# *Higgs Boson Differential Distributions from Effective Field Theory*

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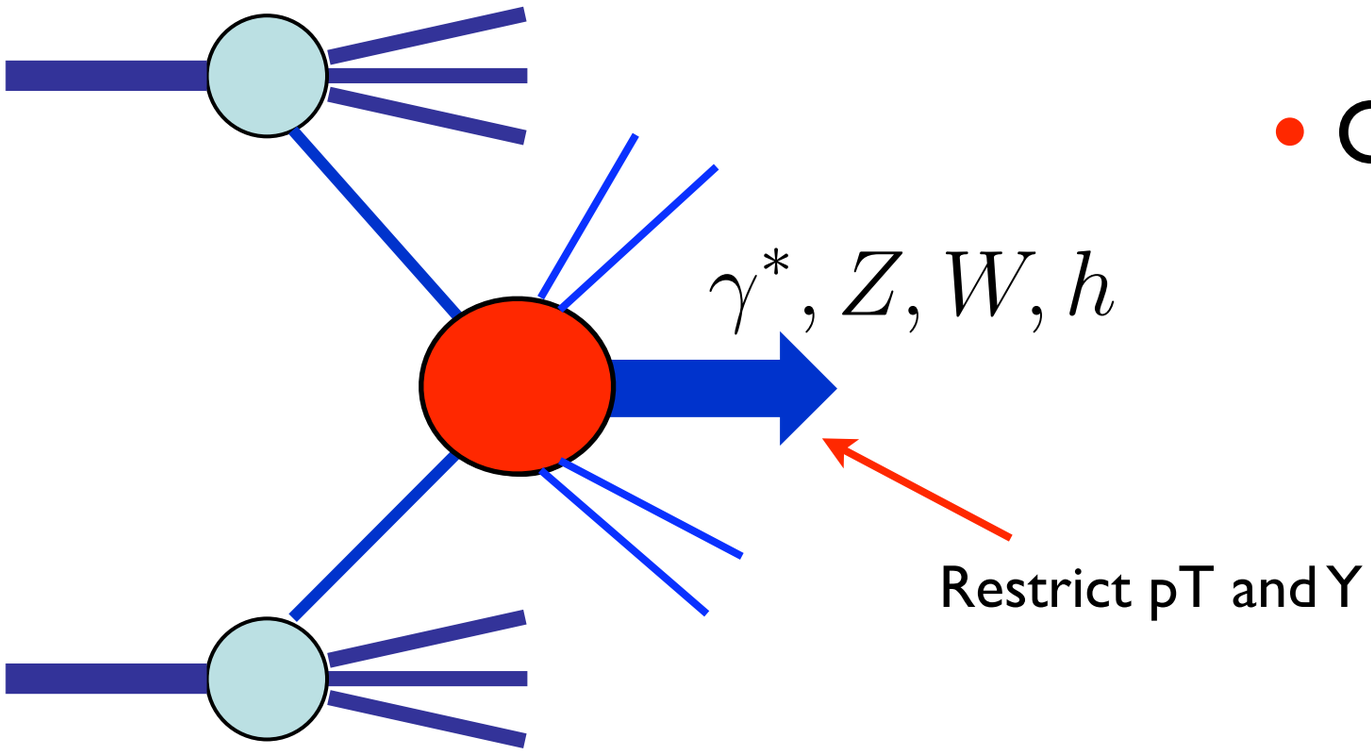
(SM, Frank Petriello) **arXiv:0911.4135, Phys.Rev.D81:093007, 2010**
**arXiv:1007.3773, Phys.Rev.D83:053007, 2011**
**arXiv:1011.0757, Phys.Rev.D84:014030, 2011**

(Ye Li, SM, Frank Petriello) **arXiv:1105.5171**

DPF Meeting, Brown University, August 9th-13th, 2011

# Transverse Momentum Spectrum

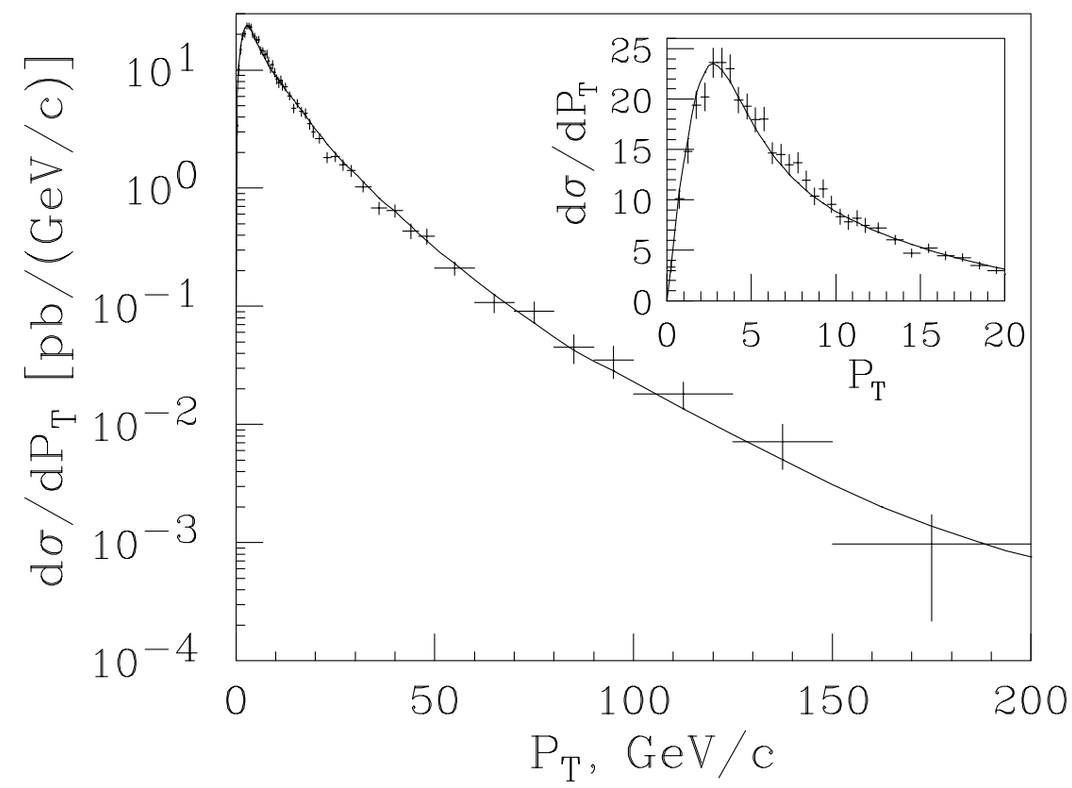



- Observable of interest

$$\frac{d^2\sigma}{dp_T^2 dY}$$

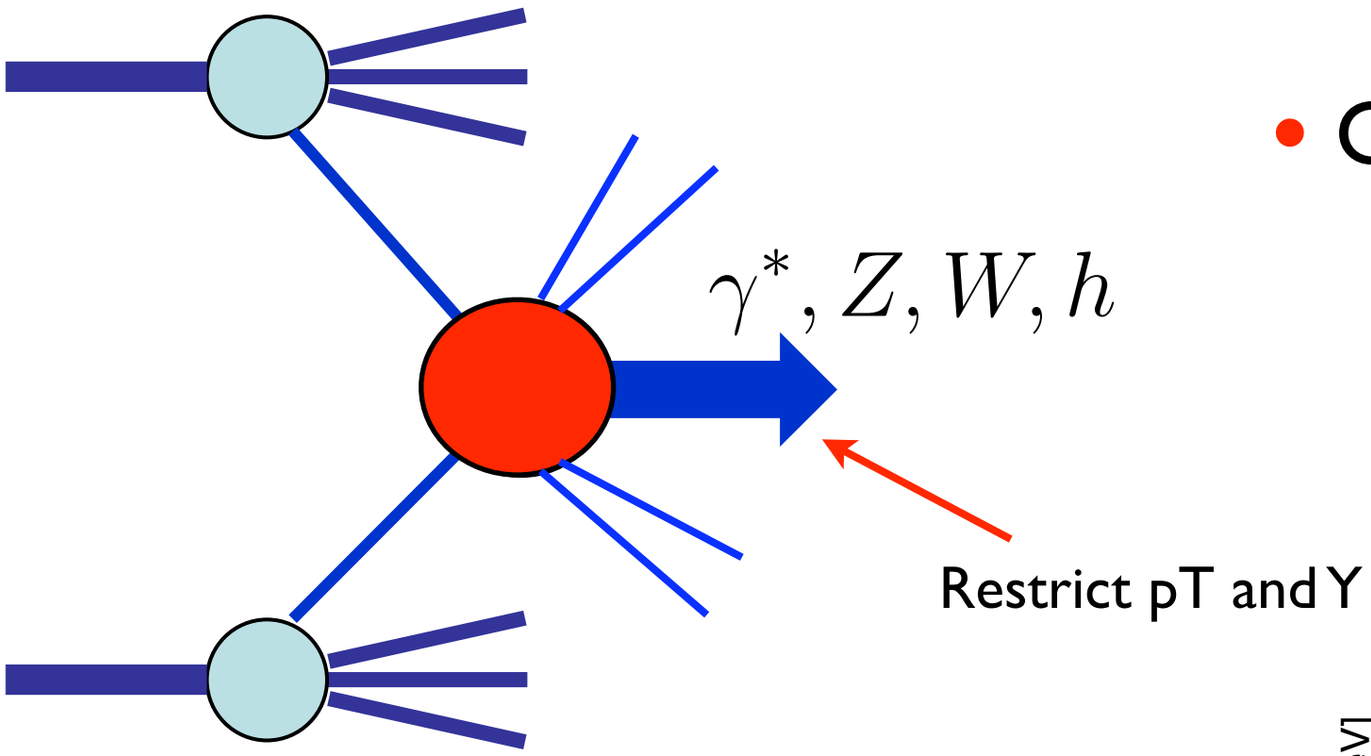
## Motivations

- Higgs Boson searches
- W-mass measurement
- Tests of pQCD
- Transverse nucleon structure




Tevatron Data
for Z-production

# Transverse Momentum Spectrum

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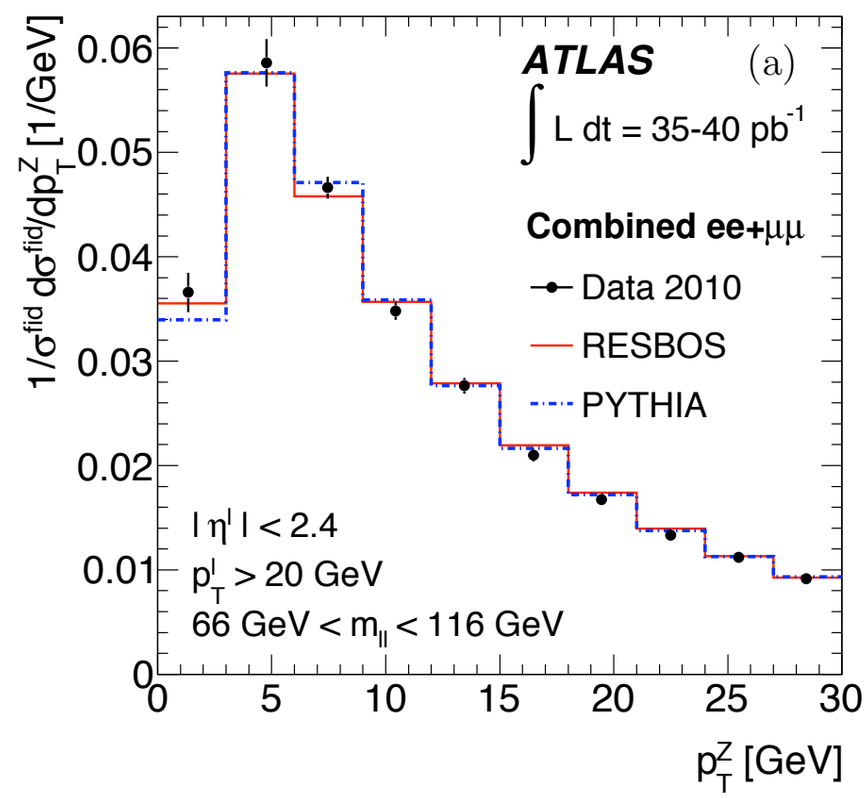
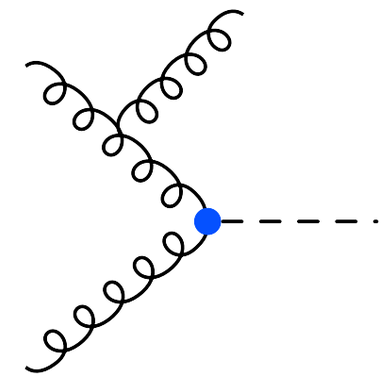



New ATLAS Data
for Z-production

# Low pT Region



- The schematic perturbative series for the pT distribution for $pp \longrightarrow h + X$

$$\frac{1}{\sigma}\frac{d\sigma}{dp_T^2} \simeq \frac{1}{p_T^2}\left[A_1\alpha_S \ln\frac{M^2}{p_T^2} + A_2\alpha_S^2 \ln^3\frac{M^2}{p_T^2} + \ldots + A_n\alpha_S^n \ln^{2n-1}\frac{M^2}{p_T^2} + \ldots\right]$$

Large Logarithms spoil perturbative convergence

- Resummation has been studied in great detail in the Collins-Soper-Sterman formalism.

(Davies, Stirling; Arnold, Kauffman; Berger, Qiu; Ellis, Veseli, Ross, Webber; Brock, Ladinsky Landry, Nadolsky; Yuan; Fai, Zhang; Catani, Emilio, Trentadue; Hinchliffe, Novae; Florian, Grazzini, Cherdnikov, Stefanis; Belitsky, Ji,.... )

- Resummation has also been studied recently using the EFT approach.

(Idilbi, Ji, Yuan; Gao, Li, Liu; SM, Petriello; Becher, Neubert)

# CSS Formalism

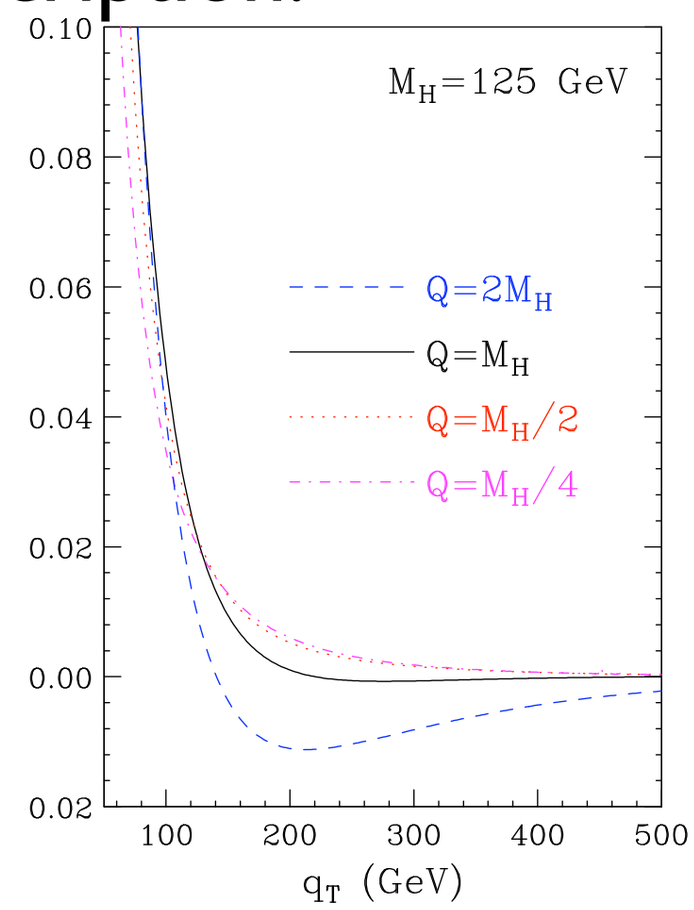
$$\frac{d^2\sigma}{dp_T\,dY} = \sigma_0 \int \frac{d^2 b_\perp}{(2\pi)^2} e^{-i\vec{p}_T\cdot\vec{b}_\perp} \sum_{a,b} \left[C_a \otimes f_{a/P}\right](x_A, b_0/b_\perp)\left[C_b \otimes f_{b/P}\right](x_B, b_0/b_\perp)$$
$$\times \exp\left\{\int_{b_0^2/b_\perp^2}^{\hat{Q}^2} \frac{d\mu^2}{\mu^2}\left[\ln\frac{\hat{Q}^2}{\mu^2} A(\alpha_s(\mu^2)) + B(\alpha_s(\mu^2))\right]\right\}.$$

Landau Pole

- Landau pole appears for any value of pT.
- Landau pole must be treated with some prescription.

(Collins, Soper, Sterman; Kulesza, Laenen, Vogelsang; Qiu, Zhang,... )

- Difficulties can arise in smoothly matching the resummed low pT cross-section with the fixed order result at large pT.

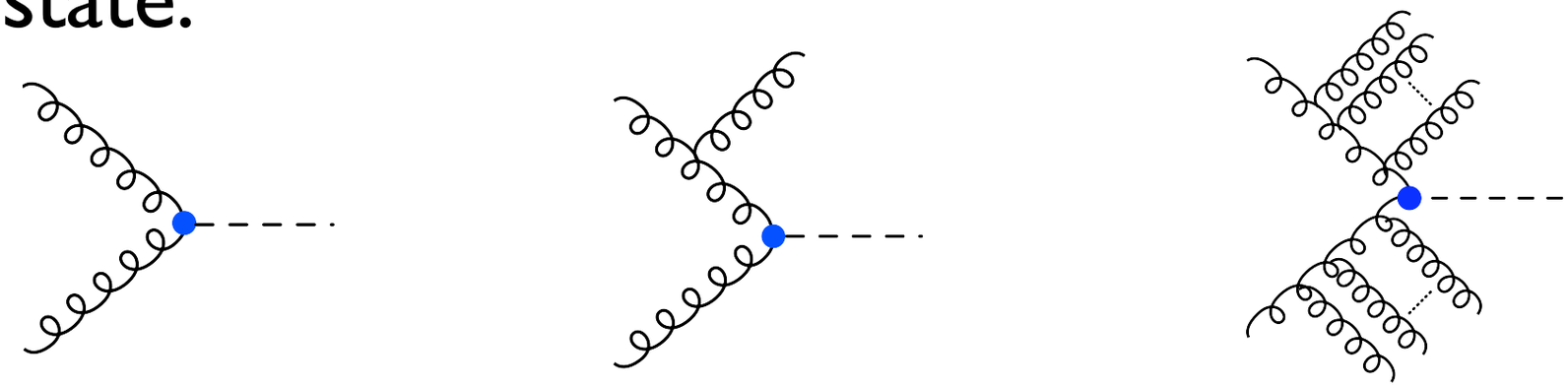



(Bozzi, Catani, de Florian, Grazzini )

# EFT Framework

# EFT framework

- Low pT region dominated by soft and collinear emissions from initial state:



pT=0 finite pT finite pT

- Soft and Collinear emissions dominate the low pT distribution:

$$p_n \sim m_h(\eta^2, 1, \eta), \quad p_{\bar{n}} \sim m_h(1, \eta^2, \eta), \quad p_s \sim m_h(\eta, \eta, \eta),$$

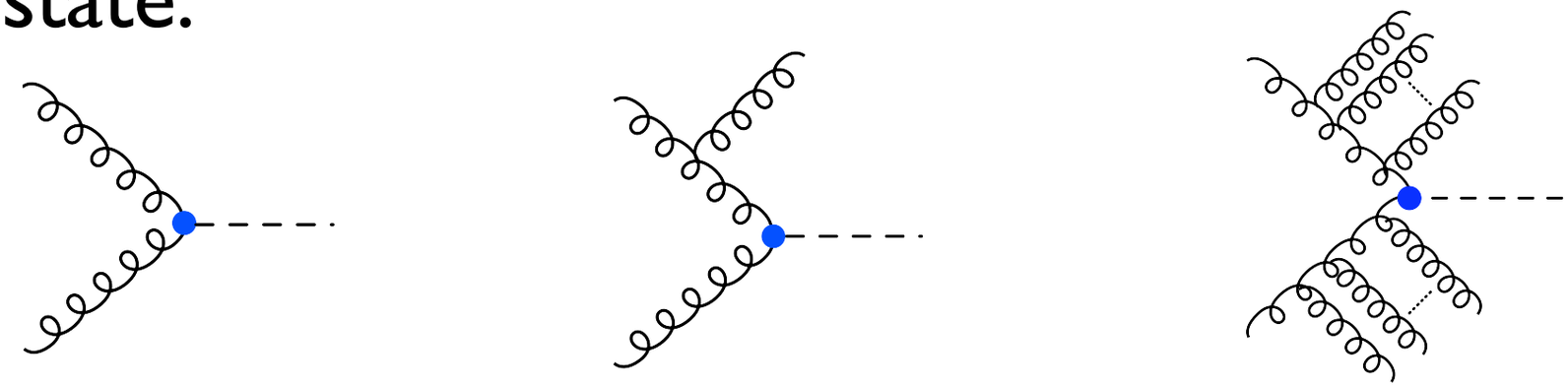
$$\eta \sim \frac{p_T}{m_h}$$

- Hierarchy of scales suggests EFT approach with well defined power counting.

$$m_h \gg p_T \gg \Lambda_{QCD}\,, \qquad p_T \sim \Lambda_{QCD}$$

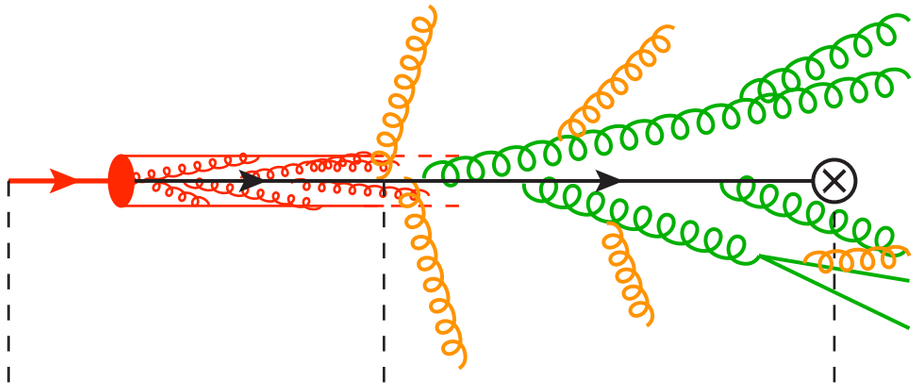
# EFT framework

- Low pT region dominated by soft and collinear emissions from initial state:



pT=0 finite pT finite pT

- Colliding parton is part of initial state pT radiation beam jet:



Initial State jet of pT radiation

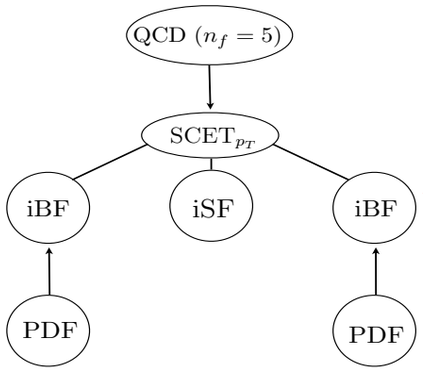
- Gives rise to impact-parameter Beam Functions (iBFs). (SM,Petriello)

Analogous beam functions arise in other processes:

(Stewart, Tackmann, Waalewijin; Fleming, Leibovich, Mehen)

- Soft recoil radiation is restricted. Gives rise to a soft function.

# SCET Cross-Section




We are here

$$\frac{d^2\sigma}{dp_T^2 dY} \sim H \otimes B_n \otimes B_{\bar{n}} \otimes S$$

Hard function

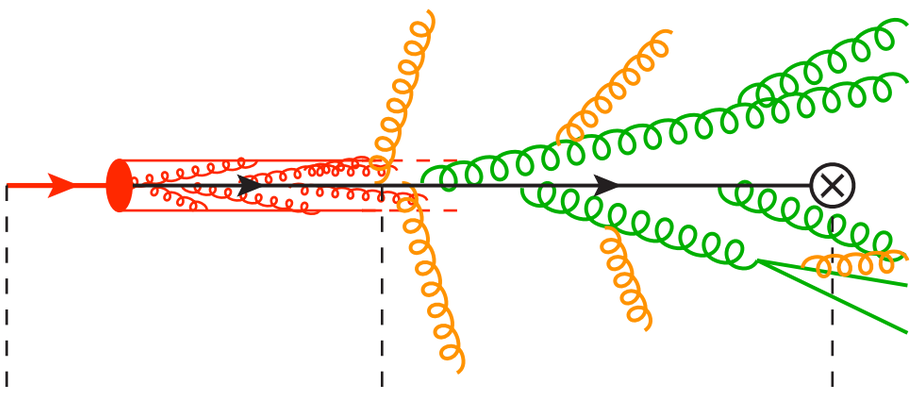
Impact-parameter Beam Functions (iBFs)

Soft function

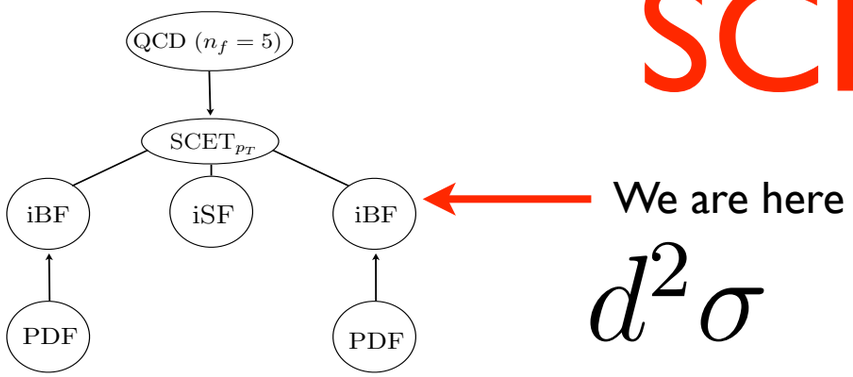
Describes collinear pT emissions

Describes soft pT emissions

iBFs: Fully Unintegrated PDFs



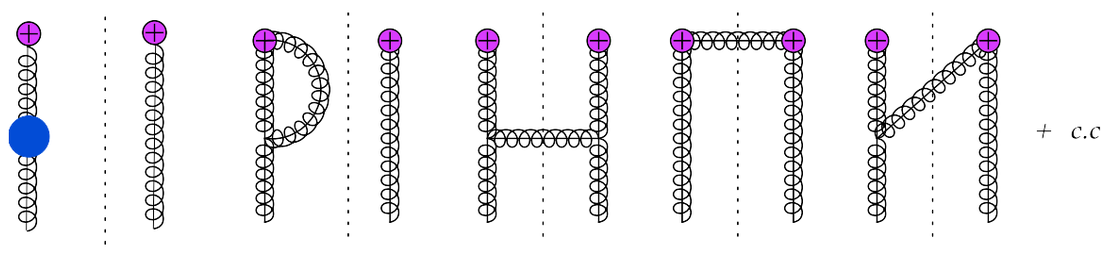
# SCET Cross-Section




$$\frac{d^2\sigma}{dp_T^2 dY} \sim H \otimes B_n \otimes B_{\bar{n}} \otimes S$$

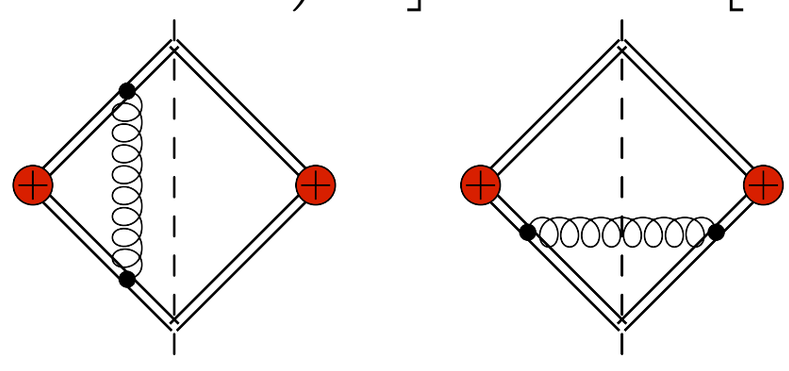
- iBF:

$$\tilde{B}_n^{\alpha\beta}(x_1, t_n^+, b_\perp, \mu) = \int \frac{db^-}{4\pi} e^{\frac{i}{2}\frac{t_n^+ b^-}{Q}} \sum_{\text{initial pols.}} \sum_{X_n} \langle p_1 | \left[ g B^A_{1n\perp\beta}(b^-, b_\perp) | X_n \rangle \right.$$
$$\times \left. \langle X_n | \delta(\bar{\mathcal{P}} - x_1 \bar{n}\cdot p_1) g B^A_{1n\perp\alpha}(0) \right] | p_1 \rangle,$$

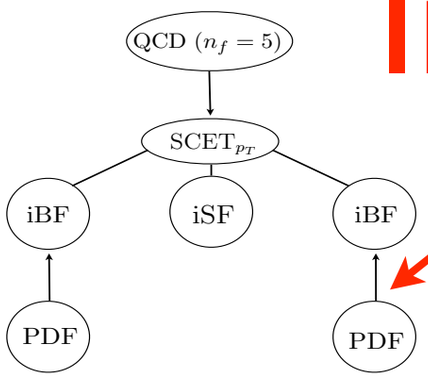



- Soft function:

$$S(b,\mu) = \sum_{X_s} \langle 0 | \bar{T} \left[ \mathrm{Tr} \left( S_{\bar{n}} T^D Y^\dagger_{\bar{n}} S_n T^C S_n^\dagger \right)(b) \right] | X_s \rangle \langle X_s | T \left[ \mathrm{Tr} \left( S_n T^C S_n^\dagger S_{\bar{n}} T^D S_{\bar{n}}^\dagger \right)(0) \right] | 0 \rangle$$



# Integrating Out the pT Scale




We are here

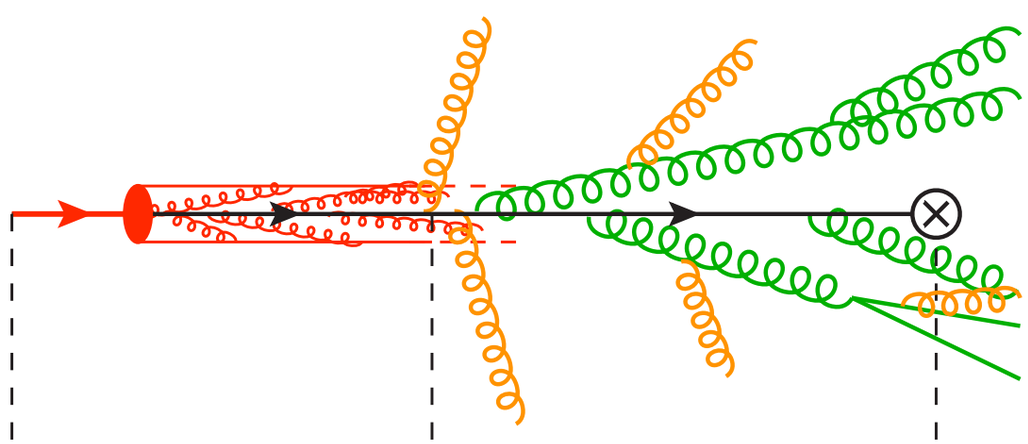
$$\frac{d^2\sigma}{dp_T^2 dY} \sim H \otimes \tilde{B}_n \otimes \tilde{B}_{\bar{n}} \otimes S^{-1}$$

iBFs are proton matrix elements
and sensitive to the
non-perturbative scale

- The iBFs are matched onto PDFs to separate the perturbative and non-perturbative scales:

$$\tilde{B}_n = \mathcal{I}_{n,i} \otimes f_i,$$

iBF  Matching coefficient  PDF



# Factorization Formula

- Factorization formula in full detail:

$$\frac{d^2\sigma}{d\mathrm{p}_T^2\, dY} = \frac{\pi^2}{4(N_c^2-1)^2 Q^2}\int_0^1 \frac{dx_1}{x_1}\int_0^1 \frac{dx_2}{x_2}\int_{x_1}^1 \frac{dx_1'}{x_1'}\int_{x_2}^1 \frac{dx_2'}{x_2'}$$
$$\times\; H(x_1,x_2,\mu_Q;\mu_T)\mathcal{G}^{ij}(x_1,x_1',x_2,x_2',p_T,Y,\mu_T) f_{i/P}(x_1',\mu_T) f_{j/P}(x_2',\mu_T)$$

Hard function.

Transverse momentum function.

PDFs.

- The transverse momentum function is a convolution of the iBF matching coefficients and the soft function:

$$\mathcal{G}^{ij}(x_1,x_1',x_2,x_2',p_T,Y,\mu_T) = \int dt_n^+ \int dt_{\bar{n}}^- \int \frac{d^2 b_\perp}{(2\pi)^2} J_0(|\vec{b}_\perp| p_T)$$

Collinear pT emissions → $\times\; \mathcal{I}^{\beta\alpha}_{n;g,i}(\frac{x_1}{x_1'}, t_n^+, b_\perp, \mu_T)\, \mathcal{I}^{\beta\alpha}_{\bar{n};g,j}(\frac{x_2}{x_2'}, t_{\bar{n}}^-, b_\perp, \mu_T)$

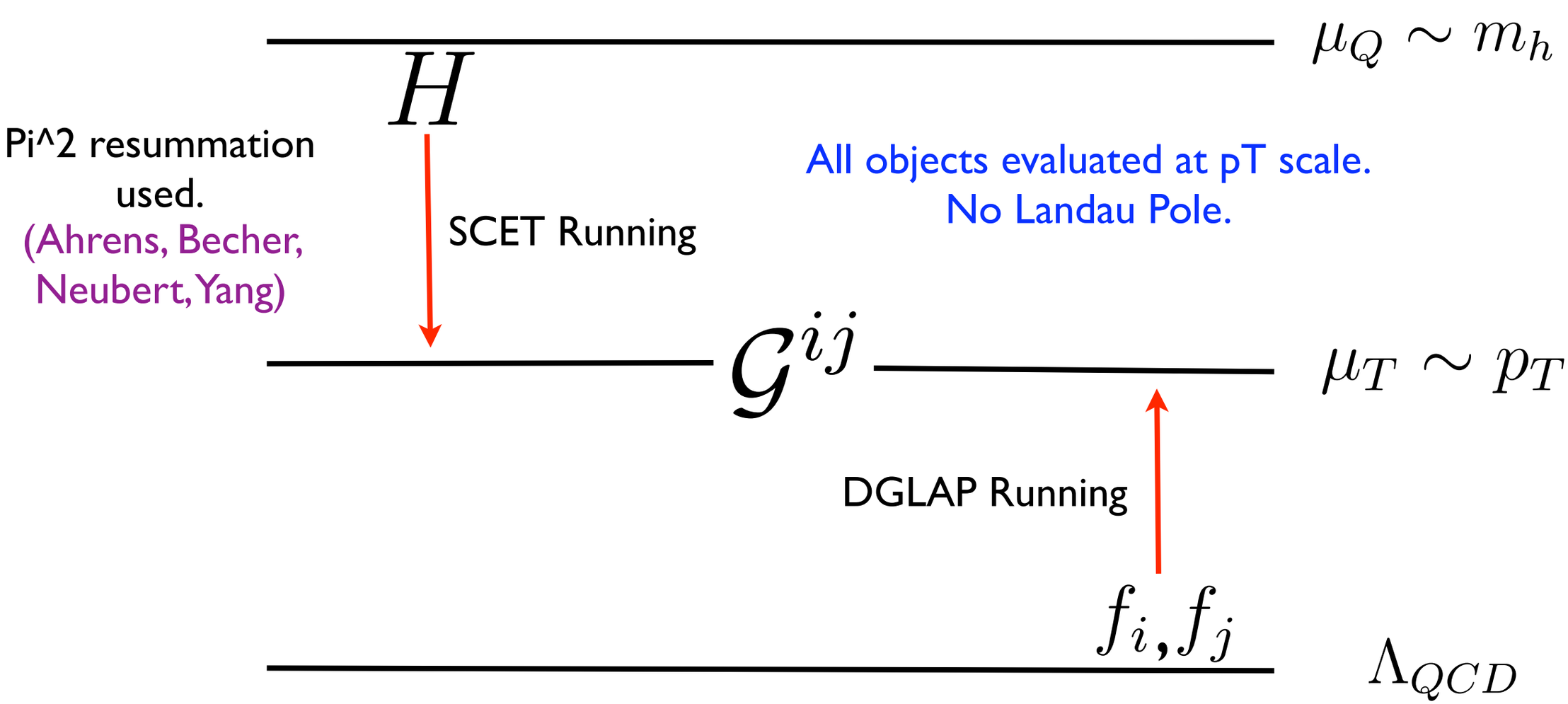
Soft pT emissions → $\times\; \mathcal{S}^{-1}(x_1 Q - e^Y\sqrt{\mathrm{p}_T^2+m_h^2} - \frac{t_{\bar{n}}^-}{Q}, x_2 Q - e^{-Y}\sqrt{\mathrm{p}_T^2+m_h^2} - \frac{t_n^+}{Q}, b_\perp, \mu_T)$

# Running

- Factorization formula:

$$\frac{d^2\sigma}{dp_T^2 dY} \sim H \otimes \mathcal{G}^{ij} \otimes f_i \otimes f_j$$

- Schematic picture of running:



# Check of NLL with Fixed Order

$$\frac{d^2\sigma_{Z,q\bar{q}}}{dp_T^2 dY} = \frac{4\pi^2}{3}\frac{\alpha}{\sin^2\theta_W} e_{q\bar{q}}^2 \frac{\alpha_s(\mu_T)}{2\pi}\frac{1}{s\,p_T^2}\Big\{2\,C_F f_{q/P}(x_A,\mu_T) f_{\bar{q}/P}(x_B,\mu_T)\ln\frac{M_Z^2}{p_T^2}$$
$$- 3\,C_F f_{q/P}(x_A,\mu_T) f_{\bar{q}/P}(x_B,\mu_T) + f_{q/P}(x_A,\mu_T)\left(P_{qq}\otimes f_{\bar{q}/P}\right)(x_B)$$
$$+ f_{\bar{q}/P}(x_B,\mu_T)\left(P_{qq}\otimes f_{q/P}\right)(x_A)\Big\}\left|\exp\left\{\frac{C_F}{4}\frac{\alpha_s}{\pi}\left[-\ln^2\frac{\mu_Q^2}{\mu_T^2} + 3\ln\frac{\mu_Q^2}{\mu_T^2}\right]\right\}\right|^2.$$

$$\frac{d^2\sigma_{Z,q\bar{q}}}{dp_T^2 dY} = \frac{4\pi^2}{3}\frac{\alpha}{\sin^2\theta_W} e_{q\bar{q}}^2 \frac{1}{s\,p_T^2}\sum_{m,n}\left(\frac{\alpha_s(\mu_R)}{2\pi}\right)^n {}_nD_m \ln^m\frac{M_Z^2}{p_T^2}$$

$$\text{leading logarithmic} \;:\; \alpha_s^n L^{2n-1},$$
$$\text{next-to-leading logarithmic} \;:\; \alpha_s^n L^{2n-2},$$
$$\text{next-to-next-to-leading logarithmic} \;:\; \alpha_s^n L^{2n-3}.$$

(Arnold, Kaufmann; Ellis)

Checked

LL ${}_1D_1 = A^{(1)} f_A f_B,$

NLL ${}_1D_0 = B^{(1)} f_A f_B + f_B\left(P_{qq}\otimes f\right)_A + f_A\left(P_{qq}\otimes f\right)_B,$

LL ${}_2D_3 = -\frac{1}{2}\left[A^{(1)}\right]^2 f_A f_B,$

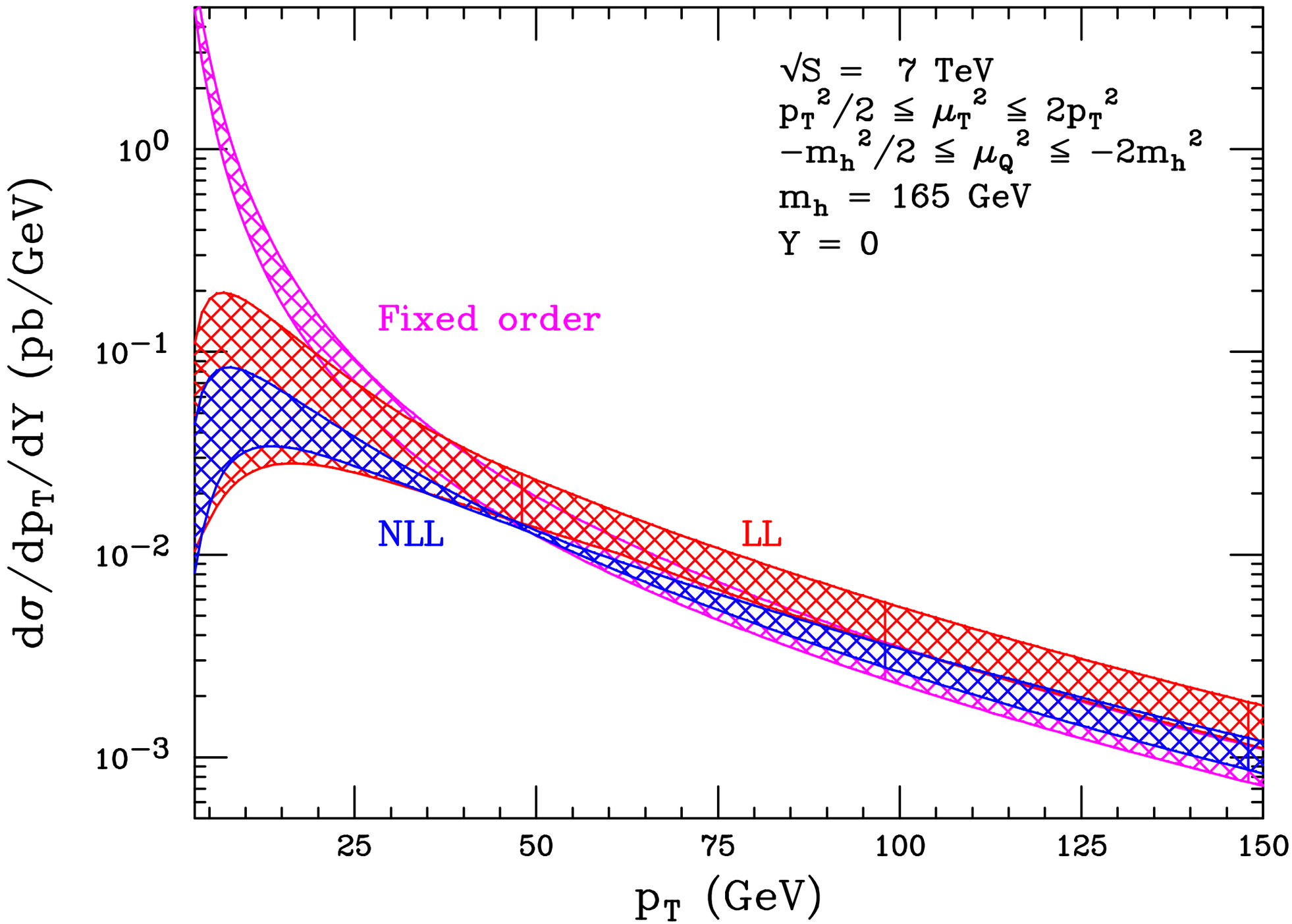
NLL ${}_2D_2 = -\frac{3}{2}A^{(1)}\left[f_B\left(P_{qq}\otimes f\right)_A + f_A\left(P_{qq}\otimes f\right)_B\right] - \left[\frac{3}{2}A^{(1)}B^{(1)} - \beta_0 A^{(1)}\right] f_A f_B,$

Requires two loop iBF and iSF

NNLL ${}_2D_1 = \Big\{-A^{(1)} f_B\left(P_{qq}\otimes f\right)_A \ln\frac{\mu_F^2}{M_Z^2} - 2B^{(1)} f_B\left(P_{qq}\otimes f\right)_A - \frac{1}{2}\left[B^{(1)}\right]^2 f_A f_B$
$+\frac{\beta_0}{2}A^{(1)} f_A f_B \ln\frac{\mu_R^2}{M_Z^2} + \frac{\beta_0}{2}B^{(1)} f_A f_B - \left(P_{qq}\otimes f\right)_A\left(P_{qq}\otimes f\right)_B$
$-f_B\left(P_{qq}\otimes P_{qq}\otimes f\right)_A + \beta_0 f_B\left(P_{qq}\otimes f\right)_A\Big\} + [A\leftrightarrow B].$

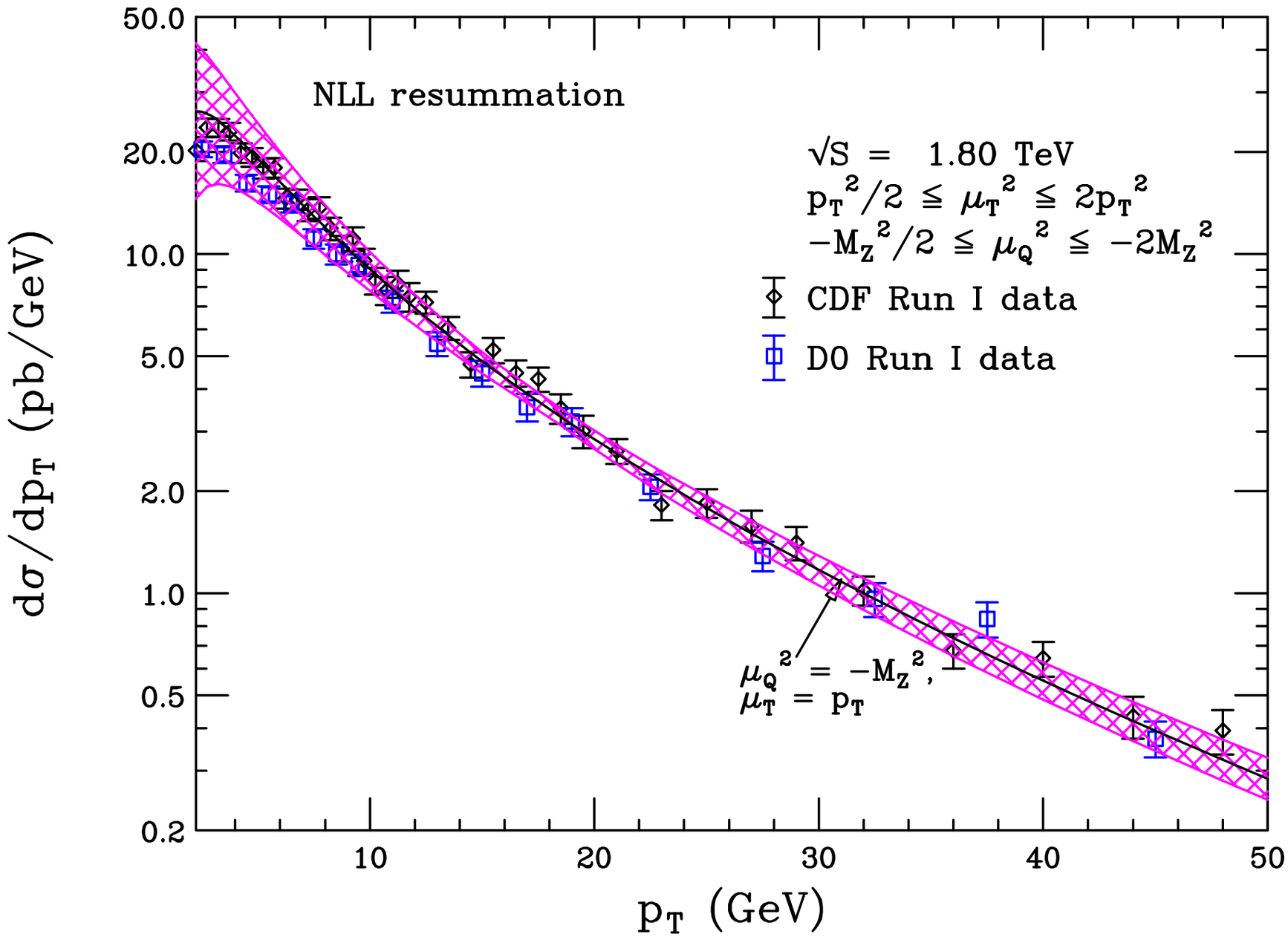
(two loop iSF known; two loop iBF in progress; Li, SM, Petriello)

# Higgs pT Distribution




- Prediction for Higgs boson pT distribution.

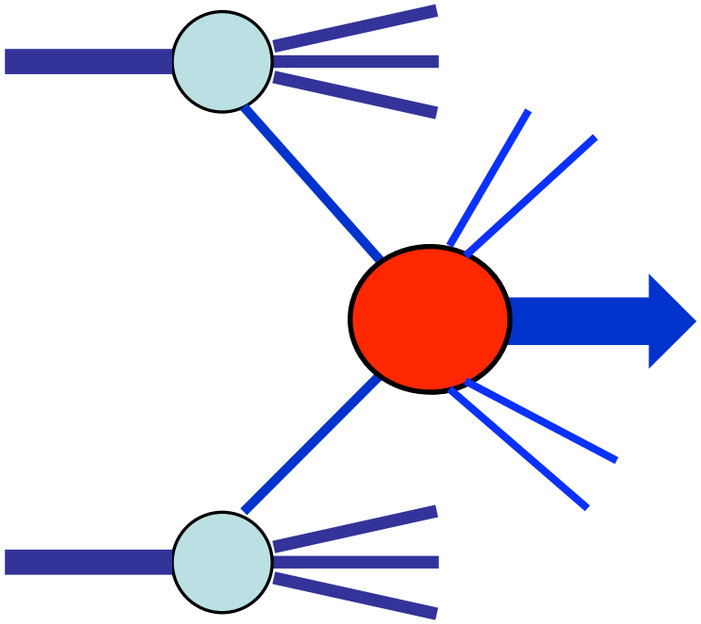
# Z-production: Comparison with Data




- Good agreement with data.
- Theory curve determined completely by perturbative functions and standard PDFs.
- Smooth matching between low and high pT regions.

# Non-Perturbative pT Region

- Non-perturbative region of pT:



$$p_T \sim \Lambda_{QCD}$$

Distribution sensitive to transverse momentum dynamics in nucleon

$$\frac{d^2\sigma}{dp_T^2 dY} \sim H \otimes \mathcal{G}^{ij} \otimes f_i \otimes f_j$$

Hard function.

Transverse momentum function (TMF).

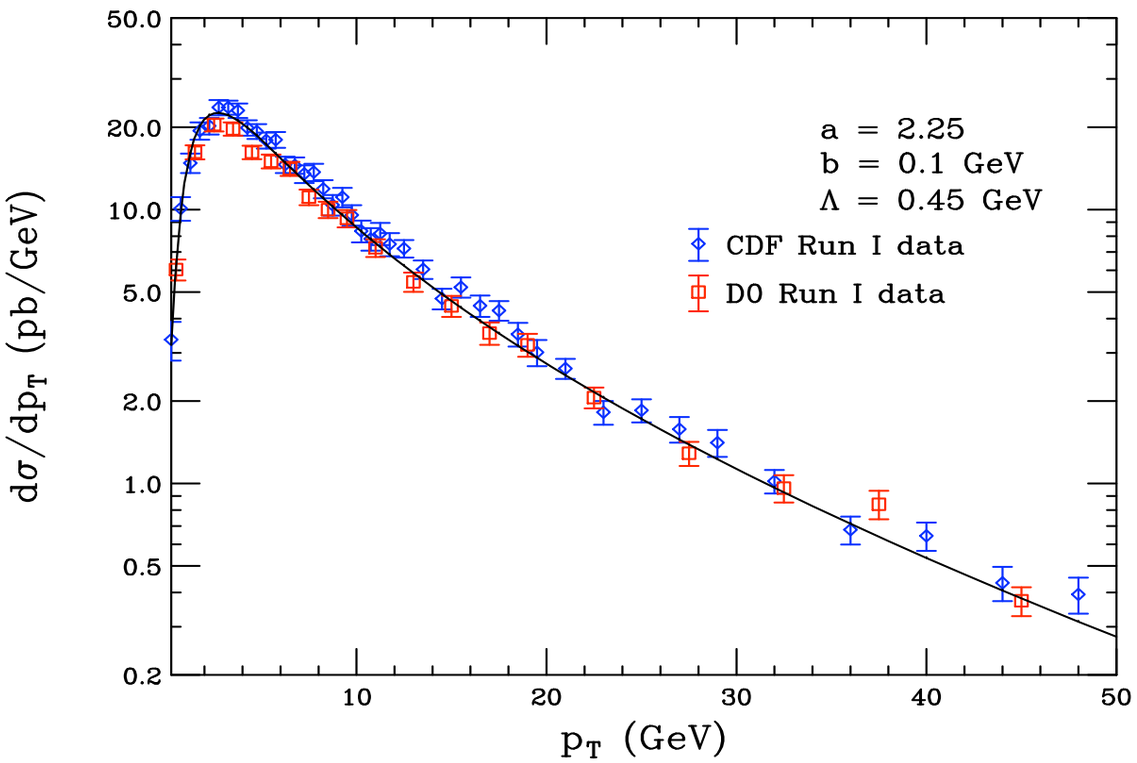
PDFs.

Field theoretically defined object

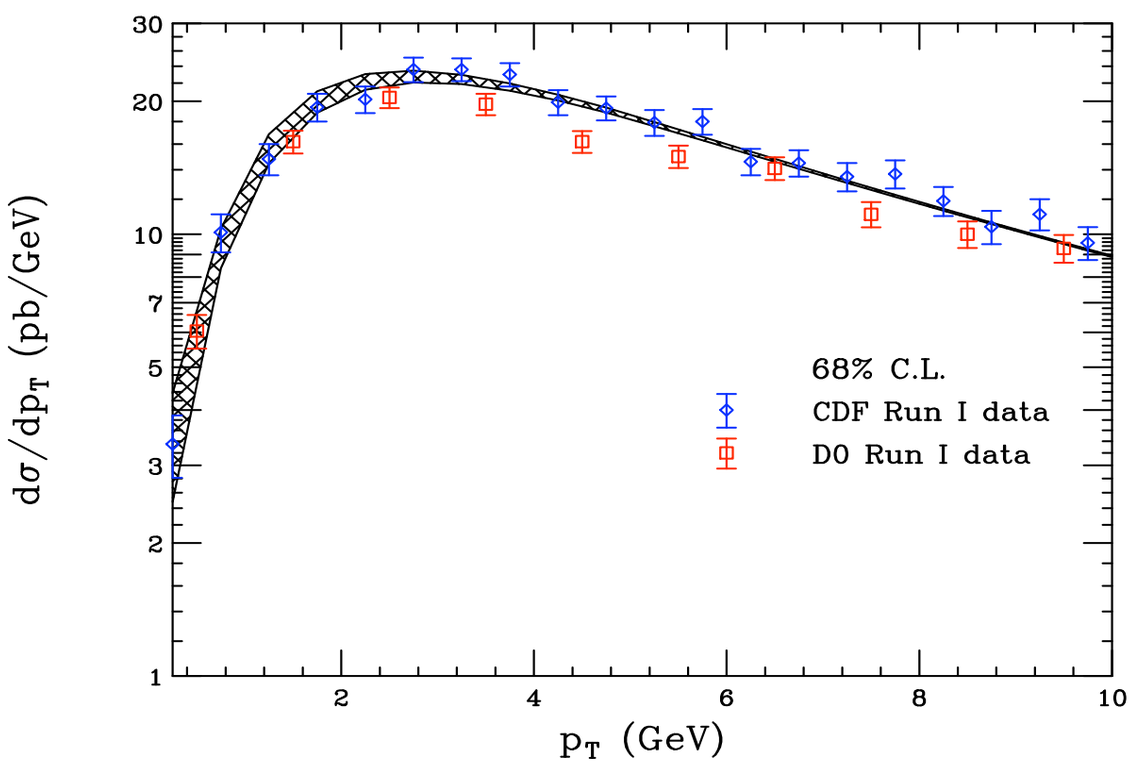
Can make non-perturbative model

Scale dependence and running known

# Including the Non-Perturbative Region

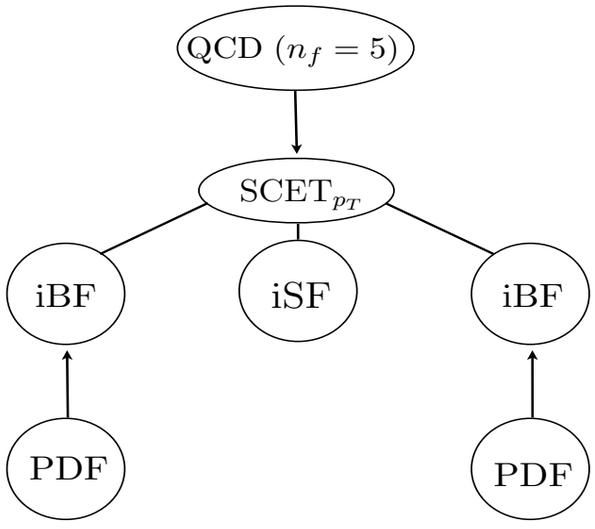



- pT spectrum including the non-perturbative region




- Model dependence restricted only to non-perturbative region as expected.

# Summary




$$\frac{d^2\sigma}{dp_T^2 dY} \sim H \otimes \mathcal{G}^{ij} \otimes f_i \otimes f_j$$

- New factorization theorem for transverse momentum distribution in terms of iBFs and iSFs.
- iBFs are fully unintegrated PDFs. Interesting objects in their own right.

PDF → TMDPDF → iBF → GPD

- Perturbative pT spectrum given in terms of PDFs and perturbatively calculable functions. Smooth matching between resummed and fixed order results. No Landau pole.
- NLO iBFs and NNLO iSF known.
  NLL resummation completed.
- Work is in progress to achieve NNLL resummation with applications to the LHC.

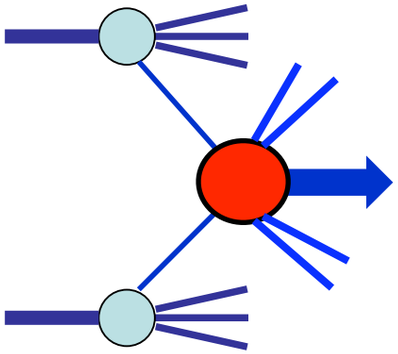
# Comparison with TMDPDF Formalism

| TMDPDF formalism | iBF formalism |
| --- | --- |
| - Rapidity divergence regulated by external regulator. | - Rapidity divergence regulated by physical residual momentum determined by kinematics. |
| - Factorization in terms of TMDPDFs. (See talk by J. Chiu for SCET formulation with TMDPDFs) | - Factorization in terms of iBFs; fully unintegrated PDFs; more differential than TMDPDFs. |
| - Cancellation of regulator dependence gives rise to Collins-Soper evolution equation for resummation. | - Resummation in terms of renormalization group equations. |

PDF $\longrightarrow$ TMDPDF $\longrightarrow$ iBF $\longrightarrow$ GPD

- iBF is another interesting probe of nucleon structure dynamics in the non-perturbative region.

# EFT framework



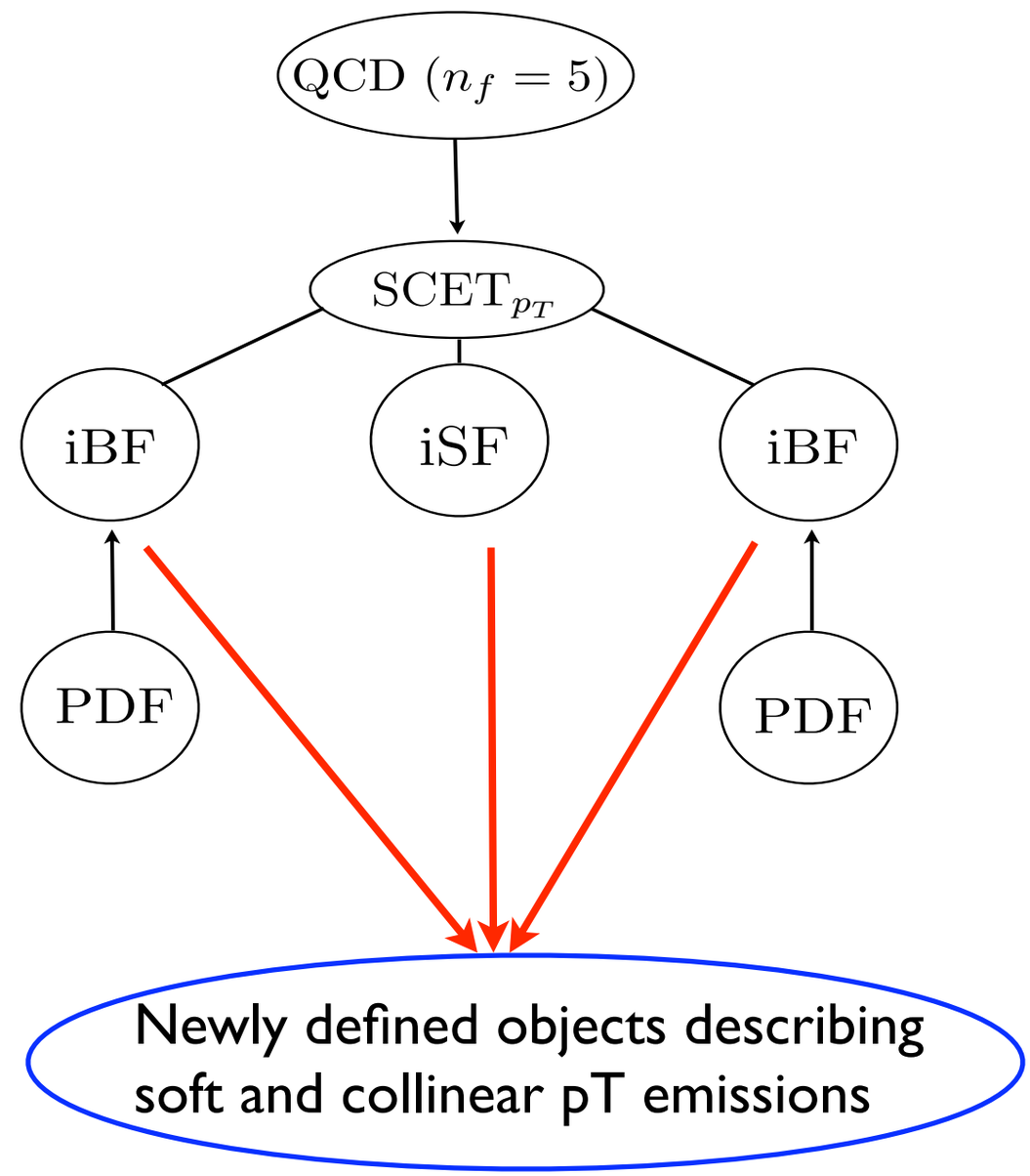
$$\mathrm{QCD}(n_f = 6) \to \mathrm{QCD}(n_f = 5) \to \mathrm{SCET}_{p_T} \to \mathrm{SCET}_{\Lambda_{QCD}}$$

Top quark integrated out.

Matched onto SCET.

Soft-collinear factorization.

Matching onto PDFs.




$$\frac{d^2\sigma}{dp_T^2 dY} \sim H \otimes \mathcal{G}^{ij} \otimes f_i \otimes f_j$$

iBF = Impact-parameter Beam Function
iSF = Inverse Soft Function

# Factorization Formula

$$\frac{d^2\sigma}{d\mathrm{p}_T^2\, dY} = \frac{\pi^2}{4(N_c^2-1)^2 Q^2}\int_0^1 \frac{dx_1}{x_1}\int_0^1 \frac{dx_2}{x_2}\int_{x_1}^1 \frac{dx_1'}{x_1'}\int_{x_2}^1 \frac{dx_2'}{x_2'}$$
$$\times\ H(x_1,x_2,\mu_Q;\mu_T)\mathcal{G}^{ij}(x_1,x_1',x_2,x_2',p_T,Y,\mu_T) f_{i/P}(x_1',\mu_T) f_{j/P}(x_2',\mu_T)$$

- One can express the formula entirely in momentum space:

$$\mathcal{G}^{ij}(x_1,x_1',x_2,x_2',p_T,Y,\mu_T) = \frac{1}{2\pi}\int dt_n^+ \int dt_{\bar{n}}^- \int d^2k_n^\perp \int d^2k_{\bar{n}}^\perp \int d^2k_s^\perp \frac{\delta(p_T - |\vec{k}_n^\perp + \vec{k}_{\bar{n}}^\perp + \vec{k}_s^\perp|)}{p_T}$$
$$\times\ \mathcal{I}^{\beta\alpha}_{n;g,i}(\frac{x_1}{x_1'}, t_n^+, k_n^\perp, \mu_T)\, \mathcal{I}^{\beta\alpha}_{\bar{n};g,j}(\frac{x_2}{x_2'}, t_{\bar{n}}^-, k_{\bar{n}}^\perp, \mu_T)$$
$$\times\ \mathcal{S}^{-1}(x_1 Q - e^Y\sqrt{\mathrm{p}_T^2+m_h^2} - \frac{t_{\bar{n}}^-}{Q}, x_2 Q - e^{-Y}\sqrt{\mathrm{p}_T^2+m_h^2} - \frac{t_n^+}{Q}, k_s^\perp, \mu_T)$$