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The Charge Radius of the Proton

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Richard J. Hill, GP PRD **82** 113005 (2010)

Richard J. Hill, GP [arXiv:1103.4617]

Form Factors

- Matrix element of EM current between nucleon states give rise to two form factors ($q = p_f - p_i$)

$$\langle p(p_f) | \sum_q e_q \bar{q} \gamma^\mu q | p(p_i) \rangle = \bar{u}(p_f) \left[\gamma_\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}}{2m} F_2(q^2) q_\nu \right] u(p_i)$$

- Sachs electric and magnetic form factors

$$G_E(q^2) = F_1(q^2) + \frac{q^2}{4m_p^2} F_2(q^2) \quad G_M(q^2) = F_1(q^2) + F_2(q^2)$$

$$G_E^p(0) = 1$$

$$G_M(0) = \mu_p \approx 2.793$$

- The slope of G_E^p

$$\langle r^2 \rangle_E^p = 6 \left. \frac{dG_E^p}{dq^2} \right|_{q^2=0}$$

determines the charge radius $r_E^p \equiv \sqrt{\langle r^2 \rangle_E^p}$

Charge radius from atomic physics

$$\langle p(p_f) | \sum_q e_q \bar{q} \gamma^\mu q | p(p_i) \rangle = \bar{u}(p_f) \left[\gamma_\mu F_1^p(q^2) + \frac{i\sigma_{\mu\nu}}{2m} F_2^p(q^2) q_\nu \right] u(p_i)$$

- For a point particle amplitude for $p + \ell \rightarrow p + \ell$

$$\mathcal{M} \propto \frac{1}{q^2} \Rightarrow U(r) = -\frac{Z\alpha}{r}$$

- Including q^2 corrections from proton structure

$$\mathcal{M} \propto \frac{1}{q^2} q^2 = 1 \Rightarrow U(r) = \frac{4\pi Z\alpha}{6} \delta^3(r) (r_E^p)^2$$

- Proton structure corrections ($m_r = m_\ell m_p / (m_\ell + m_p) \approx m_\ell$)

$$\Delta E_{r_E^p} = \frac{2(Z\alpha)^4}{3n^3} m_r^3 (r_E^p)^2 \delta_{\ell 0}$$

- **Muonic hydrogen can give the best measurement of r_E^p !**

Charge radius from atomic physics



- Lamb shift in muonic hydrogen [Pohl et al. Nature **466**, 213 (2010)]

$$r_E^p = 0.84184(67) \text{ fm}$$

- CODATA value [Mohr et al. RMP **80**, 633 (2008)]

$$r_E^p = 0.8768(69) \text{ fm}$$

extracted mainly from (electronic) hydrogen

- **5 σ discrepancy!**
- We can also extract it from electron-proton scattering data

Charge radius from scattering data

- Problem: r_E^p in literature depends on functional form of G_E^p
 r_E^p **not** stable when we include more parameters
- r_E^p from $e - p$ scattering data ($Q^2 \leq 0.04 \text{ GeV}^2$) tabulated
 in Rosenfelder [arXiv:nucl-th/9912031] (r_E^p in $10^{-18} m$)

	$k_{\max} = 1$	2	3	4	5
polynomial	836_{-9}^{+8}	867_{-24}^{+23}	866_{-56}^{+52}	959_{-93}^{+85}	1122_{-137}^{+122}
continued fraction	882_{-10}^{+10}	869_{-25}^{+26}	—	—	—
z expansion (no bound)	918_{-9}^{+9}	868_{-29}^{+28}	879_{-69}^{+64}	1022_{-114}^{+102}	1193_{-174}^{+152}
z expansion ($ a_k \leq 10$)	918_{-9}^{+9}	868_{-29}^{+28}	879_{-59}^{+38}	880_{-61}^{+39}	880_{-62}^{+39}

- Only constrained z expansion is stable and model independent

The Axial Mass

- The problem is not unique to the vector form factor!

For axial form factor m_A analogous to r_E^p

- Another discrepancy:

- Neutrino scattering: $m_A^{\text{dipole}} = 1.35 \pm 0.17 \text{ GeV}$

[MiniBooNE Collaboration, arXiv:1002.2680]

- Pion electro-production: $m_A^{\text{dipole}} = 1.07 \pm 0.02 \text{ GeV}$

[Bernard, Elouadrhiri, Meissner J. Phys. G 28, R1 (2002)]

Both use dipole ansatz for axial form factor ($F_A = F_A(0)[1 - q^2/(m_A^{\text{dipole}})^2]^{-2}$)

- Using model **independent** approach

[Bhattacharya, Hill, GP arXiv:1008.0423]

MiniBooNE: $m_A = 0.85_{-0.07}^{+0.22} \pm 0.09 \text{ GeV}$ ($m_A^{\text{dipole}} = 1.29 \pm 0.05 \text{ GeV}$)

Pion electro-production: $m_A = 0.92_{-0.13}^{+0.12} \pm 0.08 \text{ GeV}$ ($m_A^{\text{dipole}} = 1.00 \pm 0.02 \text{ GeV}$)

Discrepancy is an artifact of the use of the dipole form factor!

The recent discrepancy

- [Hill, GP PRD **82** 113005 (2010)] showed previous extractions are model dependent underestimated the error by a factor of 2 or more
- Based on a model **independent** approach using scattering data from proton, neutron and $\pi\pi$ [Hill, GP PRD **82** 113005 (2010)]
 $r_E^p = 0.871(11) \text{ fm}$
- CODATA value (extracted mainly from electronic hydrogen) [Mohr et al. RMP **80**, 633 (2008)]
 $r_E^p = 0.8768(69) \text{ fm}$
- Lamb shift in muonic hydrogen [Pohl et al. Nature **466**, 213 (2010)]
 $r_E^p = 0.84184(67) \text{ fm}$

Lamb shift in muonic hydrogen

- CREMA measured [Pohl et al. Nature **466**, 213 (2010)]

$$\Delta E = 206.2949 \pm 0.0032 \text{ meV}$$

- Comparing to the theoretical expression

[Pachucki PRA **60**, 3593 (1999), Borie PRA **71**(3), 032508 (2005)]

$$\Delta E = 209.9779(49) - 5.2262(r_E^p)^2 + 0.0347(r_E^p)^3 \text{ meV}$$

- They got

$$r_E^p = 0.84184(67) \text{ fm}$$

The Theoretical Prediction

- Is there a problem with the theoretical prediction?

[Pachucki PRA **60**, 3593 (1999), Borie PRA **71**(3), 032508 (2005)]

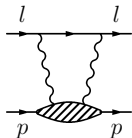
$$\Delta E = 209.9779(49) - 5.2262(r_E^p)^2 + 0.0347(r_E^p)^3 \text{ meV}$$

↑
mostly
 μ QED

↑
already
discussed

↑
where does
this term
come from?

Two-photon amplitude: “standard” calculation



- “standard” calculation: separate to proton and non-proton
 - non-proton \leftrightarrow DIS
- For proton
 - Insert form factors into vertices

$$\mathcal{M} = \int_0^\infty dq^2 f(G_E, G_M)$$

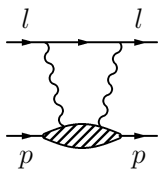
- Using a “dipole form factor”

$$G_i(q^2) \approx G_i(q^2)/G_i(0) \approx [1 - q^2/\Lambda^2]^{-2}$$

- \mathcal{M} is a function of $\Lambda \Rightarrow (r_E^p)^3$ term

- Using, $\Lambda^2 = 0.71 \text{ GeV}^2 \Rightarrow \Delta E \approx 0.018 \text{ meV}$
[Pachucki, PRA **53**, 2092 (1996)]

Two-photon amplitude: “standard” calculation



- Why is the insertion of form factors in vertices valid?
- Even if it was, result looks funny
two-photon amplitude \Leftrightarrow the charge radius
only for one parameter model for G_E and G_M
- In “standard approach” two-photon $\Rightarrow \Delta E \approx 0.018$ meV
Need **0.258(90) meV** (scattering) or **0.311(63) meV** (spec.)
to explain discrepancy

NRQED

- Model Independent approach: use NRQED

[Caswell, Lepage PLB **167**, 437 (1986); Kinoshita Nio PRD **53**, 4909 (1996); Manohar PRD **56**, 230 (1997)]

$$\begin{aligned} \mathcal{L}_e = & \psi_e^\dagger \left\{ iD_t + \frac{\mathbf{D}^2}{2m_e} + \frac{\mathbf{D}^4}{8m_e^3} + c_F e \frac{\boldsymbol{\sigma} \cdot \mathbf{B}}{2m_e} + c_D e \frac{[\boldsymbol{\partial} \cdot \mathbf{E}]}{8m_e^2} \right. \\ & + i c_S e \frac{\boldsymbol{\sigma} \cdot (\mathbf{D} \times \mathbf{E} - \mathbf{E} \times \mathbf{D})}{8m_e^2} + c_{W1} e \frac{\{\mathbf{D}^2, \boldsymbol{\sigma} \cdot \mathbf{B}\}}{8m_e^3} \\ & - c_{W2} e \frac{D^i \boldsymbol{\sigma} \cdot \mathbf{B} D^i}{4m_e^3} + c_{p'p} e \frac{\boldsymbol{\sigma} \cdot \mathbf{D} \mathbf{B} \cdot \mathbf{D} + \mathbf{D} \cdot \mathbf{B} \boldsymbol{\sigma} \cdot \mathbf{D}}{8m_e^3} \\ & \left. + i c_M e \frac{\{\mathbf{D}^i, [\boldsymbol{\partial} \times \mathbf{B}]^i\}}{8m_e^3} + c_{A1} e^2 \frac{\mathbf{B}^2 - \mathbf{E}^2}{8m_e^3} - c_{A2} e^2 \frac{\mathbf{E}^2}{16m_e^3} + \dots \right\} \psi_e \end{aligned}$$

- Need also

$$\mathcal{L}_{\text{contact}} = d_1 \frac{\psi_p^\dagger \boldsymbol{\sigma} \psi_p \cdot \psi_e^\dagger \boldsymbol{\sigma} \psi_e}{m_e m_p} + d_2 \frac{\psi_p^\dagger \psi_p \psi_e^\dagger \psi_e}{m_e m_p}$$

NRQED

- From c_i and d_i determine proton structure correction, e.g.

$$\delta E(n, \ell) = \delta_{\ell 0} \frac{m_r^3 (Z\alpha)^3}{\pi n^3} \left(\frac{Z\alpha\pi}{2m_p^2} c_D^{\text{proton}} - \frac{d_2}{m_e m_p} \right)$$

- Matching

- Operators with one photon coupling:

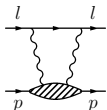
$$c_i \text{ given by } F_i^{(n)}(0)$$

- Operators with only two photon couplings:

c_{A_i} given by forward and backward Compton scattering

- d_i from two-photon amplitude

Two-photon amplitude: matching



$$\frac{1}{2} \sum_s i \int d^4x e^{iq \cdot x} \langle \mathbf{k}, s | T \{ J_{\text{e.m.}}^\mu(x) J_{\text{e.m.}}^\nu(0) \} | \mathbf{k}, s \rangle$$

$$= \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) W_1 + \left(k^\mu - \frac{k \cdot q q^\mu}{q^2} \right) \left(k^\nu - \frac{k \cdot q q^\nu}{q^2} \right) W_2$$

- Matching

$$\frac{4\pi m_r}{\lambda^3} - \frac{\pi m_r}{2m_e m_p \lambda} - \frac{2\pi m_r}{m_p^2 \lambda} \left[F_2(0) + 4m_p^2 F_1'(0) \right]$$

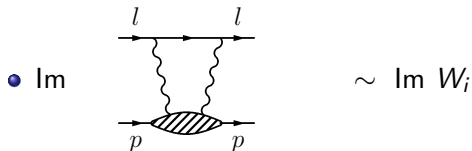
$$- \frac{2}{m_e m_p} \left[\frac{2}{3} + \frac{1}{m_p^2 - m_e^2} \left(m_e^2 \log \frac{m_p}{\lambda} - m_p^2 \log \frac{m_e}{\lambda} \right) \right] + \frac{d_2(Z\alpha)^{-2}}{m_e m_p}$$

$$= -\frac{m_e}{m_p} \int_{-1}^1 dx \sqrt{1-x^2} \int_0^\infty dQ \frac{Q^3}{(Q^2 + \lambda^2)^2 (Q^2 + 4m_e^2 x^2)}$$

$$\times \left[(1 + 2x^2) W_1(2im_p Qx, Q^2) - (1 - x^2) m_p^2 W_2(2im_p Qx, Q^2) \right]$$

d_2

- In order to determine d_2 need to know W_i



can be extracted from on-shell quantities:

Proton form factors and Inelastic structure functions

- To find W_i from $\text{Im } W_i$, need dispersion relations

Dispersion relation

- Dispersion relations ($\nu = 2k \cdot q$, $Q^2 = -q^2$)

$$W_1(\nu, Q^2) = W_1(0, Q^2) + \frac{\nu^2}{\pi} \int_{\nu_{\text{cut}}(Q^2)^2}^{\infty} d\nu'^2 \frac{\text{Im} W_1(\nu', Q^2)}{\nu'^2(\nu'^2 - \nu^2)}$$

$$W_2(\nu, Q^2) = \frac{1}{\pi} \int_{\nu_{\text{cut}}(Q^2)^2}^{\infty} d\nu'^2 \frac{\text{Im} W_2(\nu', Q^2)}{\nu'^2 - \nu^2}$$

- W_1 requires subtraction...
 - $\text{Im} W_i^P$ from form factors
 - $\text{Im} W_i^C$ from DIS
 - What about $W_1(0, Q^2)$?

$W_1(0, Q^2)$

- Can calculate in two limits: [Hill, GP, arXiv:1103.4617]

- $Q^2 \ll m_p^2$

The photon sees the proton “almost” like an elementary particle
Use NRQED to calculate $W_1(0, Q^2)$ upto $\mathcal{O}(Q^2)$ (including)

$$W_1(0, Q^2) = 2(c_F^2 - 1) + 2\frac{Q^2}{4m_p^2} (c_{A_1} + c_F^2 - 2c_F c_{W_1} + 2c_M)$$

- $Q^2 \gg m_p^2$

The photon sees the quarks inside the proton

Use OPE to find $W_1(0, Q^2) \sim 1/Q^2$ for large Q^2

- In between you will have to model!

Current calculation **pretends** there is no model dependence

How big is the model dependence?

Bound State Energy

- 1) Proton: $\text{Im } W_i^P$ using dipole form factor

$$\Delta E = -0.016 \text{ meV}$$

- 2) Continuum: $\text{Im } W_i^c$ [Carlson, Vanderhaeghen arXiv:1101.5965]

$$\Delta E = 0.0127(5) \text{ meV}$$

- 3) What about $W_1(0, Q^2)$?

“Sticking In Form Factors” (SIFF) model

$$W_1^{\text{SIFF}}(0, Q^2) = 2F_2(2F_1 + F_2) \quad F_i \equiv F_i(Q^2)$$

SIFF

- “Sticking In Form Factors” (SIFF) model

$$W_1^{\text{SIFF}}(0, Q^2) = 2F_2(2F_1 + F_2) \quad F_i \equiv F_i(Q^2)$$

Notice that for large Q^2 , $W_1^{\text{SIFF}}(0, Q^2) \propto 1/Q^8$

In contradiction to OPE

- There is **no** local Lagrangian that has a Feynman rule

$$\gamma_\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}}{2m} F_2(q^2) q_\nu$$

- Numerically using the dipole form factor

$$\Delta E^{\text{SIFF}} = 0.034 \text{ meV}$$

Model Dependence

- How big is the model dependence?

$$0.018 \text{ meV} = \underset{\substack{\uparrow \\ \text{Model independent}}}{-0.016 \text{ meV}} + \underset{\substack{\uparrow \\ \text{Model dependent}}}{0.034 \text{ meV}}$$

- The model dependent piece is the dominant one!
- Experimental discrepancy $\sim 0.3 \text{ meV}$
- It is possible that the true $W_1(0, Q^2)$ explains (or reduces) the discrepancy

Two photon amplitude: summary

- To determine two photon amplitude need
 - $\text{Im } W_i$ which can be extracted from data
 - $W_1(0, Q^2)$ which currently cannot be extracted from data
- Unlike $\text{Im } W_i$, $W_1(0, Q^2)$ **cannot** be written model independently as a sum of “proton” and “non-proton” terms
- Model independent properties of $W_1(0, Q^2)$:
 - Low Q^2 via NRQED
 - High Q^2 via OPE
 - Intermediate region poorly constrained
- Lack of theoretical control over $W_1(0, Q^2)$ introduces theoretical uncertainties not taken into account in the literature

Conclusions

- Recent discrepancy in the extraction the proton charge radius between muonic and regular hydrogen
- From **model independent** extraction of the charge radius from $e - p$ scattering data: $r_E^p = 0.871(11) \text{ fm}$
- Previous extractions have underestimated the error
- Results are compatible with CODATA value of $r_E^p = 0.8768(69) \text{ fm}$

Conclusions

- Analyzed Proton structure effects in hydrogenic bound states
Using NRQED
- Isolated model-**dependent** assumptions in previous analyses:
 $W_1(0, Q^2)$ was calculated by “Sticking In Form Factors” model
- Model **independent** calculation of $W_1(0, Q^2)$:
low Q^2 via NRQED, high Q^2 via OPE
- Possibility for a significant new effects in the two-photon amplitude
- NRQED predicts a universal shift for spin-independent energy splittings in muonic hydrogen.

Future Directions

- Analyze spin dependent effects
- Application to deuterium
- Resolution of the discrepancy?