On a Singular Solution in Higgs Field (2)

-A Representation of Certain f_0 Mesons' Masses.

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Contents

We have recently discussed the mass and the basic structure of SM Higgs boson (H^0) by obtaining asymptotic solution for their equation of motion of nonlinear Klein-Gordon type PDE.

In this paper, we'll treat with above in mind;

- Masses of glueball (GB) of ground state and of certain f0 mesons,
- Ur-SM Higgs boson (ur-H^0) which will consist of a number of GBs and/or f_0 above for respective fullerene structure,
- A representation of these f_0 mesons' masses by masses of π octet and GB,
- And transformation of ur-H^0 into H^0.

== Introductory review -SM Higgs mass formula ==

EOM of Higgs field should have a solution at the point of vacuum expectation value $\left(\phi = v\right)$, or $\varphi = 0$.

When we choose an asymptotic form for $\varphi(s)$ near s→0 as,

$$\varphi(s) \sim avs^3 \left\{1 - \exp\left(-s_0/s\right)\right\}\Big|_{s\to 0},$$

where $s \equiv \sqrt{c^2t^2 - x_i x^i}$: relativistically invariant distance from origin

Then, asymptotically, $\varphi(0) \sim 0, \ \varphi'(0) \sim 0$

And expanding near s→0;

$$\varphi(s)\Big|_{s\to 0} \approx avs^3 \left\{1 - \left(1 - s_0/s\right)\right\}\Big|_{s\to 0} \approx a_1 vs^2\Big|_{s\to 0}$$
$$= \varepsilon^2 v\Big|_{\varepsilon\to 0}, \quad \text{where } \varepsilon^2 \equiv a_1 s^2, \ a_1 \equiv as_0, \ 0 < s_0 \ll 1.$$

Higgs Mass Formula

So, $\varphi \sim \varepsilon^2 v$, ($\varepsilon \to 0$) : Asymptotic singular solution

Thus from EOM, **Higgs field mass formula** is

$$\varphi(\varepsilon_\lambda) = 0,$$

$$m_\varphi{}^2 = 2\left\{\left(\sqrt{W_\mu{}^+ W^{-\mu}}\right)^2 \Big/ \left(2\varepsilon_\lambda / g\right)^2\right\} + \left\{(\mathrm{Z}_\mu)^2 \Big/ \left(2\varepsilon_\lambda / G\right)^2\right\},$$

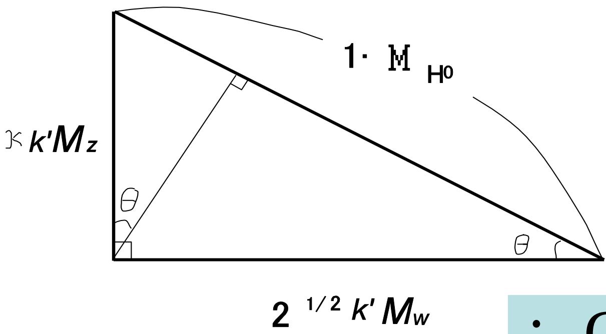
where $\varepsilon_\lambda \equiv \varepsilon$, infinitesimal Grassmann number

$$c_W \cdot \left(\sqrt{W_\mu{}^+ W^{-\mu}}\right)_0^2 \Big/ \left(2\varepsilon_\lambda / g\right)^2 \equiv m_W{}^2,$$

$$c_Z \cdot (\mathrm{Z}_\mu)_0{}^2 \Big/ \left(2\varepsilon_\lambda / G\right)^2 \equiv m_Z{}^2, \quad \text{where} \quad c_W,\ c_Z : \text{constant}$$

$$\therefore\ m_\varphi = \sqrt{2\left\{\left(\sqrt{W_\mu{}^+ W^{-\mu}}\right)_0^2 \Big/ \left(2\varepsilon_\lambda / g\right)^2\right\} + \left\{(\mathrm{Z}_\mu)_0{}^2 \Big/ \left(2\varepsilon_\lambda / G\right)^2\right\}}$$

$$= \sqrt{\frac{2m_W{}^2}{c_W} + \frac{m_Z{}^2}{c_Z}} = \sqrt{k'^2\left(2m_W{}^2 + \kappa^2 m_Z{}^2\right)}, \quad \text{where} \quad c_W \equiv \kappa^2 c_Z \equiv \frac{1}{k'^2}$$



$$\left(1\cdot \mathrm{M}_{\mathrm{H}^0}\right)^2 = \left(\sqrt{2}k'M_W\right)^2 + \left(\kappa k'M_Z\right)^2$$

$$2\mathrm{n}_W M_W;\ 2\mathrm{n}_Z M_Z$$

$$\mathrm{n}_W \mathrm{M}_{\mathrm{H}^0} + \mathrm{n}_Z \mathrm{M}_{\mathrm{H}^0} = \mathrm{N}\ \mathrm{M}_{\mathrm{H}^0}$$

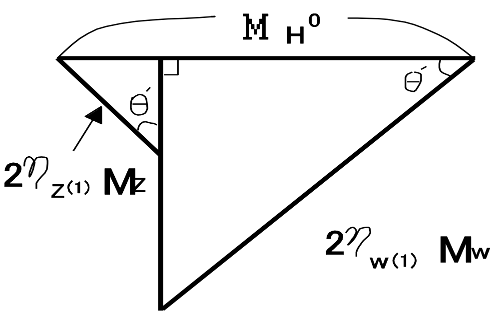
$\therefore$ Generally,

$$\mathrm{M}_{\mathrm{H}^0} = (2\eta_{W_{(1)}} M_W)\cos\theta' + (2\eta_{Z_{(1)}} M_Z)\sin\theta',$$

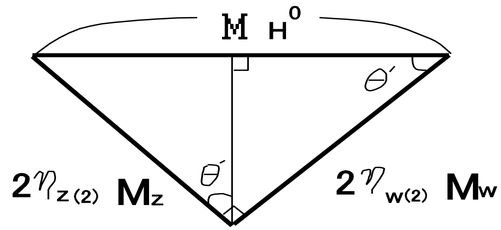
where $\eta_{W_{(1)}} \equiv \dfrac{\mathrm{n}_{W_{(1)}}}{\mathrm{N}}$, $\eta_{Z_{(1)}} \equiv \dfrac{\mathrm{n}_{Z_{(1)}}}{\mathrm{N}}$

where

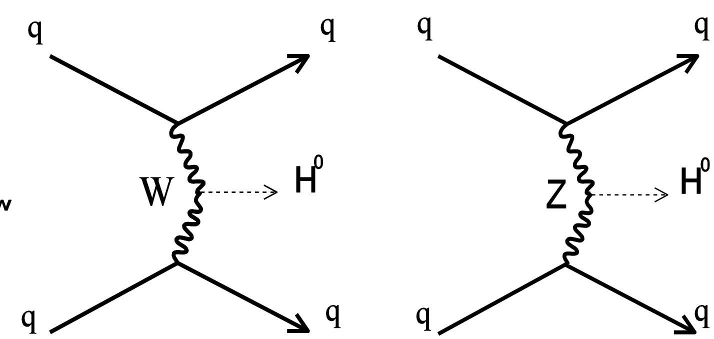
$$\gamma_W = \gamma_Z \equiv \gamma$$



(a)



(b)



$$2\eta_w \to \sqrt{2}k', \quad 2\eta_z \to \kappa k' \qquad \text{Thus,} \quad \eta_w + \eta_z \to \left(\frac{1}{\sqrt{2}} + \frac{\kappa}{2}\right)k'$$

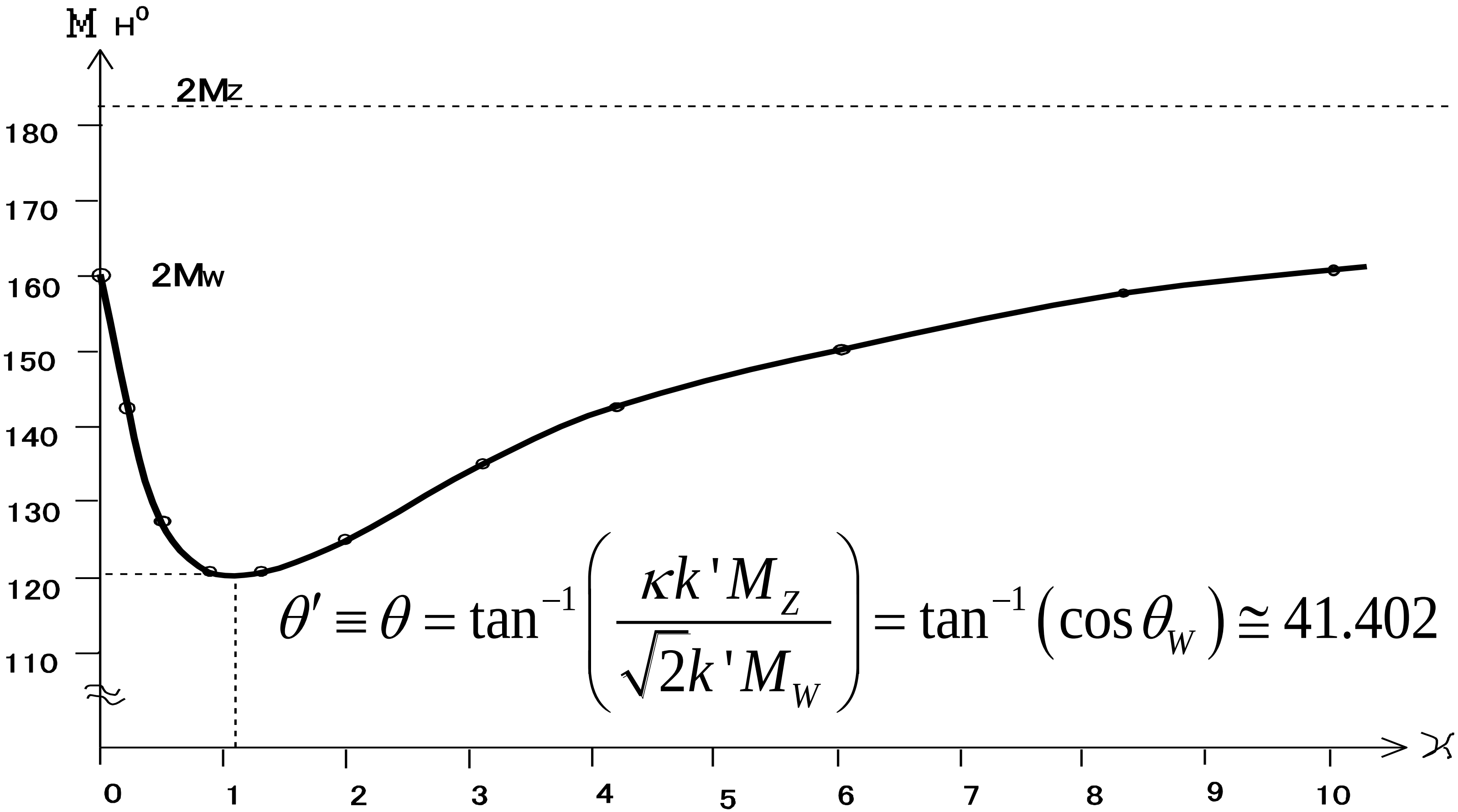
$$(\eta_w + \eta_z) \to 1 \qquad \text{Thus,} \quad k' \to \left(\frac{1}{\sqrt{2}} + \frac{\kappa}{2}\right)^{-1}$$

$$\mathrm{M}_{\mathrm{H}^0}(\kappa) = \sqrt{\left(\sqrt{2}\left(\frac{1}{\sqrt{2}} + \frac{\kappa}{2}\right)^{-1}\right)^2 M_W{}^2 + \left(\kappa\left(\frac{1}{\sqrt{2}} + \frac{\kappa}{2}\right)^{-1}\right)^2 M_Z{}^2}$$

$$\frac{d\mathrm{M}_{\mathrm{H}^0}(\kappa)}{d\kappa} \equiv 0, \quad \text{then} \quad \kappa = \sqrt{2}\left(\frac{M_W}{M_Z}\right)^2 = \sqrt{2}\cos^2\theta_W = 1.09934\cdots,$$

$$\mathrm{M}_{\mathrm{H}^0} = \frac{2M_W}{\sqrt{1+\cos^2\theta_W}} = \frac{2M_W M_Z}{\sqrt{M_W{}^2 + M_Z{}^2}}$$

$$M_{H^0} = 120.611^{+0.023}_{-0.022} \left[\mathrm{GeV}/c^2 \right]$$



$$\theta' \equiv \theta = \tan^{-1}\left(\frac{\kappa k' M_Z}{\sqrt{2} k' M_W}\right) = \tan^{-1}\left(\cos\theta_W\right) \cong 41.402 \; [\mathrm{deg}]$$

Extended EOM of Higgs scalar field (φ) from Euler-Lagrange equation

$$\lambda\varphi^3 + 3\lambda v\varphi^2 + \left[\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right) + \left\{ m_\varphi{}^2 - \frac{1}{2} g^2 W^+{}_\mu W^{-\mu} - \frac{1}{4} G^2 \left(Z_\mu\right)^2 \right\}\right]\varphi$$

$$- \left\{ g M_W W^+{}_\mu W^{-\mu} + \frac{1}{2} G M_Z \left(Z_\mu\right)^2 \right\}$$

$$+ \frac{1}{v}\left(m_{b_i}\bar{b}b + m_{c_i}\bar{c}c + m_{t_i}\bar{t}t\right) + \frac{1}{v}\left(m_{d_i}\bar{d}d + m_{u_i}\bar{u}u + m_{s_i}\bar{s}s\right)$$

$$= 0$$

Extended Higgs boson mass formula

$$m_{\varphi}^{2} = 2\left\{\left(\sqrt{W_{\mu}{}^{+}W^{-\mu}}\right)^{2} \Big/ \left(2\varepsilon_{\lambda}/g\right)^{2} - m_{q_{i(bsd)}}\left(\sqrt{\bar{q}_{bsd}q_{bsd}}\right)^{2} \Big/ \left(2\varepsilon_{\lambda}\Big/\sqrt{2\sqrt{2}G_{F}}\right)^{2}\right\}$$

$$+\left\{(Z_{\mu})^{2} \Big/ \left(2\varepsilon_{\lambda}/G\right)^{2} - 2m_{q_{i(cu)}}\left(\sqrt{\bar{q}_{cu}q_{cu}}\right)^{2} \Big/ \left(2\varepsilon_{\lambda}\Big/\sqrt{2\sqrt{2}G_{F}}\right)^{2}\right\}$$

$$-2m_{t_{i}}\left(\sqrt{\bar{t}t}\right)^{2} \Big/ \left(2\varepsilon_{\lambda}\Big/\sqrt{2\sqrt{2}G_{F}}\right)^{2},$$

where $G_{F} = 1\Big/\left(\sqrt{2}v^{2}\right)$

$$\varepsilon_{\lambda} \equiv \varepsilon;\ \ \varepsilon_{\lambda}{}^{2} = 0,$$

$$\bar{q}_{bsd}q_{bsd} \equiv \bar{b}b + \bar{s}s + \bar{d}d, \quad \bar{q}_{cu}q_{cu} \equiv \bar{c}c + \bar{u}u.$$

$$\varphi(\varepsilon_{\lambda}) = 0.$$

Top quark mass formula

We assume that the probability of m_t-decay process obeys a binominal distribution of beeing k-times in n-trials (-particles) with r_0 as decay-mode parameter.

$$
{}_nC_k \cdot r_0^{\,k}(1-r_0)^{n-k} m_t \equiv {}_nC_k \cdot r_0^{\,k-1}(1-r_0)^{n-k} m_{W(bsd)} + {}_nC_k \cdot r_0^{\,k}(1-r_0)^{n-k-1} m_{Z(cu)}
$$

$$
= {}_nC_k \cdot r_0^{\,k-1}(1-r_0)^{n-k-1}\left\{(1-r_0)m_{W(bsd)} + r_0 m_{Z(cu)}\right\},
$$

$$
\therefore \quad m_t = m_{W(bsd)}/r_0 + m_{Z(cu)}/(1-r_0)
$$

$$
0 < r_0 < 1
$$

Stationary mass value of top quark

$$\frac{dM_t}{dr_0} \equiv 0,$$

$$\therefore r_0 = \frac{-\sqrt{M_W{}^2 - M_b{}^2 - M_s{}^2 - M_d{}^2} + \left\{(M_W{}^2 - M_b{}^2 - M_s{}^2 - M_d{}^2)(M_Z{}^2 - M_c{}^2 - M_u{}^2)\right\}^{\frac{1}{4}}}{\sqrt{M_Z{}^2 - M_c{}^2 - M_u{}^2} - \sqrt{M_W{}^2 - M_b{}^2 - M_s{}^2 - M_d{}^2}}$$

$$M_t = (1/2)\left\{\left(M_W{}^2 - M_b{}^2 - M_s{}^2 - M_d{}^2\right)^{\frac{1}{4}} + \left(M_Z{}^2 - M_c{}^2 - M_u{}^2\right)^{\frac{1}{4}}\right\}^2$$

$$\simeq 171.26(6)\ \left[\mathrm{GeV/c^2}\right],\ \text{with}\ M_b = 4.68\ \mathrm{GeV/c^2}\ (\text{1S Mass}).$$

CDF / D0's experimental result : $171.3 \pm 1.1 \pm 1.2\ \mathrm{GeV/c^2}$

[C. Amsler *et al*., Physics LettersB667, 1 (2008); updated(2010)]

H^0 is expected to be a composite scalar meson

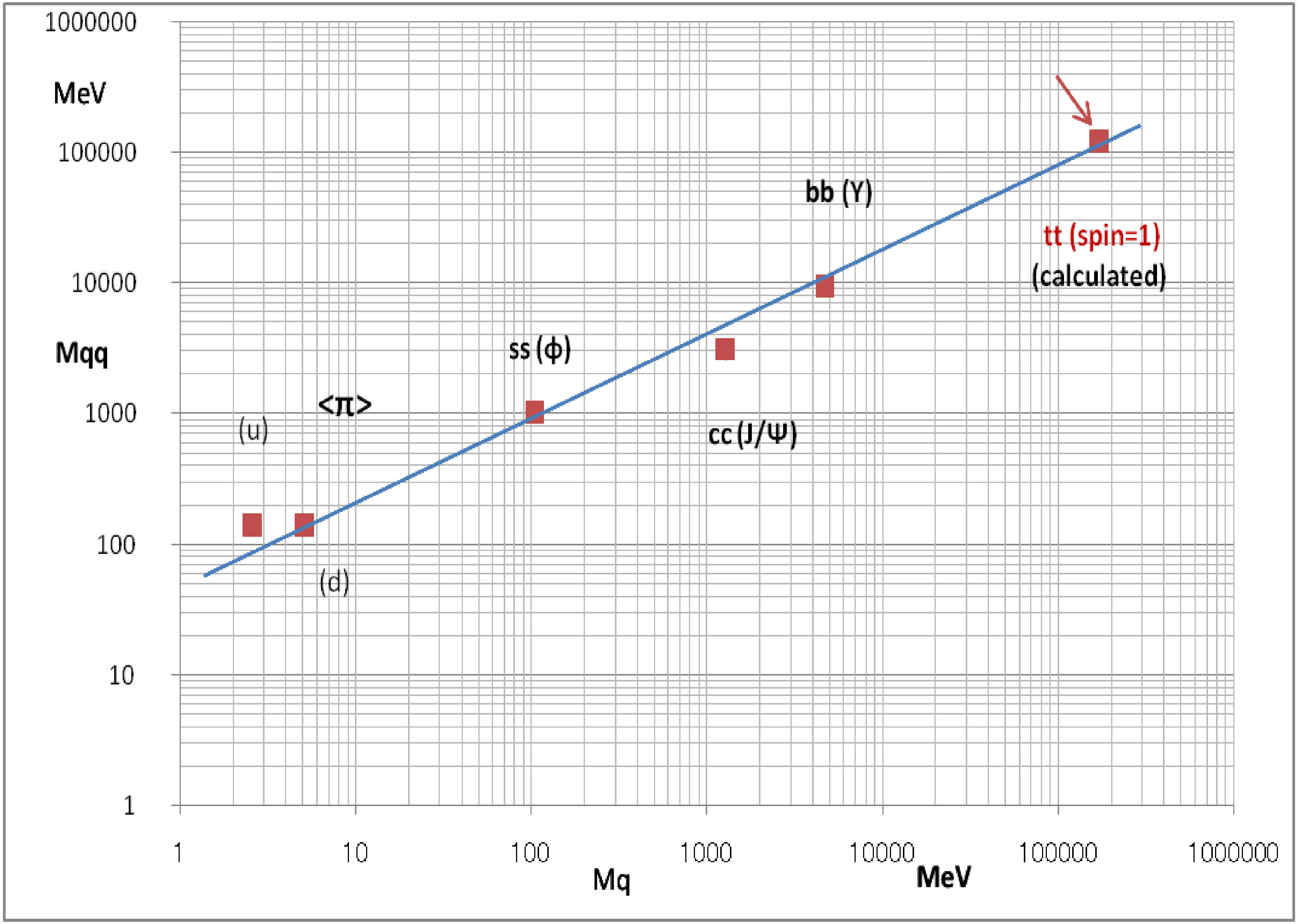
$$M_{(t\bar{t})^*} \equiv M_{H^0}{}' = M_t / \sqrt{2} = 121.10(3) \left[\mathrm{GeV/c^2}\right]$$

$$\therefore \ \Delta M \equiv M_{(t\bar{t})^*} - M_{H^0} = 0.49(2) \left[\mathrm{GeV/c^2}\right]$$

little smaller than masses of $K^{\pm,0}$ mesons, and is smaller than mass of η_0 meson.

$$\therefore \ (\mathrm{t\bar{t}})^* \rightarrow \gamma \mathrm{H}^0$$

radiative decay of 1-photon emission


1000000
MeV
100000
10000
Mqq
1000
100
10
1
1
10
100
1000
10000
100000
1000000
Mq
MeV
(u)
(d)
<π>
ss (ϕ)
cc (J/Ψ)
bb (Y)
tt (spin=1)
(calculated)

Basic structure of SM Higgs boson mass

(1) Constructed by heavy mesons' masses of all spin 0, such as

$$\left(B_S{}^0 \overline{B}_S{}^0\right)\left(B_C{}^+ B_C{}^-\right)\left(D_S{}^+ D_S{}^-\right)$$

(2) Then expected that they will form a polyhedron composed of planes of hexagon, in space. And, *'effective'* number of planes of hexagon should be

$$n_{eff.} \equiv Int\left\{\frac{M_{\mathrm{H}^0(theory)}}{M_{\left(B_S{}^0 \overline{B}_S{}^0\right)\left(B_C{}^+ B_C{}^-\right)\left(D_S{}^+ D_S{}^-\right)}}\right\} = 4.$$

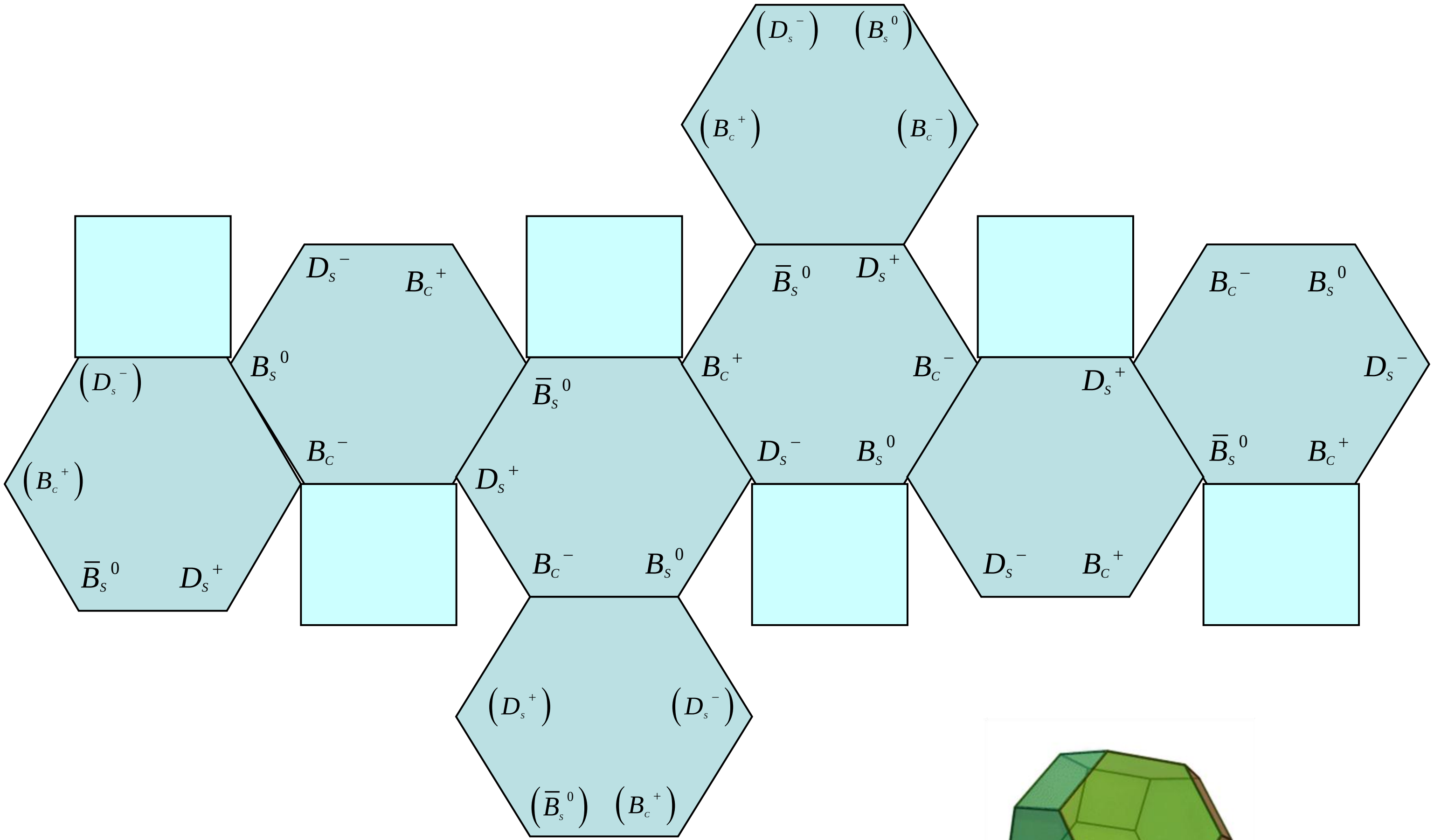
Comparison of SM Higgs boson mass values

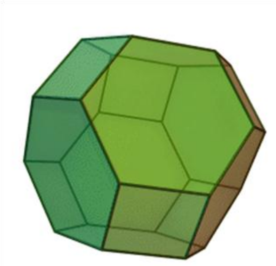
$$M_{\mathrm{H}^0}(3\eta_{c,\ldots(\text{exper. values})}) \equiv \sum_{M_i}\left[3\eta_c,\ 10\pi^+\pi^- + 4\left\{\left(B_S{}^0\overline{B}_S{}^0\right)\left(B_C{}^+B_C{}^-\right)\left(D_S{}^+D_S{}^-\right)\right\}\right]$$

$$= 120.612\ \mathrm{GeV/c^2},$$

where, $M_{\mathrm{H}^0(\text{theor. with } \mathrm{M_W}, \mathrm{M_Z})} = 120.611\ \mathrm{GeV/c^2}$

Outer shell from mass of SM Higgs boson



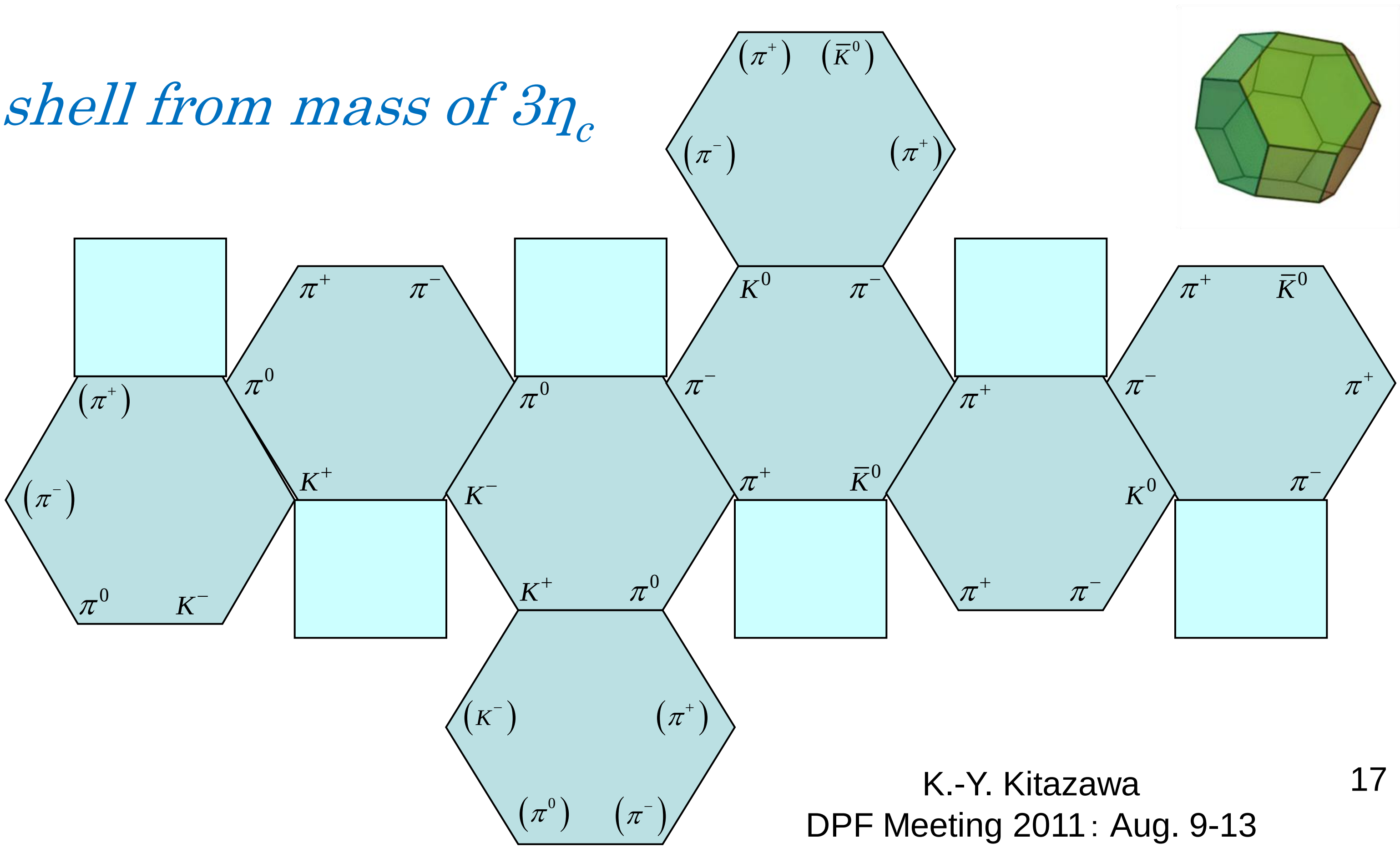


Comparison of Central meson's mass values

$$M_{3\eta_c\,(\text{theor.})} \equiv \sum_{M_i} \left[3\eta_0,\ 4\pi^+\pi^- + 2\left\{ \begin{matrix} \left(K^+K^-\right)\left(\pi^0\pi^0\right)\left(\pi^+\pi^-\right) \\ +\left(K^0\overline{K}^0\right)\left(\pi^+\pi^-\right)\left(\pi^+\pi^-\right) \end{matrix} \right\} \right] = 8940.0\ \text{MeV/c}^2,$$

$$\text{where,}\quad M_{3\eta_c\,(\text{exper.})} = 3\times\left(2980.3\pm1.2\right) = 8940.9\pm3.6\ \text{MeV/c}^2$$

Inner shell from mass of $3\eta_c$



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DPF Meeting 2011 : Aug. 9-13

== Structure of ur-SM Higgs Boson ==

-A Representation of Certain f_0 Mesons' Masses.

BETHE-SALPETER EQUATION WITH GOLDSTEIN APPROXIMATION

The general form of B-S:

$$K_B \; \phi_{Br}\left(p, P_B\right) = I_B \; \phi_{Br}\left(p', P_B\right),$$

where $K_B \equiv \left[\Delta'_{Fa}\left(\eta_a P_B + p\right)\Delta'_{Fb}\left(\eta_b P_B - p\right)\right]^{-1},$

$$I_B \equiv \int d^4 p' \, I\left(p, p'; P_B\right)$$

$\phi_{Br}\left(p, P_B\right)$: BS amplitude

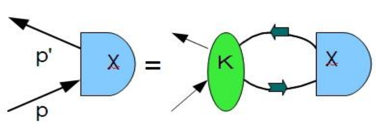
$\Delta'_{Fa}, \Delta'_{Fb}$: modified Feynman propagators

$I\left(p, p'; P_B\right)$: irreversible part of the process.

The B-S for fermion (t)-antifermion (t_bar) bound state with total four momentum

$$\left[S^{-1}\left(q+\frac{1}{2}P\right)\chi(q,P)S^{-1}\left(q-\frac{1}{2}P\right)\right]_{\alpha\beta} = \int \frac{d^4q'}{(2\pi)^4} K_{\alpha\beta;\alpha'\beta'}(q,q';P)\,\chi_{\alpha'\beta'}(q',P),$$

$$\text{where}\quad \chi_{\alpha\beta}(q,P) = \int d^4x \left\langle 0 \left| T\left[\psi_\alpha\left(\frac{1}{2}x\right)\bar{\psi}_\beta\left(-\frac{1}{2}x\right)\right]\right| P_\mu \right\rangle.$$



$$p' = q + \frac{1}{2}P,\quad p = q - \frac{1}{2}P$$

where q : relative momentum,

P : total momentum of bound state

S : fermion propagator

$\chi_{\alpha\beta}$: BS-amplitude of spinor

T : operator of time ordered

The Goldstein equation for abelian vector gluon model*, with the ladder approximation after putting $P_\mu=0$ ***and***

$$\chi_{\alpha\beta}(q,0) \equiv (\gamma_5)_{\alpha\beta} F(q)$$

$$\left(m^2-q^2\right)F(q)=\frac{\lambda}{4\pi^2 i}\int\frac{d^4q'}{-\left(q-q'\right)^2-i\varepsilon}F(q').$$

After the Wick rotation and then the Fourier transform,

$$f(r)=(mr)^{-1}K_\nu(mr),$$

$$\nu\equiv\sqrt{1-\lambda},\ \ \text{where}\ \ 0<\lambda\le 1.$$

$$\lambda\equiv(\xi+3)e^2/4\pi^2=(\xi+3)\alpha/\pi,\quad \xi:\text{gauge parameter}$$

*** K. Higashijima and A. Nishimura, Nucl. Phys., B113, 173 (1976)**

The required form of (massive) gluon propagator:

$$D(r)=\frac{\mathrm{m}^2}{\pi^2}\left\{\frac{1}{(\hat{m}r)}K_0(\hat{m}r)\right\}\ \left[\mathrm{GeV/c}^2\right]^2,$$

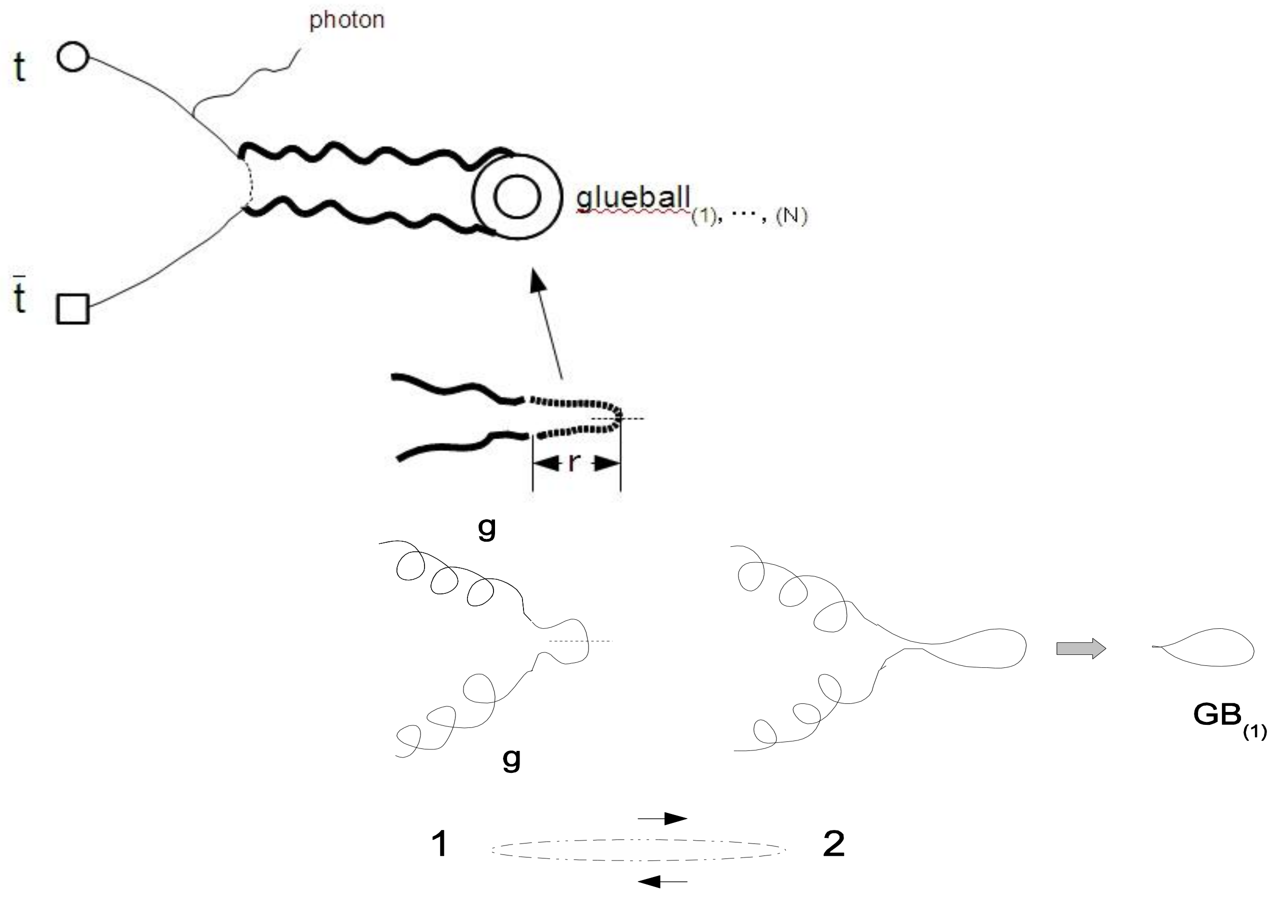
$$\text{where}\ \ [\mathrm{m}]=\mathrm{GeV/c}^2,\ [r]=\mathrm{fm},\ \hat{m}=1[\mathrm{fm}]^{-1}.$$

$\therefore$ Compton wavelength of glueball in ground state:

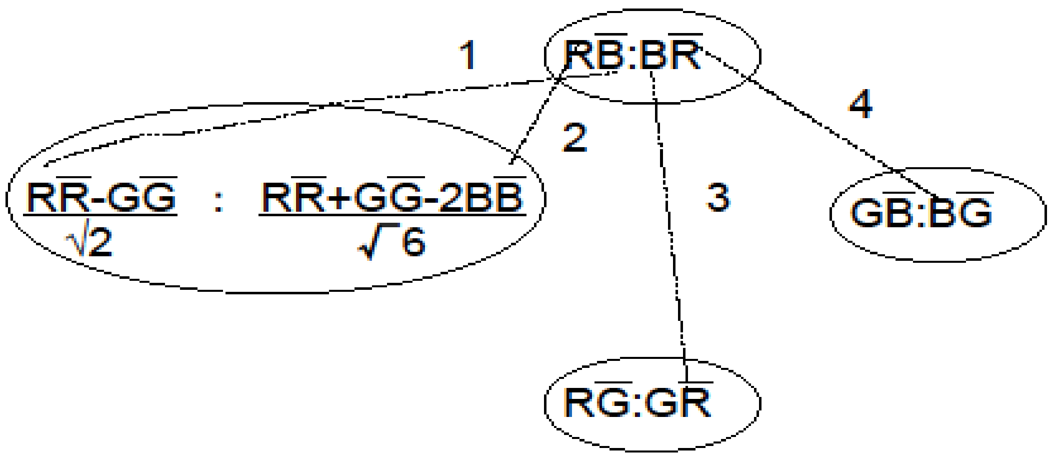
$$\lambda_c\equiv\frac{\hbar}{\mathrm{m}_{GB}c}=0.393[\mathrm{fm}],\ \text{provided}\ \ \mathrm{m}_{GB}\equiv 502.55\cong 2\mathrm{m}\left[\mathrm{MeV/c}^2\right].$$

(Thus we may put also as $2r\approx\lambda_c$.)

After all, virtual bound tt_bar decay is expected to have at last the glueball$_{(1),(2),\ldots,(N)}$ producing process:



‘Color valence’ of glueball



Color valence of glueball =4

Clustering force between glueballs = Color valence

In lattice QCD it is now believed that there might be several scalar mesons of $f_0(1370)$, $f_0(1500)$, $f_0(1710)$ all of which are supposed to have some contents of glueball (GB) of grand state.

Then we can expect similar structure of the carbon fullerenes for these scalar mesons.

Glueball (GB) Cluster as ur-SM Higgs Boson

$$M_{(f_o)i} \equiv M_{\text{ur-H}^0} / N_i = M_{\text{GB}} / \eta_i \,,$$

$$\sum_{i=1}^{3} \eta_i = 1 \,, \quad \therefore N_{GB} = \sum_{i=1}^{3} N_i \,.$$

By putting each fullerene number for these mesons as

$N_1 = 90,\ N_2 = 80,\ N_3 = 70$ **with** $\boldsymbol{M_{\text{ur-H}^0} = 120.611\,\text{GeV}/c^2}$,

$M_{GB} \approx 502.55\,\text{MeV}/c^2$, as an element of GB_{240}-fulleren e;

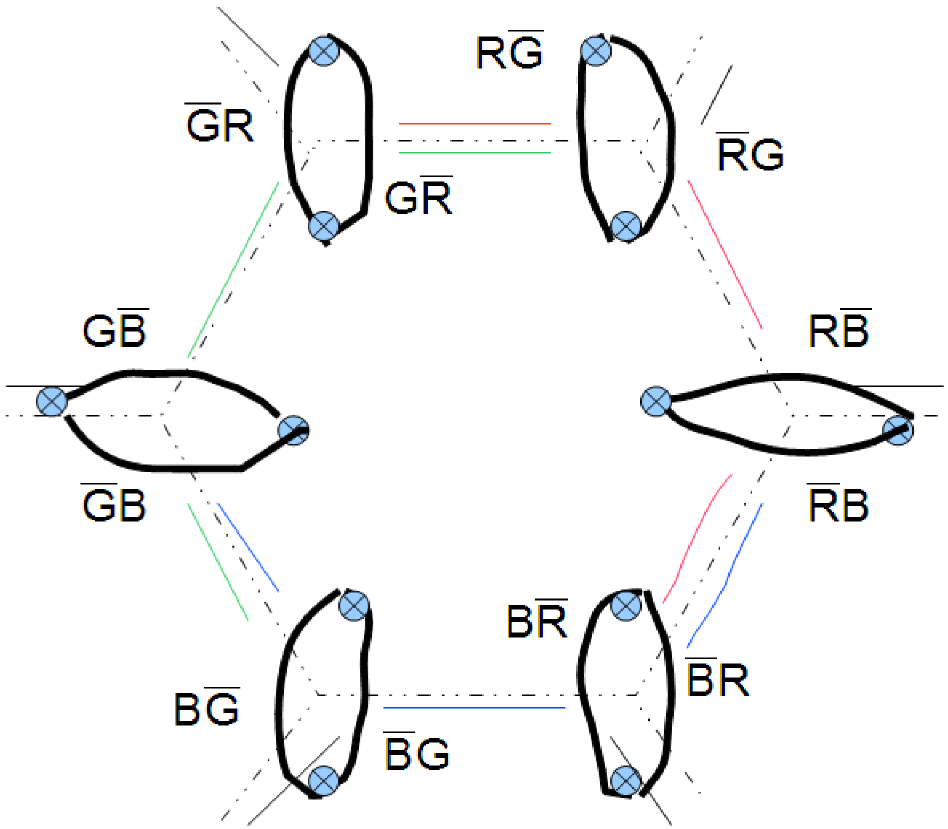
$\eta_1 \approx 0.292,\ \eta_2 \approx 0.333,\ \eta_3 \approx 0.375.$

The $f_0(1500)$ will have a glueball for each element of GB_{80} since $0.333 \times 3 = 1$.

Comparison to Experimental Mass Values of Scalar Meson

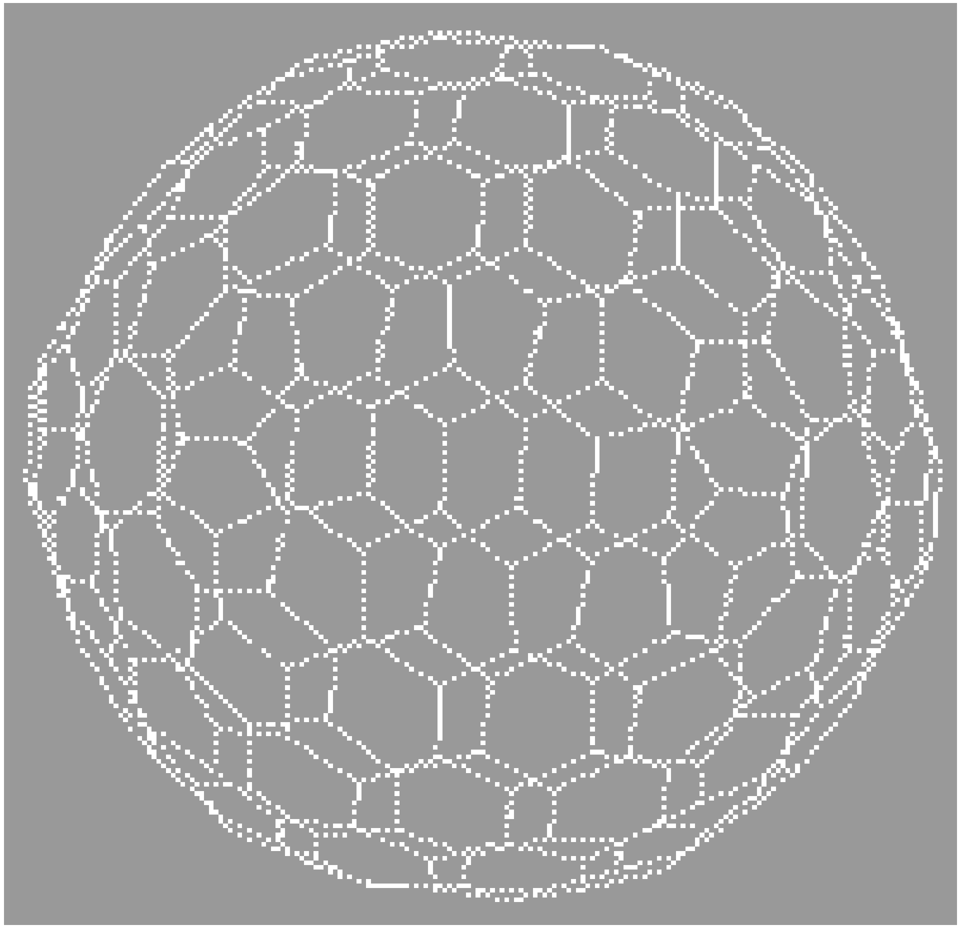
	Our Calculation	Experiment (PDG_2010)
$f_0(600) \Leftrightarrow GB$	502.55	$400 - 1200$
$f_0(1370)$	1340.1	$1200 - 1500$
$f_0(1500)$	1507.6	1505^{+6}_{-6}
$f_0(1710)$	1723.0	1720^{+6}_{-6}

A Hexagon on ur-H^0 (GB_{240})



Fullerene structure of ur-SM Higgs Boson:

$GB_{240} \Leftrightarrow$ f0(600) × 240 ≡ I_h symmetry



110-Hexagons
12-Pentagons

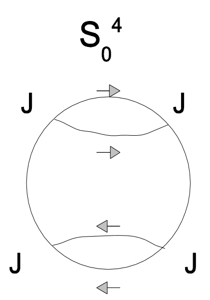
by Euler's theorem for polyhedron

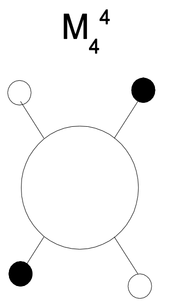
A representation of f_0 masses with π octet (light pseudoscalar meson) and GB mass

$$m_{f_0(1370)} = \left[\begin{array}{l} \mathrm{GB} + \left(\frac{3}{90}\right)\eta_0 + \left(\frac{70}{90}\right)K^0 + \\ \left(\frac{2}{3}\times\frac{70}{90}\right)K^{\pm} + \left(\frac{75}{90}\right)\pi^{\pm} + \left(\frac{40}{90}\right)\pi^0 \end{array} \right]_{m_i},$$

$$m_{f_0(1500)} = \left[3\mathrm{GB}\right]_{m_i},$$

$$m_{f_0(1710)} = \left[\mathrm{GB} + K^0 + \left(\frac{1}{3}\right)K^{\pm} + 4\pi^{\pm}\right]_{m_i}.$$





Transformation (decay) of ur-H^0 into H^0

Remind that

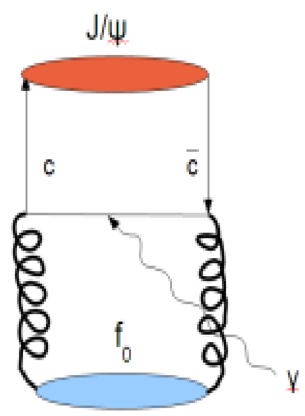
$(t\bar{t})^* \Rightarrow \underline{\underline{\gamma}} H^0$, where H^0 condensates into GB fullerene (ur-H^0), and

$$M_{H^0}(3\eta_c) \equiv \sum_{M_i} \left[3\eta_c,\ 10\pi^+\pi^- + 4\left\{ \left(B_S{}^0 \overline{B}_S{}^0 \right) \left(B_C{}^+ B_C{}^- \right) \left(D_S{}^+ D_S{}^- \right) \right\} \right].$$

1) $3\eta_c$:

$$3\{\gamma\eta_c(1S)\} \leftarrow 3\mathrm{J}/\Psi \leftarrow 3\{\gamma 6(\mathrm{f}_0(600))\},$$

$$\therefore\ 3\eta_c(1S) \leftarrow 18\mathrm{f}_0(600) \Leftrightarrow 6\mathrm{f}_0(1500).$$



2) $10\pi^+\pi^-$:

$$5\pi^+\pi^- \leftarrow \mathrm{f}_0(1500) \Leftrightarrow 3\mathrm{f}_0(600),$$

$$\therefore\ 10\pi^+\pi^- \leftarrow 6\mathrm{f}_0(600) \Leftrightarrow 2\mathrm{f}_0(1500).$$

3) $4\left\{\left(B_S{}^0\overline{B}_S{}^0\right)\left(B_C{}^+B_C{}^-\right)\left(D_S{}^+D_S{}^-\right)\right\}:$

$$\left(B_s{}^0\overline{B}_s{}^0\right)\leftarrow 22\mathrm{f}_0(600),\quad \left(B_c{}^+B_c{}^-\right)\leftarrow 24\mathrm{f}_0(600),$$

$$\left(D_s{}^+D_s{}^-\right)\leftarrow 8\mathrm{f}_0(600),$$

$$\therefore\ 4\left\{\left(B_S{}^0\overline{B}_S{}^0\right)\left(B_C{}^+B_C{}^-\right)\left(D_S{}^+D_S{}^-\right)\right\}\leftarrow 216\mathrm{f}_0(600)\Leftrightarrow 72\mathrm{f}_0(1500).$$

After all, we have the transformation under mass invariance that

$$\mathrm{H}^0\leftarrow 240\mathrm{f}_0(600)\Leftrightarrow 80\mathrm{f}_0(1500)\equiv \text{ ur-H}^0.$$

$$\Updownarrow$$

240GB

Fullerenes of Icosahedral (I_h) Symmetry

As far as carbon fullerenes, C_{20}, C_{60}, C_{80}, C_{180}, C_{240} have a common point group: I_h which is of the icosahedral symmetry. Thus we expect that GB_{80} (f0(1500)) and GB_{240} (ur-H^0) also have I_h.

Therefore, inversely, we could expect that the f0 meson which has a fullerene structure of I_h symmetry, it may consist of pure GB.