Chiral field theory of 0^{-+} glueball

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- glueball is the solution of nonperturbative QCD
- Many candidates of 0^{++}, 0^{-+}, and 2^{++} glueballs have been discovered
- A chiral field theory is applied to do systematic and quantitative study of the properties of the 0^{-+} glueball.

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$$\mathcal{L}_1 = \bar{\psi}(x)(i\gamma\cdot\partial + \gamma\cdot v + \gamma\cdot a\gamma_5 - mu(x))\psi(x) - \bar{\psi}M\psi$$
$$+\frac{1}{2}m_0^2(\rho_i^\mu\rho_{\mu i} + \omega^\mu\omega_\mu + a_i^\mu a_{\mu i} + f^\mu f_\mu + K_\mu^a\bar{K}^{a\mu} + K_1^\mu K_{1\mu} + \phi_\mu\phi^\mu + f_s^\mu f_{s\mu}) \quad (1)$$

where $a_\mu = \tau_i a_\mu^i + \lambda_a K_{1\mu}^a + (\frac{2}{3} + \frac{1}{\sqrt{3}}\lambda_8)f_\mu + (\frac{1}{3} - \frac{1}{\sqrt{3}}\lambda_8)f_{s\mu}$($i = 1, 2, 3$ and $a = 4, 5, 6, 7$), $v_\mu = \tau_i\rho_\mu^i + \lambda_a K_\mu^a + (\frac{2}{3} + \frac{1}{\sqrt{3}}\lambda_8)\omega_\mu + (\frac{1}{3} - \frac{1}{\sqrt{3}}\lambda_8)\phi_\mu$,
$u = exp\{i\gamma_5(\tau_i\pi_i + \lambda_a K^a + \lambda_8\eta_8 + \frac{1}{\sqrt{3}}\eta_0)\}$,

- m is the constituent quark mass which originates in the quark condensate, the theory has dynamical chiral symmetry breaking
- M is the current quark mass matrix, In the limit, $m_q \to 0$, the theory has global $U(3)_L \times U(3)_R$ symmetry.
- $$\rho^i_\mu = -\frac{1}{m_0^2}\bar{\psi}\gamma_\mu\tau^i\psi,$$
  $$a^i_\mu = -\frac{1}{m_0^2}\bar{\psi}\gamma_\mu\gamma_5\tau^i\psi.$$
- N_c expansion is naturally embedded
- Integrating out the quark field, the L of mesons is obtained. The theory is phenomenologically successful

- The 0^{-+} glueball χ field is expressed as the gluon operator $F\tilde{F}$

$$\mathcal{L} = -\frac{1}{4}F^{a\mu\nu}F^{a}_{\mu\nu} + F^{a}_{\mu\nu}\tilde{F}^{a\mu\nu}\chi + \frac{1}{2}G^2_{\chi}\chi\chi,$$

$$\chi = -\frac{1}{G^2_{\chi}}F_{\mu\nu}\tilde{F}^{\mu\nu}.$$

- U(1) anomaly

$$\partial_\mu(\bar{\psi}\gamma_\mu\gamma_5\psi) = \frac{3g_s^2}{(4\pi)^2}F_{\mu\nu}\tilde{F}^{\mu\nu}. \tag{2}$$

- χ-field is coupled to the quark operator

$$\mathcal{L} = -\frac{1}{4}F^{a\mu\nu}F^{a}_{\mu\nu} - \{\bar{\psi}\gamma_\mu\gamma_5\psi\partial_\mu\chi + 2i\bar{\psi}M\gamma_5\psi\chi\} + \frac{1}{2}G^2_\chi\chi\chi.$$

- Insert this L into the L of the mesons, the L of mesons and 0^{-+} glueball is constructed.
  Integrating out the quark fields, the L of the mesons and the glueball is obtained.

- In this theory the 0^{-+} mesons are Goldstone mesons.
- $\eta(1405)$ is taken as a 0^{-+} glueball

$$m^2_{\eta_8} = \frac{1}{3}\{2(m^2_{K^+} + m^2_{K^0}) - m^2_{\pi}\},$$
$$\Delta m^2_{\eta_8\eta_0} = \frac{\sqrt{2}}{9}(m^2_{K^+} + m^2_{K^0} - 2m^2_{\pi}),$$
$$\Delta m^2_{\chi\eta_8} = -\frac{\sqrt{2}}{9}\frac{f_\pi}{F}(m^2_{K^+} + m^2_{K^0} - 2m^2_{\pi}).$$

- $m^2_{\eta_0}$, $\Delta m^2_{\chi\eta_0}$, and m^2_{χ} are the three parameters of the mass matrix
- $m_{\eta(1405)}$, m_{η}, *and* $m_{\eta'}$ are taken as inputs.
- The decay width of $\eta' \to \gamma\gamma$ is taken as another input

$$\Gamma(\eta' \to \gamma\gamma) = \frac{\alpha^2}{16\pi^3}\frac{m^3_{\eta'}}{f^2_{\pi}}(2\sqrt{\frac{2}{3}}b_2 + \frac{1}{\sqrt{3}}a_2)^2.$$

- The mixing are determined

$$\begin{aligned}\eta &= 0.9742\eta_8 + 0.1593\eta_0 - 0.16\chi,\\ \eta' &= -0.1513\eta_8 + 0.8208\eta_0 - 0.551\chi,\\ \eta(1405) &= -0.003522\eta_8 + 0.5724\eta_0 + 0.8199\chi.\end{aligned}$$

- Using VMD, it is predicted

$$\Gamma(\eta(1405) \to \gamma\gamma) = \frac{\alpha^2}{16\pi^3}\frac{m^3_{\eta(1405)}}{f^2_\pi}(2\sqrt{\frac{2}{3}}b_3)^2 = 4.55\ \mathrm{keV}$$

- The branching ratio of $\eta(1405) \to \gamma\gamma$ is predicted

$$B(\eta(1405) \to 2\gamma) = 0.87(1 \pm 0.07) \times 10^{-4}$$

- The vertex of $\eta(1405)vv$ is determined by the η_0 component mainly.
- Using the VMD and the L, the vertices of $\eta(1405) \to \gamma v$ are determined

$$\Gamma(\eta(1405) \to \rho\gamma) = (0.5724)^2 \frac{3\alpha}{2\pi^4 g^2} \frac{1}{f_\pi^2} k_\rho^3,$$

$$\Gamma(\eta(1405) \to \omega\gamma) = \frac{1}{9}(0.5724)^2 \frac{3\alpha}{2\pi^4 g^2} \frac{1}{f_\pi^2} k_\omega^3,$$

$$\Gamma(\eta(1405) \to \phi\gamma) = (\frac{2}{9}0.5724)^2 \frac{3\alpha}{2\pi^4 g^2} \frac{1}{f_\pi^2} k_\phi^3.$$

- $$\Gamma(\eta(1405) \to \rho\gamma) = 0.84\mathrm{MeV},$$
  $$\Gamma(\eta(1405) \to \omega\gamma) = 90.3\mathrm{kev},$$
  $$\Gamma(\eta(1405) \to \phi\gamma) = 58.2\mathrm{kev}.$$

- Besides mass mixing there is kinetic mixing between the η_0 meson and the χ glueball.
- The mixing between the kinetic terms of the ρ^0 and the ω fields is a good example

$$\mathcal{L}_{\rho-\omega} = \{-\frac{1}{4\pi^2 g^2}\frac{1}{m}(m_d - m_u) + \frac{1}{24}e^2 g^2\}(\partial_\mu \rho_\nu - \partial_\nu \rho_\mu)(\partial_\mu \omega_\nu - \partial_\nu \omega_\mu).$$

- By calculating the S-matrix element $\langle \eta_0 | S | \chi \rangle$, the kinetic mixing is found

$$-(1 - \frac{2c}{g})^{\frac{1}{2}} \partial_\mu \eta_0 \partial_\mu \chi.$$

- The radiative decay $J/\psi \to \gamma gg$ is the best ground to search for glueballs

$$\Gamma(J/\psi \to \gamma\chi) = \frac{2^{11}}{81}\alpha\alpha_s^2(m_c)\psi_J^2(0)f_G^2\frac{1}{m_c^8}\frac{(1-\frac{m^2}{m_J^2})^3}{\{1-2\frac{m^2}{m_J^2}+\frac{4m_c^2}{m_J^2}\}^2}$$

$$\{2m_J^2 - 3m^2(1+\frac{2m_c}{m_J}) - 16\frac{m_c^3}{m_J}\}^2,$$

- The χ state is via both the mass mixing and the kinetic mixing related to the η' and the $\eta(1405)$ respectively

$$\langle \eta' | \chi(0) | 0 \rangle = -0.551 + 0.8208(1 - \frac{2c}{g})^{\frac{1}{2}} \frac{m_{\eta'}^2}{m_\chi^2 - m_{\eta'}^2} = 0.3044$$

$$\langle \eta(1405) | \chi(0) | 0 \rangle = 0.8199 + 0.0.5724(1 - \frac{2c}{g})^{\frac{1}{2}} \frac{m_G^2}{m_\chi^2 - m_G^2} = -1.7788.$$

- $B(J/\psi \to \gamma\eta') = (4.71 \pm 0.27) \times 10^{-3}$ is taken as an input

$$m_c = 1.22\mathrm{GeV},\ \ B(J/\psi \to \gamma\eta(1405)) = 3.67(1 \pm 0.06) \times 10^{-3},$$

$$m_c = 1.23\mathrm{GeV},\ \ B(J/\psi \to \gamma\eta(1405)) = 6.69(1 \pm 0.06) \times 10^{-3},$$

$$m_c = 1.24\mathrm{GeV},\ \ B(J/\psi \to \gamma\eta(1405)) = 1.13(1 \pm 0.06) \times 10^{-2}.$$

- $\eta(1405) \to \rho\pi\pi$ is an anomalous decay mode.
- Two subprocesses: (1) $\eta(1405) \to \rho\rho,\ \rho \to \pi\pi$, (2) $\eta(1405) \to \rho\pi\pi$ directly.
- The prediction of the theory is

$$\Gamma(\eta(1405) \to \rho^0\pi^+\pi^-) = 0.92\ \mathrm{MeV}.$$

$$B(\eta(1405) \to \rho^0\pi\pi) = 5.4\%.$$

- $\eta(1405) \to \rho\rho$ is the dominant process.

- Two body decay, $\eta(1405) \to a_0(980)\pi$, should be the major decay mode of $\eta(1405)$.
- The $a_0(980)$ field can be added to the Lagrangian

$$-\frac{1}{2}\bar{\psi}\{(m+\tau^i a_0^i)u + u(m+\tau^i a_0^i)\}\psi.$$

- Of course a mass term $\frac{1}{2}m_{a_0}^2 a_0^i a_0^i$ should be added too

- To the leading order in the momentum expansion the amplitude is found

$$T = -0.5724\frac{8\sqrt{2}}{\sqrt{3}f_\pi^2}\frac{1}{g}(1-\frac{1}{3\pi^2 g^2})^{-\frac{1}{2}}\{\frac{1}{3}\langle 0|\bar{\psi}\psi|0\rangle + 3m^3 g^2\}$$

- Only the η_0 component contributes
- The quark condensate

$$\frac{1}{3}\langle 0|\bar{\psi}\psi|0\rangle = -(0.24)^3 \text{ GeV}$$

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$$\Gamma(\eta(1405) \to a_0\pi) = 44 \text{ MeV}.$$

$$B(\eta(1405) \to a_0\pi) = 86(1 \pm 0.07)\%$$

- Only the η_8 component of the $\eta(1405)$ contributes to $\eta(1405) \to K^*(890)K$.
- The coefficient of η_8 of $\eta(1405)$ is determined to be 0.00352.
- the theory predicts a very small branching ratio for $\eta(1405) \to K^*(890)K$
- the experimental ratio

$$R = \frac{B(\eta(1416) \to K^*\overline{K} + c.c)}{B(\eta(1416) \to a_0\pi)} = 0.028 \pm 0.024$$

- Based on a phenomenologically successful chiral field theory mesons and the U(1) anomaly a chiral field theory of 0^{-+} glueball and mesons has been constructed.
- Systematic and quantitative study of the properties of the candidate of the 0^{-+} glueball $\eta(1405)$ have been done by this theory.
- Besides the mass mixing of η, η' $\eta(1405)$ the kinetic mixing between them has been found.
- Large $B(J/\psi \to \gamma\eta(1405))$ is via the kinetic mixing predicted.

- The prediction of the small branching ratio of $\eta(1405) \to 2\gamma$ is consistent with the fact that $\eta(1405)$ has not been found in two photon collisions.
- $\eta(1405) \to a_0(980)\pi$ is the major decay mode of $\eta(1405)$.
- A very small branching ratio of $\eta(1405) \to K^*K$ is predicted
- The $K^*\bar{K}$ is the dominant decay mode of the $\eta(1475)$.
- The separation of $\eta(1405)$ and $\eta(1475)$ in J/ψ radiative decay is the key to establish that $\eta(1405)$ is the 0^{-+} glueball.