Anisotropy flow in hydrodynamics with viscous and non-linearity corrections

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- Collaborate with Derek Teaney, in progress.
Fluctuated initial condition ⇔ experiment observations:

- **Stage I.** Information from fluctuated IC. → cumulant exp.
- **Stage II.** Fluid evolution. → viscous damping.
- **Stage III.** Freeze out. → non-linear flow coupling.
Fluctuated initial condition: MC-Glauber or MC-KLN\(^1\)

Origins of particle correlations and flow: initial geometry.

\[ \epsilon_2, \psi_2(\psi_R) \quad \epsilon_1, \psi_1 \quad \epsilon_3, \psi_3 \]

As a result of fluid response,

\[ \epsilon_n \longleftrightarrow v_n \]

\[ \{ \epsilon_n, \psi_n \} \text{ correlations} \longleftrightarrow \text{correlations at final stage.} \]

Cumulant expansion: long wavelength expansion \( W_{n,m} \)

Classify eccentricities with cumulants\(^2\):

\[
\epsilon_{n,m} \propto - \frac{W_{n,m}^c}{\langle r^m \rangle}, \quad n\psi_{n,m} = \text{atan2}(W_{n,m}^c, W_{n,m}^s) + \pi
\]

- We have two indices: \( n \to \) angle, \( m \to \) radial size.

\(^2\langle \ldots \rangle \) is average over initial distribution.
Because we have \{ n, m \}

- \( W_{1,3}^c = \frac{3}{8} \langle r^3 \cos \phi \rangle \): distortion (not displacement) of IC.
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- Cumulants w/ \( m \geq 4 \) have extra terms, e.g.,

\[
W_{4,4}^c = \frac{1}{16} \left[ \langle r^4 \cos 4\phi \rangle - 3\langle r^2 \cos 2\phi \rangle^2 \right]
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correction from lower order cumulants,

1. \( \epsilon_{4,4} \neq \epsilon_4 \)
2. \( \{\psi_{4,4}, \psi_R\} \) correlations not strong.
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The remaining significant orientation angle correlations:

\[
\{\psi_{1,3}, \psi_R\} \quad \text{and} \quad \{\psi_{1,3}, \psi_{3,3}, \psi_R\}
\]
\[ \langle \cos(2\psi_{1,3} - 2\psi_R) \rangle \leftrightarrow \langle \cos(\phi_\alpha + \phi_\beta - 2\psi_R) \rangle^3 \]

- Stronger correlation from MC-KLN than MC-Glauber.
- In the spectrum of two-particle correlations,

\[ \langle \cos(\phi_\alpha + \phi_\beta - 2\psi_R) \rangle \propto \langle \cos(2\psi_{1,3} - 2\psi_R) \rangle \times v_1^2 \]

\[ \langle \ldots \rangle \] is average over events.
\[ \langle \cos(\phi_\alpha + \phi_\beta - 2\psi_R) \rangle \text{ vs STAR}^4 \]

By MC-Glauber

\[ \langle \cos(\phi_\alpha + \phi_\beta - 2\psi_{PP}) \rangle \]

Centrality (%)

\[ x10^{-3} \]

All Particles

\[ \times 10^{-3} \]

\[ \langle \cos(\phi_\alpha + \phi_\beta - 2\psi_{PP}) \rangle \]

\[ 70 \quad 60 \quad 50 \quad 40 \quad 30 \quad 20 \quad 10 \]

\[ \times 10^{-3} \]

\[ T_{fo} \leftrightarrow 130\text{MeV} \]

\[ T_{fo} \leftrightarrow 150\text{MeV} \]

\[ T_{fo} \leftrightarrow 170\text{MeV} \]

\[ \eta/s = 1/4\pi \text{ and lattice EOS.} \]

\[ ^4 \text{B. I. Abelev et al. [STAR Collaboration], Phys. Rev. Lett. 103, 251601 (2009)} \]
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STAR data
\[ \langle \cos(\psi_{1,3} + 2\Psi_R - 3\psi_{3,3}) \rangle \longleftrightarrow \langle \cos(\phi_{\alpha} - 3\phi_{\beta} + 2\Psi_R) \rangle \]

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\[ \langle \cos(\phi_\alpha - 3\phi_\beta + 2\psi_R) \rangle \]

The effects viscosity have on flow evolution.
What effects viscosity have on flow evolution?
Anisotropy flow from viscous hydro.

- Viscous damping looks similar for both $v_1$ and $v_2$!

![Graphs showing dipole flow and elliptic flow](image)

- $T_{fo}=150\text{MeV}$, conformal EOS

**dipole flow**

- $v_n(p_T)/\varepsilon_n$ vs $p_T$ (GeV)

- $T_{fo}=150\text{MeV}$, conformal EOS

- Ideal, $1/4\pi$, and $1/2\pi$ curves

**elliptic flow**
Anisotropy flow from viscous hydro.

- Viscous damping is much stronger in $v_3$ and $v_4$.

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Anisotropy flow from viscous hydro.

Viscous damping is much stronger in $v_3$ and $v_4$.

Damping : $\{1,3\} \sim \{2,2\} < \{3,3\} \sim \{2,4\} \sim \{1,5\} < \{4,4\} \ldots$

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\( \nu_n \) from freeze-out surface

- Particle spectrum Fourier decomposition w.r.t. \( \phi_p \)

\[
E \frac{d^3 N}{dp^3} = E \frac{d^2 N}{2\pi p_t dp_t dy} \left[ 1 + \sum_{n=1}^{2} 2\nu_n(p_t) \cos[n(\phi_p - \psi_{n,p})] \right]
\]

- Borghini and Ollitrault\(^6\), w/ Cooper-Fryer freeze-out:

\[
E \frac{d^3 N}{dp^3} = \frac{\nu}{(2\pi)^3} \int_{\Sigma} p^\mu d\sigma_\mu \exp(-p \cdot u / T).
\]

When \( \frac{p_t}{T} \gg 1 \implies \nu_4(p_t) = \text{linear } \nu_4 + \frac{1}{2} \nu_2(p_t)^2. \)

\[ E \frac{d^3N}{dp^3} = E \frac{d^2N}{2\pi p_t dp_t dy} \left[ 1 + \sum_{n=1}^{\infty} 2\nu_n(p_t) \cos[n(\phi_p - \psi_{n,p})] \right] \]

Borghini and Ollitrault\textsuperscript{6}, w/ Cooper-Fryer freeze-out:

\[ E \frac{d^3N}{dp^3} = \nu \frac{1}{(2\pi)^3} \int_{\Sigma} p^\mu d\sigma_\mu \exp(-p \cdot u / T). \]

When \( \frac{p_t}{T} \gg 1 \implies \nu_4(p_t) = \text{linear } \nu_4 + \frac{1}{2} \nu_2(p_t)^2. \)

Then in analogy ...

Anisotropy flow couplings – ansatz

If we assume, (and will be tested)

- Response of flow from hydro. to cumulants is linear.
- Dominant source of non-linearity is from freeze-out.
Anisotropy flow couplings – ansatz

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\[ v_n \cos n(\phi_p - \psi_{n,p}) = v_n^0 \cos n(\phi_p - \psi_{n,m}) \]

\[ + \frac{1}{2} \sum_{i+j=n} v_i^0 v_j^0 \cos[n\phi_p - (i\psi_{i,m} + j\psi_{j,m})] \]

\[ + \frac{1}{2} \sum_{i-j=n} v_i^0 v_j^0 \cos[n\phi_p - (i\psi_{i,m} - j\psi_{j,m})] \]
Anisotropy flow couplings – ansatz

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1. \[ v_n^0 \propto \left( \frac{pT}{T} \right) \], is linear flow response.
2. Underlined terms from non-linear couplings.
3. Both flow magnitudes and orientation angles changed.
Test of this ansatz formula – a simple case w/ $\psi_{n,m} = 0$

Simplified formula: $v_n(p_t) = v^0_n(p_t) + \text{non-linear terms}$,
Test of this ansatz formula – a simple case w/ $\psi_{n,m} = 0$

Simplified formula: $v_n(p_t) = v^0_n(p_t) + \text{non-linear terms}$,

- IC: input $\epsilon_{1,3}$ and $\epsilon_{2,2}$, small $\epsilon_{3,3}$ and $\epsilon_{4,4}$... generated.
- Linear $v^0_n$: $v_1^0$, $v_2^0$, $v_3^0$, $v_4^0$. Neglect $v_5^0$... for simplicity.
Test of this ansatz formula – a simple case w/ $\psi_{n,m} = 0$

Dipole flow and elliptic flow
Test of this ansatz formula – a simple case w/ $\psi_{n,m} = 0$

Triangular flow and quadrangular flow

- Ignore non-linearity in IC and fluid evolution.
Non-linear corrections in two-particle correlations

- Two-particle correlation from one-particle spectrum

\[ \frac{dN}{d\phi} \propto 1 + \sum v_n(p_t) \cos[n(\phi - \psi_n)] \]

\[ \Longrightarrow \left\langle \frac{dN_{\text{pair},\alpha\beta}}{d\phi_\alpha d\phi_\beta} \right\rangle \propto \left[ 1 + \sum 2V_{n\Delta} \cos(n\phi_\alpha - n\phi_\beta) \right] \]

- \( V_{n\Delta} = V_{n\Delta}^0 + \delta V_{n\Delta} \).

\[ V_{n\Delta}^0 = \frac{v_{n\alpha}^0 v_{n\beta}^0}{\epsilon_n^2} \left\langle \epsilon_n^2 \right\rangle \]

\( \delta V_{n\Delta} \Leftrightarrow \) non-linear flow couplings.
Ratio of non-linear/linear: integrated flow

- Non-linearity in $v_4$ is remarkable!
- $v_5$, $v_6$ etc., expect non-linearity to be even larger!
Non-linear contribution: differential flow
Non-linear contribution: differential flow
IC w/ fluctuations characterized by cumulants expansion.
Summary

- IC w/ fluctuations characterized by cumulants expansion.
- Viscous hydro. damps $v_n$, and damping depends on $n, m$. 
IC w/ fluctuations characterized by cumulants expansion.

Viscous hydro. damps $v_n$, and damping depends on $n, m$.

Non-linear flow coupling.

1. Non-linearity generated from freeze-out.
2. Non-linearity dominate for higher order harmonic flow.
Backup slides
Possible theoretical explanations of viscosity damping on $n$.

Analytic solution to viscous hydro., from Gubser and Yarom$^7$:

- Solution with symmetries: $SO(d - 1) \times SO(1, 1) \times Z_2$.
- At very late time of hydro evolution:

$$v_n \propto v_\nu(\rho) \approx e^{-\text{const.} \frac{n}{s} l^2} (\cosh \rho)^{2/3} v_\nu(0)$$

where,

- $\rho$ is de Sitter time, $\sim \tau$.
- $l$ is the index from vector Spherical Harmonics.

Possible theoretical explanations: \( \{n, m\} \) dependence

Relation to our cumulant convention:

<table>
<thead>
<tr>
<th>( n )</th>
<th>( m ) →</th>
</tr>
</thead>
<tbody>
<tr>
<td>{1,1}</td>
<td>{1,3}</td>
</tr>
</tbody>
</table>

\{2,2\} \rightarrow \{2,4\} \rightarrow \{2,6\}  
\{3,3\} \rightarrow \{3,5\}  
\{4,4\}  

- \( \nu_2 \) and \( \nu_1 \) suppressed similarly. (\( l = 1 \))
- Stronger suppression in \( \nu_3(\ l = 2 \) and \( \nu_4(\ l = 3 \).
Comparison with viscous hydro run

We calculate:

\[-\log \left( \frac{v_n^y(p_T)}{v_n^i(p_T)} \right)/l^2 = \text{const.} \times \frac{\eta}{s} \]
Comparison with viscous hydro run

\[-\log\left(\frac{v_n v_{n,i}}{v_{n,i}^2}\right)\]

\[p_T \, (\text{GeV})\]

\[v_1, \, T_{f_0}=150\text{MeV}, \, \text{conformal EOS}\]

\[\eta/s = 1/4\pi\]

\[\eta/s = 1/2\pi\]

dipole flow

\[-\log\left(\frac{v_n v_{n,i}}{v_{n,i}^2}\right)\]

\[p_T \, (\text{GeV})\]

\[v_2, \, T_{f_0}=150\text{MeV}, \, \text{conformal EOS}\]

\[\eta/s = 1/4\pi\]

\[\eta/s = 1/2\pi\]

elliptic flow
Comparison with viscous hydro run

$\eta/s = 1/4\pi$  
$\eta/s = 1/2\pi$

$v_n$ vs. $p_T$ (GeV)

$v_3$, $T_{fo}=150\text{MeV}$, conformal EOS

$v_4$, $T_{fo}=150\text{MeV}$, conformal EOS

triangular flow

quadrangular flow