Second-Order Approximate Corrections for QCD Processes

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- **Higher-order collinear and soft corrections**
- **Factorization and Resummation**
- **Two-loop eikonal calculations**
- **Soft anomalous dimensions**
- **Master formulas for NNLO expansions**
- **Applications: Top, (charged) Higgs, W, Z, ...**

Higher-order collinear and soft corrections

Soft-gluon corrections from incomplete cancellations of infrared divergences between virtual diagrams and real diagrams with soft (low-energy) gluons

Soft corrections: $\left[\frac{\ln^k(s_4/M^2)}{s_4}\right]_+$ with $k \le 2n-1$ and s_4 distance from threshold

double collinear and soft logarithms

also purely collinear terms $\frac{1}{M^2}\ln^k(s_4/M^2)$

Soft-gluon corrections are dominant near threshold

Resum (exponentiate) these corrections

At NLL accuracy requires one-loop calculations in eikonal approximation

Recent results at NNLL – two-loop calculations completed

Essential ingredient: two-loop soft anomalous dimension

Allows NNLL resummation

Approximate NNLO cross section from expansion of resummed cross section

Factorization and Resummation

Resummation follows from factorization properties of the cross section - performed in moment space

$\sigma = (\prod \psi)\, H_{IL}\, S_{LI}\, (\prod J)$ H: **hard-scattering function**

S: **soft-gluon function**

Use RGE to evolve soft-gluon function

$$\left(\mu \frac{\partial}{\partial \mu} + \beta(g_s)\frac{\partial}{\partial g_s}\right) S_{LI} = -(\Gamma_S^\dagger)_{LB} S_{BI} - S_{LA}(\Gamma_S)_{AI}$$

Γ_S **is the soft anomalous dimension - a matrix in color space and a function of kinematical invariants** s, t, u

Resummed cross section

$$\hat{\sigma}^{res}(N) = \exp\left[\sum_i E_i(N_i)\right] \exp\left[\sum_j E'_j(N')\right] \exp\left[\sum_{i=1,2} 2\int_{\mu_F}^{\sqrt{s}} \frac{d\mu}{\mu}\, \gamma_{i/i}\left(\tilde{N}_i, \alpha_s(\mu)\right)\right]$$

$$\times\, \mathrm{tr}\left\{H(\alpha_s) \exp\left[\int_{\sqrt{s}}^{\sqrt{s}/\tilde{N}'} \frac{d\mu}{\mu}\, \Gamma_S^\dagger(\alpha_s(\mu))\right] S\left(\alpha_s\left(\frac{\sqrt{s}}{\tilde{N}'}\right)\right) \exp\left[\int_{\sqrt{s}}^{\sqrt{s}/\tilde{N}'} \frac{d\mu}{\mu}\, \Gamma_S(\alpha_s(\mu))\right]\right\}$$

collinear and soft radiation from incoming partons

$$E_i(N_i) = \int_0^1 dz \frac{z^{N_i-1}-1}{1-z} \left\{ \int_1^{(1-z)^2} \frac{d\lambda}{\lambda} A_i\left(\alpha_s(\lambda s)\right) + D_i \left[\alpha_s((1-z)^2 s)\right] \right\}$$

purely collinear: replace $\frac{z^{N-1}-1}{1-z}$ **by** $-z^{N-1}$

collinear and soft radiation from outgoing massless quarks and gluons

$$E'(N') = \int_0^1 dz \frac{z^{N'-1}-1}{1-z} \left\{ \int_{(1-z)^2}^{1-z} \frac{d\lambda}{\lambda} A_i\left(\alpha_s\left(\lambda s\right)\right) + B_i\left[\alpha_s((1-z)s)\right] + D_i \left[\alpha_s((1-z)^2 s)\right] \right\}$$

factorization scale μ_F **dependence controlled by**
$\gamma_{i/i} = -A_i \ln \tilde{N}_i + \gamma_i$

Noncollinear soft gluon emission controlled by the soft anomalous dimension Γ_S

determine Γ_S **from coefficients of ultraviolet poles in dimensionally regularized eikonal diagrams**

NNLO approximate cross sections

NLO expansion

Define $\mathcal{D}_k(s_4) \equiv \left[\frac{\ln^k(s_4/M^2)}{s_4}\right]_+$

$$\hat{\sigma}^{(1)} = \sigma^B \frac{\alpha_s(\mu_R)}{\pi} \{c_3\, \mathcal{D}_1(s_4) + c_2\, \mathcal{D}_0(s_4) + c_1\, \delta(s_4)\} + \frac{\alpha_s^{d_{\alpha_s}+1}(\mu_R^2)}{\pi} [A^c\, \mathcal{D}_0(s_4) + T_1^c\, \delta(s_4)]$$

where $c_3 = \sum_i 2\, A_i^{(1)} - \sum_j A_j^{(1)}$ **and** $c_2 = c_2^\mu + T_2$**, with** $c_2^\mu = -\sum_i A_i^{(1)} \ln\left(\frac{\mu_F^2}{M^2}\right)$

and

$$T_2 = \sum_i \left[-2\, A_i^{(1)} \ln\left(\frac{-t_i}{M^2}\right) + D_i^{(1)} - A_i^{(1)} \ln\left(\frac{M^2}{s}\right)\right] + \sum_j \left[B_j^{(1)} + D_j^{(1)} - A_j^{(1)} \ln\left(\frac{M^2}{s}\right)\right]$$

Also $A^c = \mathrm{tr}\left(H^{(0)}\Gamma_S^{(1)\,\dagger} S^{(0)} + H^{(0)} S^{(0)} \Gamma_S^{(1)}\right)$

$c_1 = c_1^\mu + T_1$**, with**

$$c_1^\mu = \sum_i \left[A_i^{(1)} \ln\left(\frac{-t_i}{M^2}\right) - \gamma_i^{(1)}\right] \ln\left(\frac{\mu_F^2}{M^2}\right) + d_{\alpha_s} \frac{\beta_0}{4} \ln\left(\frac{\mu_R^2}{M^2}\right)$$

NNLO expansion

$$\hat{\sigma}^{(2)} = \sigma^B \frac{\alpha_s^2(\mu_R)}{\pi^2} \left\{ \frac{1}{2} c_3^2 \, \mathcal{D}_3(s_4) + \left[\frac{3}{2} c_3 c_2 - \frac{\beta_0}{4} c_3 + \sum_j \frac{\beta_0}{8} A_j^{(1)} \right] \mathcal{D}_2(s_4) \right.$$

$$+ \left[c_3 c_1 + c_2^2 - \zeta_2 c_3^2 - \frac{\beta_0}{2} T_2 + \frac{\beta_0}{4} c_3 \ln\left(\frac{\mu_R^2}{M^2}\right) + \sum_i 2A_i^{(2)} - \sum_j A_j^{(2)} + \sum_j \frac{\beta_0}{4} B_j^{(1)} \right] \mathcal{D}_1(s_4)$$

$$+ \left[c_2 c_1 - \zeta_2 c_3 c_2 + \zeta_3 c_3^2 + \frac{\beta_0}{4} c_2 \ln\left(\frac{\mu_R^2}{s}\right) - \sum_i \frac{\beta_0}{2} A_i^{(1)} \ln^2\left(\frac{-t_i}{M^2}\right) \right.$$

$$+ \sum_i [\left(-2A_i^{(2)} + \frac{\beta_0}{2} D_i^{(1)}\right) \ln\left(\frac{-t_i}{M^2}\right) + D_i^{(2)} + \frac{\beta_0}{8} A_i^{(1)} \ln^2\left(\frac{\mu_F^2}{s}\right) - A_i^{(2)} \ln\left(\frac{\mu_F^2}{s}\right)]$$

$$\left. \left. + \sum_j [B_j^{(2)} + D_j^{(2)} - \left(A_j^{(2)} + \frac{\beta_0}{4}(B_j^{(1)} + 2D_j^{(1)})\right) \ln\left(\frac{M^2}{s}\right) + \frac{3\beta_0}{8} A_j^{(1)} \ln^2\left(\frac{M^2}{s}\right)] \right] \mathcal{D}_0(s_4) \right\}$$

$$+ \frac{\alpha_s^{d_{\alpha_s}+2}(\mu_R)}{\pi^2} \left\{ \frac{3}{2} c_3 A^c \, \mathcal{D}_2(s_4) + \left[\left(2c_2 - \frac{\beta_0}{2}\right) A^c + c_3 T_1^c + F^c \right] \mathcal{D}_1(s_4) \right.$$

$$\left. + \left[\left(c_1 - \zeta_2 c_3 + \frac{\beta_0}{4} \ln\left(\frac{\mu_R^2}{s}\right)\right) A^c + c_2 T_1^c + F^c \ln\left(\frac{M^2}{s}\right) + G^c \right] \mathcal{D}_0(s_4) \right\}$$

where

$$F^c = \mathrm{tr}\left[H^{(0)}\left(\Gamma_S^{(1)\,\dagger}\right)^2 S^{(0)} + H^{(0)} S^{(0)} \left(\Gamma_S^{(1)}\right)^2 + 2H^{(0)}\Gamma_S^{(1)\,\dagger} S^{(0)}\Gamma_S^{(1)}\right]$$

$$\begin{aligned} G^c \quad = \quad \mathrm{tr}\Big[& H^{(1)}\Gamma_S^{(1)\,\dagger} S^{(0)} + H^{(1)} S^{(0)}\Gamma_S^{(1)} + H^{(0)}\Gamma_S^{(1)\,\dagger} S^{(1)} + H^{(0)} S^{(1)}\Gamma_S^{(1)} \\ & + H^{(0)}\Gamma_S^{(2)\,\dagger} S^{(0)} + H^{(0)} S^{(0)}\Gamma_S^{(2)}\Big] \end{aligned}$$

and c_3**,** c_2**,** c_1**, etc are from the NLO expansion**

Two-loop universal quantities $A^{(2)}$**,** $B^{(2)}$**,** $D^{(2)}$ **known**

Two-loop process-dependent $\Gamma_S^{(2)}$ **recently calculated for several processes**

Eikonal approximation

Feynman rules for soft gluon emission simplify

$$\bar{u}(p)\,(-ig_sT_F^c)\,\gamma^\mu\,\frac{i(\not{p}+\not{k}+m)}{(p+k)^2-m^2+i\epsilon}\rightarrow\bar{u}(p)\,g_sT_F^c\,\gamma^\mu\,\frac{\not{p}+m}{2p\cdot k+i\epsilon}=\bar{u}(p)\,g_sT_F^c\,\frac{v^\mu}{v\cdot k+i\epsilon}$$

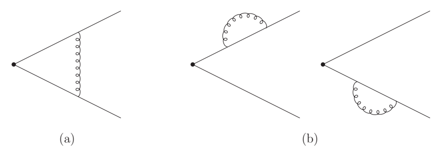
Perform calculations in momentum space and Feynman gauge

Complete two-loop results for

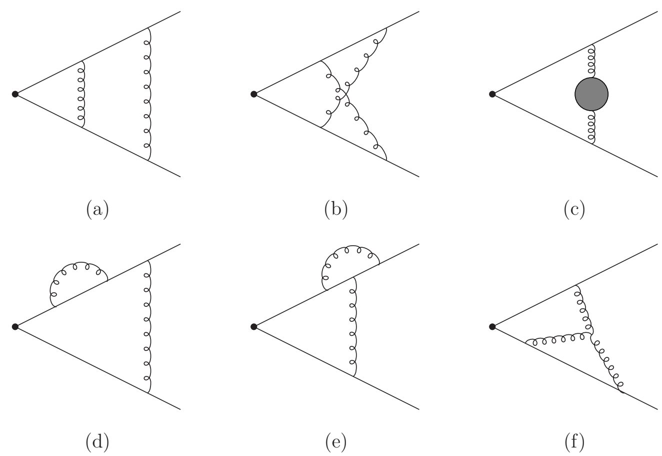
- **soft (cusp) anomalous dimension for** $e^+e^-\rightarrow t\bar{t}$
- $t\bar{t}$ **hadroproduction**
- t**-channel single top production**
- s**-channel single top production**
- $bg\rightarrow tW^-$ **and** $bg\rightarrow tH^-$
- **direct photon production**
- W **and** Z **production at large** p_T

Soft (cusp) anomalous dimension

One-loop eikonal diagrams



Two-loop eikonal vertex correction diagrams



NK, Phys. Rev. Lett. 102, 232003 (2009), arXiv:0903.2561 [hep-ph]

Color structure gets more complicated with more than two colored partons in the process - Cusp anomalous dimension an essential component of other calculations

Top-antitop production in hadron colliders

The soft anomalous dimension matrix for $q\bar{q} \rightarrow t\bar{t}$ is

$$\Gamma_{S\,q\bar{q}} = \left[\begin{array}{cc} \Gamma_{q\bar{q}\,11} & \Gamma_{q\bar{q}\,12} \\ \Gamma_{q\bar{q}\,21} & \Gamma_{q\bar{q}\,22} \end{array} \right]$$

At one loop

$$\Gamma^{(1)}_{q\bar{q}\,11} = -C_F\left[L_\beta + 1\right] \qquad \Gamma^{(1)}_{q\bar{q}\,21} = 2\ln\left(\frac{u_1}{t_1}\right) \qquad \Gamma^{(1)}_{q\bar{q}\,12} = \frac{C_F}{C_A}\ln\left(\frac{u_1}{t_1}\right)$$

$$\Gamma^{(1)}_{q\bar{q}\,22} = C_F\left[4\ln\left(\frac{u_1}{t_1}\right) - L_\beta - 1\right] + \frac{C_A}{2}\left[-3\ln\left(\frac{u_1}{t_1}\right) + \ln\left(\frac{t_1 u_1}{s m^2}\right) + L_\beta\right]$$

where $L_\beta = \frac{1+\beta^2}{2\beta}\ln\left(\frac{1-\beta}{1+\beta}\right)$ **with** $\beta = \sqrt{1-4m^2/s}$

Write the two-loop cusp anomalous dimension as $\Gamma_S^{(2)} = \frac{K}{2}\,\Gamma_S^{(1)} + C_F C_A M_\beta$**. Then at two loops**

$$\Gamma^{(2)}_{q\bar{q}\,11} = \frac{K}{2}\Gamma^{(1)}_{q\bar{q}\,11} + C_F C_A\, M_\beta \qquad \Gamma^{(2)}_{q\bar{q}\,22} = \frac{K}{2}\Gamma^{(1)}_{q\bar{q}\,22} + C_A\left(C_F - \frac{C_A}{2}\right) M_\beta$$

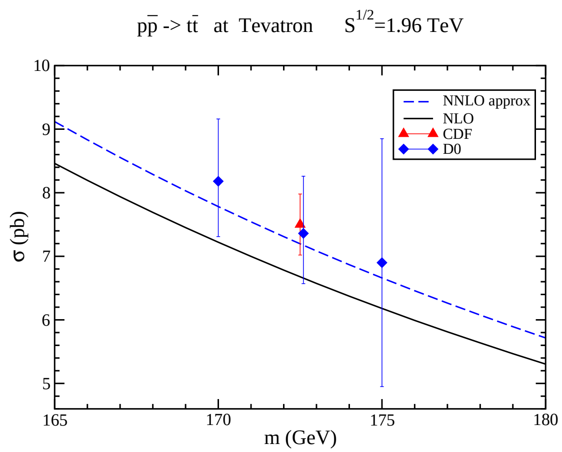
$$\Gamma^{(2)}_{q\bar{q}\,21} = \frac{K}{2}\Gamma^{(1)}_{q\bar{q}\,21} + C_A N_\beta \ln\left(\frac{u_1}{t_1}\right) \qquad \Gamma^{(2)}_{q\bar{q}\,12} = \frac{K}{2}\Gamma^{(1)}_{q\bar{q}\,12} - \frac{C_F}{2} N_\beta \ln\left(\frac{u_1}{t_1}\right)$$

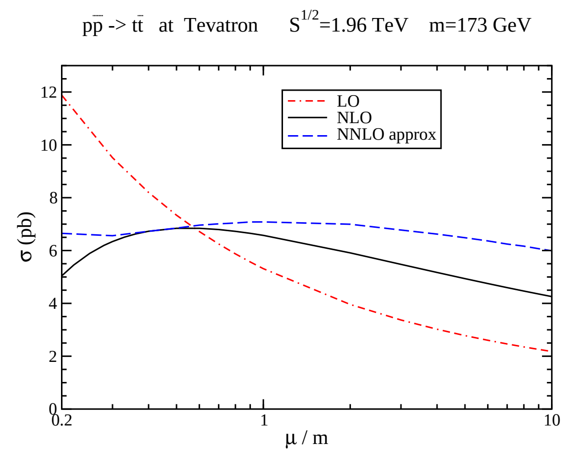
with N_β **a subset of terms of** M_β

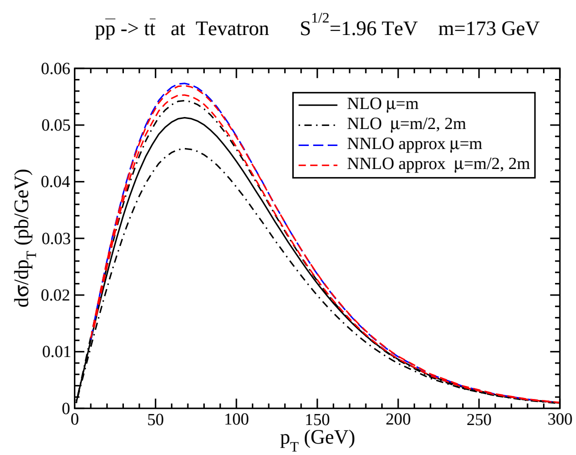
Similar results for $gg \rightarrow t\bar{t}$ channel

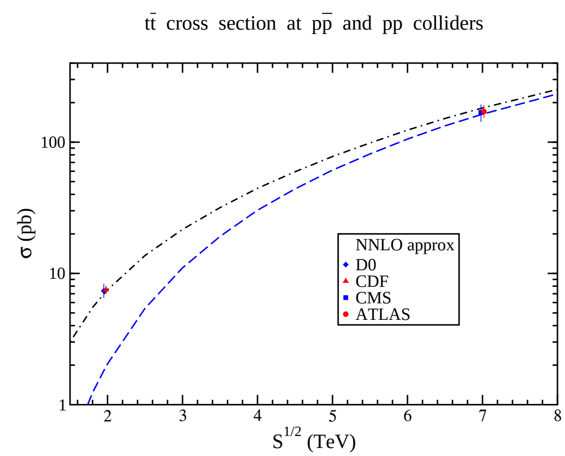
NK, Phys. Rev. D 82, 114030 (2010), arXiv:1009.4935 [hep-ph]

$t\bar{t}$ cross section at the Tevatron and LHC









Single top quark production - t channel

Dominant single top production channel at both Tevatron and LHC energies

Soft anomalous dimension for t-channel single top production

One loop

$$\Gamma_{S\,11}^{(1)} = C_F \left[\ln\left(\frac{-t}{s}\right) + \ln\left(\frac{m_t^2 - t}{m_t\sqrt{s}}\right) - \frac{1}{2}\right]$$

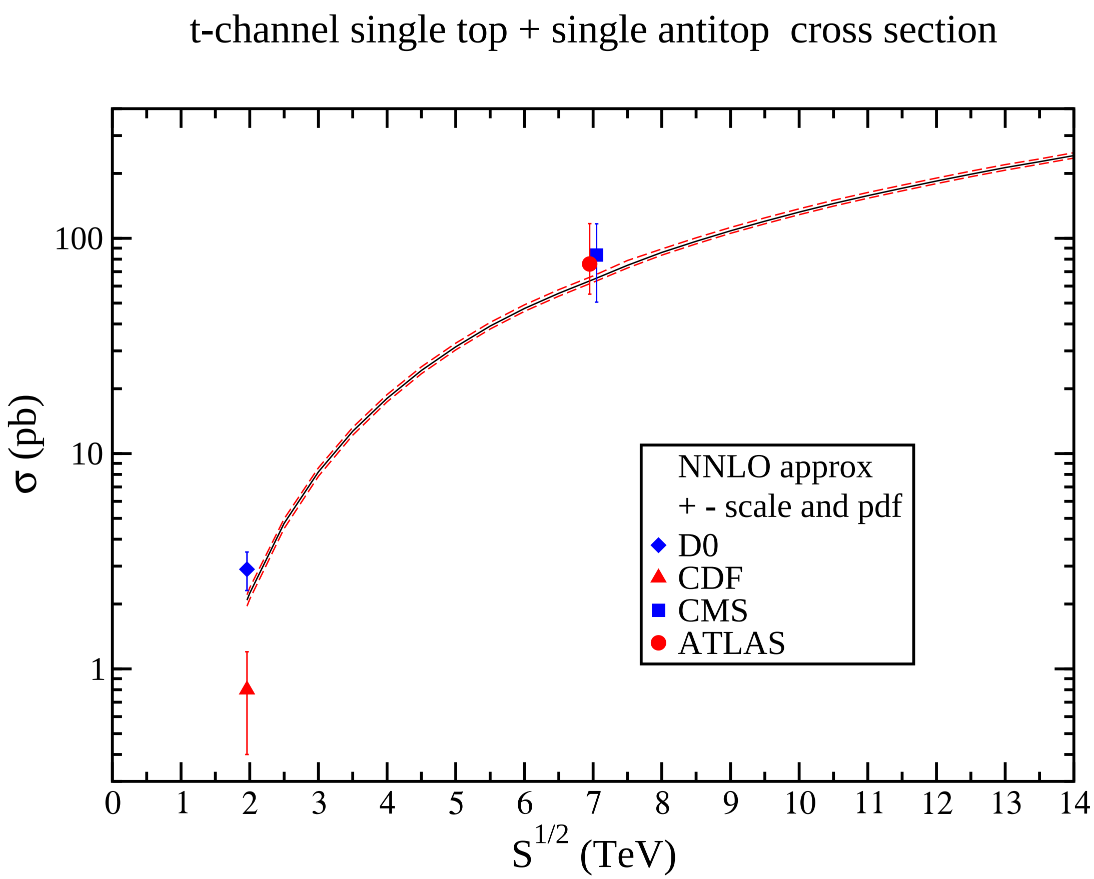
$$\Gamma_{S\,21}^{(1)} = \ln\left(\frac{u(u-m_t^2)}{s(s-m_t^2)}\right) \qquad \Gamma_{S\,12}^{(1)} = \frac{C_F}{2N_c}\,\Gamma_{S\,21}^{(1)}$$

Two loops

$$\Gamma_{S\,11}^{(2)} = \frac{K}{2}\Gamma_{S\,11}^{(1)} + C_F C_A \frac{(1-\zeta_3)}{4}$$

NK, Phys. Rev. D 83, 091503(R) (2011), arXiv:1103.2792 [hep-ph]

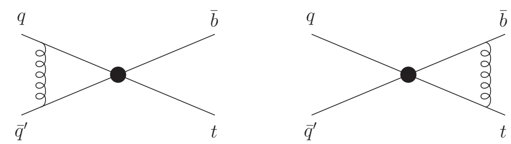
t-channel combined cross section versus energy



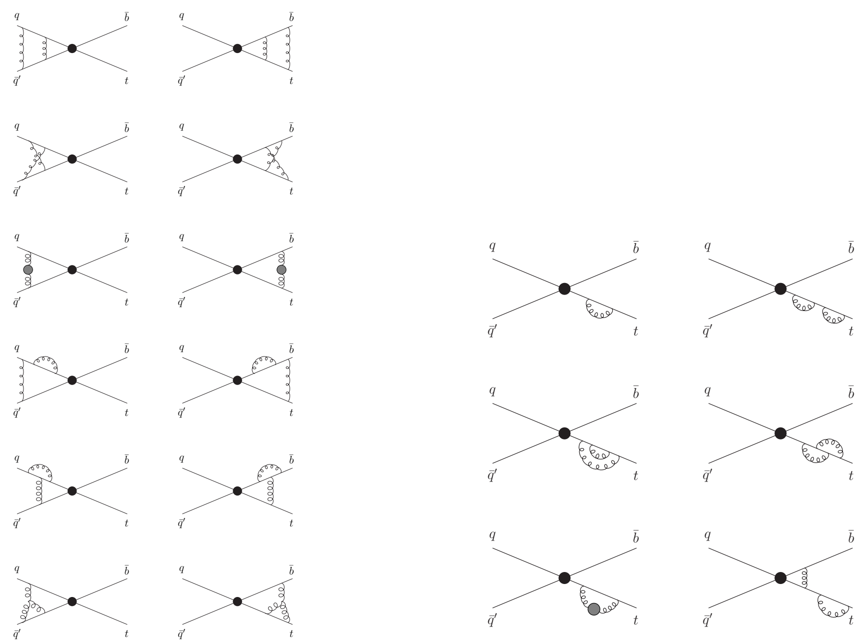
Tevatron and LHC results are consistent with theory

Single top quark production - s channel

One-loop eikonal diagrams



Two-loop eikonal diagrams



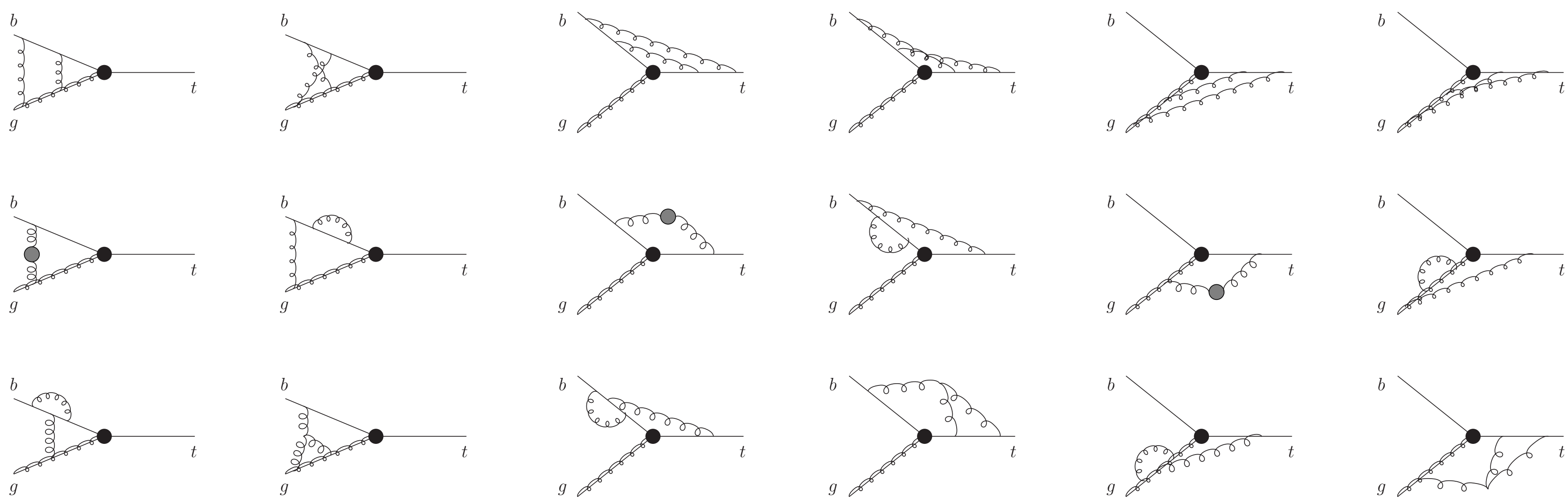
Soft anomalous dimension for s-channel single top production

$$\Gamma_{S\,11}^{(1)} = C_F \left[\ln \left(\frac{s - m_t^2}{m_t \sqrt{s}} \right) - \frac{1}{2} \right] , \qquad \Gamma_{S\,11}^{(2)} = \frac{K}{2} \Gamma_{S\,11}^{(1)} + C_F C_A \frac{(1-\zeta_3)}{4}$$

NK, Phys. Rev. D 81, 054028 (2010), arXiv:1001.5034 [hep-ph]

Associated production of a top quark with a W^- or H^-

Two-loop eikonal diagrams (+ top-quark self-energy graphs)



Soft anomalous dimension for $bg \to tW^-$ (or $bg \to tH^-$)

$$\Gamma_{S,\,tW^-}^{(1)} = C_F \left[\ln\left(\frac{m_t^2 - t}{m_t\sqrt{s}}\right) - \frac{1}{2}\right] + \frac{C_A}{2}\ln\left(\frac{m_t^2 - u}{m_t^2 - t}\right)$$

$$\Gamma_{S,\,tW^-}^{(2)} = \frac{K}{2}\Gamma_{S,\,tW^-}^{(1)} + C_F C_A \frac{(1-\zeta_3)}{4}$$

NK, Phys. Rev. D 82, 054018 (2010), arXiv:1005.4451 [hep-ph]

W, Z, and direct photon production at large p_T

Threshold corrections dominate at large transverse momentum
Two loop soft anomalous dimensions for NNLL resummation

For $qg \to Wq$ or $qg \to Zq$ or $qg \to \gamma q$

$$\Gamma^{(1)}_{S,\,qg\to Wq} = C_F \ln\left(\frac{-u}{s}\right) + \frac{C_A}{2}\ln\left(\frac{t}{u}\right)$$

$$\Gamma^{(2)}_{S,\,qg\to Wq} = \frac{K}{2}\Gamma^{(1)}_{S,\,qg\to Wq}$$

For $q\bar{q} \to Wg$ or $q\bar{q} \to Zg$ or $q\bar{q} \to \gamma g$

$$\Gamma^{(1)}_{S,\,q\bar{q}\to Wg} = \frac{C_A}{2}\ln\left(\frac{tu}{s^2}\right)$$

$$\Gamma^{(2)}_{S,\,q\bar{q}\to Wg} = \frac{K}{2}\Gamma^{(1)}_{S,\,q\bar{q}\to Wg}$$

Summary

- **Collinear and soft logarithms**
- **Factorization, RGE, and resummation**
- **Two-loop soft anomalous dimension matrices**
- **NNLL resummation**
- **Master formula for NNLO expansions**
- **Results for top quark production,** W/Z **production, and other processes**