A New Algorithm for Estimating Quark Propagation in Lattice QCD

Keisuke Jimmy Juge
University of the Pacific

John Bulava (DESY → CERN)
Justin Foley (U. of Utah)
You-Cyuan Jhang (CMU)
David Lenkner (CMU)
Colin Morningstar (CMU)
Michael Peardon (TCD)
Chik-Him Wong (CMU → UCSD)

Outline

- Introduction
- Hadron Spectroscopy
  - Lattice QCD
- Stochastic LapH Algorithm
  - in a nut-shell
- Some Results
- Future/Summary

Introduction

- Goal:

  To deterimine the low-lying hadron spectrum from first principals Lattice QCD calculations

- Require:

  - large, dynamical lattices with "light" quarks (in this study: Mpi > 240 MeV , $V < (5\ \mathrm{fm})^3$)
  - inclusion of the multi-particle spectrum (we can no longer ignore the contributions from disconnected diagrams)

- Focus: multi-particle spectrum and disconnected diagrams in addition to the single-particle spectrum obtained in the past couple of years

Introduction

- Examples of the single-particle spectrum obtained in the past couple of years: (no longer quenched)

J. Bulava et al.	Phys.Rev.D79, 2009
C. Gattringer et al.	Phys.Rev.D79, 2009
J. Dudek et al.	Phys.Rev.Lett.103, 2009
C. Alexandrou et al.	Phys.Rev.D80, 2009
S. Aoki et al.	Phys.Rev.D81, 2010
J. Dudek et al.	Phys.Rev.D82, 2010
J. Bulava et al.	Phys.Rev.D82, 2010

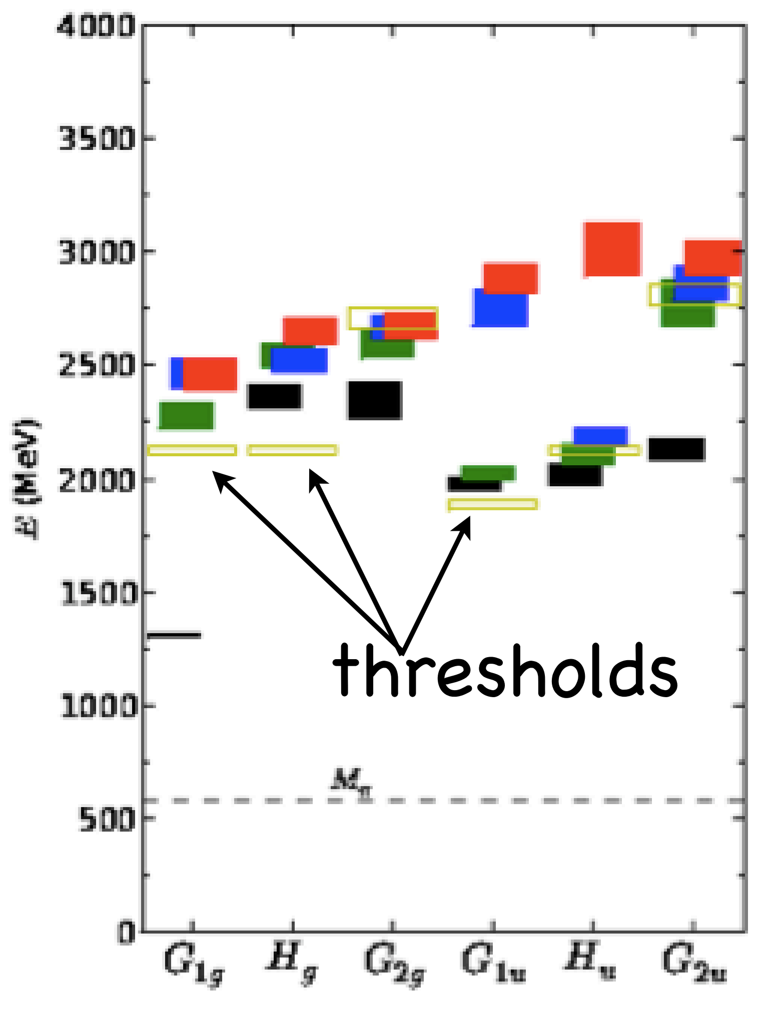
and more ...

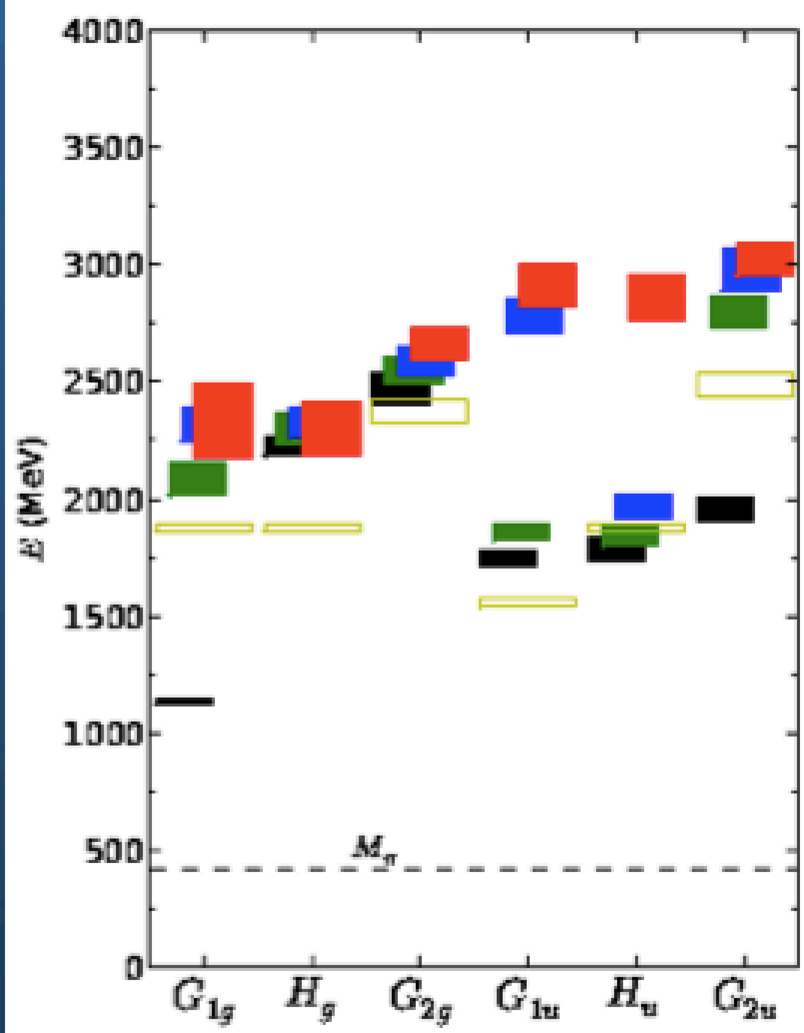
- Why not multi-particle states ?

  simple answer: computationally expensive

Nucleon Spectrum

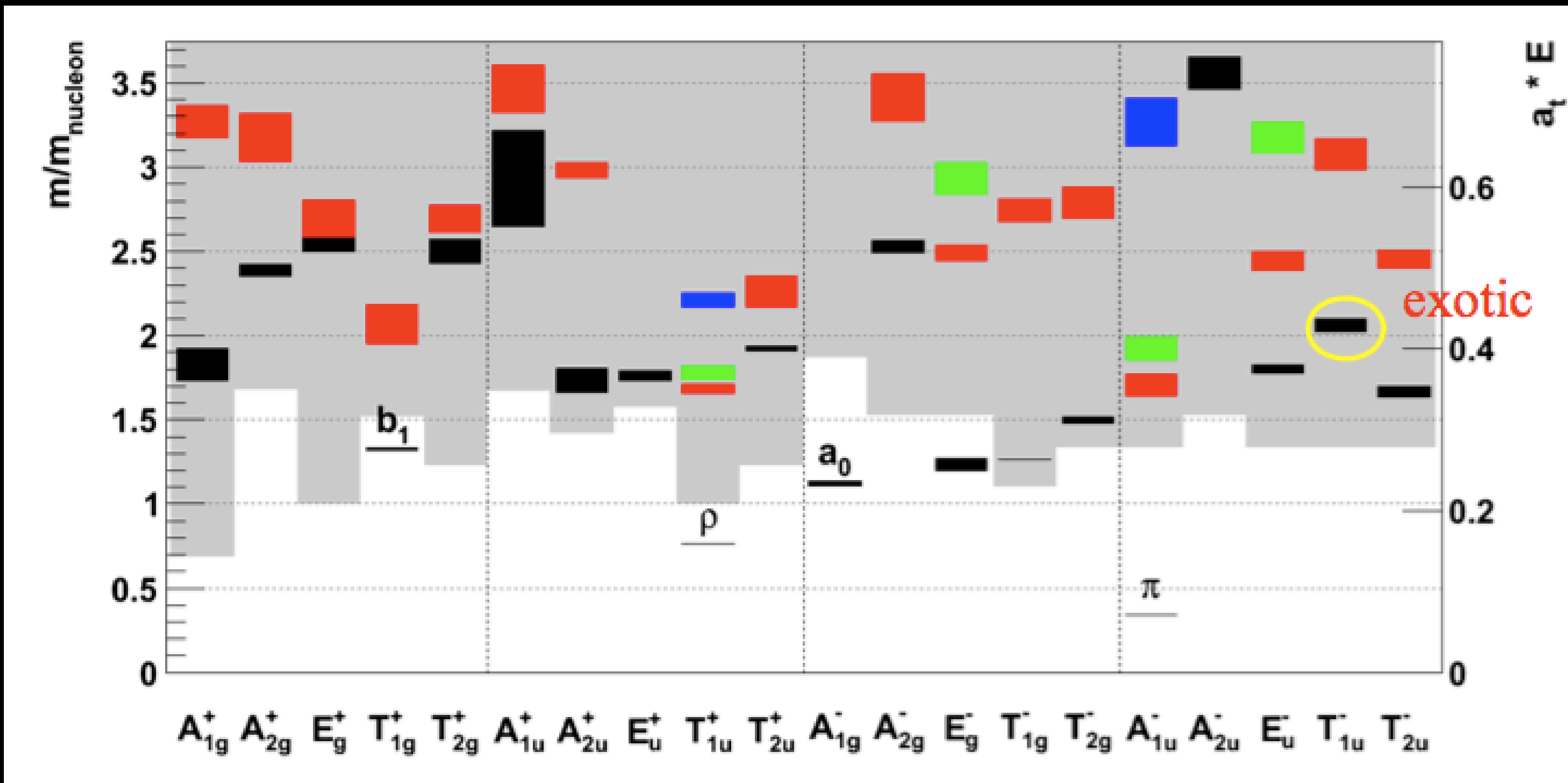
J. Bulava etal Mpi=415 MeV $(2.5\mathrm{fm})^3$
PRD79 2009





Isovector Mesons

24^3x128 170 cfgs Mpi=390MeV



Lattice Hadron Spectroscopy

Euclidean Correlation Functions

$$C(t,t_0) = \langle \mathcal{O}_\pi(t)\mathcal{O}_\pi^\dagger(t_0)\rangle \rightarrow A_0 e^{-M_\pi(t-t_0)} + A_1 e^{-E^{(1)}(t-t_0)} + \cdots$$

$$C(t,t_0) = \langle \qquad \sum_{\vec{x}_0} \bar{\psi}(\vec{x}_0,t_0)\gamma_5\psi(\vec{x}_0,t_0)\rangle$$

Lattice Hadron Spectroscopy

Euclidean Correlation Functions

$$C(t,t_0) = \langle \mathcal{O}_\pi(t)\mathcal{O}_\pi^\dagger(t_0)\rangle \rightarrow A_0 e^{-M_\pi(t-t_0)} + A_1 e^{-E^{(1)}(t-t_0)} + \cdots$$

$$C(t,t_0) = \langle \qquad \sum_{\vec{x}_0} \bar{\psi}(\vec{x}_0,t_0)\gamma_5\psi(\vec{x}_0,t_0)\rangle$$

$$C(t,t_0) = \langle \sum_{\vec{x}} \bar{\psi}(\vec{x},t)\gamma_5\psi(\vec{x},t) \sum_{\vec{x}_0} \bar{\psi}(\vec{x}_0,t_0)\gamma_5\psi(\vec{x}_0,t_0)\rangle$$

Lattice Hadron Spectroscopy

Euclidean Correlation Functions

$$C(t,t_0) = \langle \mathcal{O}_\pi(t)\mathcal{O}_\pi^\dagger(t_0)\rangle \rightarrow A_0 e^{-M_\pi(t-t_0)} + A_1 e^{-E^{(1)}(t-t_0)} + \cdots$$

$$C(t,t_0) = \langle \qquad \sum_{\vec{x}_0} \bar{\psi}(\vec{x}_0,t_0)\gamma_5\psi(\vec{x}_0,t_0)\rangle$$

$$C(t,t_0) = \langle \sum_{\vec{x}} \bar{\psi}(\vec{x},t)\gamma_5\psi(\vec{x},t) \sum_{\vec{x}_0} \bar{\psi}(\vec{x}_0,t_0)\gamma_5\psi(\vec{x}_0,t_0)\rangle$$

$$C(t,t_0) \simeq \frac{1}{N_{\rm cfg}} \sum \mathrm{Tr}\left[\gamma_5 M^{-1}(\vec{x},t;\vec{x}_0,t_0)\gamma_5 M^{-1}(\vec{x}_0,t_0;\vec{x},t)\right]$$

Quark Propagator $M^{-1}(\vec{x},t;\vec{x}_0,t_0)$

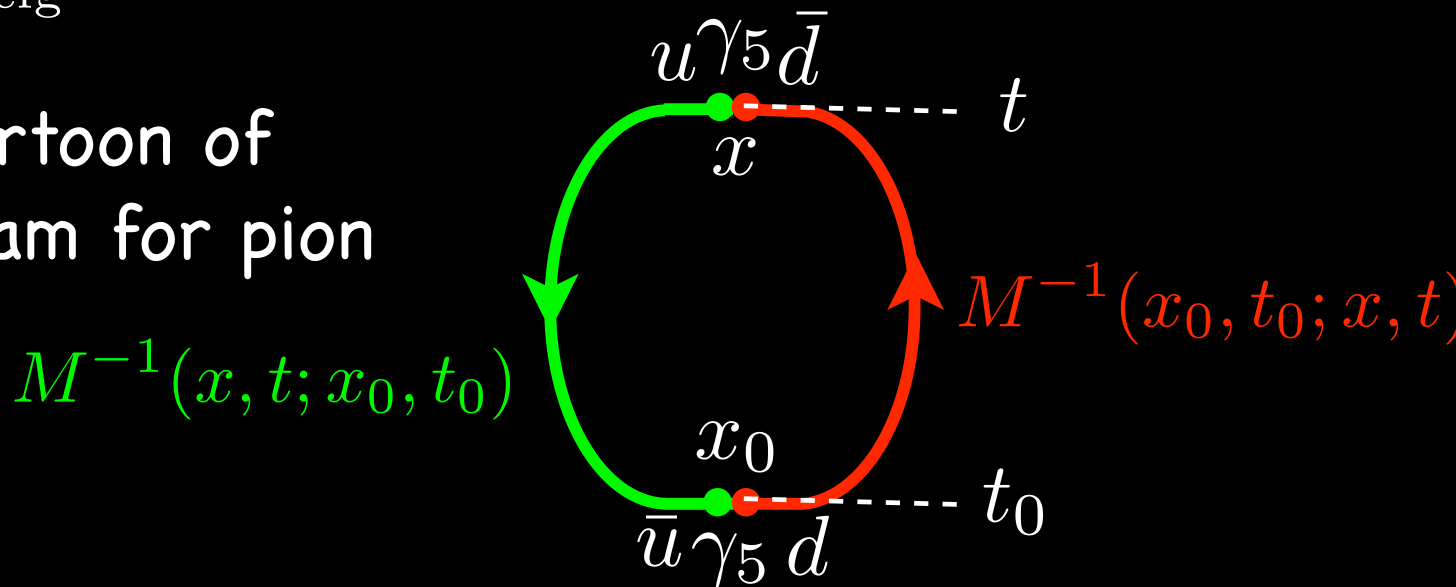
Expensive!!!

from matrix inversion $M\psi(\vec{x},t) = \rho(\vec{x}_0,t_0)$

Lattice Hadron Spectroscopy

$$C(t, t_0) \simeq \frac{1}{N_{\rm cfg}} \sum \mathrm{Tr}\left[\gamma_5 M^{-1}(\vec{x}, t; \vec{x}_0, t_0) \gamma_5 M^{-1}(\vec{x}_0, t_0; \vec{x}, t)\right]$$

cartoon of diagram for pion

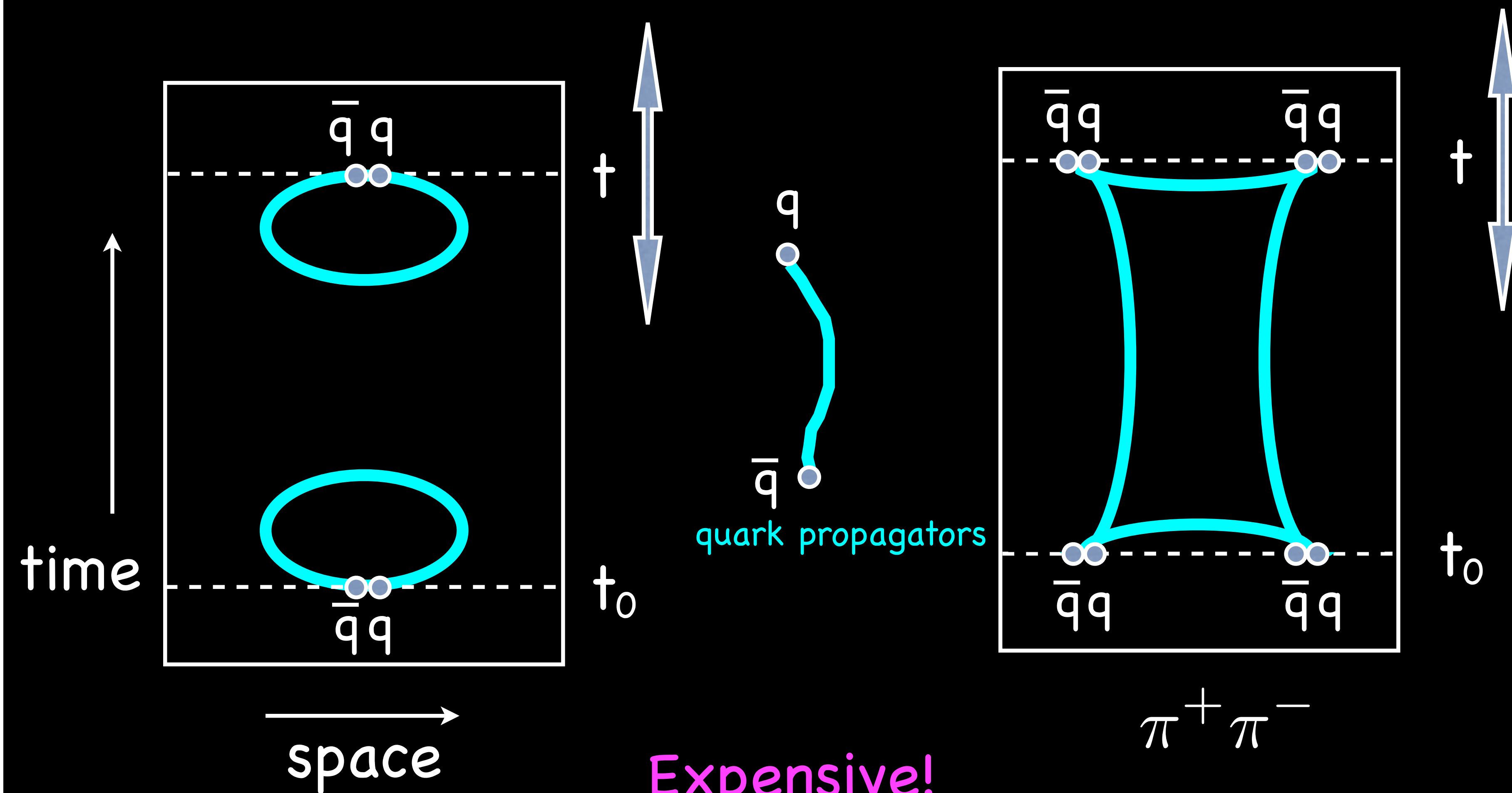


$$M^{-1}(x_0, t_0; x, t) = \gamma_5 M^{-1}(x, t; x_0, t_0)^\dagger \gamma_5$$

Masses can be extracted from the large time behaviour of the correlation function $C(t,t_0)$ by inverting from a single site (x_0,t_0)

The difficulty with simulating multi-particle states and isosinglet states :

In a nutshell … "t-to-t" diagrams



Each quark propagator requires the inversion of a N^2 matrix where $N=(3 \times 4 \times V_s \times L_t)$

One must do this for all spatial sites on the lattice (all-to-all quark propagators)

Example: $V=24^3$ $T=128$

Total Number of inversions = N = 21,233,664

Each inversion of the 21,233,664 x 21,233,664 matrix becomes expensive as the quarks get lighter

$\longleftrightarrow$ Typical calculation: $\underset{N_c}{3} \times \underset{N_\gamma}{4} \times \underset{N_{timeslices}}{10} = 120$ inversions

$\Rightarrow$ Multi-hadron simulations and disconnected diagrams are difficult to evaluate

Stochastic LapH Algorithm

(C. Morningstar et al. PRD 2011)

Exact evaluation of the all-to-all propagators is unnecessary because we are making a Monte-Carlo estimates of the low energy correlation functions which have "gauge noise"

Stochastic LapH Algorithm

(C. Morningstar et al. PRD 2011)

Exact evaluation of the all-to-all propagators is unnecessary because we are making a Monte-Carlo estimates of the low energy correlation functions which have "gauge noise"

1. We are interested in the low-momentum modes of the fields (M. Peardon etal. PRD 2009)

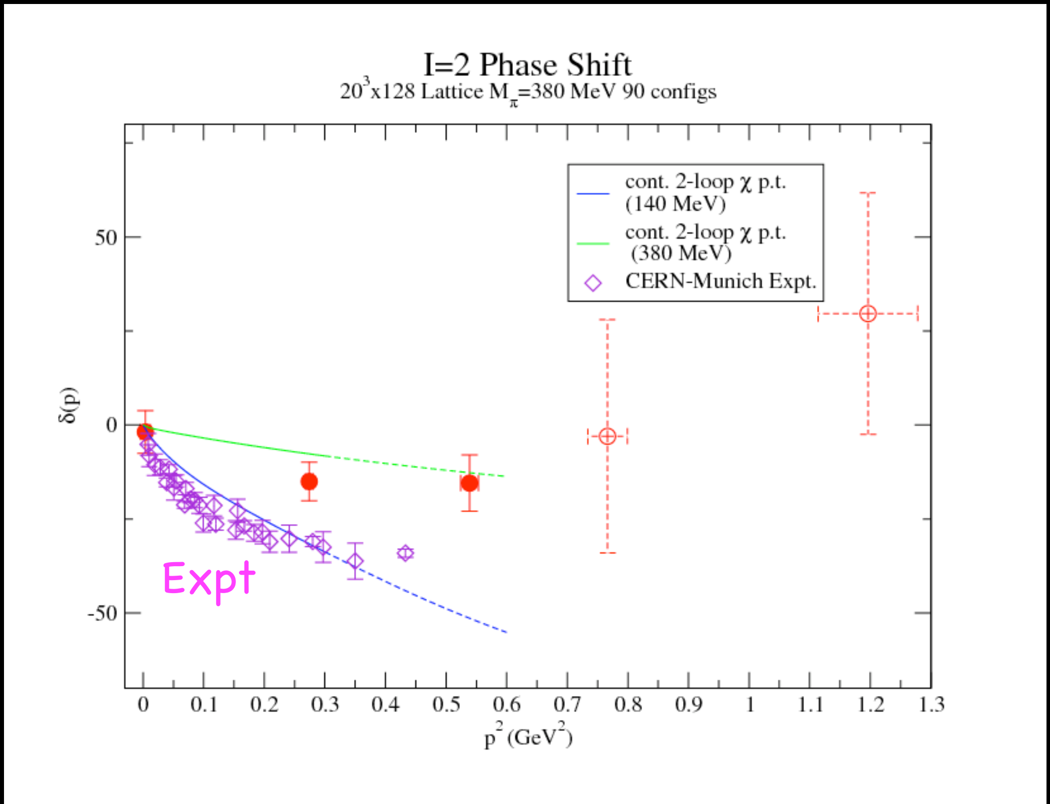
- "smeared fields" $e^{-\vec{p}^2/\Lambda^2}\psi \rightarrow \left(1 + \frac{\sigma}{n}\vec{\nabla}^2\right)^n \psi$
- Laplacian operator (eigenvector rep) $-\vec{\nabla}^2 = \sum_i^{N_{ev}} \frac{1}{\lambda_i} v^{(i)} v^{(i)\dagger}$
- momentum cutoff -> Heaviside step function

Distillation (LapH)

- Single-timeslice to all quark propagators
- Can give mesons/baryons momentum

If one only has connected diagrams to evaluate, then this can give phaseshifts, threshold params ...

(J. Dudek et al., S. Prelovsek et al., D. Richards et al.)



$(2.5\ \mathrm{fm})^3$

Mpi=390 MeV

90 configs

J. Bulava et al. (2010)

2. V scaling problem ...

low modes needed increases with V,

but we need to make the volume bigger ...

3. Can't do disconnected diagrams unless one computes the perambulators from all timeslices

   a) disconnected diagrams typically have larger gauge noise

   b) anisotropic dynamical lattices

      fine temporal lattice spacing ... Nt=256

   ⟶ exact evaluation can be wasteful

4. Stochastic method tends to be noisy

   - need many noise sources to average over

$$\frac{1}{\sqrt{N}}$$

Stochastic LapH Algorithm

1. Solve for Nev lowest lying eigenvectors of the Laplacian on each timeslice $v^{(\alpha)}(\vec{x},t)$

$$-\nabla^2 v^{(\alpha)}(\vec{x},t) = \lambda_\alpha v^{(\alpha)}(\vec{x},t) \quad \alpha = 1 \dots N_{ev}$$

2. Solve for the solution of the Dirac equation using each eigenvector as the source $\varphi^{(\alpha)}(\vec{x},t)$

$$(\not{D} + m)\,\varphi^{(\alpha)}(\vec{x},t) = v^{(\alpha)}(\vec{x},t)$$

3. Introduce a Z4 noise field in the LapH subspace

$$\varrho^{[\alpha k \tau]}(\vec{x},t)$$

α eigenvector index

k spin index

τ timeslice index

4. Dilute the noise according to some scheme

- eigenvector indices (full: $\alpha = 1 \ldots N_{ev}$)
- spin indices (full: $k = 1 \ldots 4$)
- time indices (full: $\tau = 0 \ldots (N_t - 1)$)

5. Solve the Dirac equation with $\varrho^{[\alpha k \tau]}(\vec{x}_0, t_0)$ as the source

$$(\not{D} + m)\,\varphi^{[i]}(\vec{x}, t) = \varrho^{[i]}(\vec{x}_0, t_0)$$

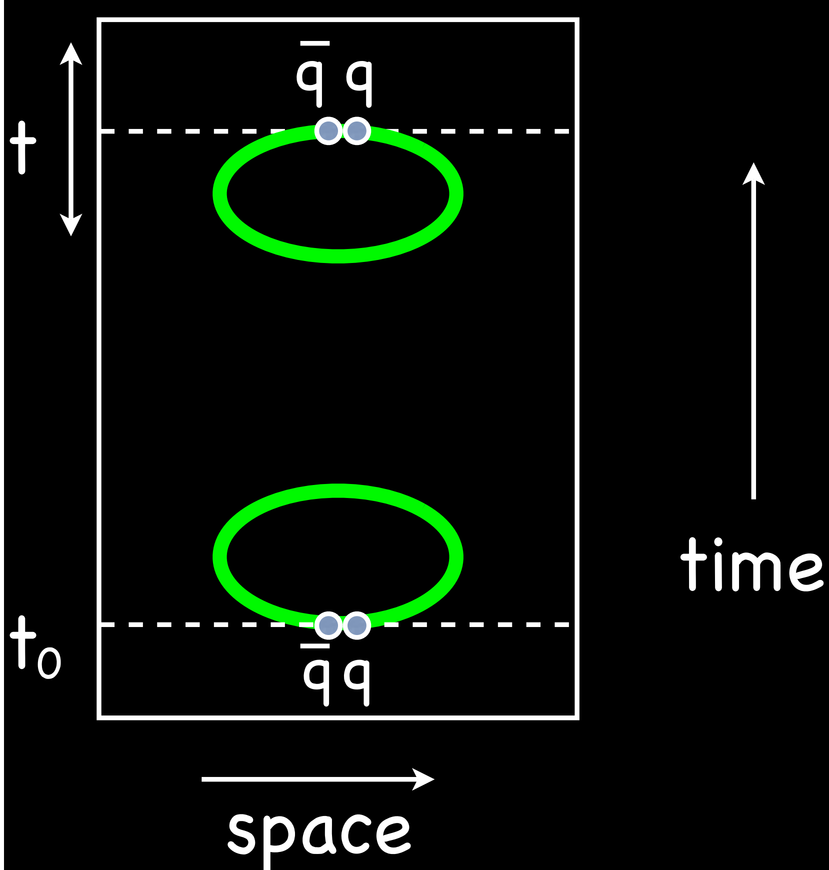
6. Quark propagator:

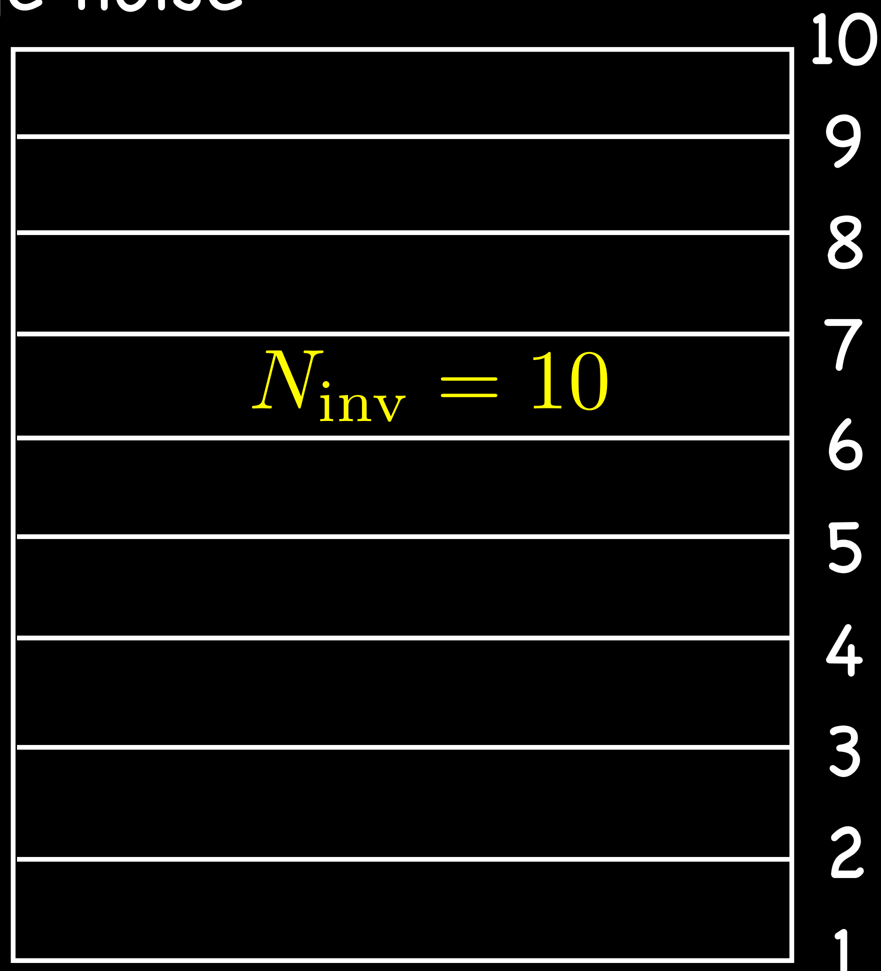
$$\ldots.\sum_i \varphi^{[i]}(\vec{x}, t) \otimes \varrho^{[i]}(\vec{x}_0, t_0)$$

But now, t_0 can be anytime ... “all-to-all”

Now we can evaluate all of these "t-to-t" diagrams

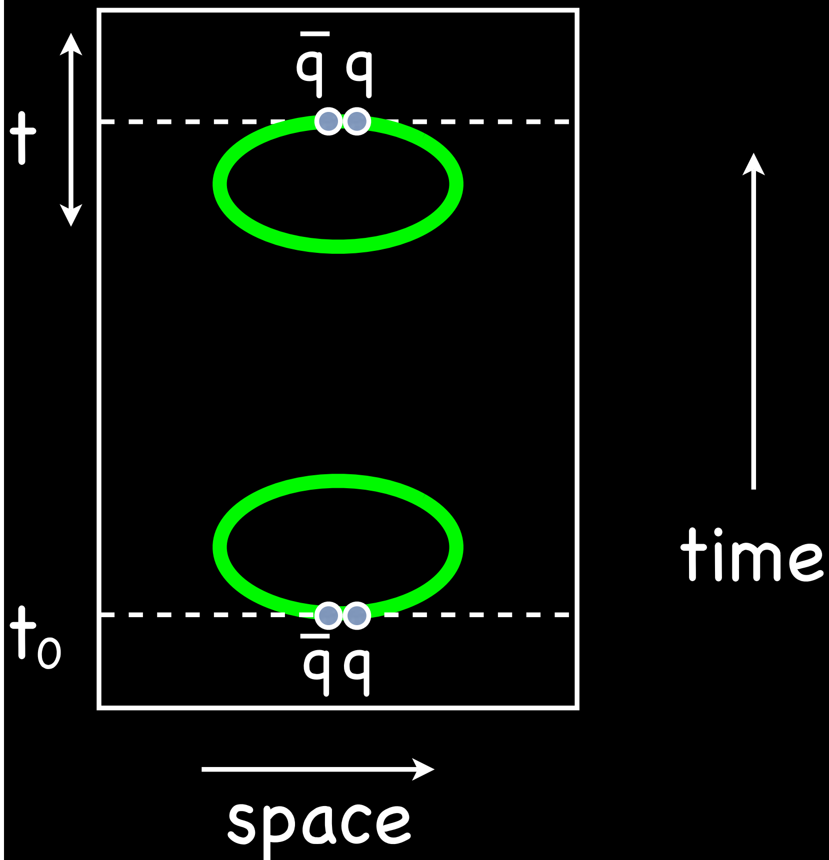
... and with a judicious choice of dilution schemes, we can make the errors due to the stochastic estimation to be < gauge noise

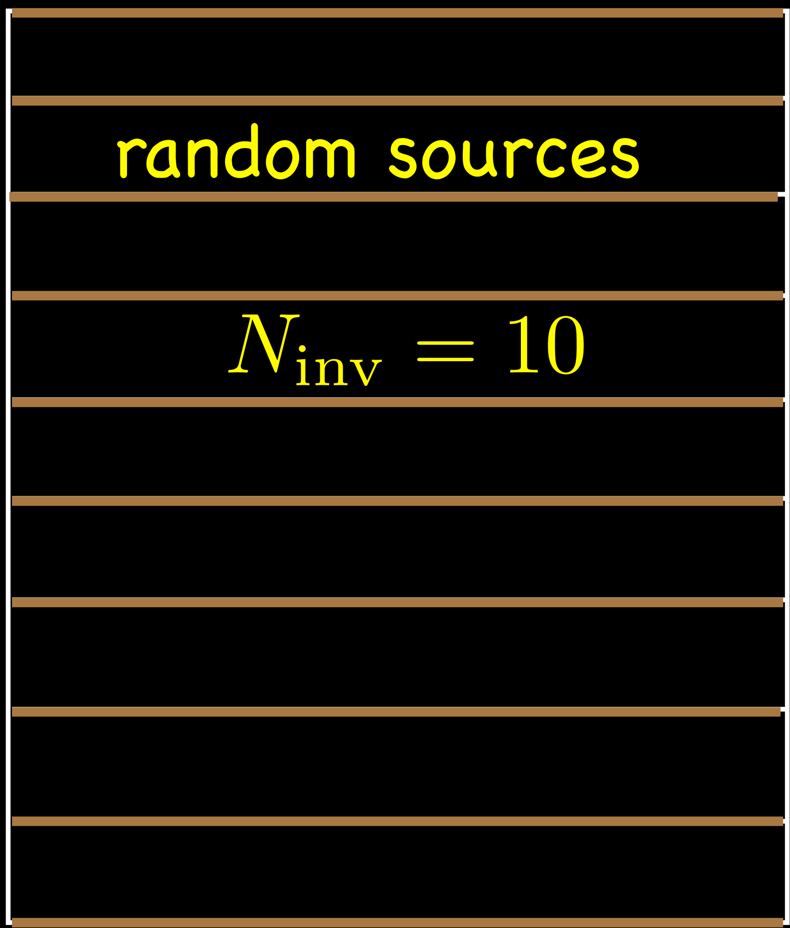




Now we can evaluate all of these "t-to-t" diagrams

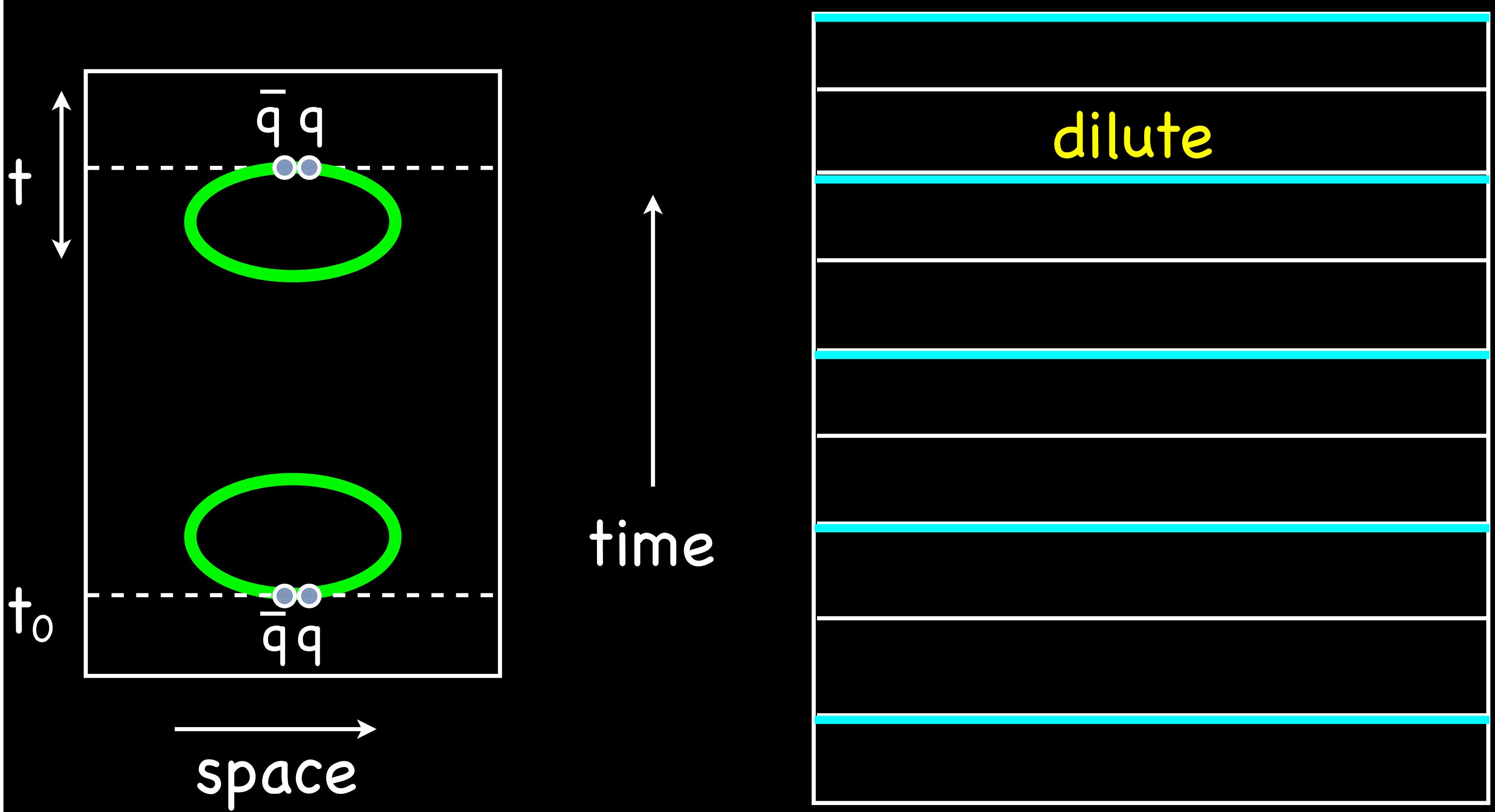
... and with a judicious choice of dilution schemes, we can make the errors due to the stochastic estimation to be < gauge noise





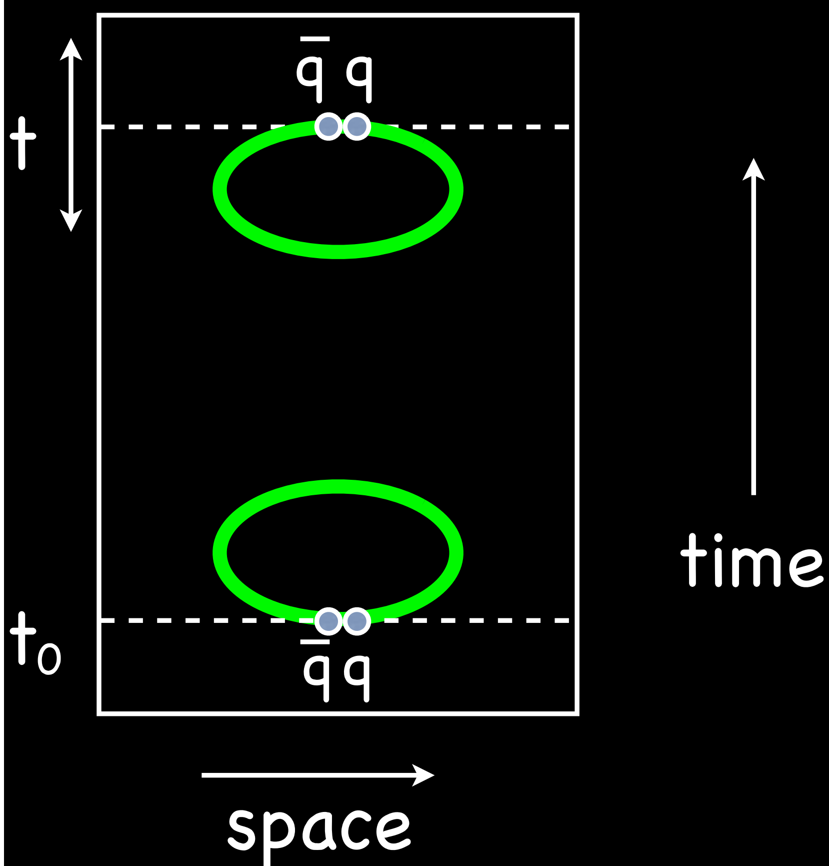
Now we can evaluate all of these “t-to-t” diagrams

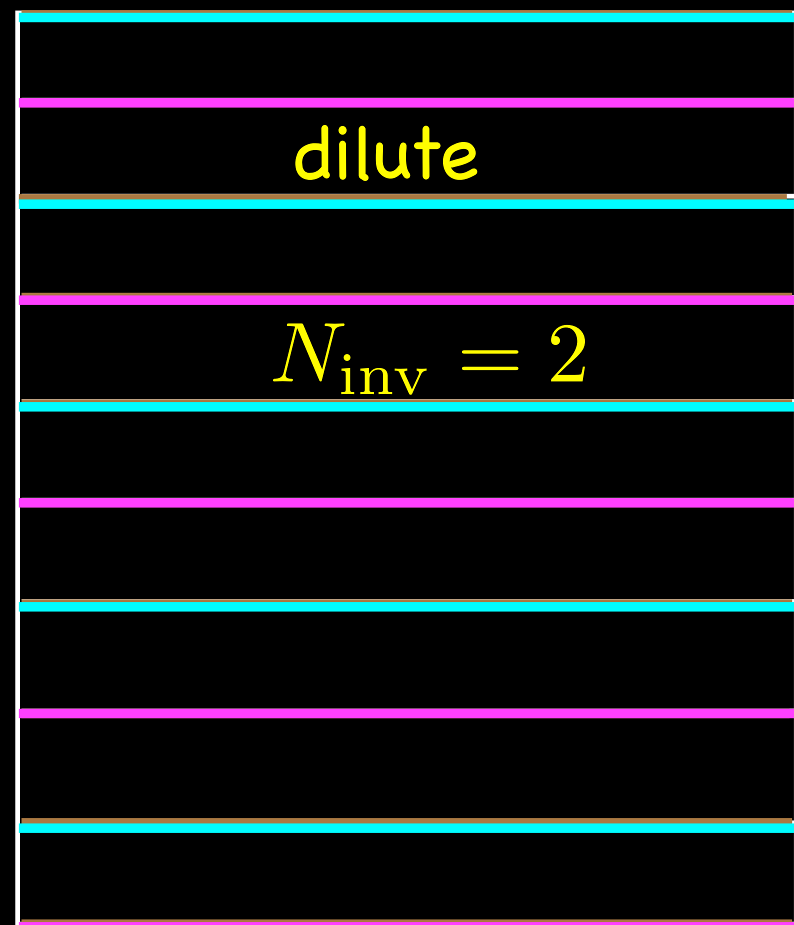
... and with a judicious choice of dilution schemes, we can make the errors due to the stochastic estimation to be < gauge noise



Now we can evaluate all of these "t-to-t" diagrams

... and with a judicious choice of dilution schemes, we can make the errors due to the stochastic estimation to be < gauge noise





Examples:

1. isospin-2 pi-pi scattering length
2. rho to pi-pi decay (mixing diagram)
3. isospin-0 pi-pi correlation function
4. isoscalar mixing
   - glueball to pi-pi decay (mixing diagram)

Simulation Parameters

Nf=2+1 anisotropic lattices

R.Edwards, B.Joo and H-W.Lin
Phys.Rev.D78 014505

as/at=3.5

$24^3 \times 128$ Mpi $\simeq$ 240 MeV mL $\simeq$ 3.4
112 eigenvectors
550 configs

$24^3 \times 128$ Mpi $\simeq$ 390 MeV mL $\simeq$ 5.6
580 configs

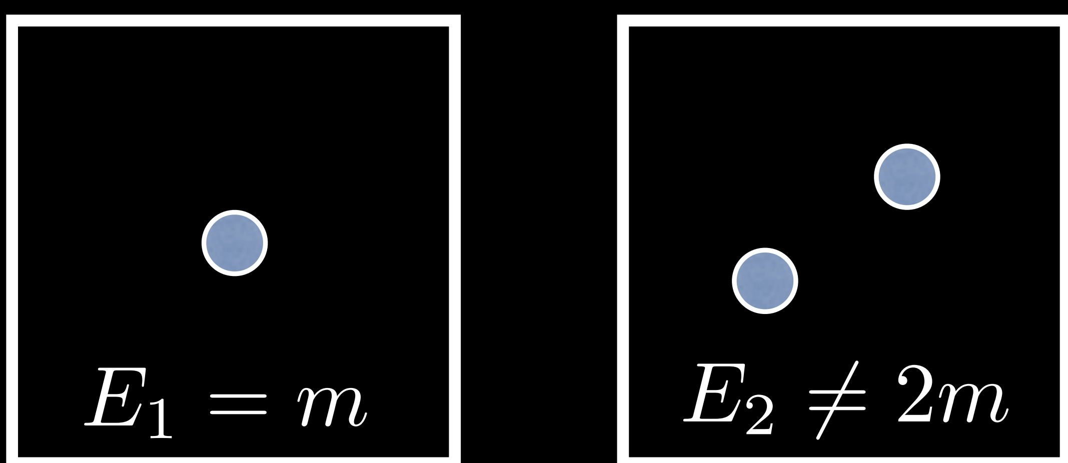
$20^3 \times 128$ Mpi $\simeq$ 390 MeV mL $\simeq$ 5

$16^3 \times 128$ Mpi $\simeq$ 390 MeV mL $\simeq$ 3.9

I=2 pi-pi Scattering Length

M.Luscher, Com.Math.Phys.(1986)
Nucl.Phys.B354(1991)

The scattering length can be obtained from a Euclidean lattice calculation of the energy shift of a two particle state in a finite box.



$$E_2 = 2m + \boxed{\delta E}$$

$$\downarrow$$

$$a_0$$

I=2 pi-pi Scattering Length

M.Luscher, Com.Math.Phys.(1986)
Nucl.Phys.B354(1991)

$$(a_t \delta E) = -\frac{1}{\xi^2} \frac{4\pi \tilde{a}_0}{(a_t m)\left(\frac{L}{a_s}\right)^3} \left[1 + c_1 \frac{\tilde{a}_0}{\left(\frac{L}{a_s}\right)} + c_2 \frac{\tilde{a}_0^2}{\left(\frac{L}{a_s}\right)^2} \right]$$

from simulation

$$c_1 = -2.837297$$

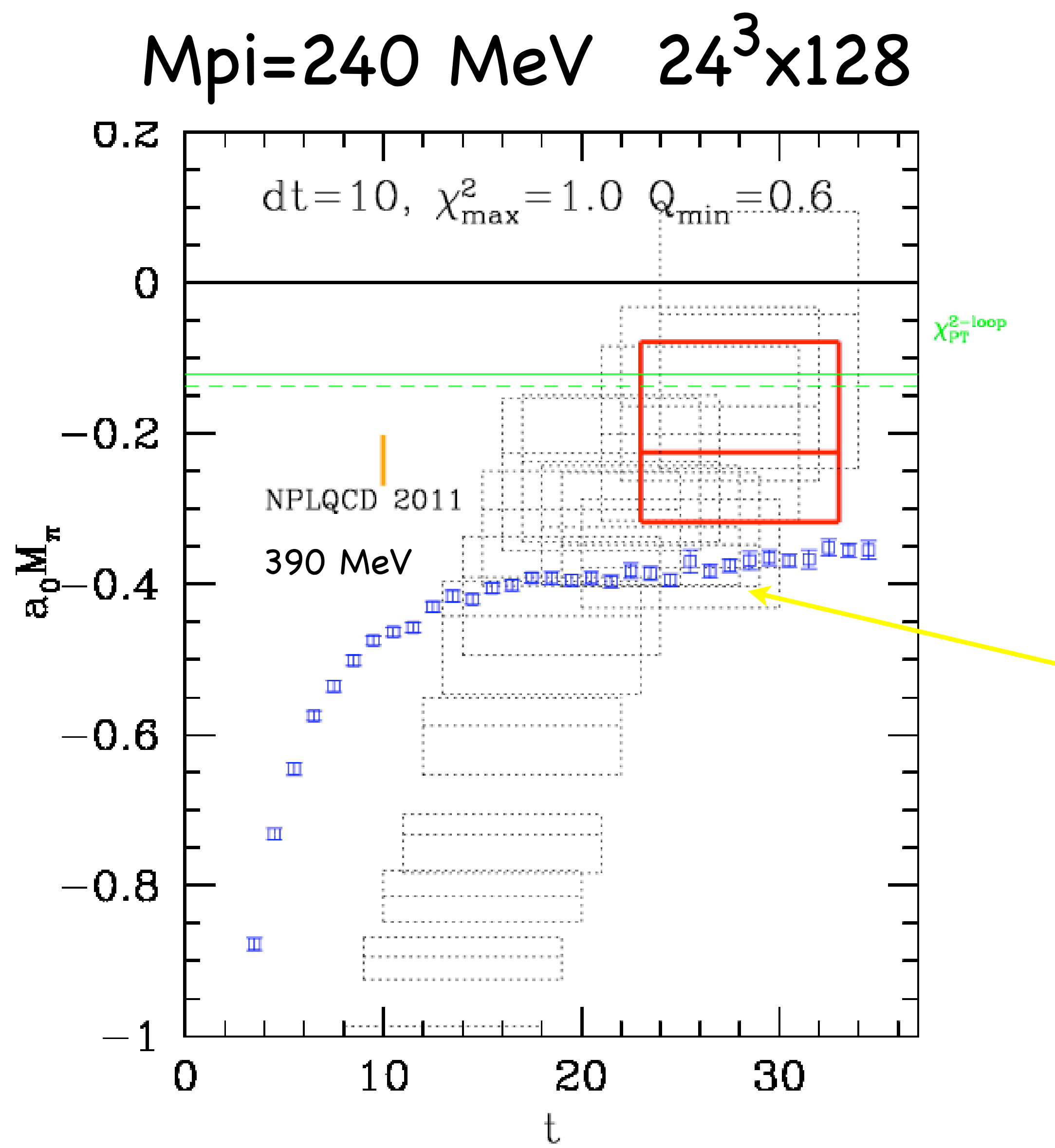
$$c_2 = 6.375183$$

$$\tilde{a}_0 = \frac{a_0}{a_s} \qquad \frac{L}{a_s} = 24 \qquad \xi = \frac{a_s}{a_t} \simeq 3.5$$

$$a_t m = \text{pion mass}$$

$$a_t \delta E = (a_t E_{\pi\pi} - 2m_\pi) = \text{energy shift}$$

I=2 pi-pi Scattering Length



- Need the "better" $\gamma 5$ operators so that that we can reduce the number of fitting parameters (running)

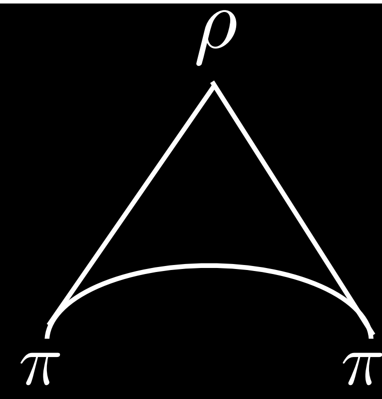
$(-M_{\rm eff})$ and shifted vertically

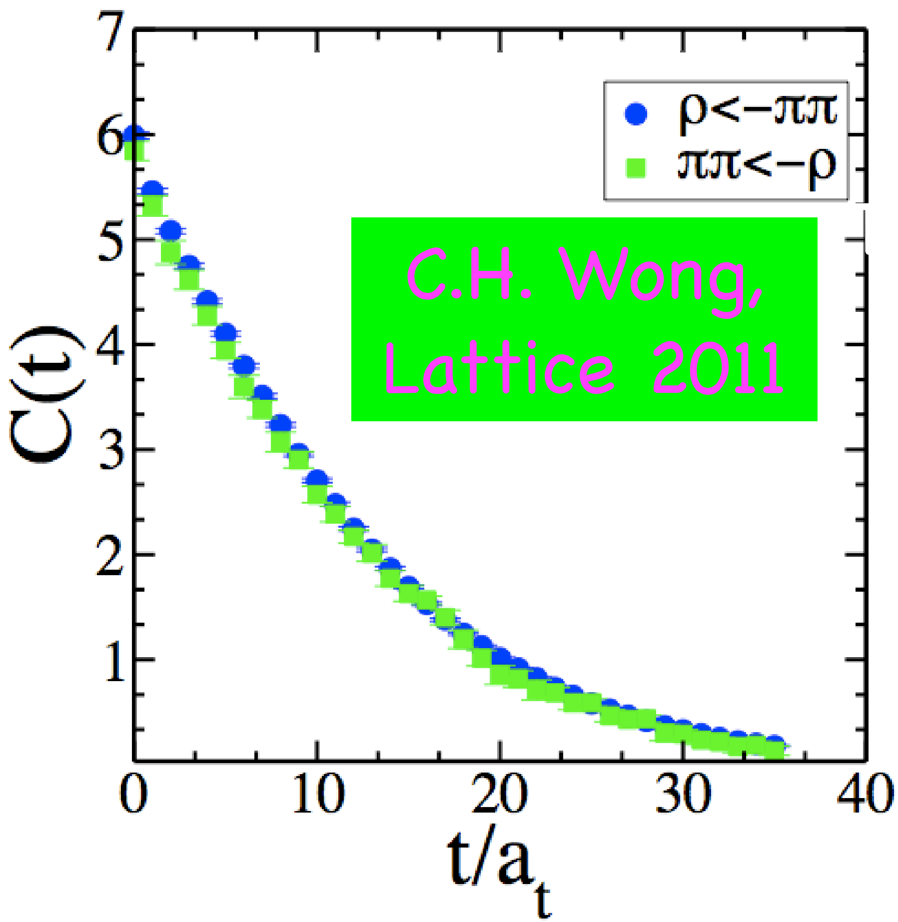
$\gamma 5$ operator simulation is now running ...

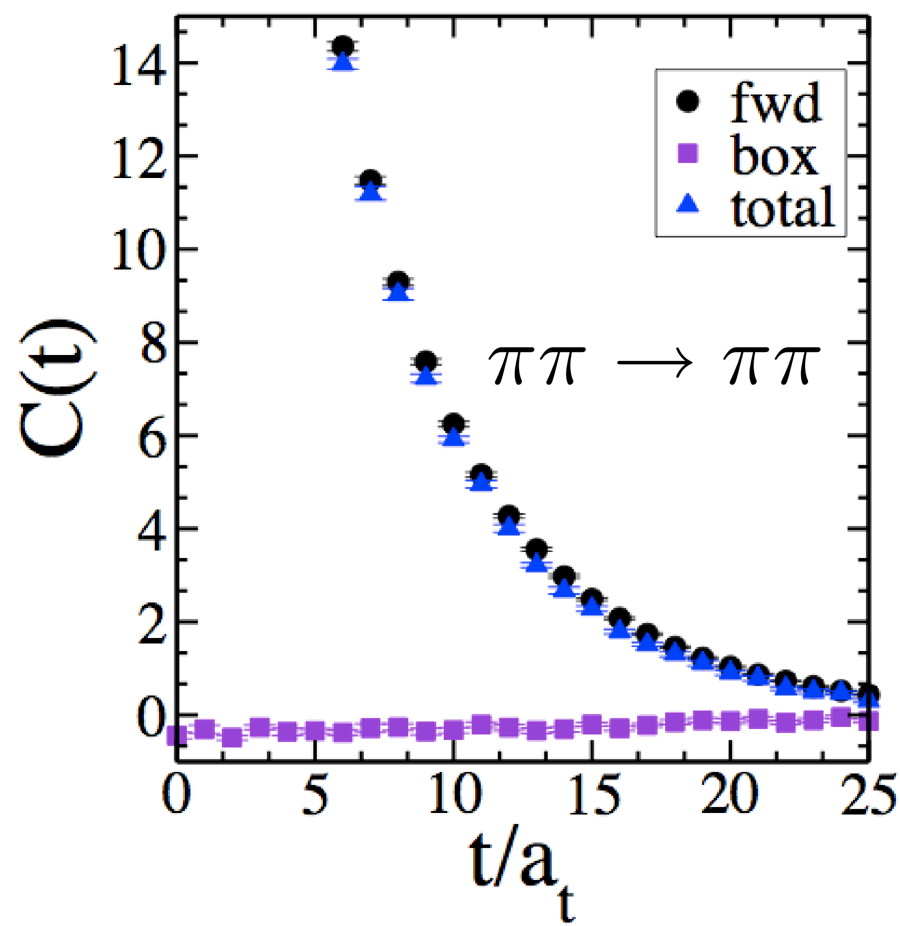
I=1 rho to pi-pi

$24^3 \times 128$ Mpi=240 MeV

E_0= 0.855(13) GeV E_1= 1.168(23) GeV

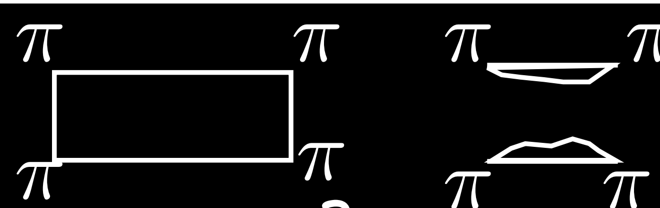






C.H. Wong, Lattice 2011

I=0 pi-pi



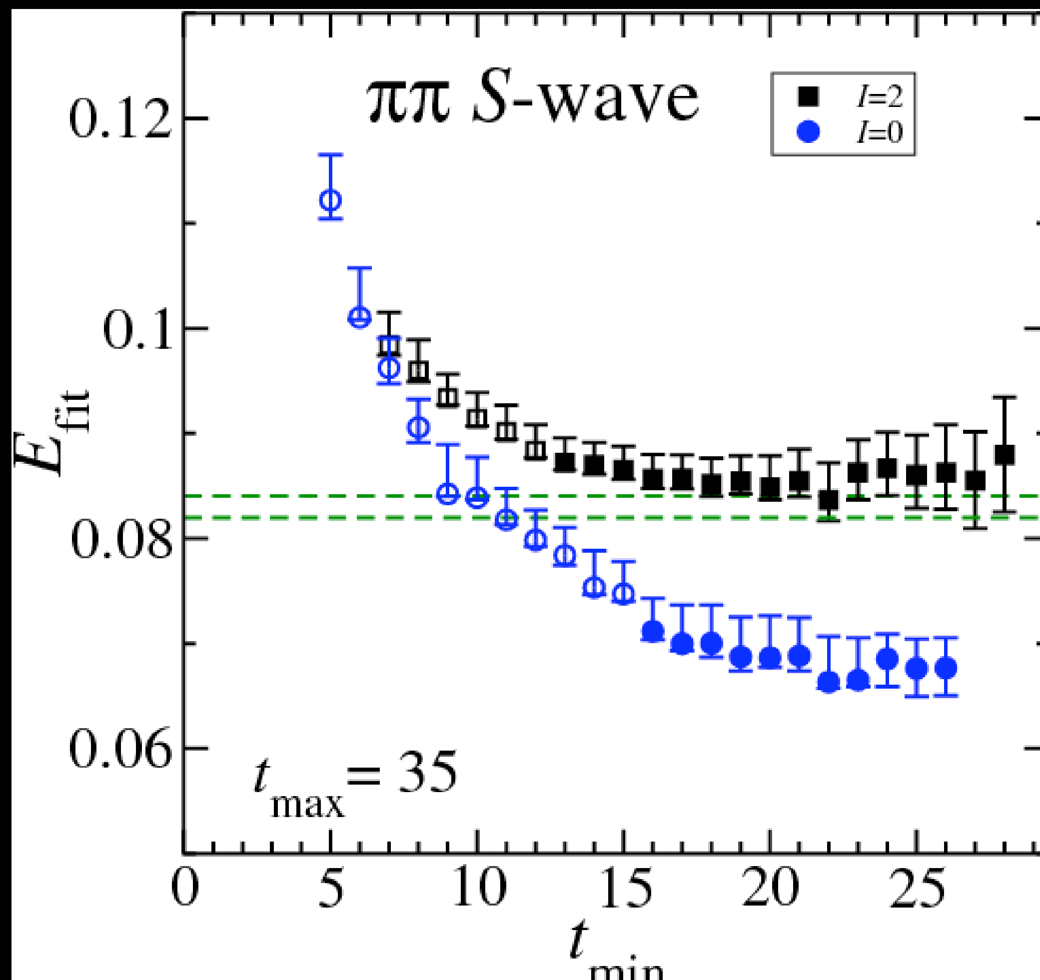
$24^3 \times 128$

M=240 MeV

"tmin plot"

- Despite the inclusion of the box diagram, the errors are comparable to the I=2 case

- Inserting finite momenta operators proceeds in the same way (the same building blocks as the I=2 case with stochastic LapH propagators)



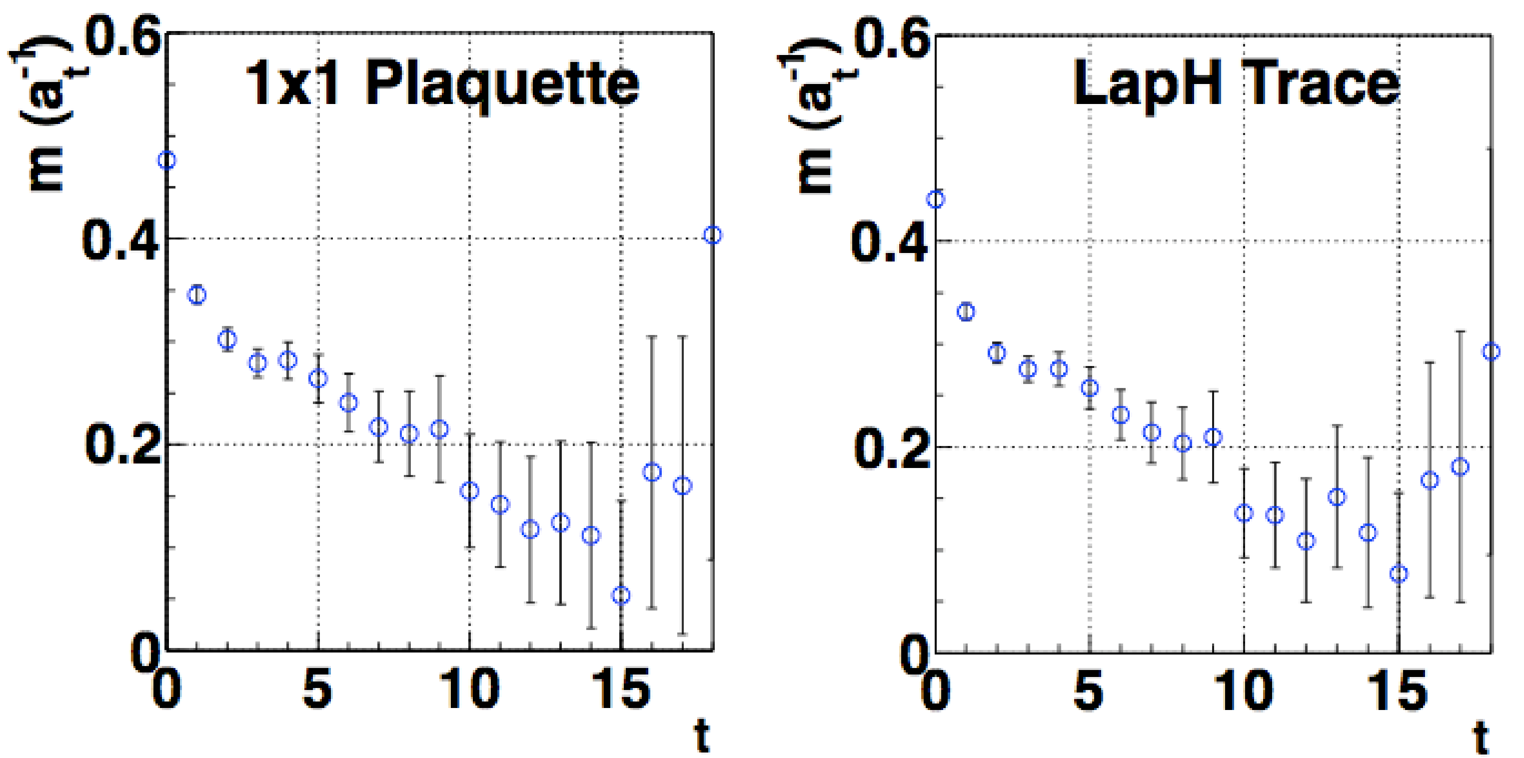
Glueballs

D. Lenkner : Lattice 2011

$24^3 \times 128$ Mpi=240 MeV

New glueball operator : trace of LapH operator

0^{++} $\longleftrightarrow$ trace of plaquette



Isoscalar Mixings

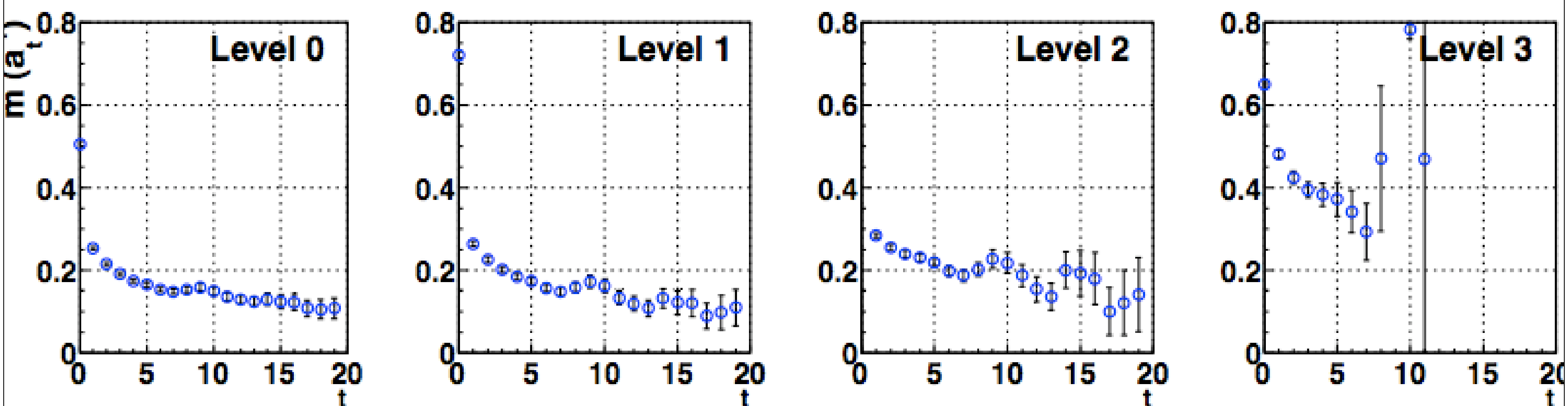
D. Lenkner : Lattice 2011

$16^3 \times 128$ Mpi = 400 MeV 100 cfgs

Diagonalized levels in the isoscalar channel

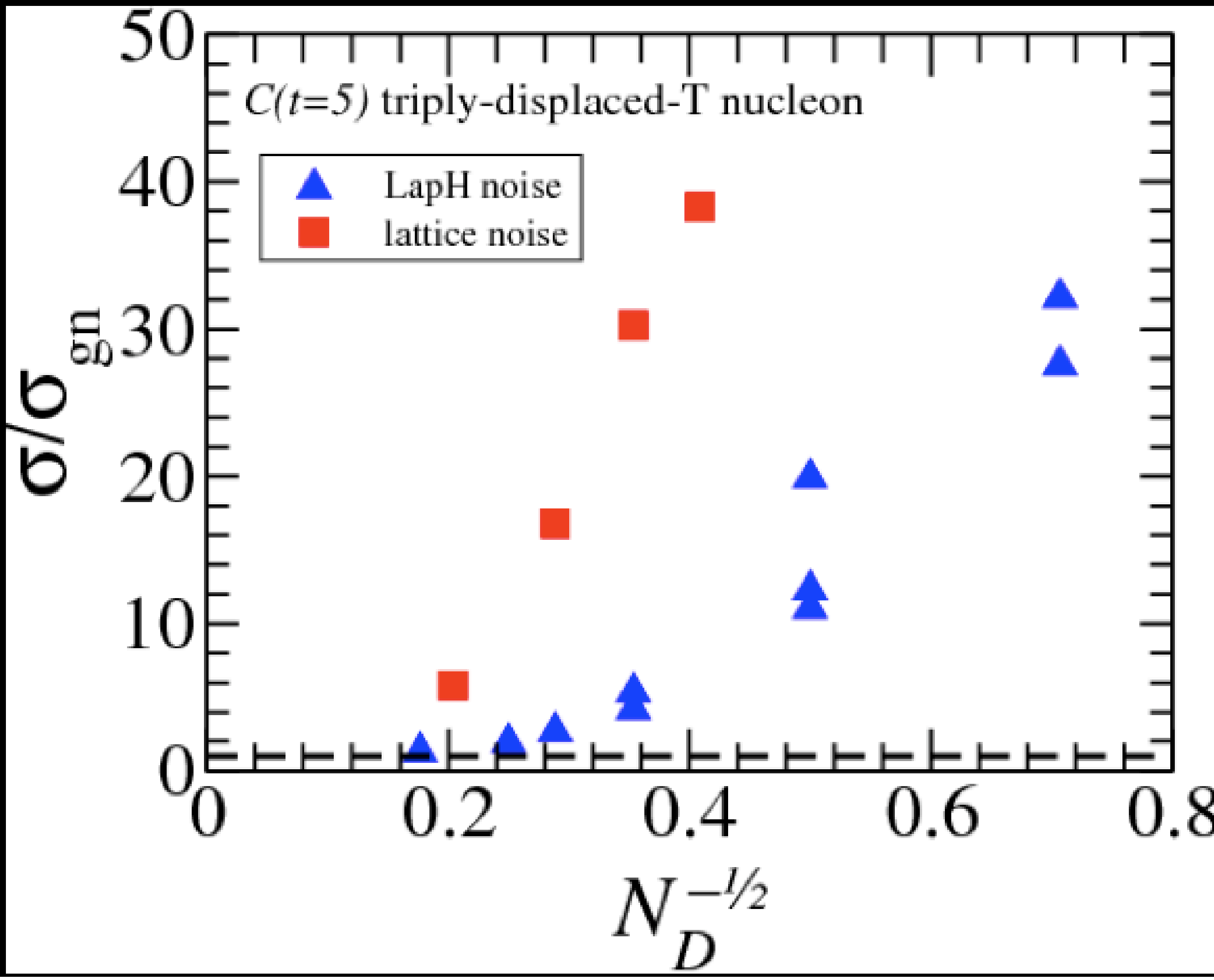
Mixing of scalar mesons, multiparticles and glueballs!

(preliminary)



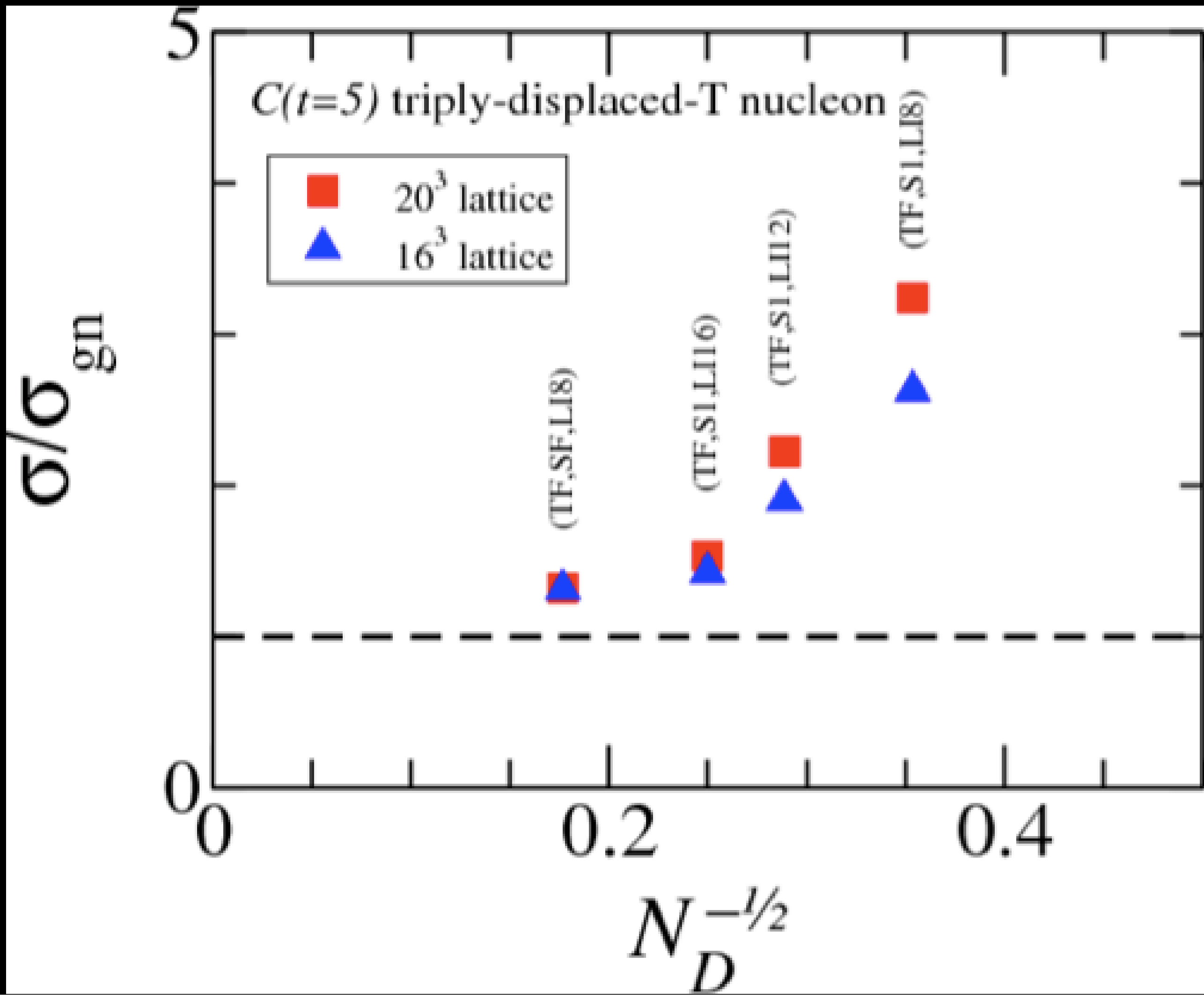
Nucleons

Example of how the errors are reduced for a nucleon correlation function (at time t=5)



Volume Scaling

The number of inversions required to achieve similar errors on 20^3 and 16^3 lattices

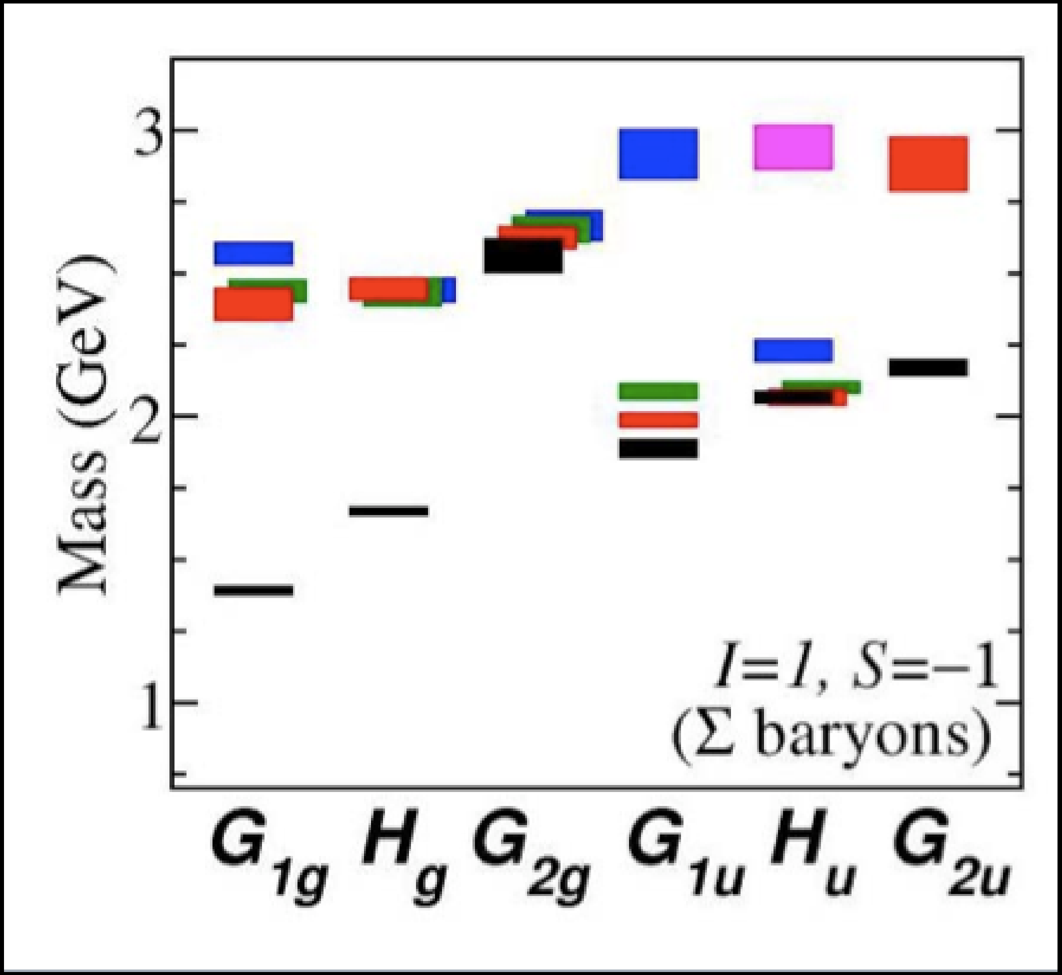


more results to come ...

single hadron operator pruning
has been done on small volumes ...
(PRD79)

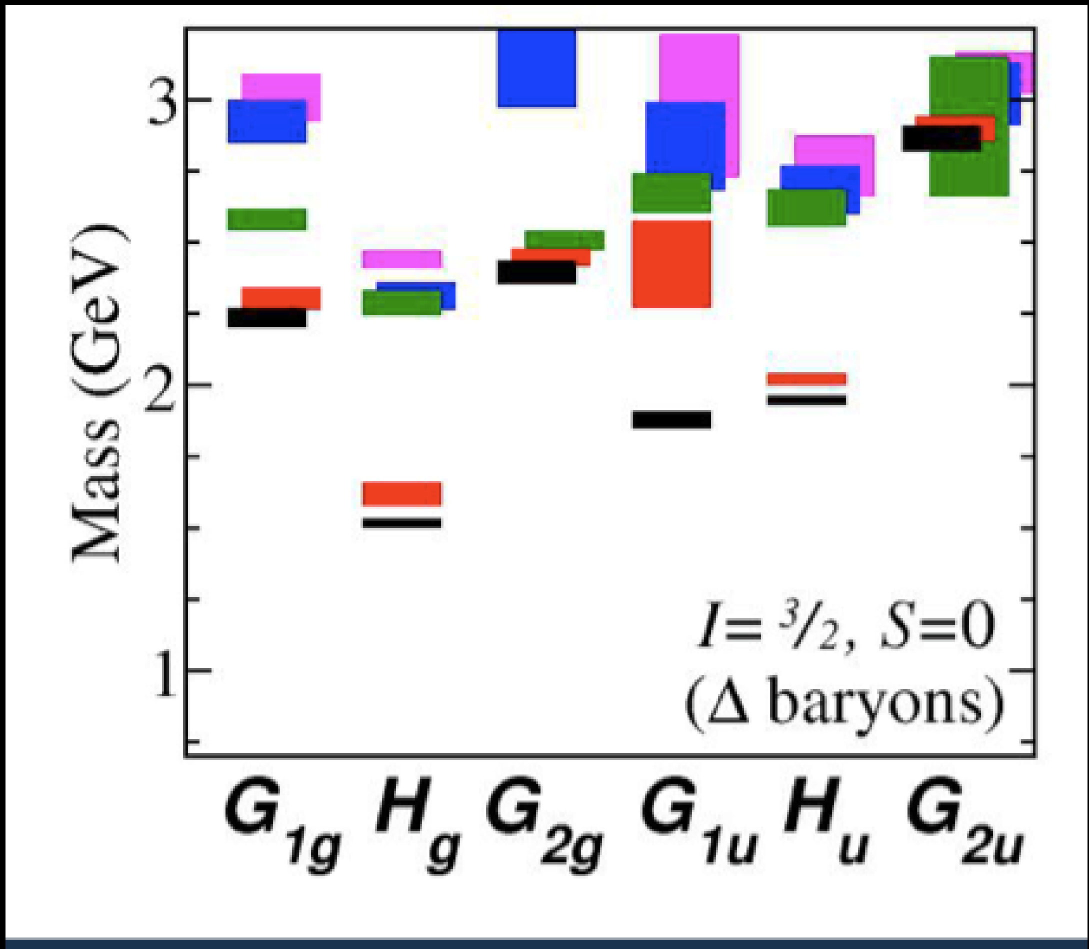
Sigma

16^3x128 100 cfgs

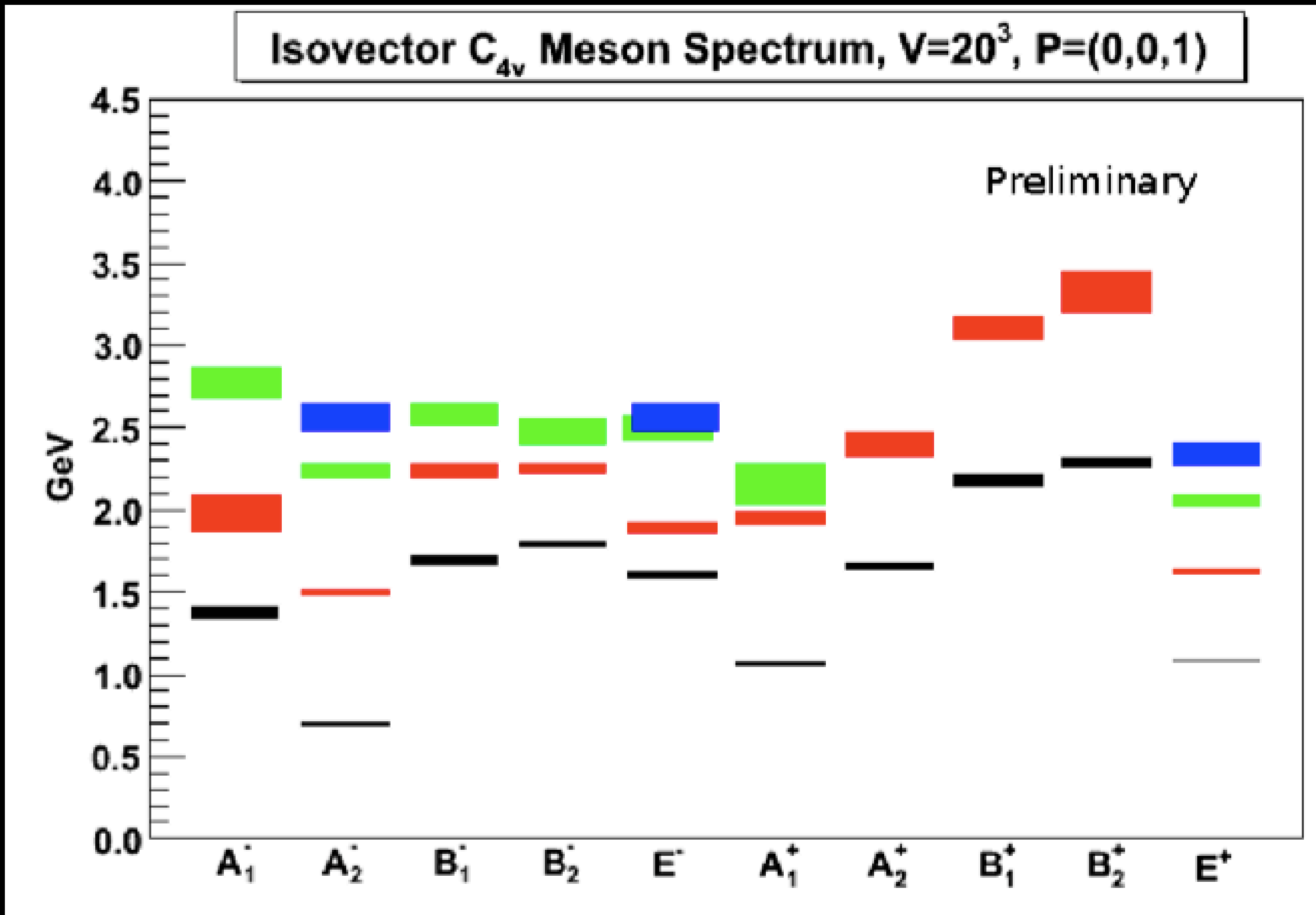


Delta

$16^3 \times 128$ 100 cfgs



Moving Mesons



Conclusions

- Stochastic LapH algorithm works like a charm!
  - no volume scaling problem (important for light q)
  - two-particle states
    - finite momenta operators are easy to implement
    - phase shifts are accessible (if V is big enough)
  - mixing between single-particle and two-particle
  - t-to-t diagrams do not appear to be a problem!
  - a new glueball operator

Excitation spectra with multi-particle states within reach ...