The Fermion Sign Problem at Finite Density and Large N Orbifold Equivalence

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Sign Problem

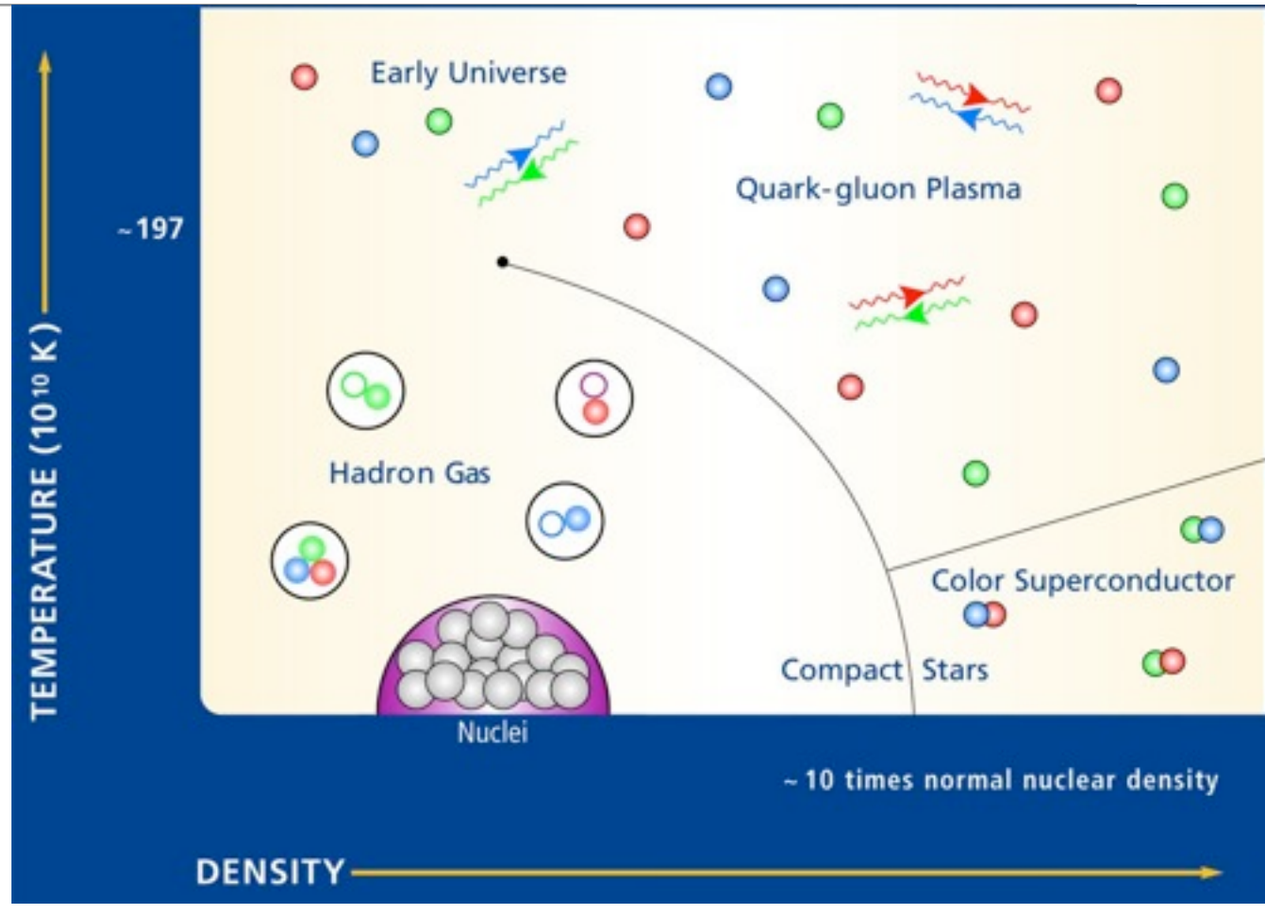
- QCD at finite baryon density:
  nuclei, nuclear matter, neutron stars, ...

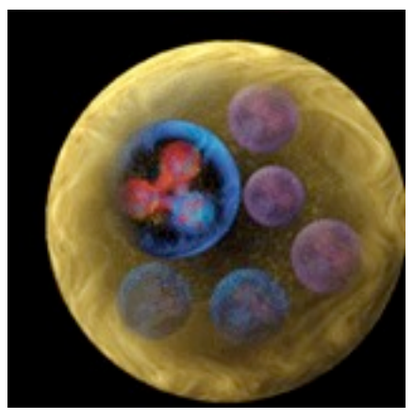
- Impasse: sign problem

$$\sum_{\{A_\mu\}} \mathrm{Det}\left(\not{D} + m + \mu_B \gamma_4\right) e^{-S[A_\mu]}$$

Importance sampling

$$\mathrm{Det}(0) > 0 \qquad \mathrm{Det}(\mu_B) \in \mathbb{C}$$





- Attempt to solve QCD sign problem
- Build nuclei three quarks at a time
- Model QCD at finite density
- QCD-like theories without sign problems

Sign Problem & Orbifold Equivalence

- QCD & QCD-like theories at finite density
  How like QCD are QCD-like theories?

$$SU(3) + \mu_B \longrightarrow SU(N_c) + \mu_B$$

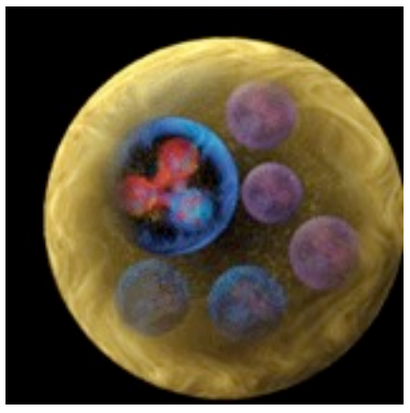
- Large N_c orbifold equivalence at finite baryon density
  New insight from large N_c limit

- Perturbative vs. non-perturbative equivalence
  Breakdown... double trace deformation?

E.g.

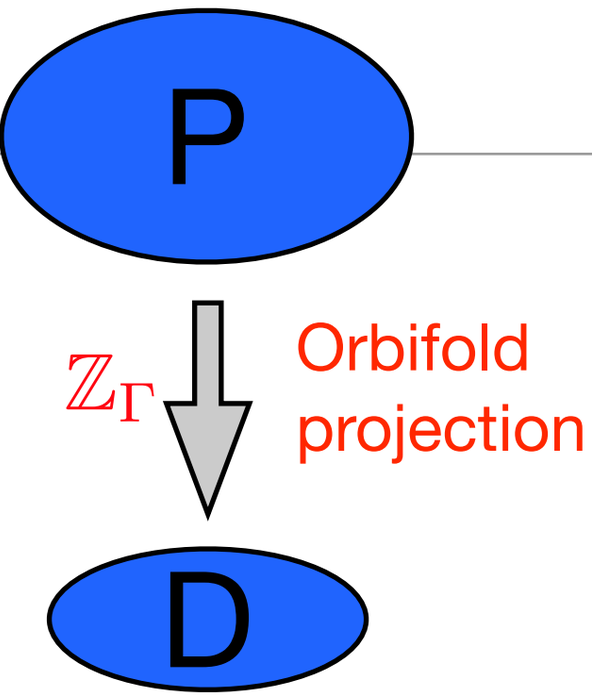
$$SU(2) + \mu_B$$
$$SU(N_c) + \mu_I \ *$$
$$SU(N_c)_{\rm Adj} + \mu_B$$
$$SO(2N_c) + \mu_B \ *$$
$$Sp(2N_c) + \mu_B$$
...

Cherman, Hanada and Robles-Llana, PRL **106**: 091603 (2011)
Cherman and Tiburzi, JHEP **1106**: 34 (2011)
Hanada and Yamamoto, arXiv:1103.5480



Orbifolding Gauge Theories

- Discrete global symmetry of (Parent) theory $\mathbb{Z}_\Gamma$
- Form (Daughter) theory from invariant *fields*



- In general, gauge and matter content differ between Parent & Daughter theories

Large Nc Orbifold Equivalence

Karchu, Silverstein, PRL **80**: 4855 (1998)
Bershadsky, Johansen, NuPhB **536**: 141 (1998)

$$\langle \ldots \phi_P^{Q_\Gamma=0} \ldots \rangle \overset{N_c\to\infty}{=\!=} \langle \ldots \varphi_D \ldots \rangle$$

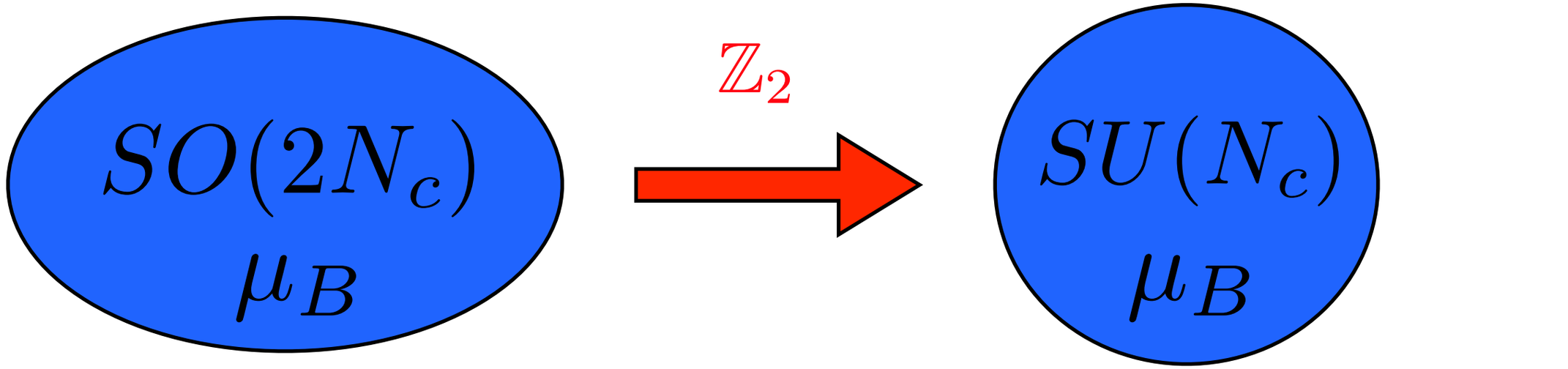
- Planar equivalence of neutral operator correlation functions (these are generally a subset of the correlators in the Daughter theory)

Embed Sign Problem in Orbifold Projection

- Start with sign-problem-free theory, project onto sign-problematic theory

$$\mathbb{Z}_2 : \begin{pmatrix} 0 & 1_{N_c} \\ -1_{N_c} & 0 \end{pmatrix} \otimes e^{i\pi/2} \in SO(2N_c) \otimes U(1)_B$$

- Baryon number transformation **must** be involved



- Orbifold equivalence holds to all orders in perturbation theory...

Cherman, Hanada and Robles-Llana, PRL **106**: 091603 (2011)

Non-Perturbative Effects

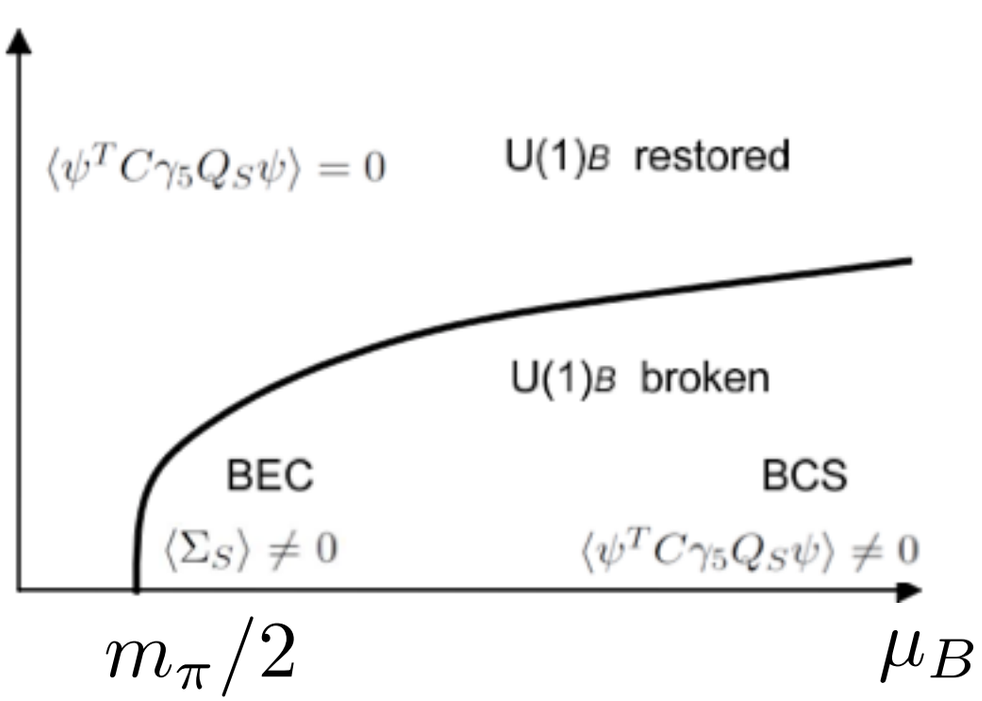
Baryon number can be spontaneously broken in SO(N_c) theories

QCD-type baryons: N_c valence fermions $\quad \epsilon_{ijk\ldots}\,\psi_i\psi_j\psi_k\cdots$

Non QCD-type baryons: 2 valence fermions $\quad \psi_i^T\psi_i$ $\quad$ * are Goldstone modes along with pions @ zero density

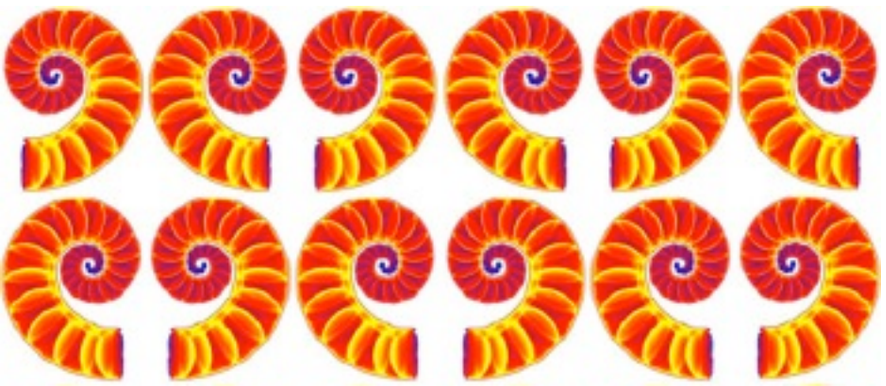
$\mathbb{Z}_2$

$$\psi_i^T\psi_i \longrightarrow -\psi_i^T\psi_i$$



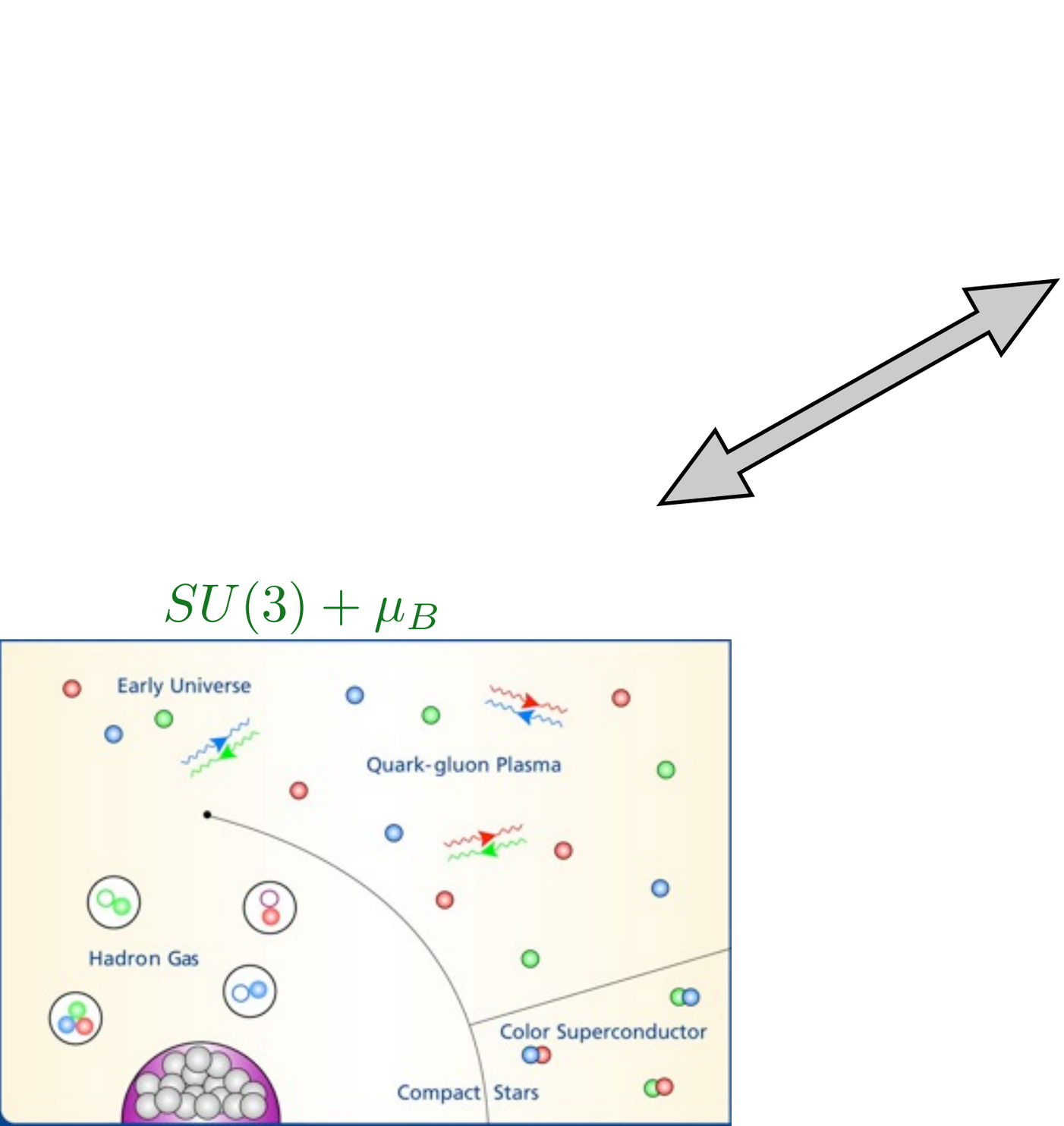
These baryonic pions condense @

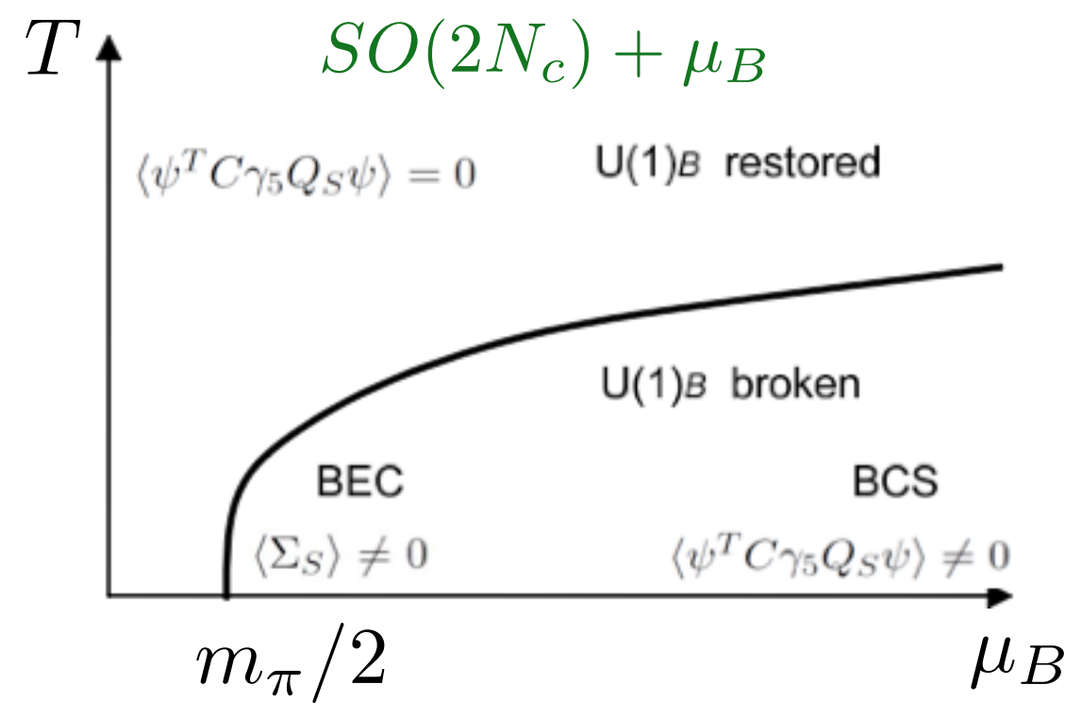
$$\mu_B = \frac{m_\pi}{2}$$

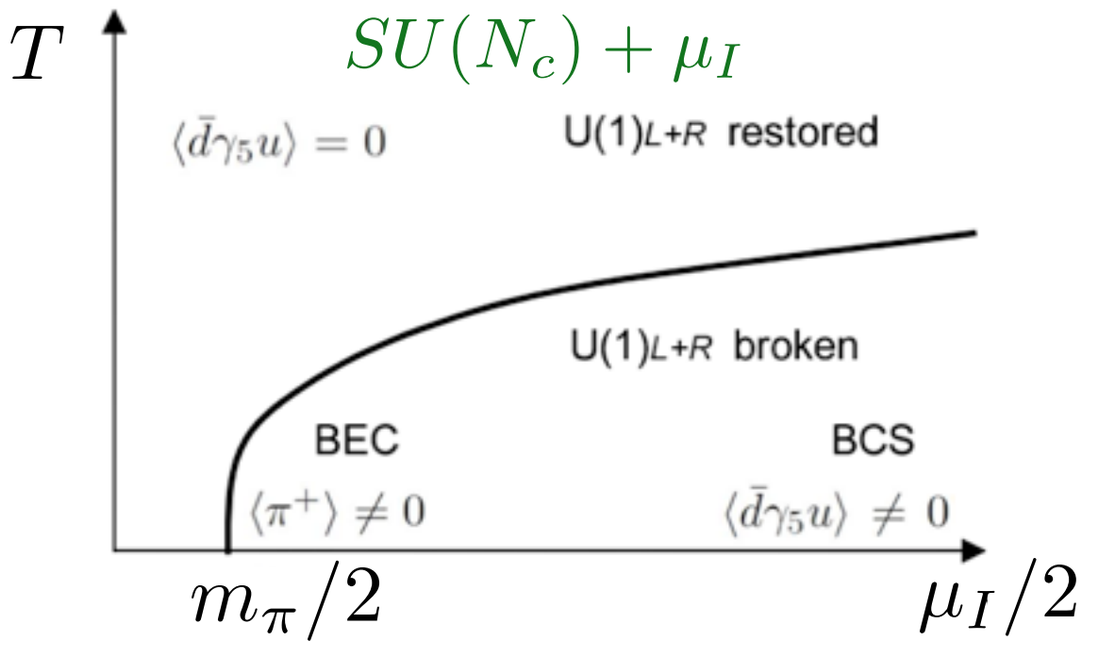


No conservation of $\mathbb{Z}_2$ charge means no QCD equivalence

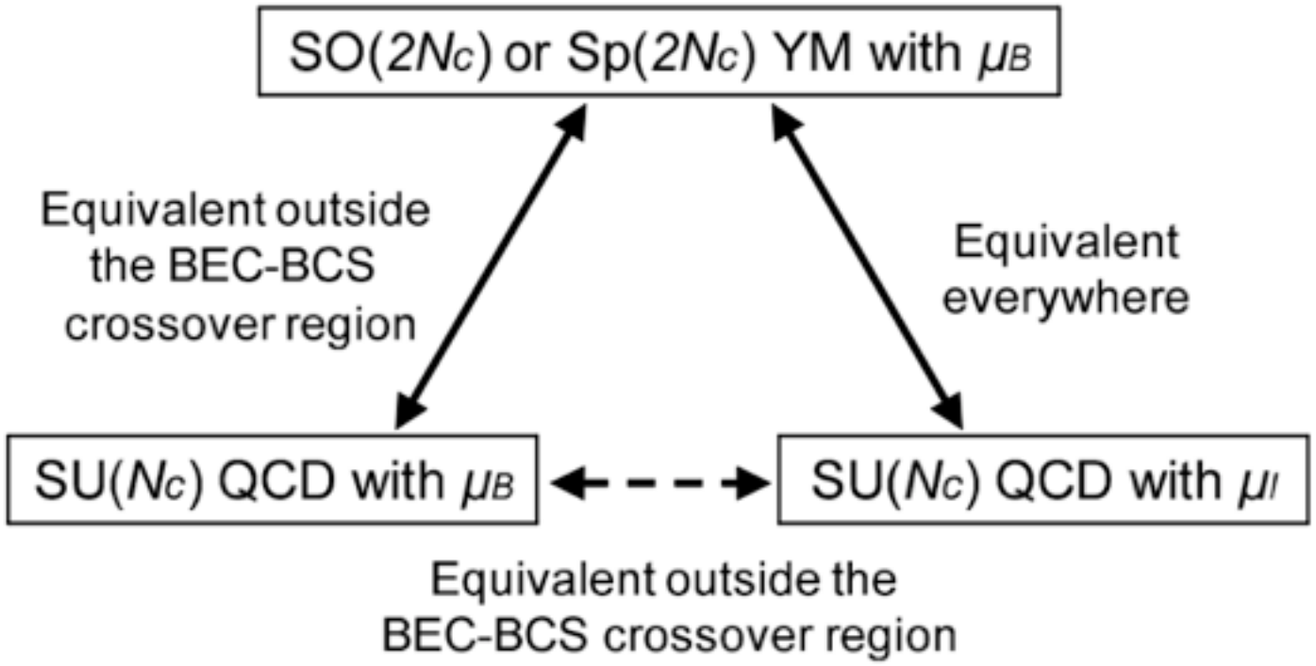
Non-Perturbative Equivalences

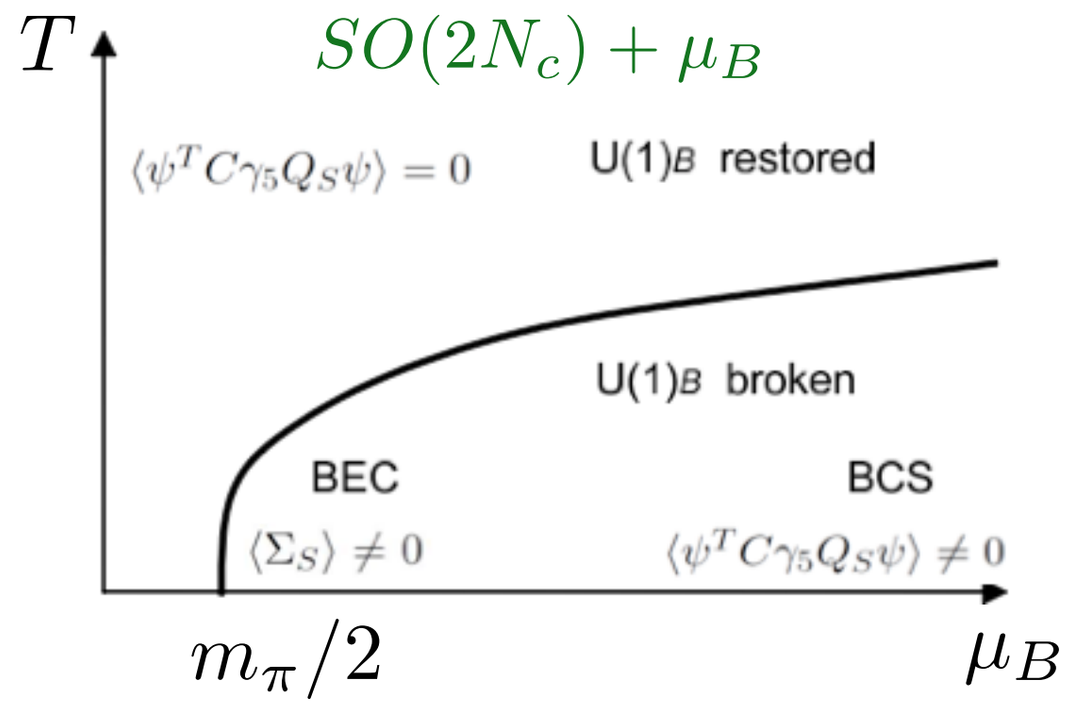




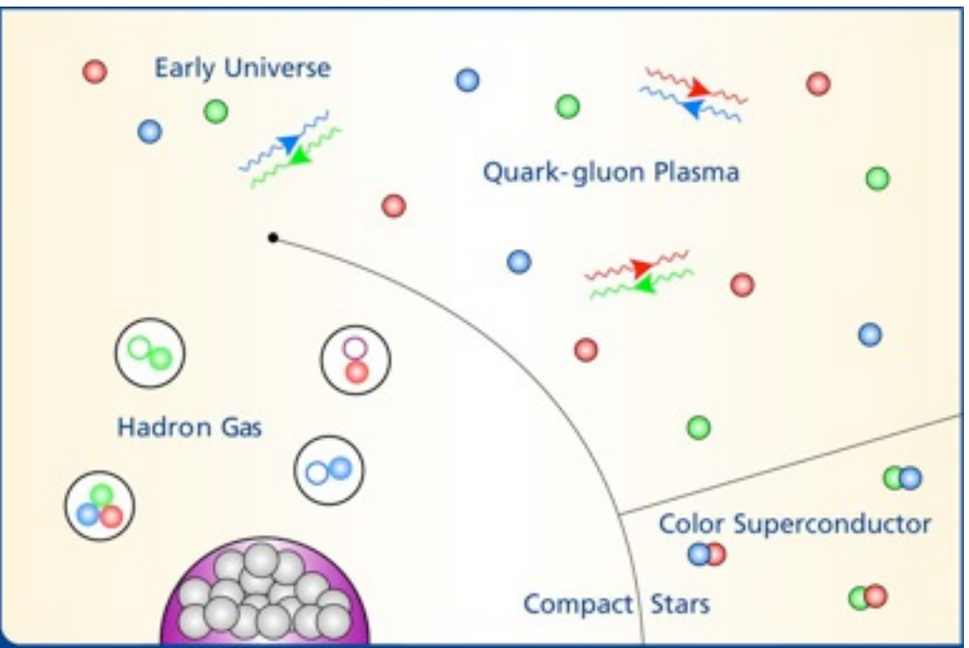


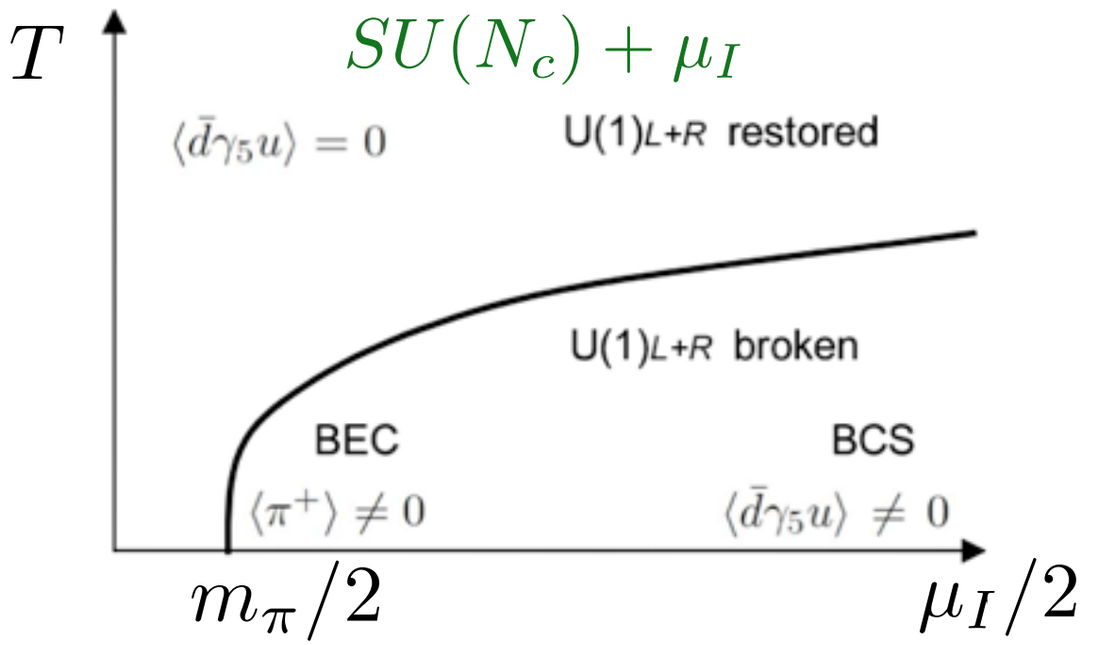
Non-Perturbative Equivalences



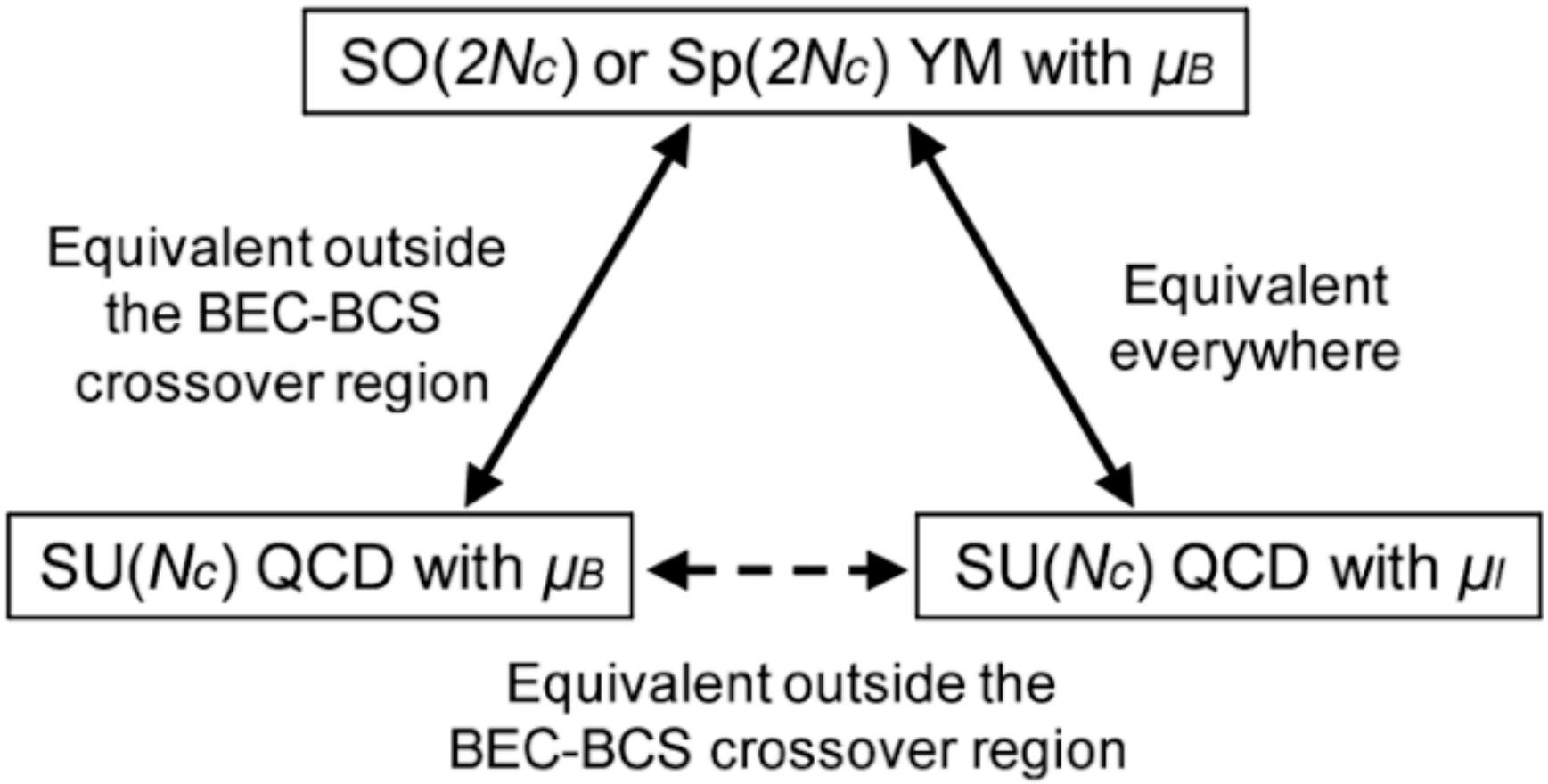


$SU(3) + \mu_B$





Non-Perturbative Equivalences (Conjectured)

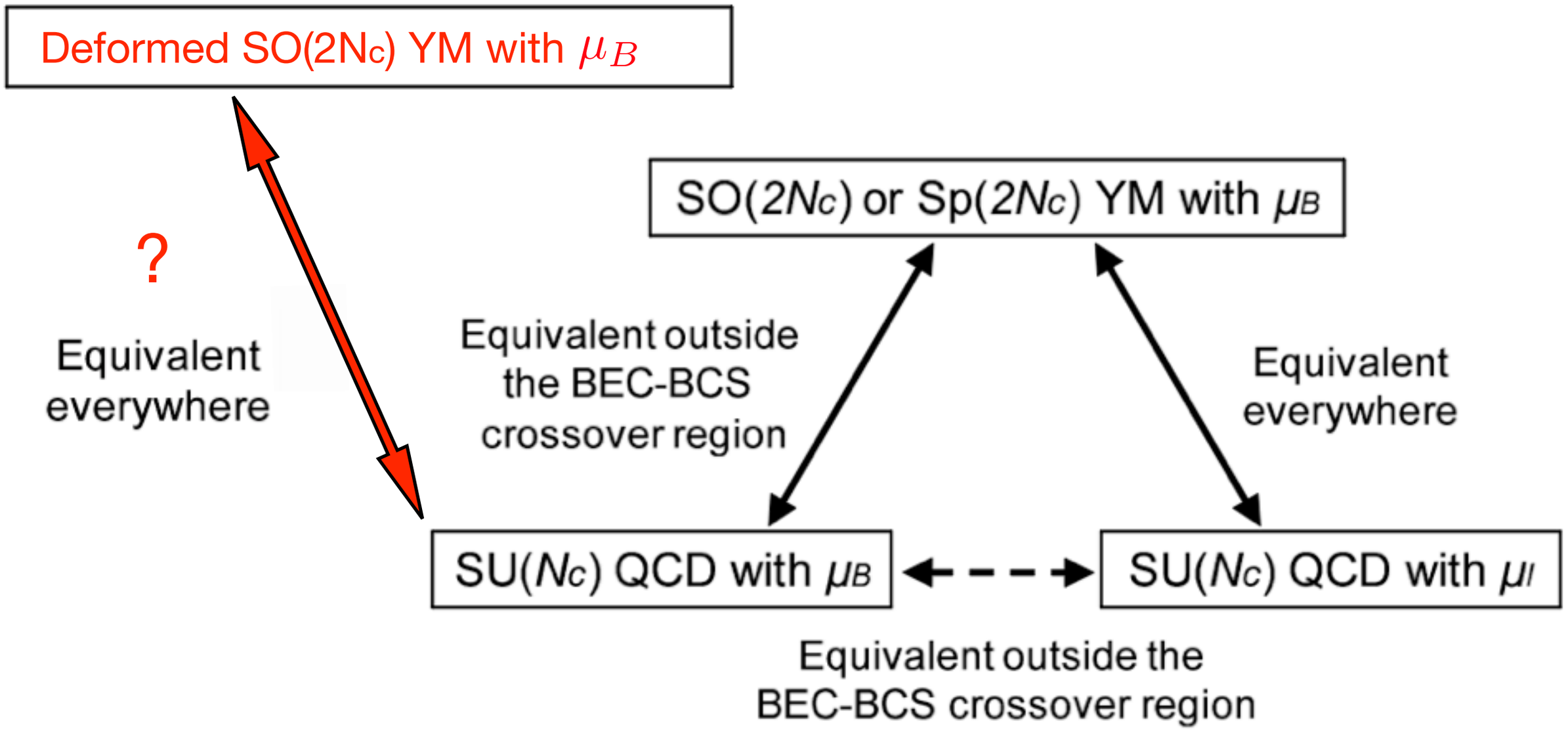


Non-perturbative checks:

ChPT — Cherman and Tiburzi, JHEP **1106**: 34 (2011)

ChRMT, BCS gap — Hanada and Yamamoto, arXiv:1103.5480

Non-Perturbative Equivalences: A New Hope?



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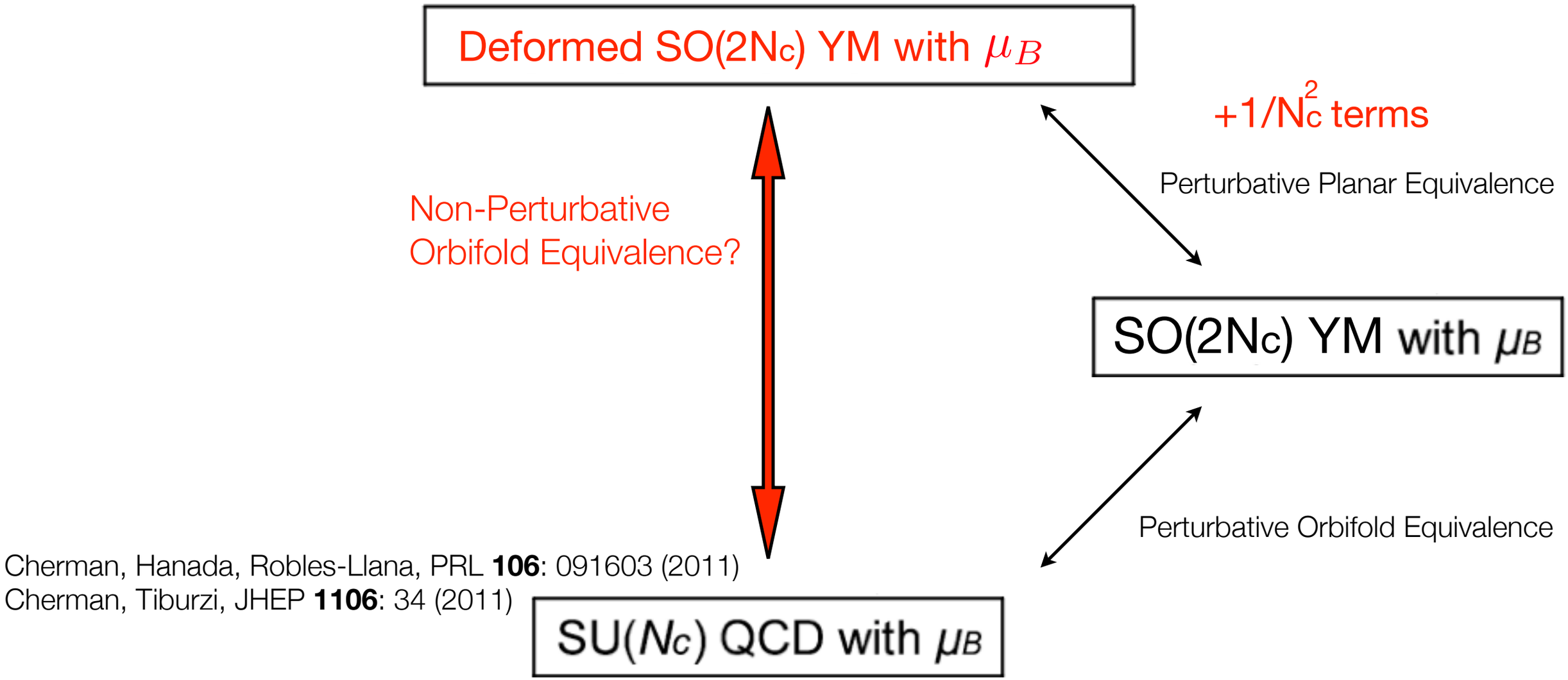


Cherman, Hanada, Robles-Llana, PRL **106**: 091603 (2011)
Cherman, Tiburzi, JHEP **1106**: 34 (2011)

Inspiration: Progress in volume reduction of YM using deformations

Ünsal and Yaffe, PRD **78**: 065035 (2008)

Being tested on the lattice

Bringoltz and Sharpe, PRD **80**: 065031 (2009)

Hietanen and Narayanan, JHEP **1001**: 79 (2010)

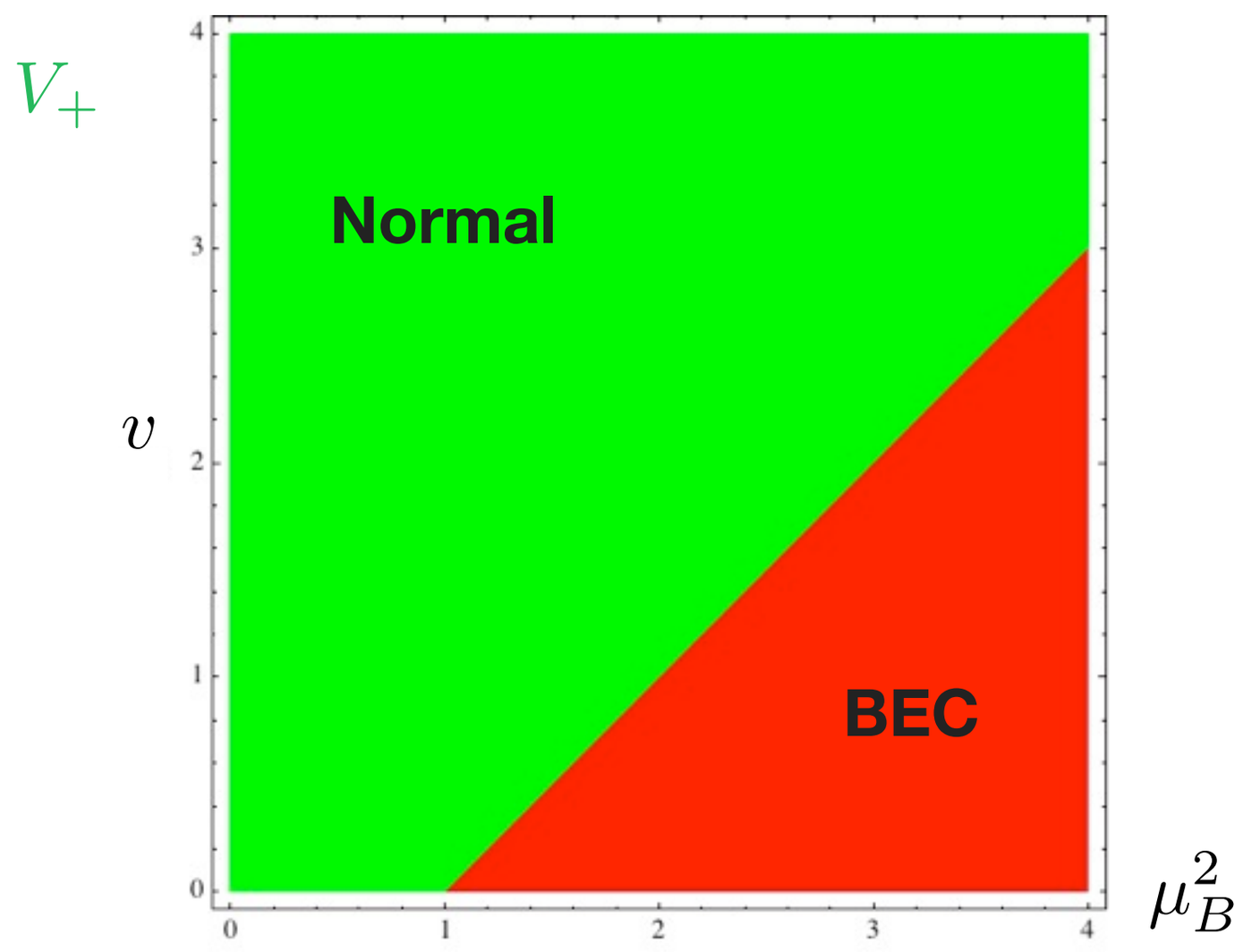
Phases of the Deformed Theory

Cherman and Tiburzi, JHEP **1106**: 34 (2011)

- Make scalar diquark scattering repulsive to penalize condensation

$$V_\pm = v\left[\left|\psi^T C\gamma_5\psi\right|^2 \pm \left|\psi^T C\psi\right|^2\right]$$ without BEC, U(1)B remains unbroken

- Treat effects of deformation systematically using low-energy EFT



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- Treat effects of deformation systematically using low-energy EFT

V_-

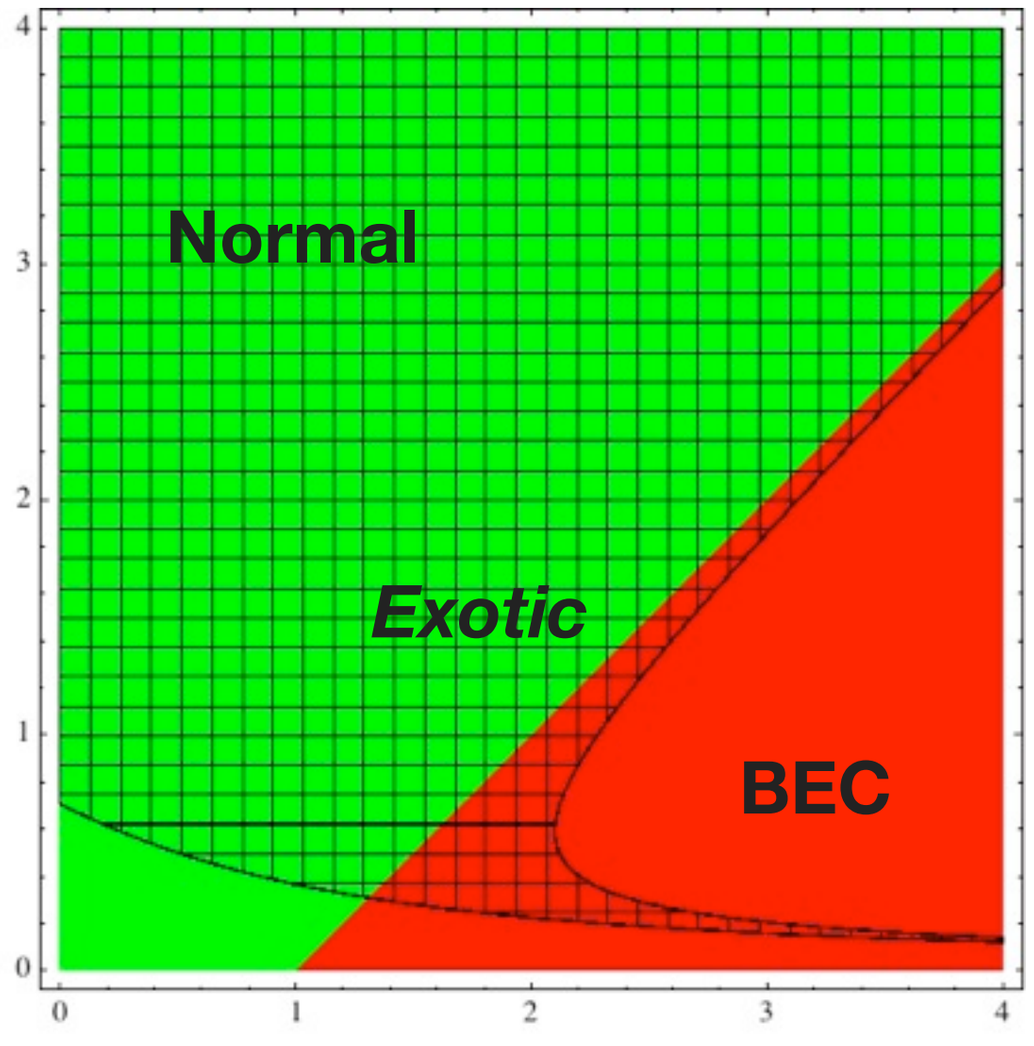
Exotic phase

Large Nc artifact

Flavor singlet Goldstone

$\langle\eta'\rangle \neq 0$

But always metastable



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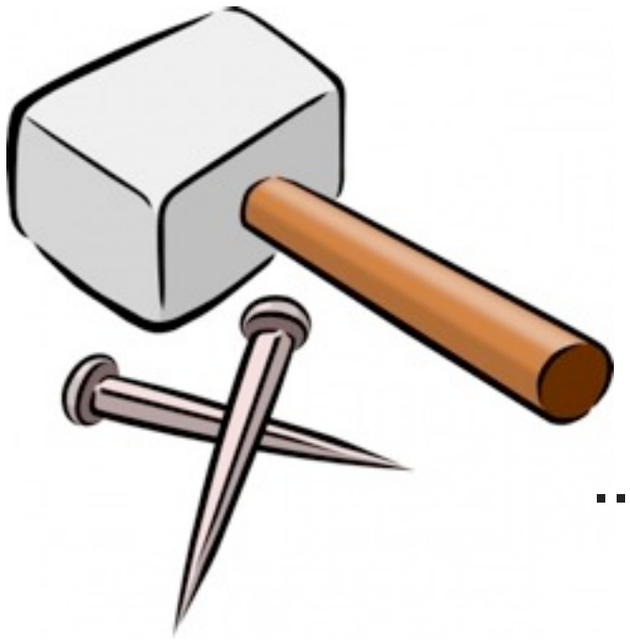
Both deformations work (at least in limited reach of the EFT)



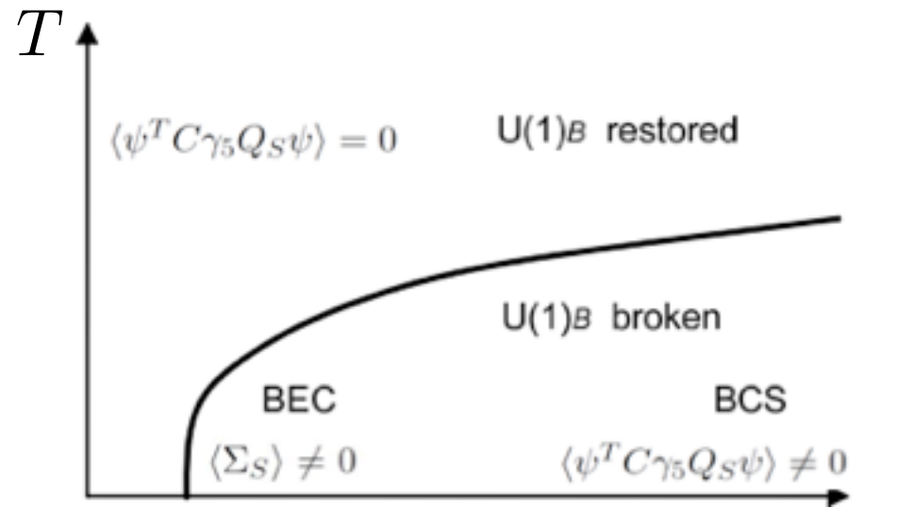
...of course, few non-perturbative tools are available!

A New Hope or a New Hoax?

- Large N_C orbifold equivalences organize information about sign-problem and sign-problem-free theories

Chiral transition at finite density from orbifold equivalence

Hanada and Yamamoto, arXiv:1103.5480



- Deformations needed to relate sign-free theories to sign-plagued theories inside the BEC-BCS region & EFT analysis shows efficacious

Cherman, Tiburzi, JHEP **1106**: 34 (2011)

- Interesting physics lies beyond reach of EFT: need lattice Monte Carlo

Monte Carlo requires auxiliary fields to include deformations

$$e^{-x^4} = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{+\infty} e^{-y^2 - 2iyx^2}\, dy$$

Only known how to implement V- deformation sign-problem-free in chiral limit