Direct CP Asymmetry in $\bar{B} \rightarrow X_{s,d} \gamma$

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$\bar{B} \to X_s \gamma$ in the SM

- $b \to s \gamma$ is a flavor changing neutral current (FCNC)
  In SM no FCNC at tree level
  Arises as a loop effect:

![Diagram](image)


gives rise to the operator:

$$Q_{7\gamma} = -\frac{e}{8\pi^2} m_b \bar{s} \sigma_{\mu\nu} F^{\mu\nu} (1 + \gamma_5) b$$

part of the effective Hamiltonian

$$\mathcal{H}_{\text{eff}} \ni \frac{G_F}{\sqrt{2}} C_{7\gamma} Q_{7\gamma}$$
Constraints on New Physics: $\bar{B} \rightarrow X_s \gamma$

- $\bar{B} \rightarrow X_s \gamma$ is an important probe of new physics
- $b \rightarrow s \gamma$ can have contribution from new physics e.g. SUSY
  (only one diagram shown):

\[
\begin{array}{c}
  b \\
  \tilde{u} \tilde{W} \\
  \tilde{u} \gamma \\
  s \\
\end{array}
\]

leads to same operator, modifies $C_7 \gamma$
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  - $b \to s \gamma$ can have contribution from new physics e.g. SUSY
    - (only one diagram shown):
      
      \[
      \begin{array}{c}
      b \\
      \tilde{W} \\
      \tilde{u} \\
      \gamma \rightarrow \\
      \tilde{u} \rightarrow \\
      s \\
      \end{array}
      \]

    - leads to same operator, modifies $C_{7\gamma}$

- But $Q_{7\gamma}$ is not the whole story...
How To Make a Photon?

- Produce it directly...

\[
Q_{\gamma} = \frac{-e}{8\pi^2} m_b \bar{s}s \sigma_{\mu\nu} F^{\mu\nu} (1 + \gamma_5) b
\]

Or make a gluon or a quark pair

\[
Q_{g} = \frac{-e}{8\pi^2} m_b \bar{s}s \sigma_{\mu\nu} G^{\mu\nu} (1 + \gamma_5) b
\]

and convert them to a photon

But it will cost you...

\[
\alpha_s or \frac{1}{m_b} \gamma\gamma \frac{hc}{A1}
\]
How To Make a Photon?

- Produce it directly...

\[ Q_{7\gamma} = \frac{-e}{8\pi^2} m_b \bar{s} \sigma_{\mu\nu} F^{\mu\nu} (1 + \gamma_5) b \]

- Or make a gluon or a quark pair

\[ Q_{8g} = \frac{-e}{8\pi^2} m_b \bar{s} \sigma_{\mu\nu} G^{\mu\nu} (1 + \gamma_5) b \]
\[ Q_{1q} = (\bar{q} b)_{V-A} (\bar{s} q)_{V-A} \quad (p = u, c) \]

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How To Make a Photon?

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  \[ Q_1^q = (\bar{q}b)_{V-A}(\bar{s}q)_{V-A} \quad (p = u, c) \]

  and convert them to a photon

- But it will cost you...

  \[ \alpha_s \quad \text{or} \quad \frac{1}{m_b} \]
Effective Hamiltonian

For $\bar{B} \to X_s \gamma$ need Effective Hamiltonian

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \left( C_1 Q_1^p + C_2 Q_2^p + \sum_i C_i Q_i + C_7 \gamma Q_7 \gamma + C_8 g Q_8 g \right) + \text{h.c.}$$

- At leading power only $Q_7 \gamma - Q_7 \gamma$ contribute
- At higher orders need other $Q_i - Q_j$ contributions
Effective Hamiltonian

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- At leading power only $Q_{7\gamma} - Q_{7\gamma}$ contribute
- At higher orders need other $Q_i - Q_j$ contributions
- Most important: $Q_{7\gamma}, Q_{8g},$ and $Q_1$

$$Q_{7\gamma} = \frac{-e}{8\pi^2} m_b \bar{s}s \sigma_{\mu\nu} (1 + \gamma_5) F^{\mu\nu} b$$

$$Q_{8g} = \frac{-g_s}{8\pi^2} m_b \bar{s}s \sigma_{\mu\nu} (1 + \gamma_5) G^{\mu\nu} b$$

$$Q_1^q = (\bar{q} b)_{V-A}(\bar{s} q)_{V-A} \quad (q = u, c)$$

- Ratio of Wilson coefficients:

$$C_1 : C_{7\gamma} : C_{8g} = 3 : 1 : \frac{1}{2}$$
**CP asymmetry**

- **Measured** [Heavy Flavor Averaging Group, arXiv:1010.1589]

\[
A_{Xs\gamma} = \frac{\Gamma(\bar{B} \to X_s\gamma) - \Gamma(B \to X_s\bar{\gamma})}{\Gamma(\bar{B} \to X_s\gamma) + \Gamma(B \to X_s\bar{\gamma})} = -(1.2 \pm 2.8)\%
\]

Experiments measure \( A_{Xs\gamma}(E_\gamma \geq E_0) \) where \( 1.9 \leq E_0 \leq 2.1 \) GeV
CP asymmetry

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Experiments measure \( A_{X_s \gamma}(E_\gamma \geq E_0) \) where \( 1.9 \leq E_0 \leq 2.1 \text{ GeV} \)

- Theoretically, thought to be of a perturbative origin, e.g.

We need

- "Weak" (CP odd) phase: CKM phase \( \propto \lambda^2 \), \( C_i \) of \( H_{\text{eff}} \) (not in SM)
- "Strong" (CP even) phase: \( \alpha_s \) suppressed
The perturbative prediction

\[ A^\text{dir}_{\bar{X}_s \gamma}(E_0) = \alpha_s \left\{ \frac{40}{81} \text{Im} \frac{C_1}{C_{7\gamma}} - \frac{8}{9} \left[ \nu(z) + b(z, \delta) \right] \text{Im}\left[ (1 + \epsilon_s) \frac{C_1}{C_{7\gamma}} \right] \right. \]

\[ - \frac{4}{9} \text{Im} \frac{C_{8g}}{C_{7\gamma}} + \frac{8z}{27} b(z, \delta) \frac{\text{Im}\left[(1 + \epsilon_s) C_1 C_{8g}^* \right]}{|C_{7\gamma}|^2} + \frac{16z}{27} \tilde{b}(z, \delta) \left| \frac{C_1}{C_{7\gamma}} \right|^2 \text{Im} \epsilon_s \left\} \]

where \( \delta = (m_b - 2E_0)/m_b, \) \( z = (m_c/m_b)^2 \)

and \( \epsilon_s = (V_{ub}V_{us}^*)/(V_{tb}V_{ts}^*) \approx \chi^2 (i\bar{\eta} - \bar{\rho}) \)
CP asymmetry

- Taking $m_c^2 = O(m_b \Lambda_{QCD})$, expand $z, \delta = O(\Lambda_{QCD}/m_b)$

  $$A^{\text{dir}}_{X_s\gamma} = \alpha_s \left\{ \frac{40}{81} \text{Im} \frac{C_1}{C_7\gamma} - \frac{4}{9} \text{Im} \frac{C_{8g}}{C_{7\gamma}} \right. \nonumber$$

  $$- \frac{40\Lambda_c}{9m_b} \text{Im} \left[ (1 + \epsilon_s) \frac{C_1}{C_{7\gamma}} \right] + O\left(\frac{\Lambda_{QCD}^2}{m_b^2}\right) \right\}$$

  where $\Lambda_c(m_c, m_b) \approx 0.38$ GeV

- In the SM triple suppression:
  $\alpha_s, \text{Im}(\epsilon_s) \sim \lambda^2$, and $(m_c/m_b)^2 \sim \Lambda_{QCD}/m_b$
CP asymmetry in the Standard Model

- In SM asymmetry around 0.5%
  

- A dedicated analysis [Hurth, Lunghi, Porod NPB 704, 56 (2005)]

\[ A_{Xs\gamma} = (0.44 \pm 0.15 - 0.10 \pm 0.03 + 0.19 - 0.09)\% \]

- Compared to measured, \[ A_{Xs\gamma} = -1.2 \pm 2.8 \% \], room for new physics

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  \[ A_{X_s\gamma}^{SM} = (0.44 \pm 0.15 \pm 0.19)\% \]
  errors: \( m_c/m_b \), CKM parameters, and scale
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  \[ \mathcal{A}_{X_s \gamma}^{SM} = \left( 0.44^{+0.15}_{-0.10} \pm 0.03^{+0.19}_{-0.09} \right)\% \]
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SM asymmetry around 0.5%

Measured, $A_{X_s\gamma} = -(1.2 \pm 2.8)\%$, room for new physics

One of the goals of the super B factories:

- Super B in Italy
- Belle 2 in Japan
Table 2-2. The expected precision of some of the most important measurements that can be performed at SuperB. For comparison we put the reach of the B Factories at 2 ab⁻¹. Numbers quoted as percentages are relative precisions. Measurements marked (†) will be systematics limited, and those marked (*) will be theoretically limited, with 75 ab⁻¹. Note that in many of these cases, there exist data driven methods of reducing the errors. See the text for further discussion of each measurement.

<table>
<thead>
<tr>
<th>Observable</th>
<th>B Factories (2 ab⁻¹)</th>
<th>SuperB (75 ab⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_{CP}(b \rightarrow s\gamma) )</td>
<td>0.012 (†)</td>
<td>0.004 (†)</td>
</tr>
</tbody>
</table>

SuperB CONCEPTUAL DESIGN REPORT
**Technical Design Report**
arXiv:1011.0352

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## CP asymmetry in Future B factories

<table>
<thead>
<tr>
<th>Observable</th>
<th>Belle 2006 ($\sim 0.5$ ab$^{-1}$)</th>
<th>SuperKEKB (5 ab$^{-1}$)</th>
<th>SuperKEKB (50 ab$^{-1}$)</th>
<th>$^\dagger$LHCb (2 fb$^{-1}$)</th>
<th>$^\dagger$LHCb (10 fb$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{CP}(B \to X_s \gamma)$</td>
<td>0.058</td>
<td>0.01</td>
<td>0.005</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

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Gil Paz  (The University of Chicago & Wayne State University)
Resolved Photon Contributions

...but the theory prediction is not complete

- Soft $Q_u, c_1$
- $b_s$
Resolved Photon Contributions

...but the theory prediction is not complete

- What is missing are the resolved photon contributions

[Lee, Neubert, GP PRD 75, 114005 (2007);
Benzke, Lee, Neubert, GP JHEP 1008 099 (2010)]
Resolved Photon Contributions

...but the theory prediction is not complete

- What is missing are the resolved photon contributions
  [Lee, Neubert, GP PRD 75, 114005 (2007);
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- Resolved photon contributions probe photon’s hadronic structure
  Arise from $Q_i - Q_j$ contributions, other than $Q_7\gamma - Q_7\gamma$, e.g.

\[
\begin{aligned}
    \text{soft} \\
    b \rightarrow u, c \\
    s \rightarrow Q_7^{u,c}
\end{aligned}
\]
Resolved Photon Contributions

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  \[
  \begin{array}{c}
  b \\
  u, c \\
  s \\
  \end{array}
  \]

  \[
  Q_{u,c}^7
  \]

- Give non-perturbative $\mathcal{O}(\Lambda_{QCD} / m_b)$ corrections to $\Gamma(B \rightarrow X_s \gamma)$
  Unlike $\Gamma(B \rightarrow X_{c,u} \nu \bar{\nu})$, where non-perturbative $= \mathcal{O}(\Lambda_{QCD}^2 / m_b^2)$
  Currently largest irreducible error of $\sim 5\%$ on $\Gamma(B \rightarrow X_s \gamma)$
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    u, c \\
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    Q_{u,c}^{u,c}
  \end{array}
  \]

  Give non-perturbative $O(\Lambda_{QCD}/m_b)$ corrections to $\Gamma(\bar{B} \to X_s \gamma)$
  Unlike $\Gamma(\bar{B} \to X_{c,u}\ell\bar{\nu})$, where non-perturbative $= O(\Lambda_{QCD}^2/m_b^2)$
  Currently largest irreducible error of $\sim 5\%$ on $\Gamma(\bar{B} \to X_s \gamma)$

- Calculated from a systematic study using
  Soft Collinear Effective Theory (SCET)
Resolved Photon Contributions

Are they important for the CP asymmetry?
Resolved Photon Contributions

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- For $\Gamma(\bar{B} \rightarrow X_s \gamma)$
  - Direct photon contributions are $\mathcal{O}(1)$ effect
  - Resolved photon contributions are $\mathcal{O}(\Lambda_{QCD}/m_b)$ effect

$\Rightarrow$ Potentially big effect!
Resolved Photon Contributions

Are they important for the CP asymmetry?

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  Direct photon contributions are $\mathcal{O}(1)$ effect
  
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- For $A_{X_s \gamma}$
  
  Direct photon contributions are $\alpha_s$ suppressed
  
  Resolved photon contributions are $\mathcal{O}(\Lambda_{QCD}/m_b)$ effect
Resolved Photon Contributions

Are they important for the CP asymmetry?

- For $\Gamma(\bar{B} \rightarrow X_s \gamma)$
  Direct photon contributions are $O(1)$ effect
  Resolved photon contributions are $O(\Lambda_{QCD}/m_b)$ effect

- For $A_{X_s\gamma}$
  Direct photon contributions are $\alpha_s$ suppressed
  Resolved photon contributions are $O(\Lambda_{QCD}/m_b)$ effect

⇒ Potentially big effect!
Resolved Photon Contributions

\[ A_{X_s \gamma} \sim \begin{array}{cc}
\bar{J} & \otimes \ h \\
\uparrow & \uparrow \\
\text{Calc. in PT} & \text{Non pert.}
\end{array} \]

The non perturbative functions \( h_{ij} \) are

\[ h_{17}(\omega_1) \quad \text{F.T. of} \quad \langle \bar{B} | \bar{b}(0) \cdots G(s\bar{n}) \cdots b(0) | \bar{B} \rangle \]

\[ h_{78}^{(1)}(\omega_1, \omega_2) \quad \text{F.T. of} \quad \langle \bar{B} | \bar{b}(0) \cdots b(0) \sum_q e_q \bar{q}(r\bar{n}) \cdots q(s\bar{n}) | \bar{B} \rangle \]

Cannot be extracted from data, must be modeled
Resolved Photon Contributions

- How can a dimension 5 and 6 operators

\[ \langle \bar{B} | \bar{b}(0) \cdots b(0) \sum_q e_q \bar{q}(r \bar{n}) \cdots q(s \bar{n}) | \bar{B} \rangle, \langle \bar{B} | \bar{b}(0) \cdots G(s \bar{n}) \cdots b(0) | \bar{B} \rangle \]

give \( \Lambda_{QCD} / m_b \) corrections to dimension 3 operator \( \langle \bar{B} | \bar{b}(0) b(0) | \bar{B} \rangle \) ?
Resolved Photon Contributions

- How can a dimension 5 and 6 operators
  \[ \langle \bar{B} | \bar{b}(0) \cdots b(0) \sum_q e_q \bar{q}(r\bar{n}) \cdots q(s\bar{n}) | \bar{B} \rangle, \langle \bar{B} | \bar{b}(0) \cdots G(s\bar{n}) \cdots b(0) | \bar{B} \rangle \]

  give \( \Lambda_{QCD}/m_b \) corrections to dimension 3 operator \( \langle \bar{B} | \bar{b}(0)b(0) | \bar{B} \rangle \)?

- Define \( \bar{n}, n = (1, 0, 0, \pm 1) \Rightarrow n \cdot p = E + |\vec{p}|, \bar{n} \cdot p = E - |\vec{p}| \)

\[ \frac{1}{\bar{n} \cdot p} \approx 1, \bar{n} \cdot p \approx 1 \]

Soft functions are convoluted with jet functions \( \bar{J} \) which enhance their contribution.
Resolved Photon Contributions

- How can a dimension 5 and 6 operators
  \[ \langle \bar{B} | \bar{b}(0) \cdots b(0) \sum_q e_q \bar{q}(r\bar{n}) \cdots q(s\bar{n}) | \bar{B} \rangle, \langle \bar{B} | \bar{b}(0) \cdots G(s\bar{n}) \cdots b(0) | \bar{B} \rangle \]
  give \( \Lambda_{QCD}/m_b \) corrections to dimension 3 operator \( \langle \bar{B} | \bar{b}(0)b(0) | \bar{B} \rangle \)?

- Define \( \bar{n}, n = (1, 0, 0, \pm 1) \Rightarrow n \cdot p = E + |\vec{p}|, \bar{n} \cdot p = E - |\vec{p}| \)

\[ \frac{\phi}{p^2} = \frac{n \cdot \vec{p} \vec{n}}{n \cdot \vec{p} n \cdot \vec{p}} \approx \frac{1}{\bar{n} \cdot \vec{p} n} \approx \frac{1}{\Lambda_{QCD}} \]
Resolved Photon Contributions

- How can a dimension 5 and 6 operators
  
  \[
  \langle \bar{B} | \bar{b}(0) \cdots b(0) \sum_q e_q \bar{q}(r\bar{n}) \cdots q(s\bar{n}) | \bar{B} \rangle, \quad \langle \bar{B} | \bar{b}(0) \cdots G(s\bar{n}) \cdots b(0) | \bar{B} \rangle
  \]
  
give \( \Lambda_{QCD}/m_b \) corrections to dimension 3 operator \( \langle \bar{B} | \bar{b}(0)b(0) | \bar{B} \rangle \)?

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- Soft functions are convoluted with jet functions \( \bar{J} \) which enhance their contribution

\[
\frac{\rho}{p^2} = \frac{n \cdot p \frac{\eta}{2} + \bar{n} \cdot p \frac{\eta}{2}}{\bar{n} \cdot p \cdot n \cdot p} \approx \frac{1}{\bar{n} \cdot p} \frac{\eta}{2} \approx \frac{1}{\Lambda_{QCD}}
\]
Strong Phase

• For a non-zero asymmetry we need
  ▶ “Weak” (CP odd) phase: CKM phase $\propto \lambda^2$, $C_i$ of $\mathcal{H}_{\text{eff}}$ (not in SM)
  ▶ “Strong” (CP even) phase: What is it?
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  - "Weak" (CP odd) phase: CKM phase $\propto \lambda^2$, $C_i$ of $H_{\text{eff}}$ (not in SM)
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- Resolved photon contributions

\[ Q_{7\gamma} - Q_{8g} \quad \text{and} \quad Q_1 - Q_{7\gamma} \]

- $h_{ij}$ are real due to PT and heavy quark symmetry
  - $J$ are complex: arise from uncut propagators and loops
Resolved Photon contributions to CP asymmetry

- At lowest order in $\alpha_s$ and $\mathcal{O}(\Lambda_{QCD}/m_b)$

\[
\mathcal{A}_{X_s\gamma}^{\text{res}} = \frac{\pi}{m_b} \left\{ \text{Im} \left[ (1+\epsilon_s) \frac{C_1}{C_{7\gamma}} \right] \tilde{\Lambda}_u^{17} - \text{Im} \left[ \epsilon_s \frac{C_1}{C_{7\gamma}} \right] \tilde{\Lambda}_c^{17} \right\} + \text{Im} \left[ \epsilon_s \frac{C_1}{C_{7\gamma}} \right] \tilde{\Lambda}_u^{17} + \text{Im} \left[ \frac{C_{8g}}{C_{7\gamma}} 4\pi\alpha_s \tilde{\Lambda}^{\tilde{B}}_{78} \right],
\]

with

\[
\tilde{\Lambda}_u^{17} = \frac{2}{3} h_{17}(0)
\]

\[
\tilde{\Lambda}_c^{17} = \frac{2}{3} \int_{4m_c^2/m_b}^{\infty} \frac{d\omega_1}{\omega_1} f \left( \frac{m_c^2}{m_b \omega_1} \right) h_{17}(\omega_1)
\]

\[
\tilde{\Lambda}^{\tilde{B}}_{78} = 2 \int_{-\infty}^{\infty} \frac{d\omega_1}{\omega_1} \left[ h_{78}^{(1)}(\omega_1, \omega_1) - h_{78}^{(1)}(\omega_1, 0) \right]
\]

where

\[
f(x) = 2x \ln \frac{1 + \sqrt{1 - 4x}}{1 - \sqrt{1 - 4x}}
\]

- Model $h_{ij}$ as in

[Benzke, Lee, Neubert, GP JHEP 1008 099 (2010)]
We need to estimate

\[ \tilde{\Lambda}_{\bar{B}} = 2 \int_{-\infty}^{\infty} \frac{d\omega_1}{\omega_1} \left[ h_{78}(1)(\omega_1, \omega_1) - h_{78}(1)(\omega_1, 0) \right] \]

Recall

\[ h_{78}^{(1)}(\omega_1, \omega_2) \quad \text{F.T. of} \quad \langle \bar{B} | \bar{b}(0) \cdots b(0) \sum_q e_q \bar{q}(r \bar{n}) \cdots q(s \bar{n}) | \bar{B} \rangle \]
\[ \tilde{\Lambda} = 2 \int_{-\infty}^{\infty} \frac{d\omega_1}{\omega_1} \left[ h_{78}^{(1)}(\omega_1, \omega_1) - h_{78}^{(1)}(\omega_1, 0) \right] \]

Recall
\[ h_{78}^{(1)}(\omega_1, \omega_2) \quad \text{F.T. of} \quad \langle \bar{B} | \bar{b}(0) \cdots b(0) \sum_q e_q \bar{q}(r \bar{n}) \cdots q(s \bar{n}) | \bar{B} \rangle \]

Fierz and use VIA
\[ \tilde{\Lambda}_{78} \bigg|_{\text{VIA}} = e_{\text{spec}} \frac{2f_B^2 M_B}{9} \int_0^\infty d\omega_1 \frac{[\phi_+^B(\omega_1, \mu)]^2}{\omega_1} \]

where \( \phi_+^B(\omega_1) \) is B-meson LCDA ("wave function")

Using [Lee, Neubert PRD 72, 094028 (2005)] to constrain the integral
\[ \tilde{\Lambda}_{78} \bigg|_{\text{VIA}} \in e_{\text{spec}}[17 \text{ MeV}, 190 \text{ MeV}] \]
We need to estimate

\[ \tilde{\Lambda}_{17}^c = \frac{2}{3} \int_{4m_c^2/m_b}^{\infty} \frac{d\omega_1}{\omega_1} f\left(\frac{m_c^2}{m_b \omega_1}\right) h_{17}(\omega_1) \]

- \( h_{17}(\omega_1) \) has support over hadronic range
  - The convolution starts at \( 4m_c^2/m_b \approx 1 \text{ GeV} \)
  - \( \Rightarrow \) small overlap

\[ -9 \text{ MeV} < \tilde{\Lambda}_{17}^c < +11 \text{ MeV} \]
\[ \tilde{\Lambda}_{17}^c \text{ and } \tilde{\Lambda}_{17}^u \]

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  \(-9 \text{ MeV} < \tilde{\Lambda}_{17}^c < +11 \text{ MeV} \)

- We need to estimate
  \[ \tilde{\Lambda}_{17}^u = \frac{2}{3} h_{17}(0) \]
  We find
  \(-330 \text{ MeV} < \tilde{\Lambda}_{17}^u < +525 \text{ MeV} \)
  Asymmetric range since \( h_{17}(\omega_1) \) normalization is \( 2\lambda_2 \approx 0.24 \text{ GeV}^2 \)
  Same result as naive dimensional analysis
Correct CP asymmetry in the SM

- Including both direct and resolved contributions using $\mu = 2 \text{ GeV}$ for the factorization scale

$$A_{X_s \gamma}^{\text{SM}} \approx \pi \left| \frac{C_1}{C_7} \right| \text{Im} \epsilon_s \left( \frac{\tilde{\Lambda}_{17}^u - \tilde{\Lambda}_{17}^c}{m_b} + \frac{40\alpha_s}{9\pi} \frac{\Lambda_c}{m_b} \right)$$

$$= \left( 1.15 \times \frac{\tilde{\Lambda}_{17}^u - \tilde{\Lambda}_{17}^c}{300 \text{ MeV}} + 0.71 \right) \%.$$
Correct CP asymmetry in the SM

- Including both direct and resolved contributions using \( \mu = 2 \text{ GeV} \) for the factorization scale

\[
\mathcal{A}_{X_s \gamma}^{\text{SM}} \approx \pi \left| \frac{C_1}{C_{7\gamma}} \right| \text{Im} \epsilon_s \left( \frac{\tilde{\Lambda}_u^{17} - \tilde{\Lambda}_c^{17}}{m_b} + \frac{40 \alpha_s}{9\pi} \frac{\Lambda_c}{m_b} \right)
\]

\[
= \left( 1.15 \times \frac{\tilde{\Lambda}_u^{17} - \tilde{\Lambda}_c^{17}}{300 \text{ MeV}} + 0.71 \right) \%
\]

- CP asymmetry dominated by non-perturbative effects!

\(-0.6\% < \mathcal{A}_{X_s \gamma}^{\text{SM}} < 2.8\%\)

Only value below \(-2\%\) could be interpreted as sign of new physics
Correct CP asymmetry Beyond the SM

\[
\frac{A_{X_s\gamma}}{\pi} \approx \left[ \left( \frac{40}{81} - \frac{40}{9} \frac{\Lambda_c}{m_b} \right) \frac{\alpha_s}{\pi} + \frac{\tilde{\Lambda}_{17}^c}{m_b} \right] \text{Im} \frac{C_1}{C_{7\gamma}} \\
- \left( \frac{4\alpha_s}{9\pi} - 4\pi\alpha_s e_{\text{spec}} \frac{\tilde{\Lambda}_{78}}{m_b} \right) \text{Im} \frac{C_{8g}}{C_{7\gamma}} \\
- \left( \frac{\tilde{\Lambda}_{17}^u - \tilde{\Lambda}_{17}^c}{m_b} + \frac{40}{9} \frac{\Lambda_c}{m_b} \frac{\alpha_s}{\pi} \right) \text{Im} \left( \epsilon_s \frac{C_1}{C_{7\gamma}} \right)
\]

- **New** test of physics beyond the SM

\[
A_{X_s^-\gamma} - A_{X_s^0\gamma} \approx 4\pi^2 \alpha_s \frac{\tilde{\Lambda}_{78}}{m_b} \text{Im} \frac{C_{8g}}{C_{7\gamma}} \approx 12\% \times \frac{\tilde{\Lambda}_{78}}{100 \text{ MeV}} \text{Im} \frac{C_{8g}}{C_{7\gamma}}
\]

Should be looked at by experiments!
Comments on $\bar{B} \rightarrow X_d \gamma$

- $\bar{B} \rightarrow X_d \gamma$ analogous to $\bar{B} \rightarrow X_s \gamma$:
  replace $\epsilon_s$ by $\epsilon_d = (V_{ub} V_{ud}^*)/(V_{tb} V_{td}^*) = (\bar{\rho} - i \bar{\eta})/(1 - \bar{\rho} + i \bar{\eta})$

- CP asymmetry for $\bar{B} \rightarrow X_d \gamma$ in SM
  enhanced by $\text{Im}(\epsilon_d)/\text{Im}(\epsilon_s) \approx -22$

- Including resolved photon contributions

  $$-62\% < A_X^{\text{SM}} < 14\%$$
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- CP asymmetry for $\bar{B} \to X_d\gamma$ in SM
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- Including resolved photon contributions
  
  $$-62\% < A_{X_d\gamma}^{SM} < 14\%$$

- Untagged CP asymmetry for $\bar{B} \to X_{s+d}\gamma$ vanishes in SM
  up to tiny U-spin breaking corrections


- Even after including resolved photon effects
Summary: Direct CP Asymmetry in $\bar{B} \rightarrow X_{s,d} \gamma$

- Direct CP asymmetry in $\bar{B} \rightarrow X_{s,d} \gamma$ is a probe of new physics
- Its precise measurement is one of the goals of future B factories
Summary: Direct CP Asymmetry in $\bar{B} \rightarrow X_{s,d} \gamma$

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Summary: Direct CP Asymmetry in $\bar{B} \rightarrow X_{s,d} \gamma$

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- How well can we calculate $A_{X_s\gamma}$? Worse than we thought...

Worse than we thought...

Resolved photon contributions, which are non-perturbative, are the dominant effect in the SM

$-0.6 < A_{X_s\gamma} < 2.8$ compared to $A_{X_s\gamma} \approx 0.5$ from perturbative effects alone

New test of physics beyond the SM

$A_{X_s\gamma} - A_{X_0\gamma} \approx 4\pi^2 \alpha_s \tilde{\Lambda}_{78} m_b Im C_8 g C_7 \gamma \approx 12\% \times \tilde{\Lambda}_{78} 100$ MeV Im $C_8 g C_7 \gamma$
Summary: Direct CP Asymmetry in $\bar{B} \to X_{s,d} \gamma$

- Direct CP asymmetry in $\bar{B} \to X_{s,d} \gamma$ is a probe of new physics. Its precise measurement is one of the goals of future B factories.

- How well can we calculate $A_{X_{s}\gamma}$? Worse than we thought...

- Resolved photon contributions, which are non-perturbative, are the dominant effect in the SM.


  $-0.6\% < A_{X_{s}\gamma}^{SM} < 2.8\%$

  compared to $A_{X_{s}\gamma}^{SM} \approx 0.5\%$ from perturbative effects alone.

- New test of physics beyond the SM

  $A_{X_{s}\gamma}^{-} - A_{X_{s}\gamma}^{0} \approx 4\pi^2 \alpha_s \frac{\tilde{\Lambda}_{78}}{m_b} \text{Im} \frac{C_{8g}}{C_{7\gamma}} \approx 12\% \times \frac{\tilde{\Lambda}_{78}}{100 \text{ MeV}} \text{Im} \frac{C_{8g}}{C_{7\gamma}}$