Triangle Anomaly in QGP : Chiral Magnetic Effect and Chiral Magnetic Wave

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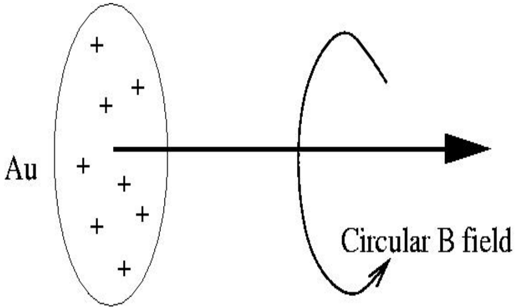
A table of recent interesting discoveries

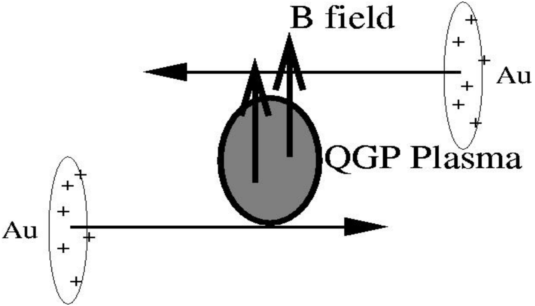
	$T < T_c$	$T > T_c$ (RHIC,LHC)
$B = 0$	- gauged WZW term $\pi^0 \to 2\gamma$, etc - $\mu \neq 0$, Quarkyonic spiral	- chiral vortex current ($\mu \neq 0$) $J^\mu \sim \epsilon^{\mu\nu\alpha\beta} u_\nu \partial_\alpha u_\beta$ - chiral shear wave ($\mu \neq 0$)
$B \neq 0$	- meson(pion) suppercurrent ($\mu \neq 0$) $\partial\pi \neq 0$ "chiral spiral" - chiral magnetic spiral ($\mu \neq 0$) $J^1 + iJ^2 \neq 0$	- chiral magnetic effect ($\mu \neq 0$) $\vec{J} \sim \vec{B}\mu$ -chiral magnetic wave ($\mu = 0$) $\omega \sim v_\chi k$

- Important difference between chiral magnetic effect(CME) and chiral magnetic wave(CMW) will be that CMW exists even in the average neutral plasmas
- Magnetic field is an important ingredient in both

Why is magnetic field in quark-gluon plasma relevant ?



Huge magnetic field in the middle



- Reference : Kharzeev-McLarren-Warringa
- eB depends on impact parameter, collision energy, as well as time during evolution
- Maximum eB is typically of order of $eB \sim (0.1 - 4)\ m_\pi^2 \sim 10^{18}\ G$. Astrophysical eB from neutron stars is about $10^{15}\ G$
- Kinematically, it lasts about $2\ fm$. But, Lenz's law in QGP makes it stay longer, even to freezeout time in optimistic scenarios

Scale comparisons

$$\text{Scale comparison : } T \sim 200\ MeV,\ \mu_B \sim 30\ MeV,\ \sqrt{eB} \sim 140\ MeV$$

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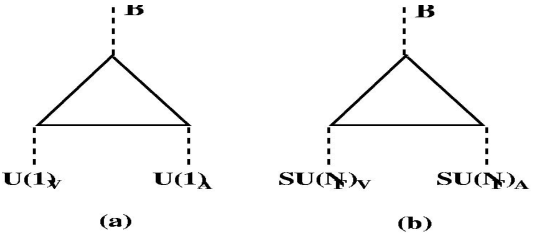
UPSHOT : Effects from this huge magnetic field may be important

Chiral Magnetic Effect

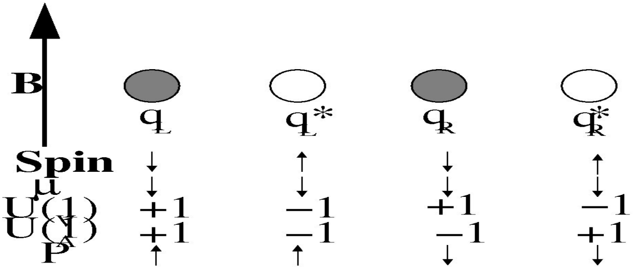


Our starting point is the recent discovery of Chiral magnetic effects (Kharzeev-McLarren-Warringa, Fukusima-Kharzeev-Warringa, Son-Newman)

$$\vec{j}_V = \frac{N_c e\vec{B}}{2\pi^2}\mu_A \quad , \quad \vec{j}_A = \frac{N_c e\vec{B}}{2\pi^2}\mu_V \quad , \qquad (1)$$

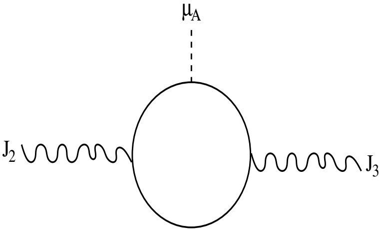
Charge current along the magnetic field is induced by chemical potential

Heuristic quasi-particle picture



- Quantize free massless chiral quarks fields ψ_L and ψ_R
- ψ_L has charges $B = +1$ and $A = +1$, and ψ_R has $B = +1$ and $A = -1$
- Massless quarks/antiquarks have definite helicities, so their motions are correlated with their spin alignment

Weak coupling computation

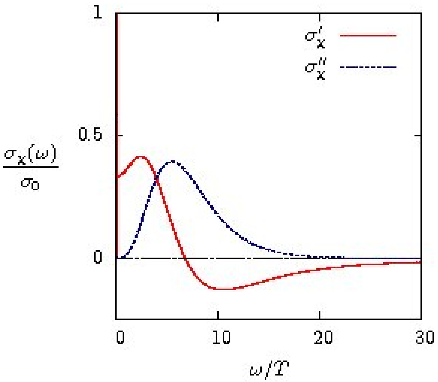


- Regard the magnetic field $B = F_{12}$ linearly, and think of linear response framework
- Observe the current J^3 in response of $B = \partial_1 A_2 - \partial_2 A_1$. Note that A_μ couples to J_μ
- This gives us $J^3 = \sigma_\chi B$ where the chiral magnetic conductivity σ_χ is

$$\sigma_\chi \sim \lim_{\omega\to 0} \lim_{k_1\to 0} \frac{1}{k_1} \langle J_3(k,\omega) J_2(k,\omega)\rangle \quad (2)$$

- One can also speak about frequency dependent chiral magnetic conductivity $\sigma_\chi(\omega)$

Weak coupling result



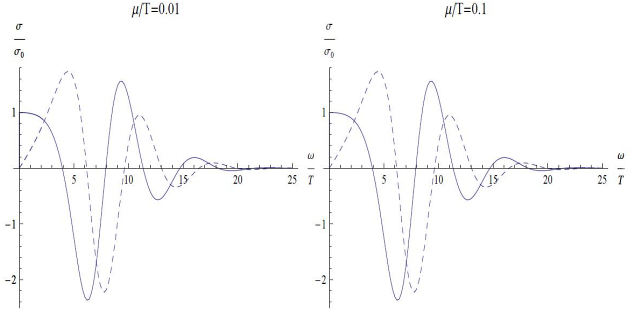
It was done by Kharzeev and Warringa

- Some ambiguity regarding order of limits for $k \to 0$ and $\omega \to 0$
- There is a sudden drop from $\omega = 0$ to $\omega \neq 0$
- It is not completely clear yet how to resolve these ambiguities

Holographic computation

Holographic chiral magnetic conductivity was computed in the Sakai-Sugimoto model (H.-U.Y)

In the case of Sakai-Sugimoto model, the necessary Chern-Simons term is already present, and we can compute things more reliably



Solid line: $\mathrm{Re}\sigma(\omega)$, Dashed line : $\mathrm{Im}\sigma(\omega)$

Chiral Magnetic Wave (Kharzeev and H.-U.Y)

- Consider general deconfined QCD plasma with applied magnetic field $\vec{B} = B\hat{x}^1$. The plasma can be neutral in general.
- We will treat electromagnetism as non-dynamical external environment
- Let's start from the basic chiral magnetic effects

$$\vec{j}_V = \frac{N_c e\vec{B}}{2\pi^2}\mu_A \quad , \quad \vec{j}_A = \frac{N_c e\vec{B}}{2\pi^2}\mu_V \quad , \tag{3}$$

$$\begin{pmatrix} \vec{j}_V \\ \vec{j}_A \end{pmatrix} = \frac{N_c e\vec{B}}{2\pi^2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} \mu_V \\ \mu_A \end{pmatrix} \quad . \tag{4}$$

- We are interested in small fluctuations out of neutral plasma, so let's expand $\mu_{V,A}$ in terms of small charge densities $j^0_{V,A}$ linearly

$$\begin{pmatrix} \mu_V \\ \mu_A \end{pmatrix} = \begin{pmatrix} \frac{\partial \mu_V}{\partial j^0_V} & \frac{\partial \mu_V}{\partial j^0_A} \\ \frac{\partial \mu_A}{\partial j^0_V} & \frac{\partial \mu_A}{\partial j^0_A} \end{pmatrix}\begin{pmatrix} j^0_V \\ j^0_A \end{pmatrix} \equiv \begin{pmatrix} \alpha_{VV} & \alpha_{VA} \\ \alpha_{AV} & \alpha_{AA} \end{pmatrix}\begin{pmatrix} j^0_V \\ j^0_A \end{pmatrix} \quad . \tag{5}$$

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CMW Derivation continued

- Recall that chemical potentials are defined as $\mu_i = \frac{\partial \mathcal{F}}{\partial j_i^0} \quad , \quad i = V, A$ where $\mathcal{F}$ is free energy, so

$$\alpha_{ij} = \frac{\partial^2 \mathcal{F}}{\partial j_i^0 \partial j_j^0} \quad . \tag{6}$$

are in fact susceptabilities

- By parity invariance, $\alpha_{AV} = \alpha_{VA} = 0$. In a deconfined phase, one also has $\alpha_{VV} \sim \alpha_{AA} \equiv \alpha$, so at the end one arrives at

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- Diagonalize in terms of chiral basis

$$j_L^\mu \equiv \frac{1}{2}\left(j_V^\mu - j_A^\mu\right) \quad , \quad j_R^\mu \equiv \frac{1}{2}\left(j_V^\mu + j_A^\mu\right) \quad . \tag{8}$$

One can also rewrite

$$\alpha = \frac{1}{2}\left(\frac{\partial \mu_L}{\partial j_L^0}\right) = \frac{1}{2}\left(\frac{\partial^2 \mathcal{F}}{\partial j_L^0 \partial j_L^0}\right) = \frac{1}{2}\left(\frac{\partial \mu_R}{\partial j_R^0}\right) = \frac{1}{2}\left(\frac{\partial^2 \mathcal{F}}{\partial j_R^0 \partial j_R^0}\right) \quad . \tag{9}$$

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Continued...

The semi-final result is

$$\vec{j}_{L,R} = \mp \left(\frac{N_c e \vec{B} \alpha}{2\pi^2} \right) j^0_{L,R} - D_L \frac{\vec{B}(\vec{B} \cdot \vec{\nabla})}{B^2} j^0_{L,R} + \cdots \quad , \tag{10}$$

with a longitudinal diffusion constant D_L. This should be taken as a hydrodynamic constitutive equation in long wave-length expansion
A few comments are

- What we are claiming is that the above result is valid for arbitrarily large magnetic field B. Several previous literature considered only linear in B
- Note that α will be a non-trivial, non-linear function of B as well as temperature, coupling constant, etc. We will see this explicitly later for infinitely large B case and holographic QCD computation
- It might also be interesting to consider transverse diffusion coefficient D_T

New propagating modes out of this

Now, plug the above in the conservation laws $\partial_\mu j^\mu_{L,R} = 0$, and consider only longitudinal derivative ∂_1, which results in

$$\left(\partial_0 \mp \frac{N_c e B \alpha}{2\pi^2}\partial_1 - D_L \partial_1^2\right) j^0_{L,R} = 0 \quad . \tag{11}$$

- This is a directional (chiral) wave with velocity

$$v_\chi = \frac{N_c e B \alpha}{2\pi^2} = \frac{N_c e B}{4\pi^2}\left(\frac{\partial \mu_L}{\partial j^0_L}\right) = \frac{N_c e B}{4\pi^2}\left(\frac{\partial \mu_R}{\partial j^0_R}\right) \quad . \tag{12}$$

- The momentum space dispersion relation looks as

$$\omega = \mp v_\chi k - i D_L k^2 + \cdots \quad , \tag{13}$$

  The propagating leading part would be absent if there was no anomaly or magnetic field B, and it is essentially important to have them to find the propagating behavior

Large B limit and 1+1 dimensional reduction

We will consider one particular weak coupling limit, $eB \to \infty$

- At first, the expression

$$v_\chi = \frac{N_c eB}{4\pi^2}\left(\frac{\partial \mu_L}{\partial j^0_L}\right) \tag{14}$$

looks worrisome in view of causality

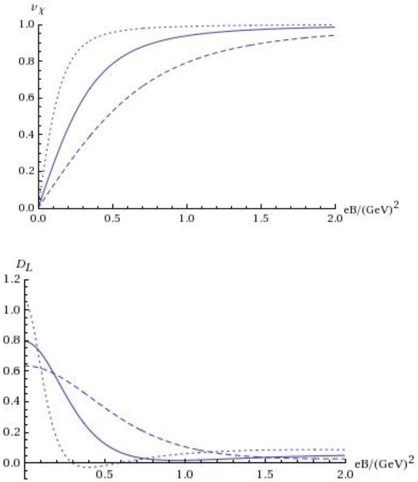
- It is natural to expect that the quarks are sitting in the lowest Landau Levels, as the gap $\Delta = \sqrt{eB} \gg T$. We have an effective 1+1 dimensional reduction.
- For a chemical potential μ_L, 1+1 dimensional free fermion density is simply $\frac{\mu_L}{(2\pi)}$. We also have transverse space density of state of $\frac{N_c eB}{(2\pi)}$, so that the net 3-dimensional density j^0_L is

$$j^0_L = \left(\frac{\mu_L}{(2\pi)}\right)\left(\frac{N_c eB}{(2\pi)}\right) = \frac{N_c eB}{4\pi^2}\mu_L \tag{15}$$

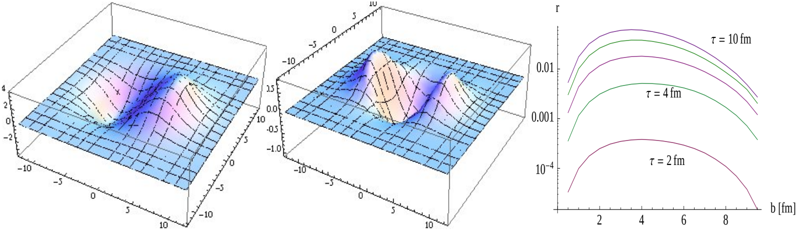
or $\left(\frac{\partial \mu_L}{\partial j^0_L}\right) = \frac{4\pi^2}{N_c eB}$ in the limit of $eB \to \infty$. This gives

$$v_\chi \to 1 \quad , \quad eB \to \infty \tag{16}$$

Results from Sakai-Sugimoto model



Application to heavy-ion experiments (Burnier, Kharzeev, Liao, H.-U.Y)



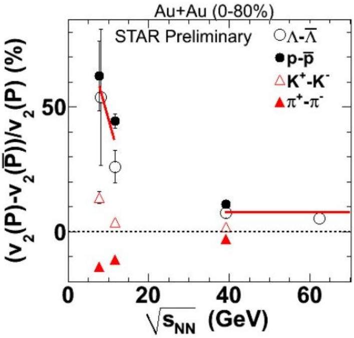
- Created plasma has a net baryonic density
- Baryonic density is a sum of equal amount of left anf right-handed chiral charges

$$\rho_V = \rho_L + \rho_R \tag{17}$$

- According to CMW, ρ_L and ρ_R move to opposite direction due to their chirality
- Net result is electric quadrupole deformation, which eventually leads to charge dependent elliptic flow $v_2^\pm$ for pions

$$v_2^\pm = v_2 \pm 0.15r \tag{18}$$

Experimental observation of $v_2^{\pm}$



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