Three-Pion Decays of the Tau Lepton, the $a_1(1260)$ Properties, and the $a_1\rho\pi$ Lagrangian

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My personal motivation of this study

Thermal photons and dileptons from the hadronic systems

- P.L. and L. Van Hove, PLB 245, 605 (1990), photons from cold QGP;
- J. Kapusta, P.L., D. Seibert, PRD 44, 2774 (1991), hadron gas shines as brightly as QGP at the same T;
- L. Xiong, E. Shuryak, G. Brown, PRD 46, 3798 (1992), reaction $\pi\rho \to a_1 \to \pi\gamma$, simple $\boldsymbol{a_1\rho\pi}$ Lagrangian;
- C. Song, PRC 47, 2861 (1993), a more detailed study, two sets of the $\boldsymbol{a_1\rho\pi}$ Lagrangian parameters, photon yields differred by more than two orders of magnitude;
- C. Gale and P.L., PRD 49, 3338 (1994), dileptons from meson gas.

Main properties of $a_1(1260)$

$$I^G = 1^-, J^P = 1^+$$

PDG	m_{a_1} (MeV)	Γ_{a_1} (MeV)	$a_1 \to \rho\pi$
1980	1100 to 1300	$\approx$ 300	dominant
1982	1275$\pm$30	315$\pm$45	dominant
1986	1275$\pm$28	316$\pm$45	dominant
1988	1260$\pm$30	300 to 600	dominant
1990	1260$\pm$30	350 to 500	dominant
1992	1260$\pm$30	$\approx$ 400	dominant
1994	1230$\pm$40	$\approx$ 400	dominant
1998	1230$\pm$40	250 to 600	dominant
2000	1230$\pm$40	250 to 600	seen
2010	1230$\pm$40	250 to 600	seen

$a_1(1260)$ in low-energy processes

- Hadronic processes

  diffractive production $\pi^- p \to \pi^- \pi^- \pi^+ p$
  charge-exchange reaction $\pi^- p \to \pi^+ \pi^- \pi^0 n$
  central production $pp \to p_{fast}(\pi^+ \pi^- \pi^0) p_{slow}$
  decays, e.g. $\rho(1700) \to a_1 \pi$

- Electromagnetic processes

  $e^+ e^- \to \pi^+ \pi^- \pi^+ \pi^-$
  $e^+ e^- \to \pi^+ \pi^- \pi^0 \pi^0$

- Weak decays

  $\tau^- \to \nu_\tau \pi^- \pi^- \pi^+$
  $\tau^- \to \nu_\tau \pi^- \pi^0 \pi^0$
  $\tau^- \to \nu_\tau \pi^- \pi^- \pi^+ \pi^0$
  $D^+ \to K_S^0 a_1^+$, etc.

$\tau^- \to \nu_\tau \pi^- \pi^- \pi^+$ & $\tau^- \to \nu_\tau \pi^- \pi^0 \pi^0$

Many different models, each characterized by its choice of the $a_1 \rho \pi$ Lagrangian (or vertex) and the a_1 propagator.

The a_1 mass and width are parameters entering the a_1 propagator.

Problems:

- The same model applied to different data $\Rightarrow$ incompatible results
- Different models applied to the same data $\Rightarrow$ incompatible results

The aim:

- To find a model which would be compatible with all (or most of) data sets.

Phenomenological $a_1\rho\pi$ Lagrangian

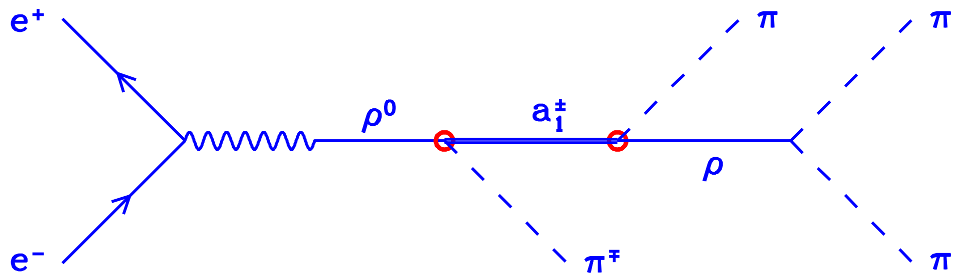
J. Wess, B. Zumino, Phys. Rev. 163, 1727 (1967)

$$\mathcal{L} = \frac{g_{a_1\rho\pi}}{\sqrt{2}}\left(\mathcal{L}_1 \cos\theta + \mathcal{L}_2 \sin\theta\right)$$

$$\mathcal{L}_1 = \boldsymbol{A}^\mu \cdot (\boldsymbol{V}_{\mu\nu} \times \partial^\nu \boldsymbol{\phi}), \quad \mathcal{L}_2 = \boldsymbol{V}_{\mu\nu} \cdot (\partial^\mu \boldsymbol{A}^\nu \times \boldsymbol{\phi})$$

	$\sin\theta$
Xiong, Shuryak, Brown, PRD 46, 3798 (1992)	0
C. Song, PRC 47, 2861 (1993), solution I	0.217
C. Song, PRC 47, 2861 (1993), solution II	0.631
Janssen, Holinde, Speth, PRC 49, 2763 (1994)	1
Faessler, Fuchs, Krivoruchenko, PRC 61, 35206	1
Turbide, Rapp, Gale, IJMPA 19, 5351 (2004)	0.558

Lagr. mixing angle from $e^+e^- \to 4\pi$



Eight diagrams in the $\pi^+\pi^-\pi^+\pi^-$ case,
four in the $\pi^+\pi^-\pi^0\pi^0$ case,
+ diagrams without a_1 (model dependent)

Only $\pi^+\pi^-\pi^+\pi^-$:
P.L. & J.Juráň, Phys. Rev. D 76, 094030 (2007)

$$\sin\theta = 0.460 \pm 0.003$$

Both channels:
J.Juráň & P.L., Phys. Rev. D 78, 017501 (2008)

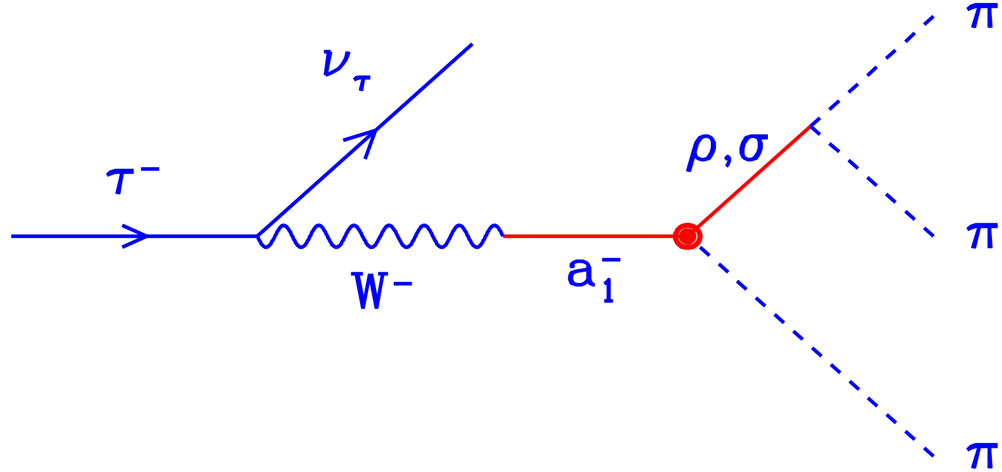
$$\sin\theta = 0.466 \pm 0.005$$

Model of $\tau^- \to 3\pi + \nu_\tau$

$$\tau^- \to a_1(1260)^- + \nu_\tau$$

followed by

$$a_1^- \to \rho(770) + \pi \to 3\pi \quad \text{or} \quad a_1^- \to \sigma(600) + \pi^- \to 3\pi$$



2 diagrams with ρ both in the $\pi^-\pi^+\pi^-$ and $\pi^-\pi^0\pi^0$ cases
2 diagrams with σ in the $\pi^-\pi^+\pi^-$ case
1 diagram with σ in the $\pi^-\pi^0\pi^0$ case

Main ingredients of the model

- Two-component $a_1\rho\pi$ Lagrangian shown above;
- Running mass $M_{a_1}(s)$ in the a_1 propagator ($s = p^2$)

$$-iG_{a_1}^{\mu\nu}(p) = \frac{-g^{\mu\nu} + p^\mu p^\nu / m_{a_1}^2}{s - M_{a_1}^2(s) + i m_{a_1}\Gamma_{a_1}(s)} \; ;$$

- $\Gamma_{a_1}(s)$ is a sum of the following decay widths:
  $a_1 \to \rho + \pi \to 3\pi, \quad a_1 \to \bar{K}^* + K \to K\bar{K}\pi,$
  $a_1 \to \sigma + \pi \to 3\pi, \quad a_1 \to K^* + \bar{K} \to K\bar{K}\pi;$
- Running mass from a once-subtracted dispersion relation with $\Gamma_{a_1}(s)$ as input

$$M_{a_1}^2(s) = M_{a_1}^2(0) - \frac{s}{\pi}\mathrm{P}\int_{9m_\pi^2}^{\infty} \frac{m_{a_1}\Gamma_{a_1}(s')}{s'(s'-s)} ds' \; ;$$

Main ingredients of the model, cont.

- $M^2_{a_1}(m^2_{a_1}) = m^2_{a_1}$ ... by fixing the subtraction constant;
- $\frac{d}{ds}M^2_{a_1}(m^2_{a_1}) = 0$ ... by fixing $x = (g_{a_1\sigma\pi}g_{\sigma\pi\pi})^2$, i.e. $\Gamma(a_1 \to 3\pi$ via $\sigma)$ (applicable for $\sin\theta \lesssim 0.5$);
- The same value of x is used in the τ decay diagrams with σ in the intermediate state;
- Running mass in the $\rho(770)$ propagator from P.L., Phys. Rev. D **60**, 053007 (1999) and its updated version;
- $\sigma(600)$ propagator with fixed mass $m_\sigma =$ 500 MeV and energy dependent width normalized to 329 MeV at m_σ. These values are weighted averages of values from E791 Coll., PRL 86, 770 (2001) and CLEO Coll., PRL 89, 251802 (2002);

Main ingredients of the model, cont.

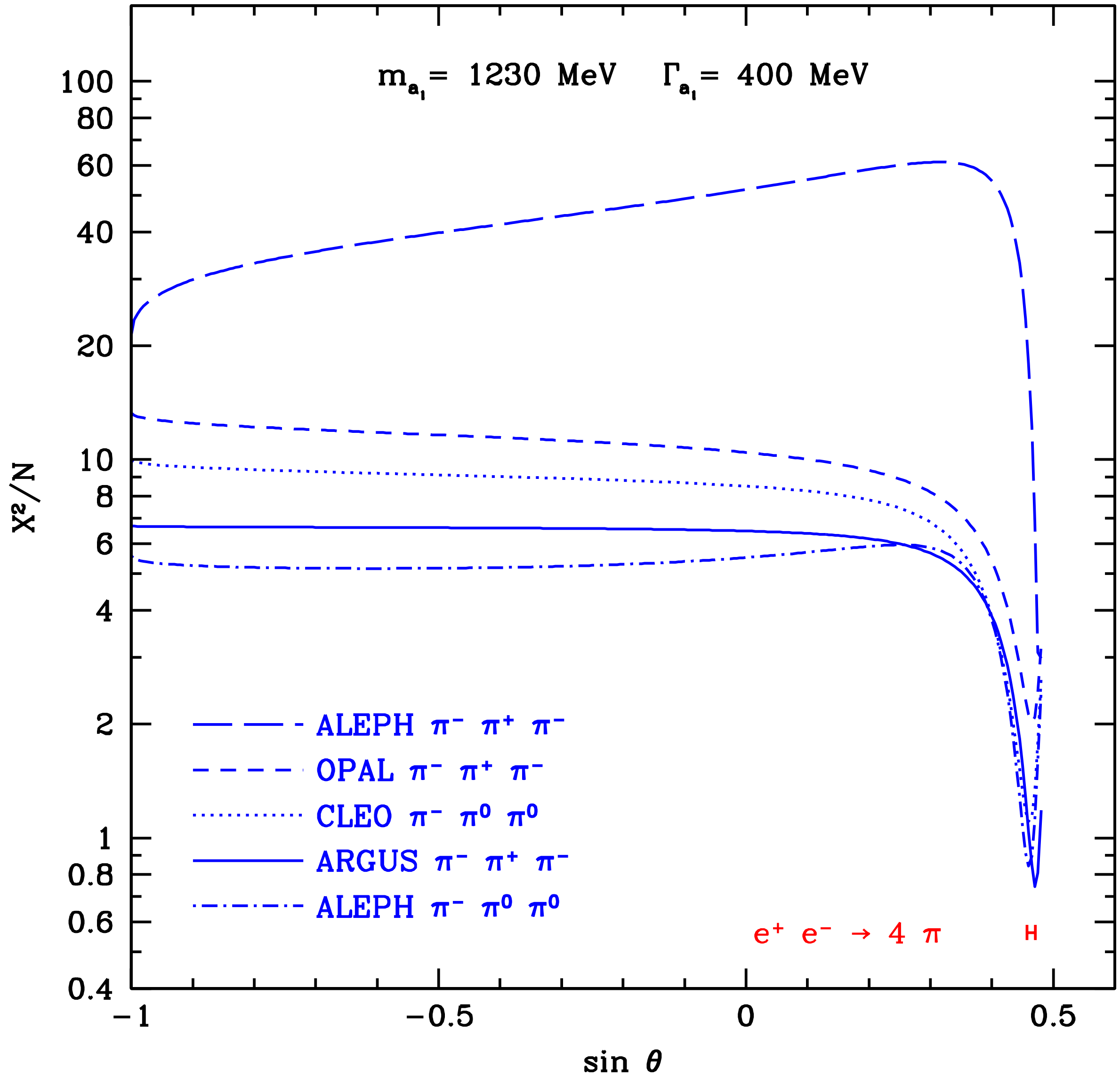
- Form factor in the strong interaction vertices taken from the chromoelectric flux-tube breaking model of Kokoski and Isgur [Phys. Rev. D **35**, 907 (1987)].

Experimental data explored

We use the three-pion mass distributions from the following sets of data:

- H. Albrecht et al. (ARGUS Coll.), Z. Phys. C **58**, 61 (1993) ($\pi^{\pm}\pi^{+}\pi^{-}$),
- K. Ackerstaff et al. (OPAL Coll.), Z. Phys. C **75**, 593 (1997) ($\pi^{\pm}\pi^{+}\pi^{-}$),
- D. Asner et al. (CLEO Coll.), Phys. Rev. D **61**, 012002 (1999) ($\pi^{\pm}\pi^{0}\pi^{0}$),
- S. Schael et al. (ALEPH Coll.), Physics Reports **421**, 191 (2005) ($\pi^{\pm}\pi^{+}\pi^{-}$),
- S. Schael et al. (ALEPH Coll.), Physics Reports **421**, 191 (2005) ($\pi^{\pm}\pi^{0}\pi^{0}$).

Individual χ^2/N versus $\sin\theta$



Results I

Data	χ^2/NDF	CL	m_{a_1}	Γ_{a_1}	$\sin\theta$
ALEPH$_1$	$119/111$	28.3	1220 ± 20	418 ± 40	0.460 ± 0.004
ALEPH$_2$	$51/111$	100	1256 ± 10	443 ± 15	0.466 ± 0.004
All sets	$358/321$	7.7	1232 ± 25	431 ± 25	0.463 ± 0.005
$e^+e^- \to \pi^+\pi^-\pi^+\pi^-$					0.460 ± 0.003
$e^+e^- \to \pi^+\pi^-\pi^+\pi^-$ & $\pi^+\pi^-\pi^0\pi^0$					0.466 ± 0.005

ALEPH$_1$ = ALEPH 2005, $\pi^-\pi^+\pi^-$ mode,

ALEPH$_2$ = ALEPH 2005, $\pi^-\pi^0\pi^0$ mode,

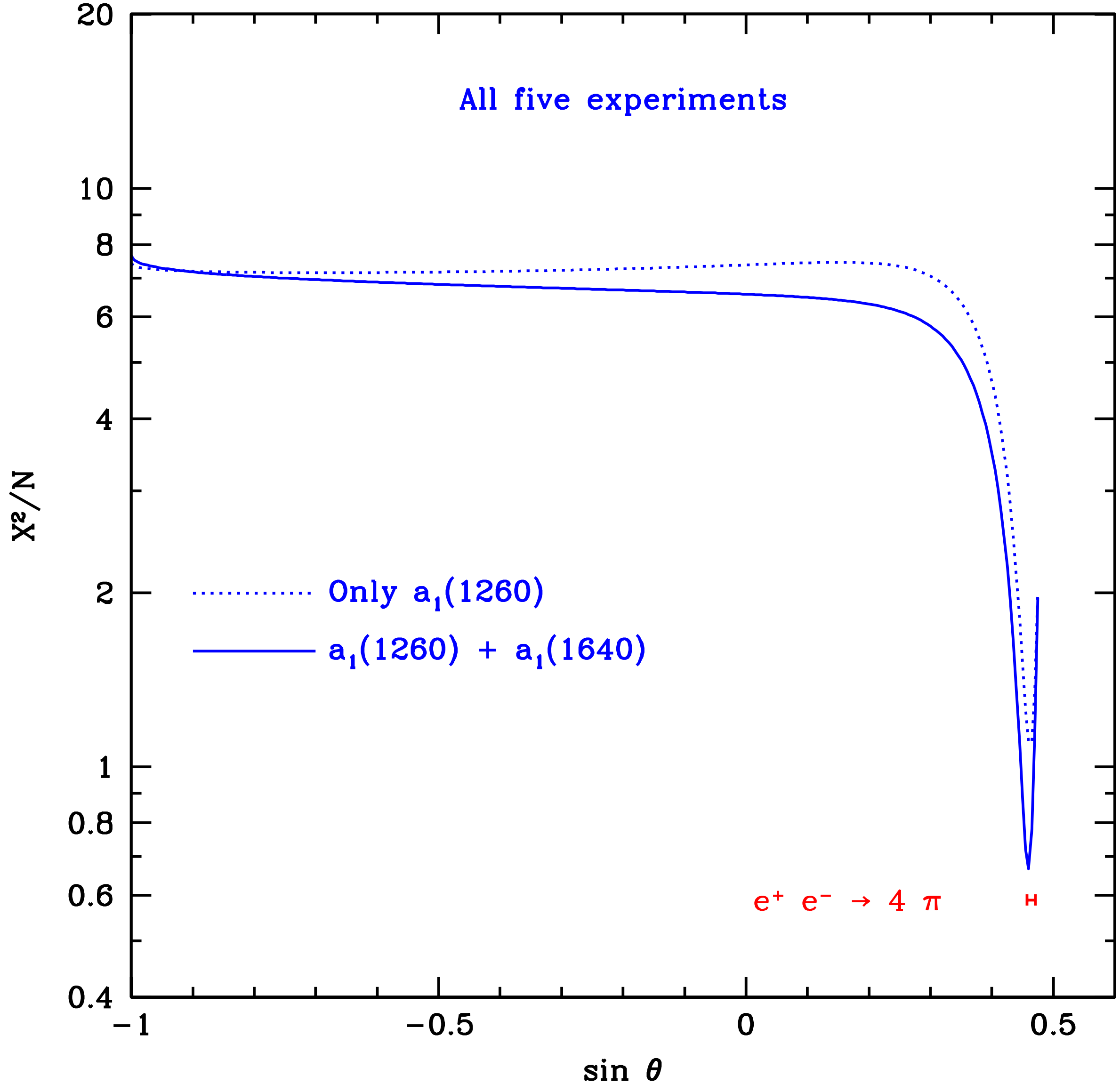
All sets = ARGUS + OPAL + CLEO + ALEPH$_1$ + ALEPH$_2$.

Results II: $a_1(1640)$ included

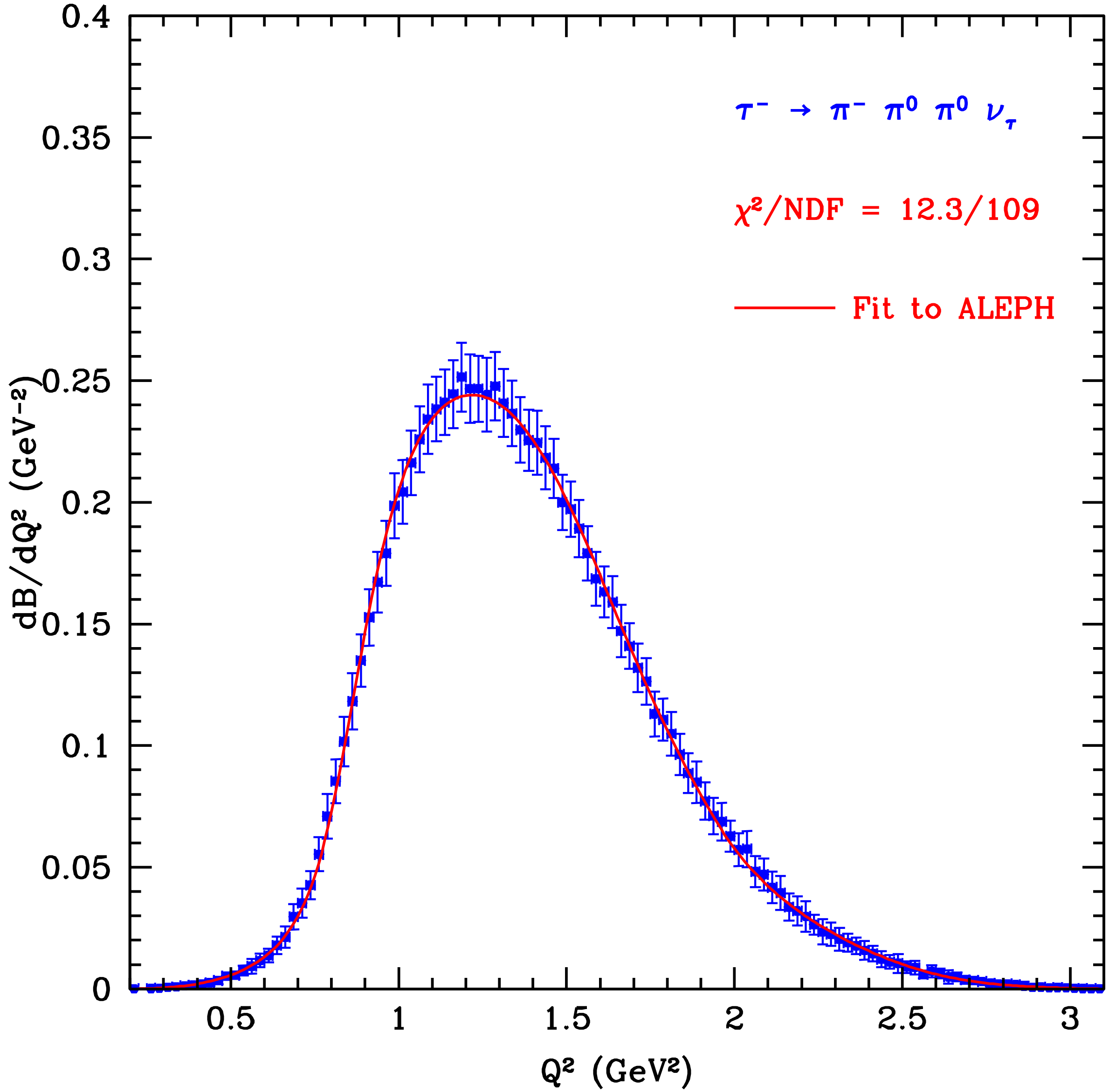
From PDG 2008:
$m_{a_1'} = 1647$ MeV
$\Gamma_{a_1'} = 254$ MeV.

Data	χ^2/NDF	CL	m_{a_1}	Γ_{a_1}	$\sin\theta$
ALEPH$_1$	$31/109$	100	1218 ± 19	418 ± 30	0.457 ± 0.004
ALEPH$_2$	$12/109$	100	1255 ± 18	455 ± 15	0.457 ± 0.006
All sets	$220/318$	100	1233 ± 18	431 ± 20	0.459 ± 0.004
$e^+e^- \to \pi^+\pi^-\pi^+\pi^-$					0.460 ± 0.003
$e^+e^- \to \pi^+\pi^-\pi^+\pi^-$ & $\pi^+\pi^-\pi^0\pi^0$					0.466 ± 0.005

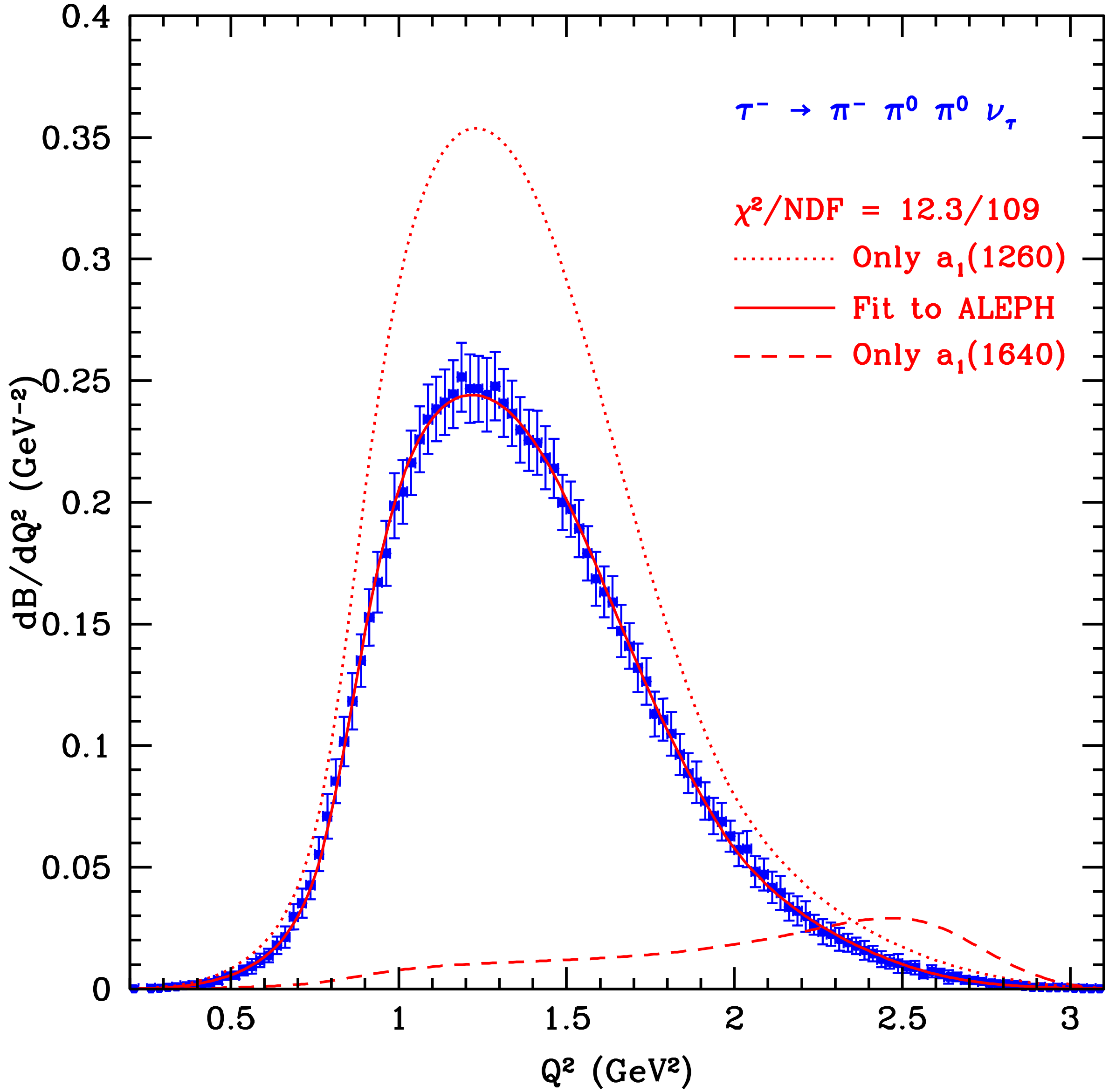
Total χ^2/N versus $\sin\theta$



$a_1(1260) + a_1(1640)$ fit to ALEPH data



$a_1(1260) + a_1(1640)$ fit to ALEPH data



Summary

- Our phenomenological Lagrangian leads to good fits of both $e^+e^- \to 4\pi$ and $\tau \to \nu_\tau + 3\pi$ data with the same value of the mixing parameter $\sin\theta$;
- The existence of the $a_1(1640)$ resonance is confirmed with the mass and width compatible with the PDG values;
- An explanation is found why the $a_1(1640)$ resonance is not in seen as a bump or shoulder in the 3π mass spectrum;
- From the common fit to the data from five experiments we have obtained the following results:
  Mass of the $a_1(1260)$ $m_{a_1} = (1233 \pm 18)$ MeV;
  Width of the $a_1(1260)$ $\Gamma_{a_1} = (431 \pm 20)$ MeV;
  Lagrangian mixing parameter $\sin\theta = 0.459 \pm 0.004$.