Anisotropy flow in hydrodynamics with viscous and non-linearity corrections

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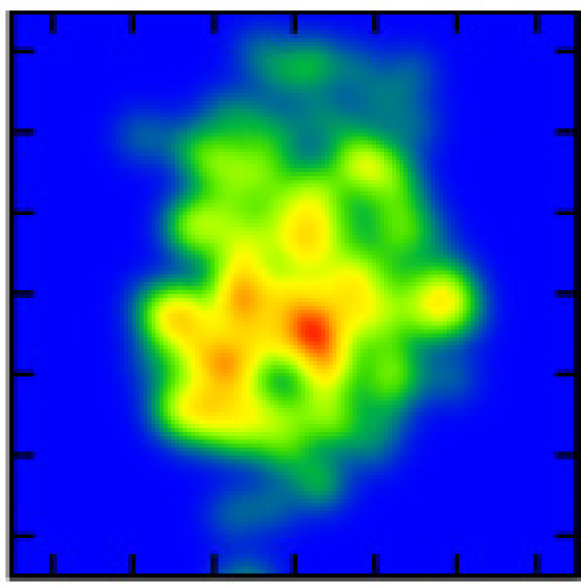
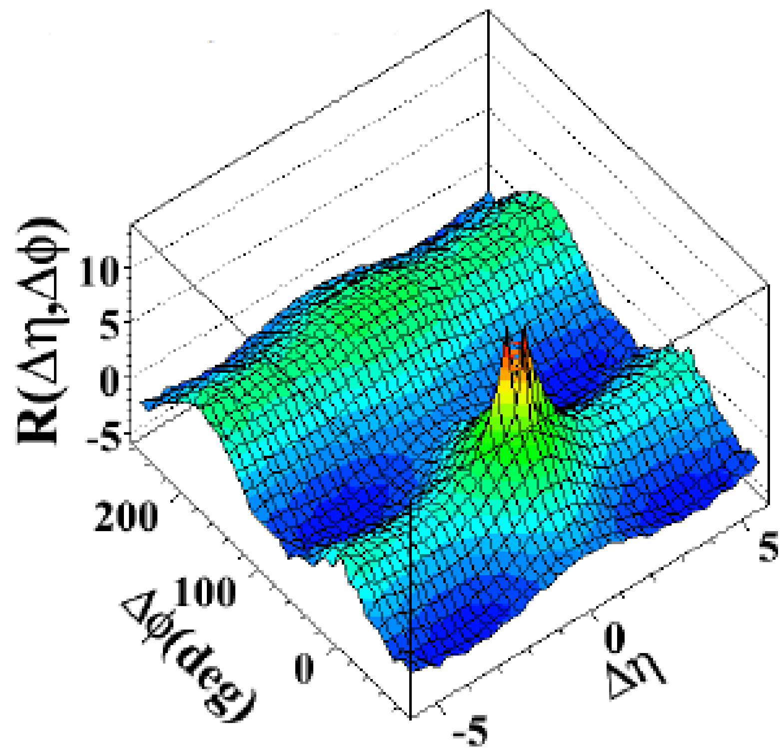
- Collaborate with Derek Teaney, Phys.Rev.C83:064904,2011
- Collaborate with Derek Teaney, in progress.

Fluctuated initial condition ⇔ experiment observations:

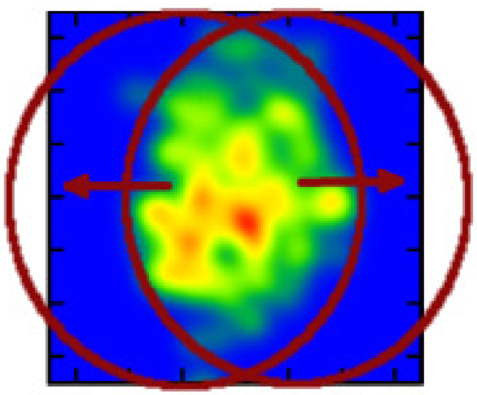




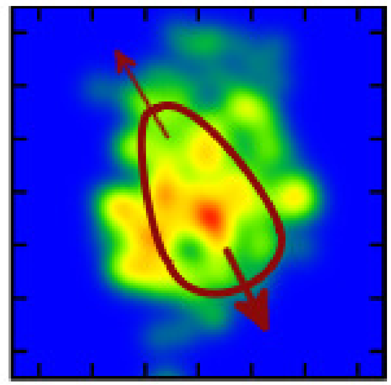
- **Stage I**. Information from fluctuated IC. → cumulant exp.
- **Stage II**. Fluid evolution. → viscous damping.
- **Stage III**. Freeze out. → non-linear flow coupling.

Fluctuated initial condition: MC-Glauber or MC-KLN[1]

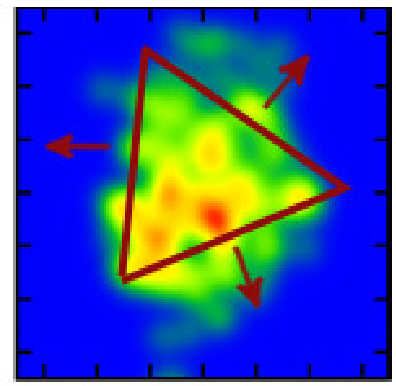
Origins of particle correlations and flow: initial geometry.



$\epsilon_2, \psi_2(\psi_R)$



ϵ_1, ψ_1



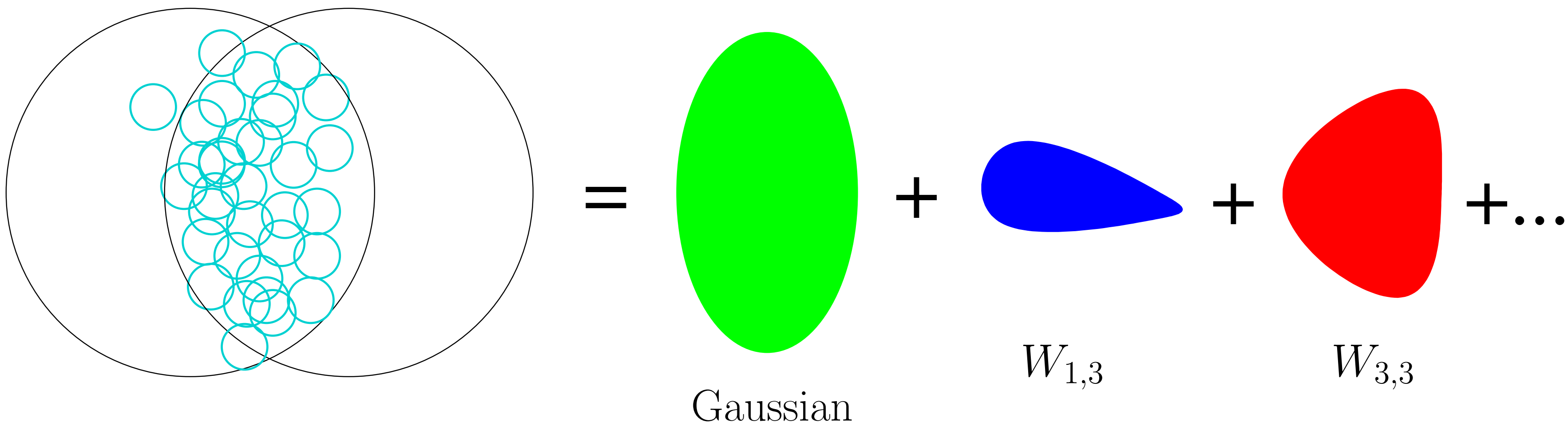
ϵ_3, ψ_3

As a result of fluid response,

- $\epsilon_n \longleftrightarrow v_n$
- $\{\epsilon_n, \psi_n\}$ correlations $\longleftrightarrow$ correlations at final stage.

[1] MC-Glauber: Alver, B. et all, arXiv:0805.4411. MC-KLN: H. Drescher and Y. Nara, PhysRevC.76.041903.

Cumulant expansion: long wavelength expansion $W_{n,m}$



- Classify eccentricities with cumulants[2]:

$$\epsilon_{n,m} \propto -\frac{W^c_{n,m}}{\langle r^m \rangle}, \qquad n\psi_{n,m} = \mathrm{atan2}(W^c_{n,m}, W^s_{n,m}) + \pi$$

- We have two indices : $n \rightarrow$angle, $m \rightarrow$ radial size.

[2] $\langle \ldots \rangle$ is average over initial distribution.

Because we have $\{n, m\}$

- $W^c_{1,3} = \frac{3}{8}\langle r^3 \cos\phi\rangle$: distortion (not displacement) of IC.

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$$W^c_{4,4} = \frac{1}{16}\left[\langle r^4 \cos 4\phi\rangle - \underline{3\langle r^2 \cos 2\phi\rangle^2}\right]$$

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correction from lower order cumulants,

1. $\epsilon_{4,4} \neq \epsilon_4$
2. $\{\psi_{4,4}, \psi_R\}$ correlations not strong.

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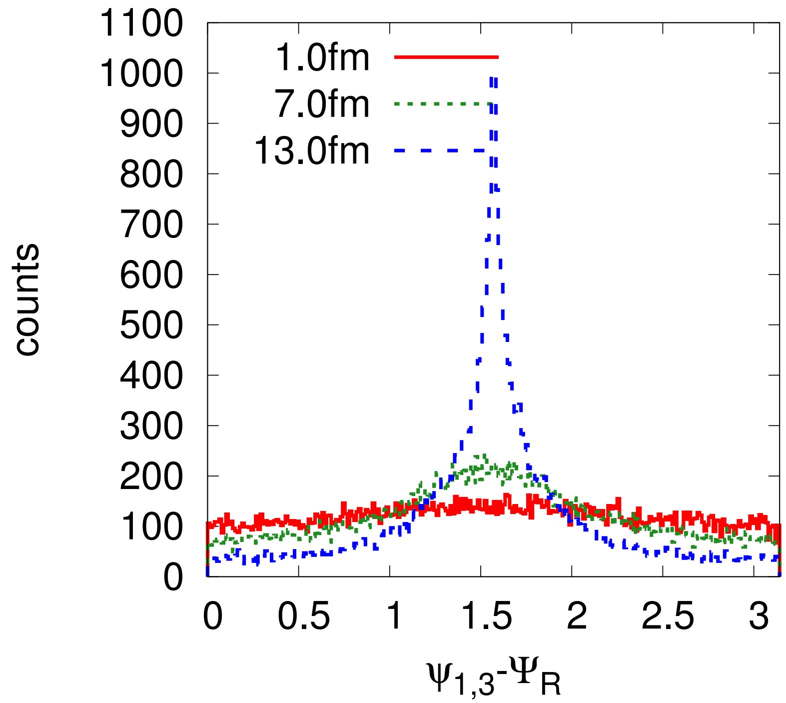
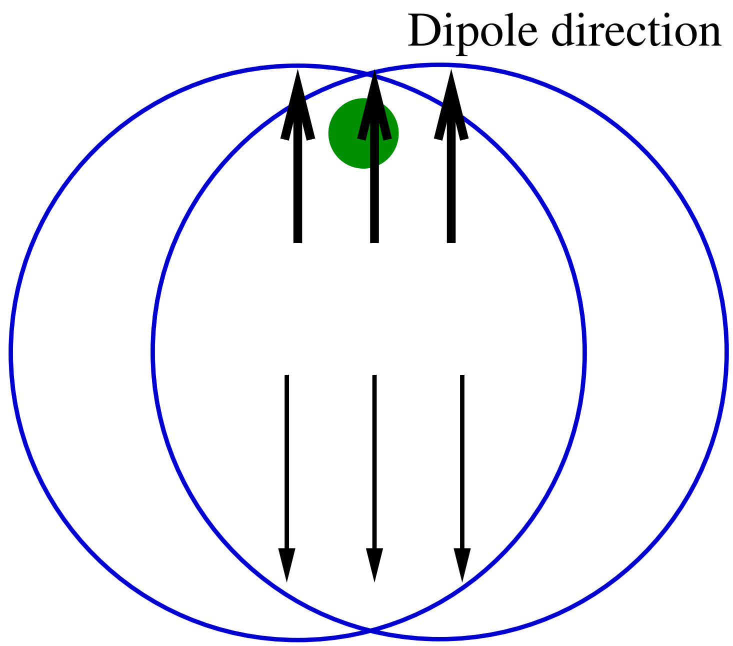
The remaining significant orientation angle correlations:

$$\{\psi_{1,3}, \psi_R\} \qquad \text{and} \qquad \{\psi_{1,3}, \psi_{3,3}, \psi_R\}$$

$\langle\cos(2\psi_{1,3} - 2\Psi_R)\rangle \longleftrightarrow \langle\cos(\phi_\alpha + \phi_\beta - 2\Psi_R)\rangle$[3]





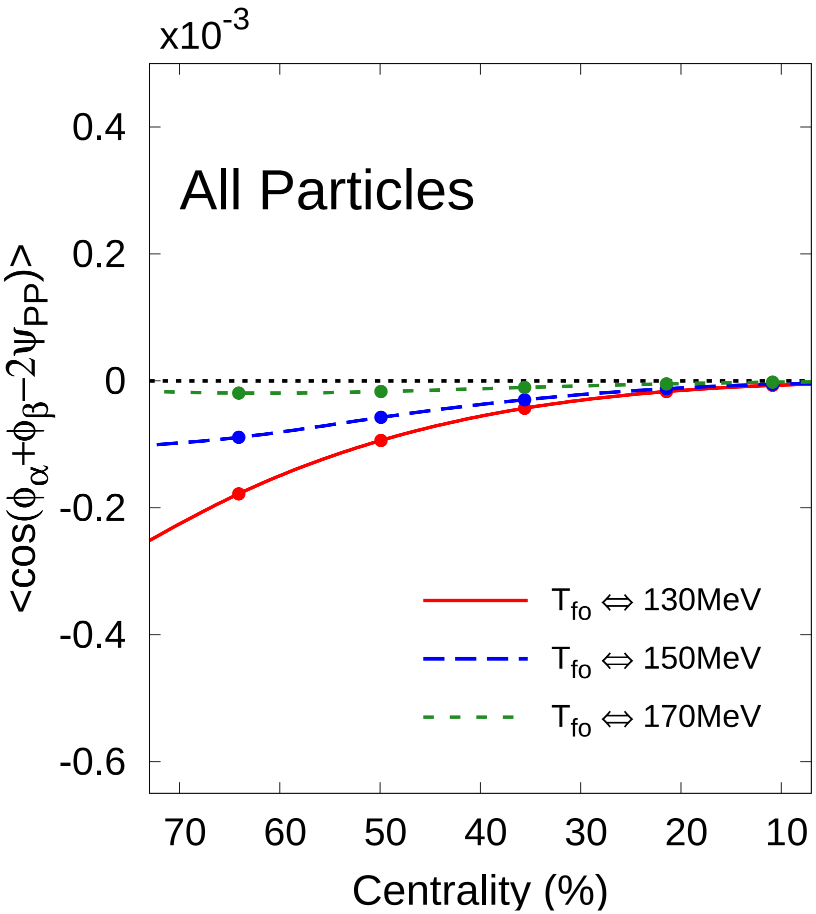
- Stronger correlation from MC-KLN than MC-Glauber.
- In the spectrum of two-particle correlations,

$$\langle\cos(\phi_\alpha + \phi_\beta - 2\Psi_R)\rangle \propto \langle\cos(2\psi_{1,3} - 2\Psi_R)\rangle \times v_1^2$$

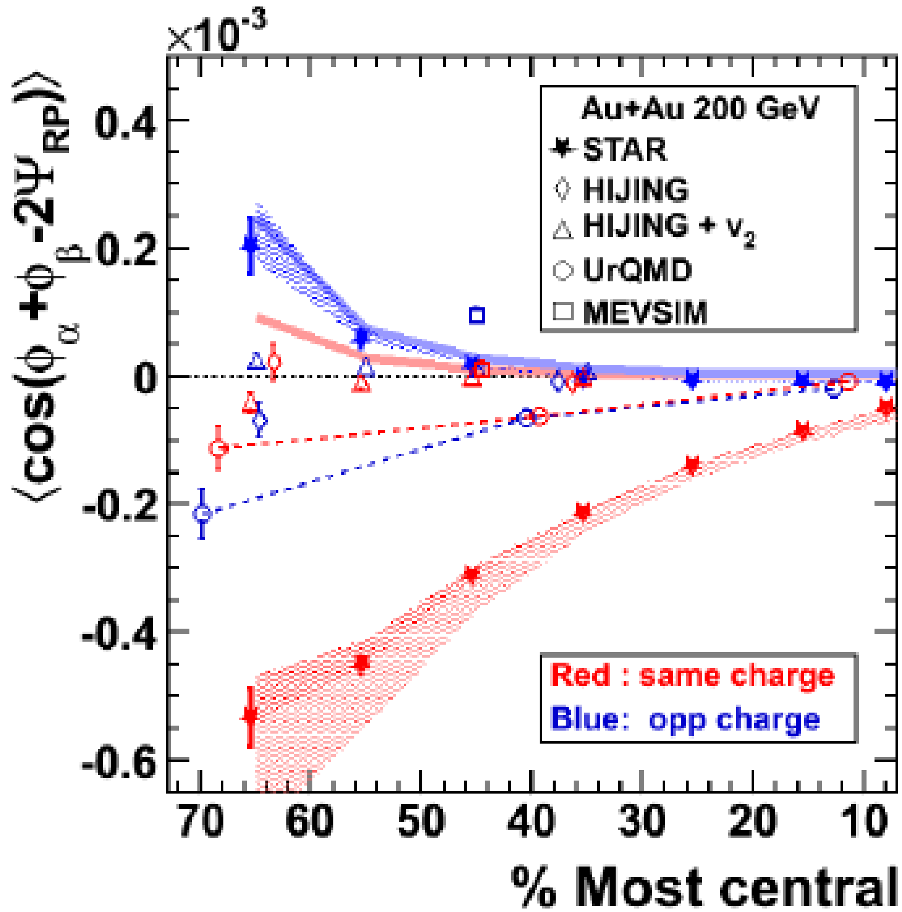
[3] $\langle\ldots\rangle$ is average over events.

$\langle\cos(\phi_\alpha + \phi_\beta - 2\Psi_R)\rangle$ vs STAR[4]

By MC-Glauber



STAR data

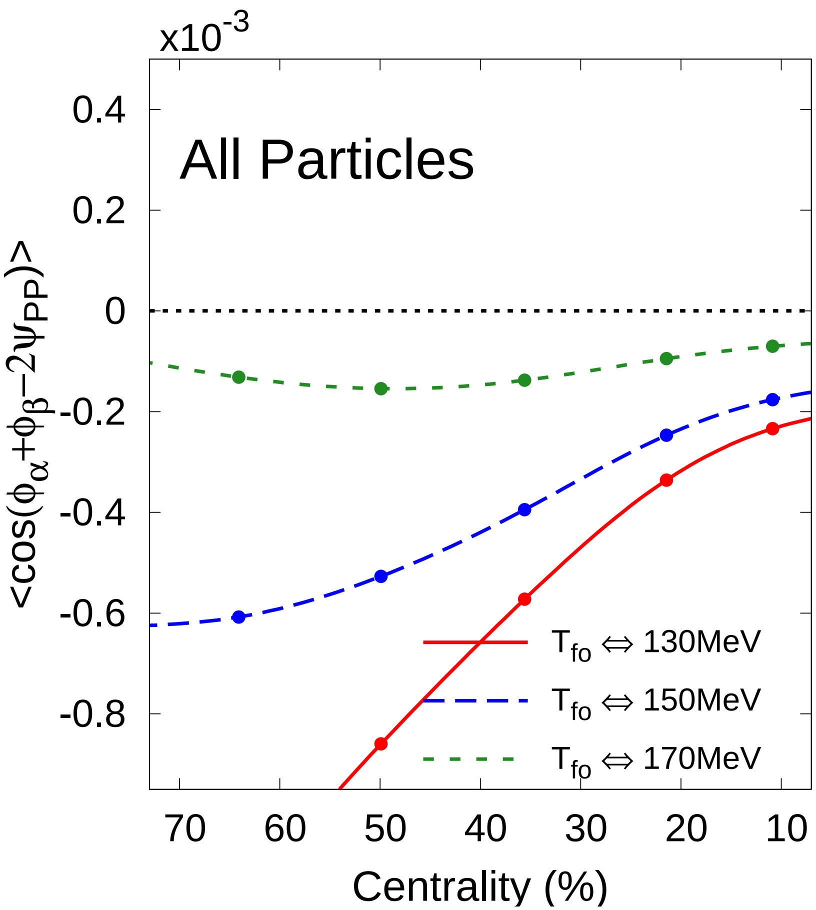


- $\eta/s = 1/4\pi$ and lattice EOS.

[4] B. I. Abelev et al. [STAR Collaboration], Phys. Rev. Lett. 103, 251601 (2009)

$\langle \cos(\phi_\alpha + \phi_\beta - 2\Psi_R) \rangle$

By MC-KLN



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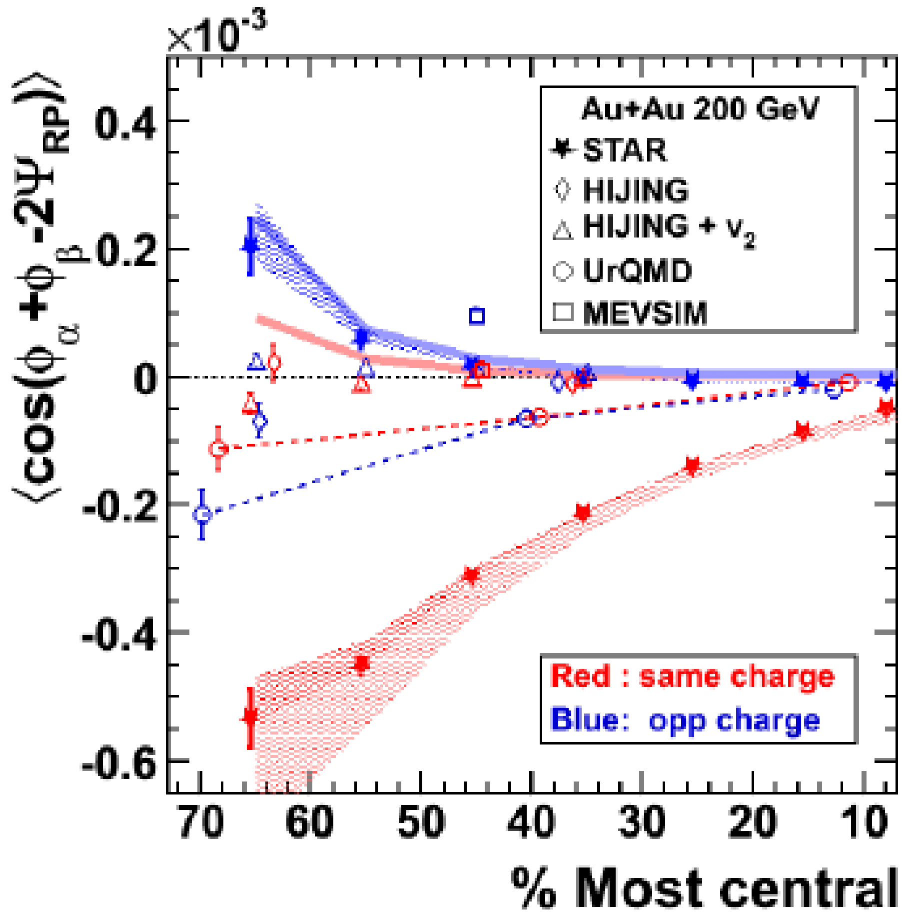

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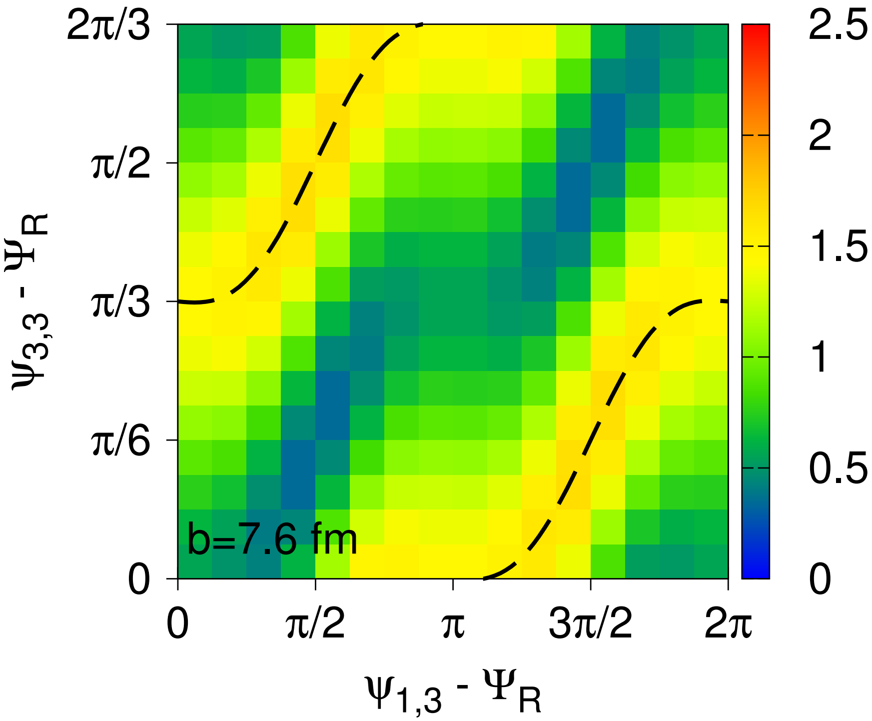
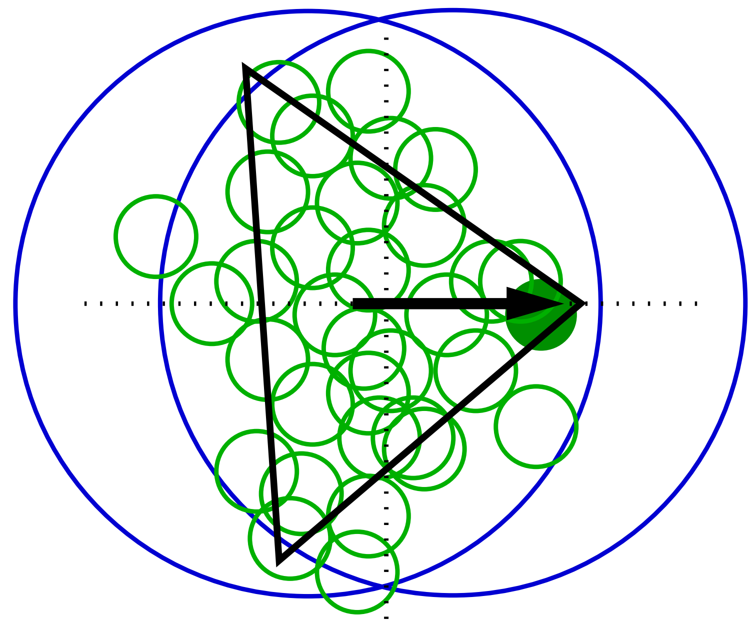
$\langle\!\langle \cos(\psi_{1,3} + 2\Psi_R - 3\psi_{3,3}) \rangle\!\rangle \longleftrightarrow \langle\!\langle \cos(\phi_\alpha - 3\phi_\beta + 2\Psi_R) \rangle\!\rangle$





Dipole and triangular

- Stronger correlation from MC-KLN than MC-Glauber.
- In the spectrum of two-particle correlations,

$$\langle\!\langle \cos(\phi_\alpha - 3\phi_\beta + 2\Psi_R) \rangle\!\rangle \propto \langle\!\langle \cos(\psi_{1,3} + 2\Psi_R - 3\psi_{3,3}) \rangle\!\rangle \times v_1 v_3$$

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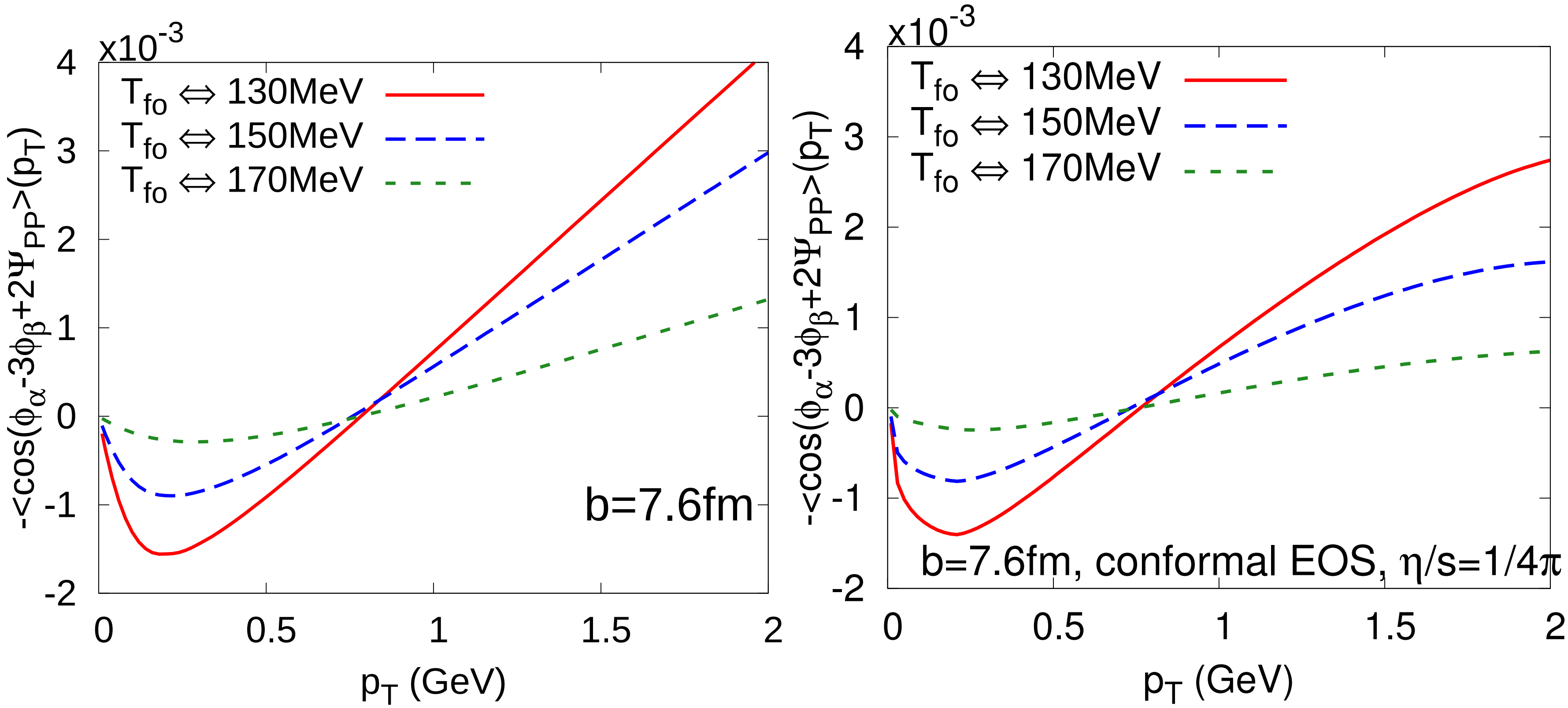

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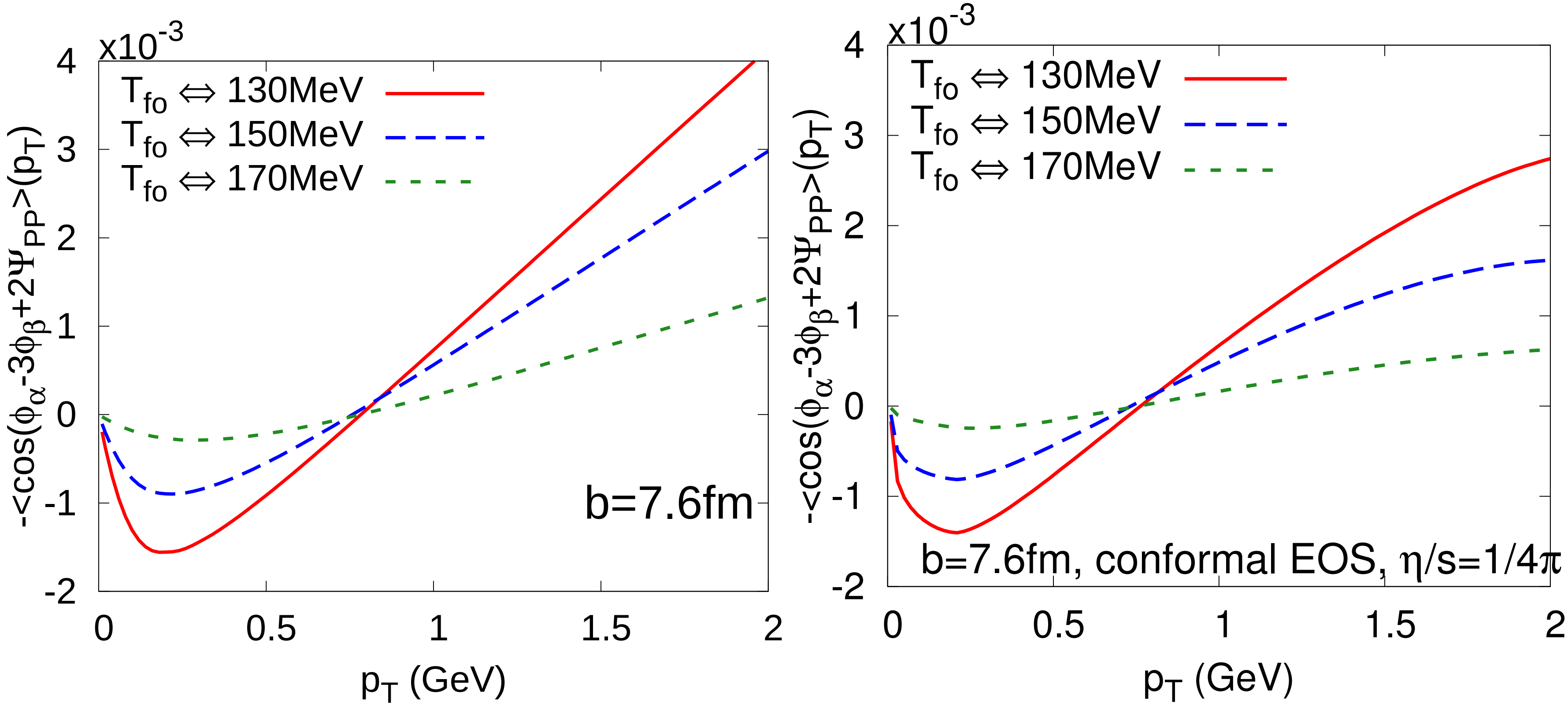
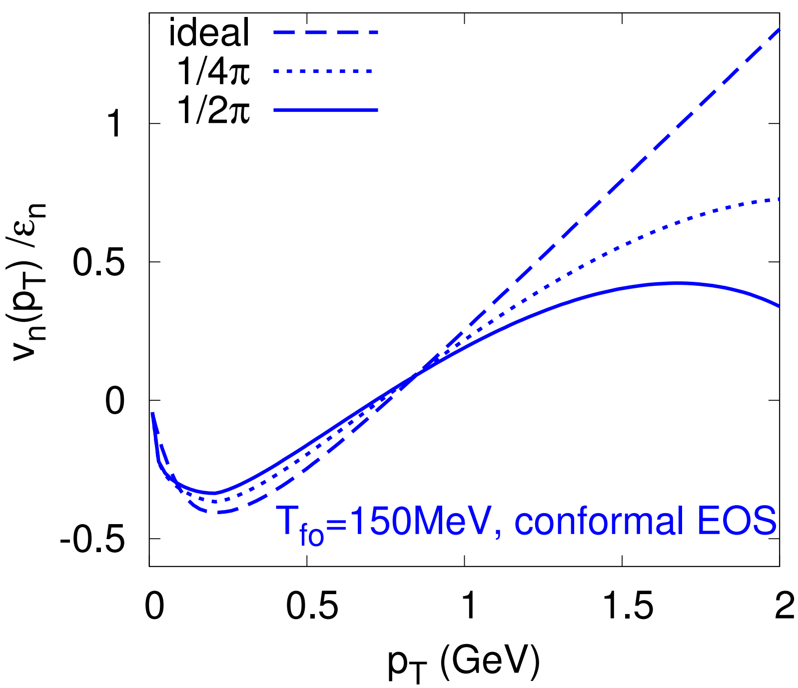
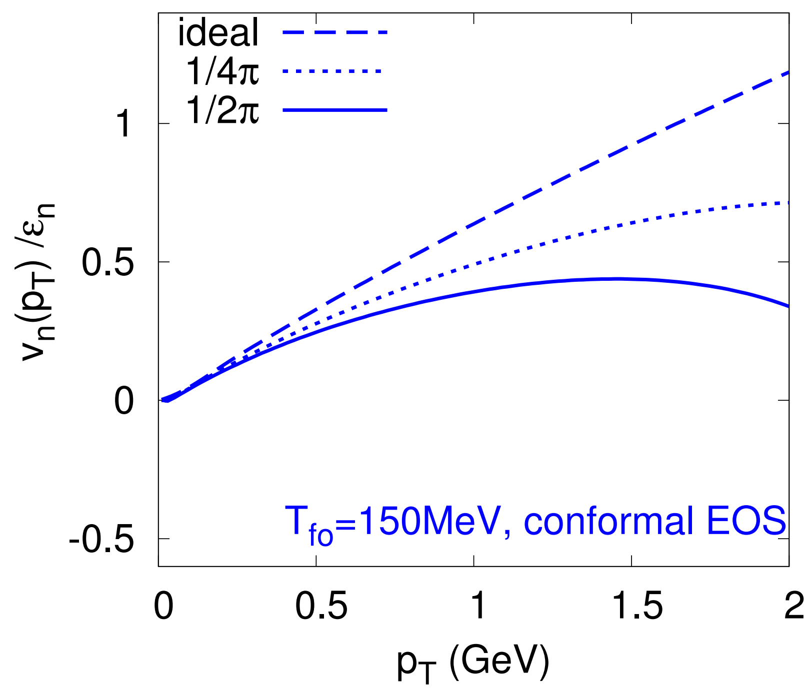


What effects viscosity have on flow evolution?

Anisotropy flow from viscous hydro.



dipole flow



elliptic flow

- Viscous damping looks similar for both v_1 and v_2!

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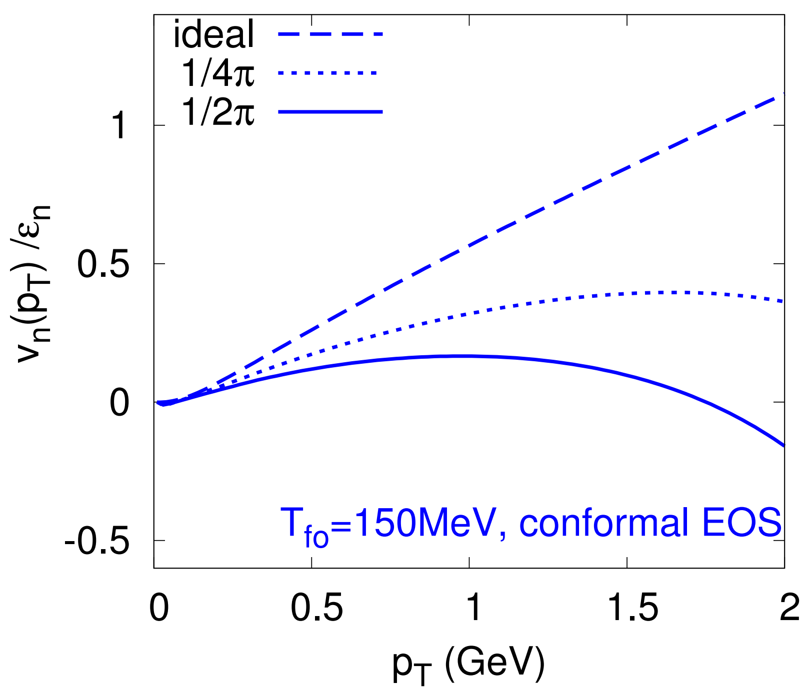
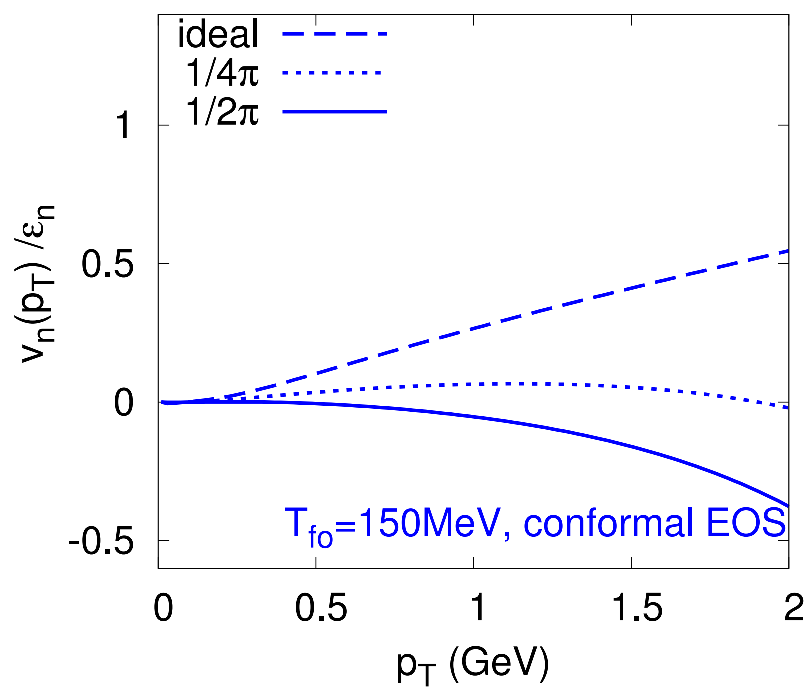


triangular flow



quadrangular flow

- Viscous damping is much stronger in v_3 and v_4.

[5] S. Gubser and A. Yarom, Nucl.Phys.B846:469-511,2011.

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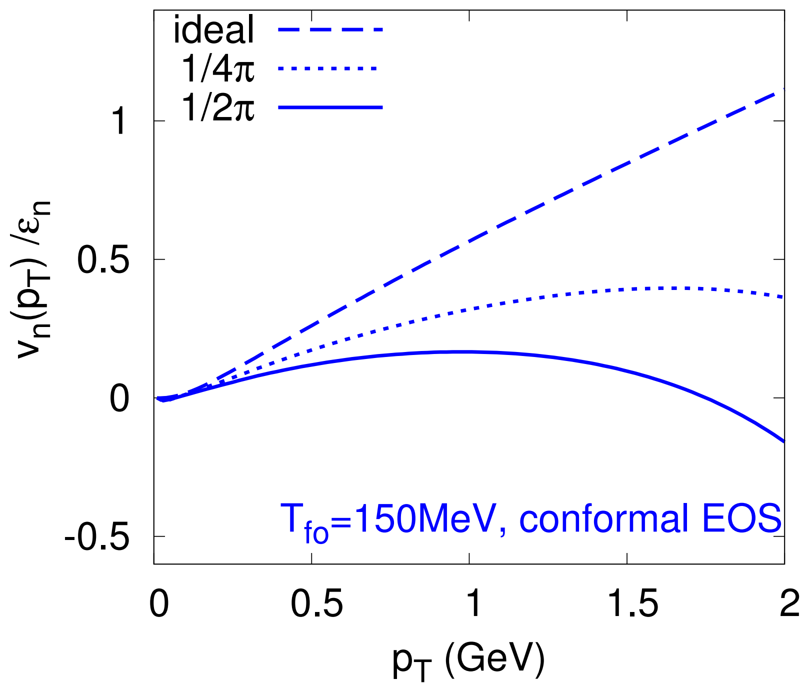

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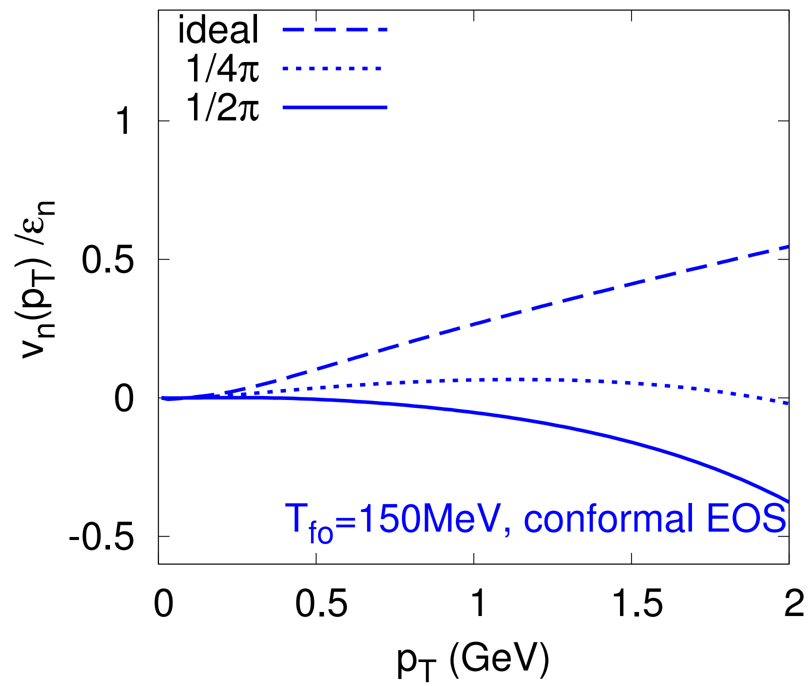

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- Viscous damping is much stronger in v_3 and v_4.

$$\boxed{\text{Damping}: \{1,3\} \sim \{2,2\} < \{3,3\} \sim \{2,4\} \sim \{1,5\} < \{4,4\} \ldots}$$ [5]

[5] S. Gubser and A. Yarom, Nucl.Phys.B846:469-511,2011.

v_n from freeze-out surface

- Particle spectrum Fourier decomposition w.r.t. ϕ_p

$$E\frac{d^3N}{dp^3} = E\frac{d^2N}{2\pi p_t dp_t dy}\left[1 + \sum_{n=1} 2v_n(p_t)\cos[n(\phi_p - \psi_{n,p})]\right]$$

- Borghini and Ollitrault[6], w/ Cooper-Fryer freeze-out:

$$E\frac{d^3N}{dp^3} = \frac{\nu}{(2\pi)^3}\int_\Sigma p^\mu d\sigma_\mu exp(-p\cdot u/T).$$

When $\frac{p_t}{T} \gg 1 \Longrightarrow v_4(p_t) = \text{linear } v_4 + \frac{1}{2}v_2(p_t)^2$.

[6] Phys.Lett. B642 (2006) 227-231

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Then in analogy ...

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Anisotropy flow couplings – ansatz

If we assume,(and will be tested)

- Response of flow from hydro. to cumulants is linear.
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v_n \cos n(\phi_p - \psi_{n,p}) = & v_n^0 \cos n(\phi_p - \psi_{n,m}) \\
& \underline{+ \frac{1}{2} \sum_{i+j=n} v_i^0 v_j^0 \cos[n\phi_p - (i\psi_{i,m} + j\psi_{j,m})]} \\
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\end{aligned}
$$

1. $v_n^0 \propto \frac{p_T}{T}$, is linear flow response.
2. Underlined terms from non-linear couplings.
3. Both flow magnitudes and orientation angles changed.

Test of this ansatz formula – a simple case w/ $\psi_{n,m} = 0$

Simplified formula: $v_n(p_t) = v_n^0(p_t) +$ non-linear terms,

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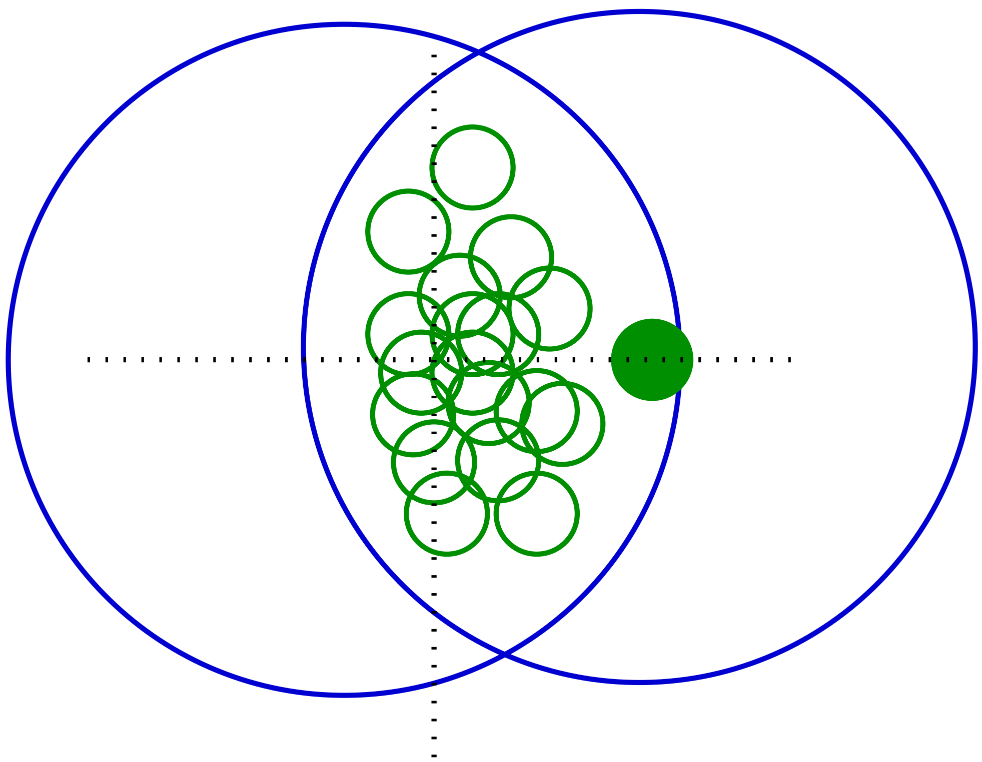
- IC : input $\epsilon_{1,3}$ and $\epsilon_{2,2}$, small $\epsilon_{3,3}$ and $\epsilon_{4,4} \dots$ generated.
- Linear v_n^0: v_1^0, v_2^0, v_3^0 v_4^0. Neglect $v_5^0 \dots$ for simplicity.



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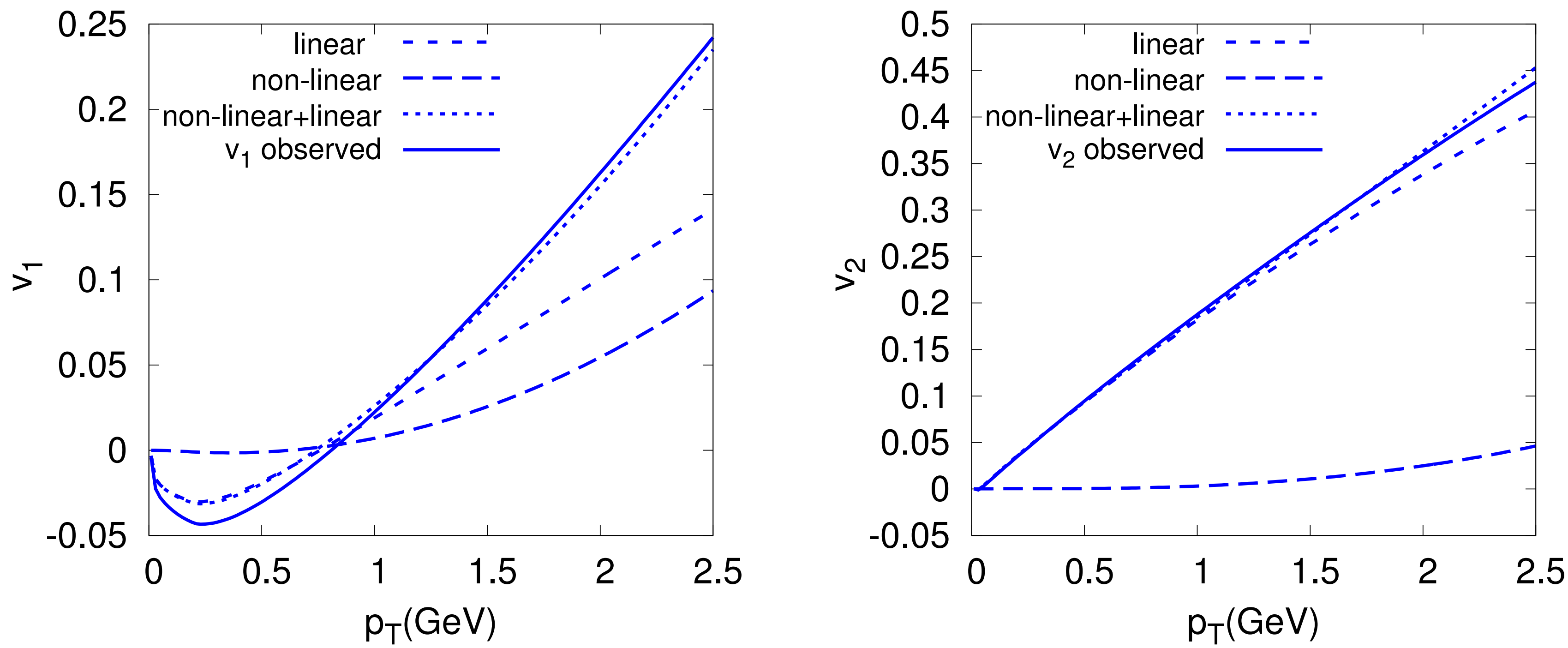

Dipole flow and elliptic flow

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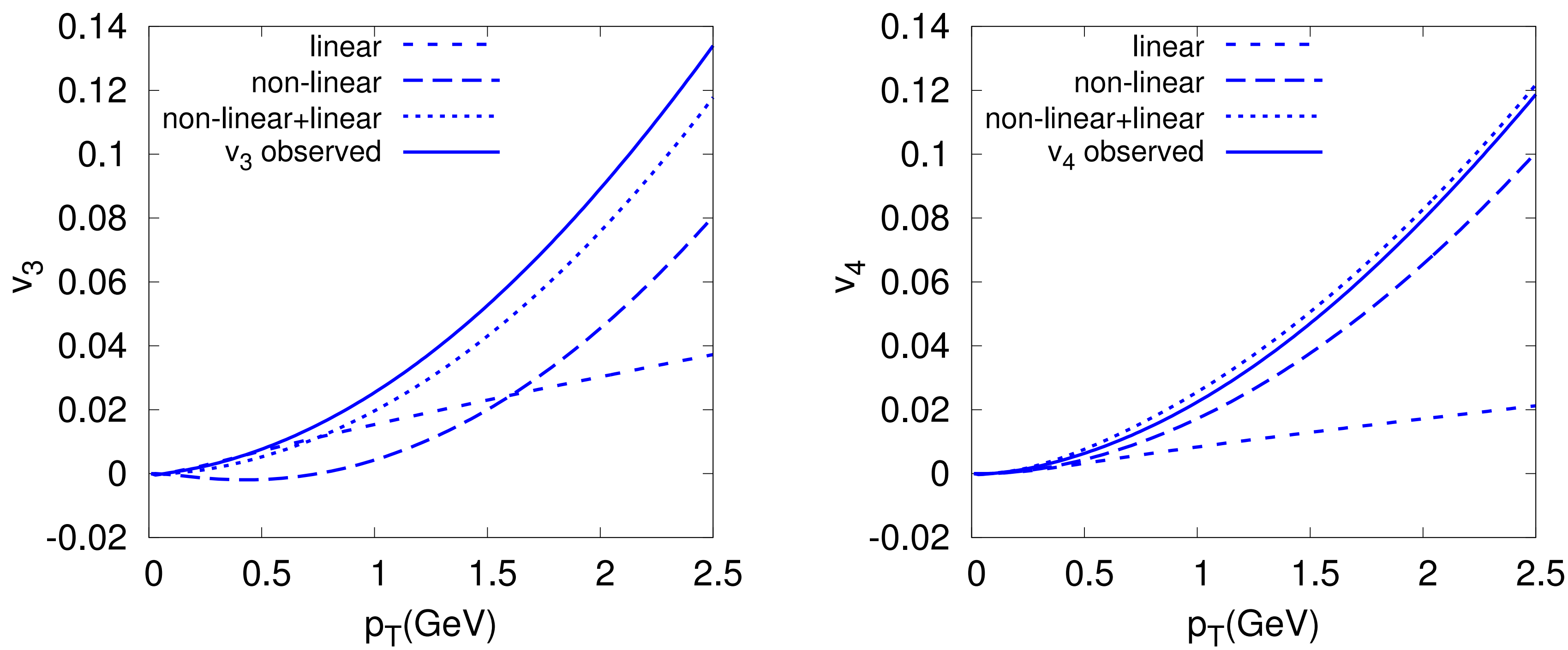


Triangular flow and quadrangular flow

- Ignore non-linearity in IC and fluid evolution.

Non-linear corrections in two-particle correlations

- Two-particle correlation from one-particle spectrum

$$\frac{dN}{d\phi} \propto 1 + \sum v_n(p_t)\cos[n(\phi - \psi_n)]$$

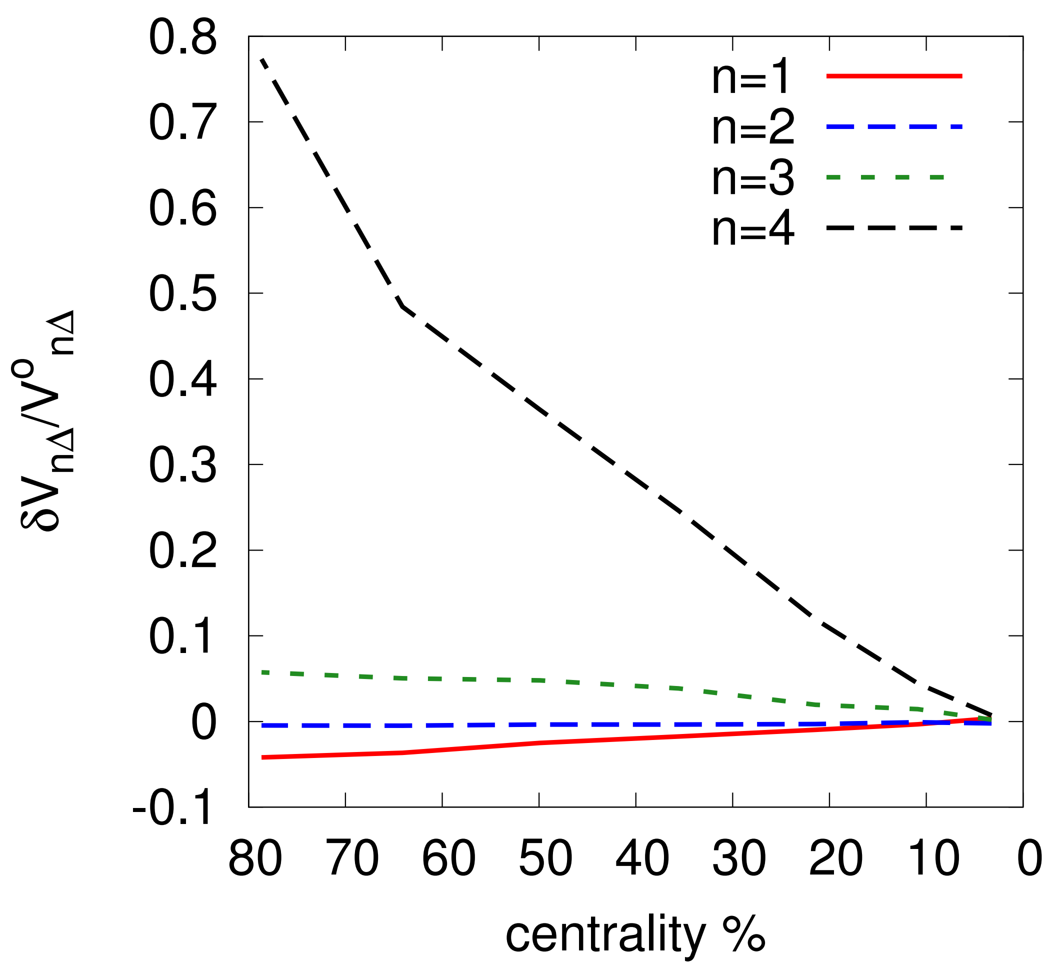
$$\Longrightarrow \quad \left\langle\!\!\left\langle \frac{dN_{\mathrm{pair},\alpha\beta}}{d\phi_\alpha d\phi_\beta} \right\rangle\!\!\right\rangle \propto \left[1 + \sum 2V_{n\Delta}\cos(n\phi_\alpha - n\phi_\beta)\right]$$

- $V_{n\Delta} = V_{n\Delta}^0 + \delta V_{n\Delta}$.

$$V_{n\Delta}^0 = \frac{v_{n\alpha}^0 v_{n\beta}^0}{\epsilon_n^2} \langle\!\langle \epsilon_n^2 \rangle\!\rangle$$

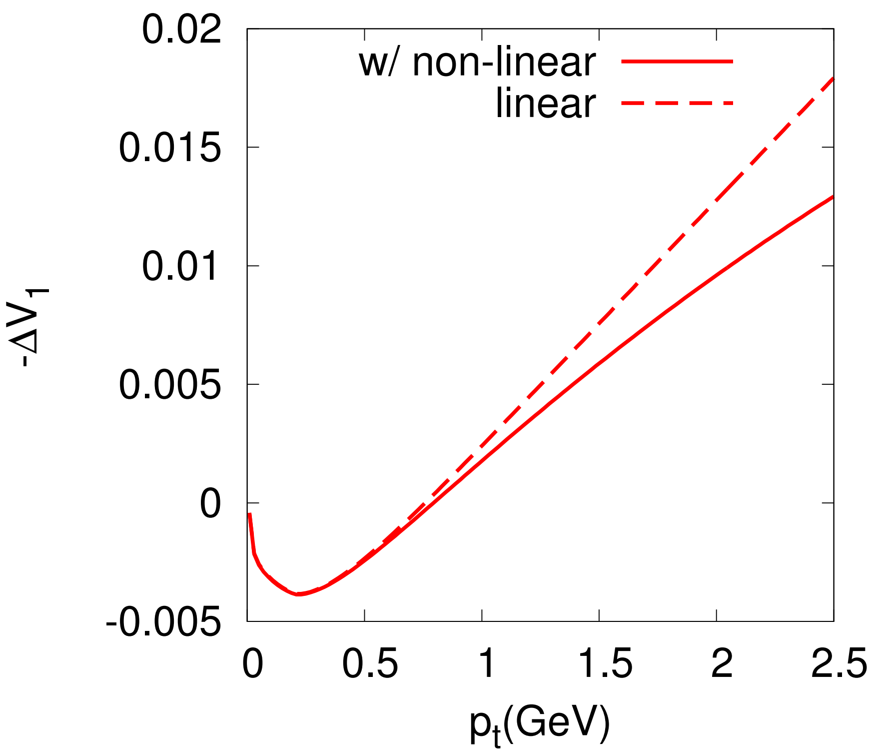
$\delta V_{n\Delta} \Leftrightarrow$ non-linear flow couplings.

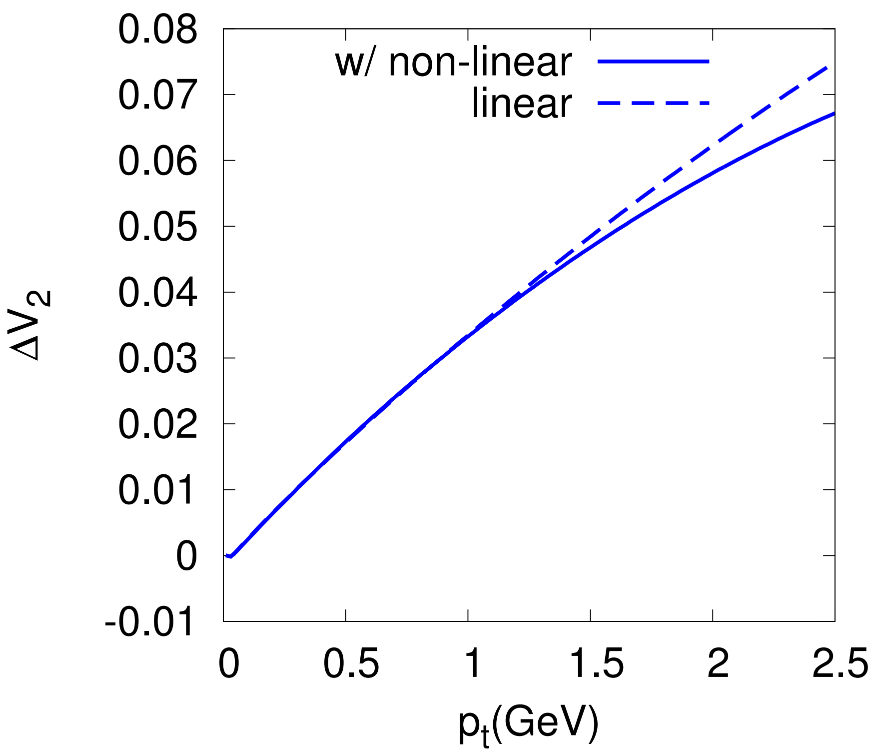
Ratio of non-linear/linear: integrated flow



- Non-linearity in v_4 is remarkable!
- v_5, v_6 etc., expect non-linearity to be even larger!

Non-linear contribution: differential flow





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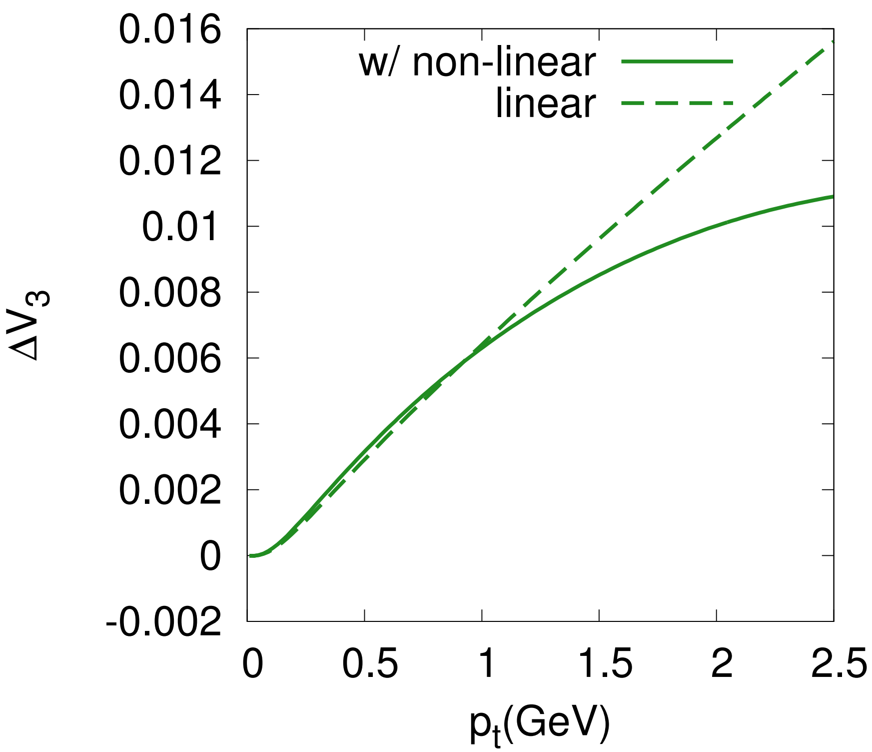

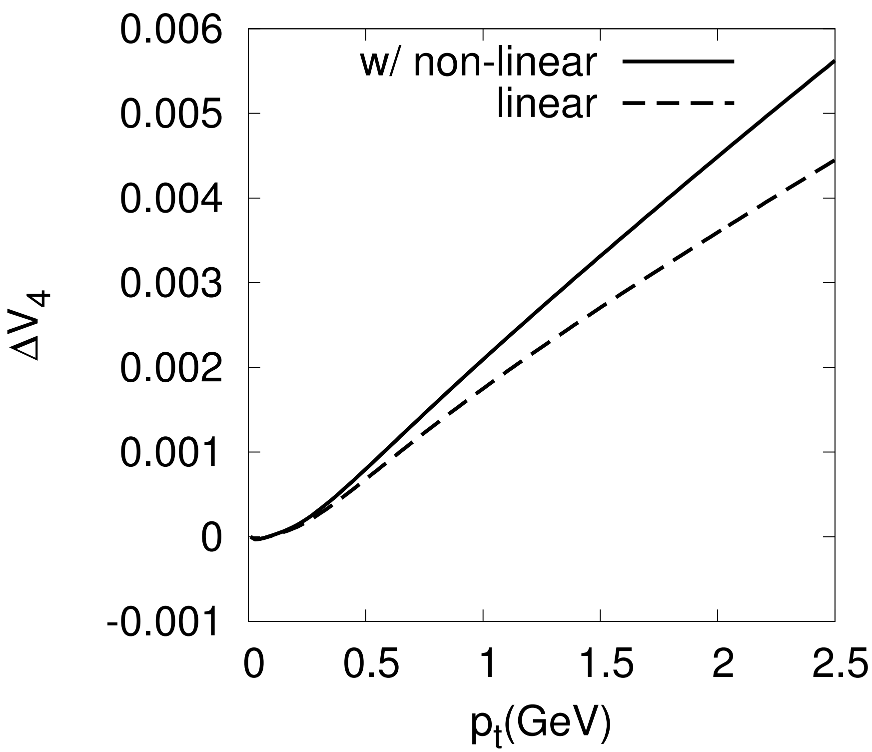

Summary

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- Viscous hydro. damps v_n, and damping depends on n, m.
- Non-linear flow coupling.
  1. Non-linearity generated from freeze-out.
  2. Non-linearity dominate for higher order harmonic flow.

Backup slides

Possible theoretical explanations of viscosity damping on v_n

Analytic solution to viscous hydro., from Gubser and Yarom[7]:

- Solution with symmetries: $SO(d-1) \times SO(1,1) \times \mathbf{Z}_2$.
- At very late time of hydro evolution:

$$v_n \propto v_\nu(\rho) \approx e^{-const.\frac{\eta}{s} l^2} (\cosh\rho)^{2/3} v_\nu(0)$$

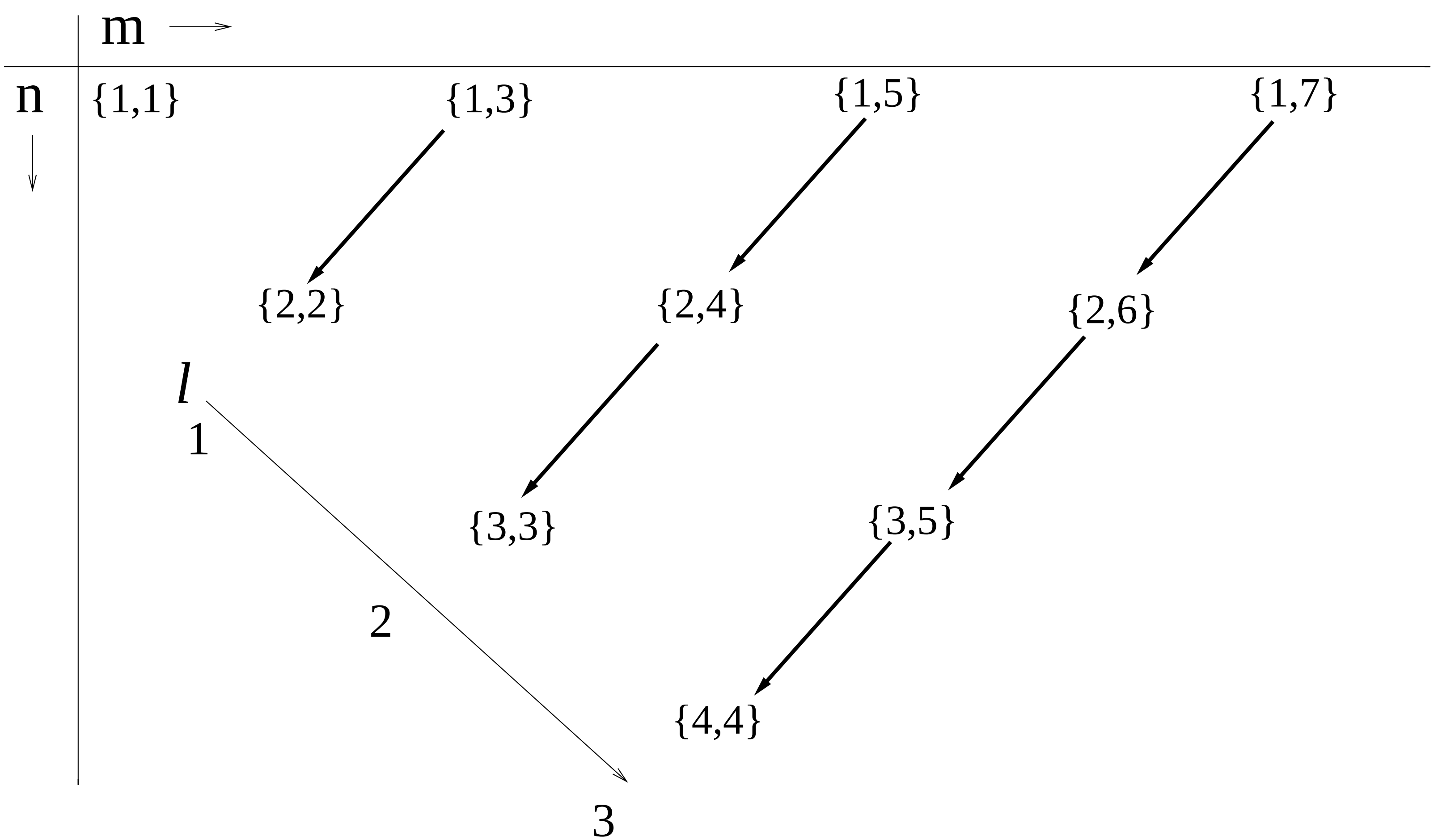
  where,

  - ρ is de Sitter time, $\sim \tau$.
  - l is the index from vector Spherical Harmonics.

[7] Nucl.Phys.B846:469-511,2011.

Possible theoretical explanations: $\{n, m\}$ dependence

Relation to our cumulant convention:



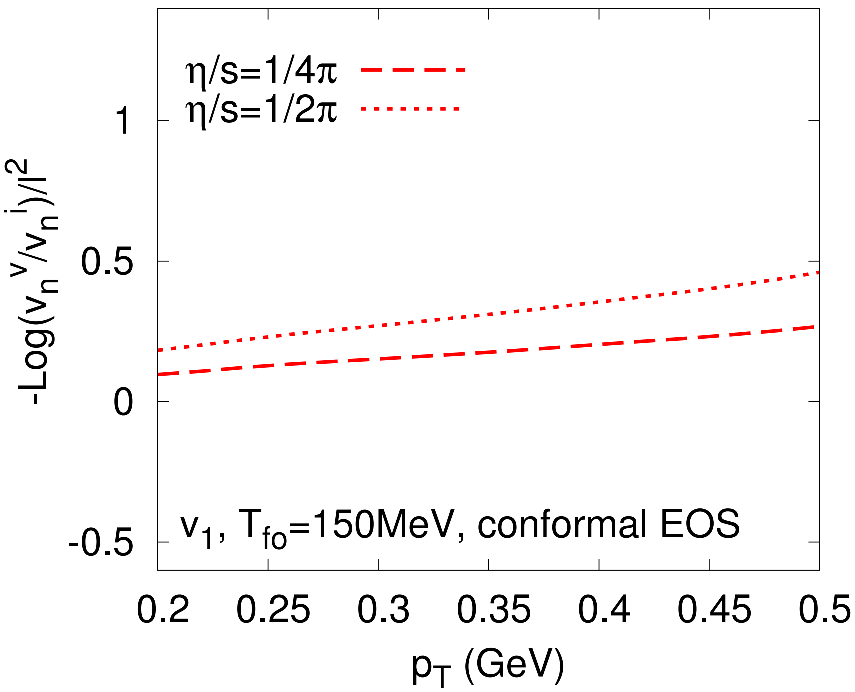
- v_2 and v_1 suppressed similarly. $(l = 1)$
- Stronger suppression in $v_3(l = 2)$ and $v_4(l = 3)$.

Comparison with viscous hydro run

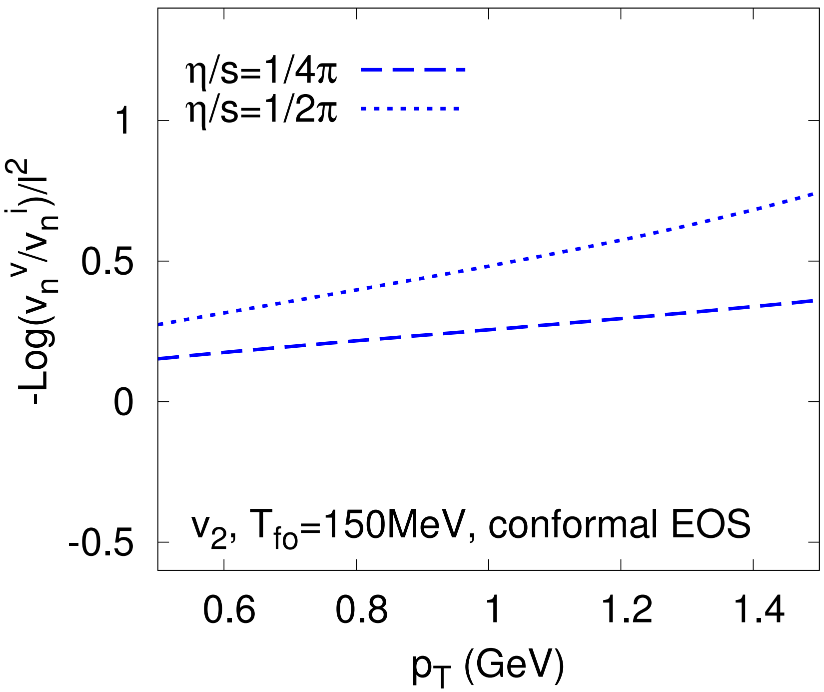
We calculate:

$$-log(\frac{v_n^v(p_T)}{v_n^i(p_T)})/l^2 = \text{const.} \times \frac{\eta}{s}$$

Comparison with viscous hydro run



dipole flow



elliptic flow

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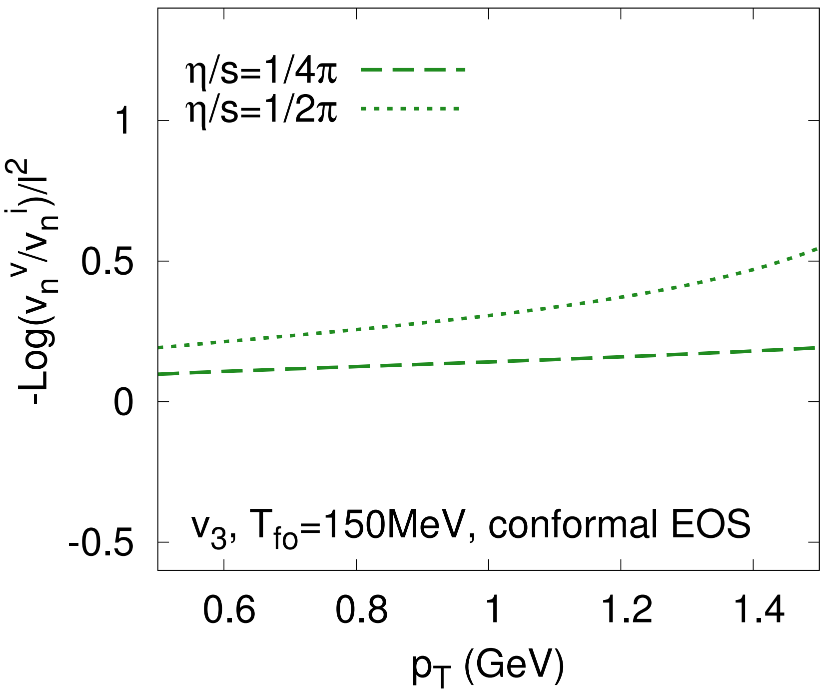
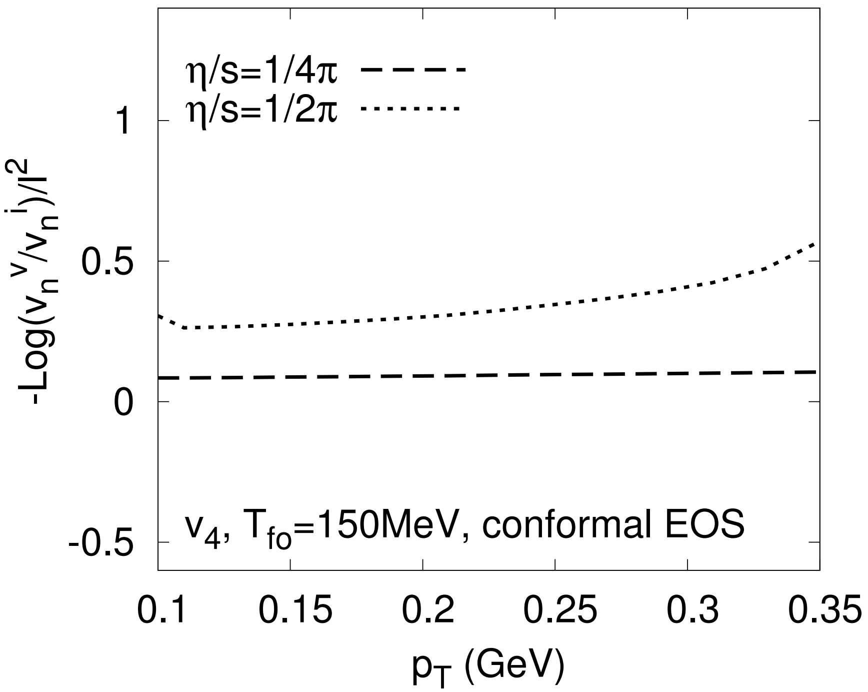


triangular flow



quadrangular flow