-g} \left( m_{pl}^2 R - \frac{1}{48} F^2 - \frac{1}{2} (\partial \varphi)^2 + \frac{\mu}{24} \varphi^* F \right)
\]

\[
F_{\mu\nu\lambda\rho} = \partial_{[\mu} A_{\nu\lambda\rho]} \quad \text{U}(1) \text{ gauge symmetry: } \delta A_{\mu\nu\lambda} = \partial_{[\mu} \Lambda_{\nu\lambda]},
\]

\[
\varphi \text{ periodic: } \varphi \rightarrow \varphi + f_{\varphi}
\]

F does not propagate.

\[
F_{\mu\nu\lambda\rho} = n e^2 \epsilon_{\mu\nu\lambda\rho}; \; n \in \mathbb{Z}
\]

U(1) quantized

\[n\] can jump across domain walls/membranes

\[
H_{\text{tree}} = \frac{1}{2} p_{\phi}^2 + \frac{1}{2} (p_A + \mu \phi)^2 + \text{grav}.
\]

\[
p_A = n e^2
\]

Good model for inflation: fits data well if \( \mu \sim 10^{-6} m_{pl} \)

+ observable GW
Does this survive quantum corrections?

Can we get quadratic inflation or does the potential flatten?

(1) Direct corrections to $V(\varphi)$

Periodicity of $\varphi \Rightarrow$ quantum corrections to $S$ must be

$$\delta V \sim \Lambda^4 \sum_{n>1} c_n \cos(n\varphi/f_\varphi)$$

$$f_\varphi \ll m_{pl}$$

Monodromy potential modulated by periodic effects

$$V_{corr} \ll \frac{1}{2} \mu^2 \varphi^2 \Rightarrow \Lambda^4 \ll M_{gut}^4$$

$$\eta = m_{pl}^2 \frac{V''}{V} \ll 1 \Rightarrow \frac{\Lambda^4}{f^2_\varphi} \ll \frac{V}{m_{pl}^2} = H^2$$

Example: feasible if $\Lambda \sim .1 \ M_{gut}, \ f > .01 \ m_{pl}$
Caveat: moduli stabilization

In any string theory: couplings in $V$ will depend on moduli $\psi$

$$V = V_0(\psi) + \frac{1}{2} \mu^2 \left( \frac{\psi}{m_{pl}} \right) \varphi^2 + \Lambda^4 \sum_n c_n \left( \frac{\psi}{m_{pl}} \right) \cos \left( \frac{n \varphi}{f_\varphi} \right)$$

Periodic corrections change sign many times since $f_\varphi \ll m_{pl}$

Moduli must be stabilized by different effects than instantons coupling to inflaton

$$M^2_\psi \equiv V_0''(\psi) \gg \frac{\Lambda^4}{m^2_{pl}}$$

Large $\varphi \gg m_{pl}$ sources potential for $\psi$

Stability requires $M^2_\psi \gg \mu^2 \varphi^2 / m^2_{pl} \sim \mu^2 / \epsilon \sim H^2$
(2) Indirect corrections to $V(\varphi)$

Additional corrections must respect periodicity of $\varphi$

⇒ corrections to dynamics of four-form $F$

$$S_{class} = \int d^4x \sqrt{g} \left( m^2_{pl} R - \frac{1}{48} F^2 - \frac{1}{2} (\partial \varphi)^2 + \frac{\mu}{24} \varphi^* F \right)$$

Consider $\delta \mathcal{L} = \sum_n d_n \frac{F^{2n}}{M^{4n-4}}$

Integrate out $F$: $F \sim \mu \varphi + \ldots$

$$\delta V_{eff} = V_{class} \times \left( \sum_{n=1}^{d_n+1} d_n + \frac{V^{n}_{class}}{M^{4n}} \right)$$

Safe if: $M^4 \gg V_{class} \sim M^4_{gut}$
Bounds on membrane tension from quantum stability

Transitions occur by bubble nucleation. Let:

- $T =$ tension of bubble wall
- $E =$ energy difference between branches

Decay probability: \[ \Gamma \sim \exp \left( -\frac{27\pi^2 T^4}{2 E^3} \right) \] (thin wall)

Rate should be slow compared to time scale of inflation

\[ \Rightarrow \text{Phenomenological bound on } T \]

\[ \Gamma \ll 1 \Rightarrow T^{1/3} \gg \left( \frac{2}{27\pi^2 N^3} \right)^{1/4} V^{1/4} \]

Let: \[ f_\phi \sim 0.1 \, m_{pl}; \quad N \sim 100; \quad V \sim M_{gut}^4 \]

\[ T \gg (0.2V^3)^{1/4} \sim (0.9M_{gut})^3 \]

Borderline; should check against explicit models

N.B. $E$ larger for large $V$; transitions more likely early in inflation
V. Conclusions

- PNGB-driven inflation in UV-complete theory must use monodromy mechanism

- Interesting observational signals if a single branch-changing or mass-changing bubble nucleates early within our horizon?

- General issue: monodromy inflation does not seem *parametrically* safe. Should we worry?
  
  Perhaps this is interesting:
  
  - Implies number of e-foldings could be close to lower bound
  - Implications for measurements of curvature, pre-inflation transients

- Other interesting applications of axion monodromy
  
  Kerr black holes; axion condensation via Penrose process. Instability/disappearance of branch can lead to observable axion decays

Kaloper and AL, in progress

Dubovsky and Gorbenko