

# Two-loop corrections to $W$ and $Z$ boson production at high $p_T$

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- Partonic production channels at LO
- NLO corrections
- Soft-gluon corrections
- Two-loop calculations and NNLL resummation
- Approximate NNLO  $p_T$  distribution at LHC and Tevatron

In collaboration with R. J. Gonsalves

## **$W$ and $Z$ production at large $p_T$ - parton processes**

$W$  and  $Z$  hadroproduction useful in testing the SM and in estimates of backgrounds to Higgs production and new physics (new gauge bosons)

$p_T$  distribution falls rapidly as  $p_T$  increases

### **Partonic channels at LO**

$$q(p_a) + g(p_b) \longrightarrow W(Q) + q(p_c)$$

$$q(p_a) + \bar{q}(p_b) \longrightarrow W(Q) + g(p_c)$$

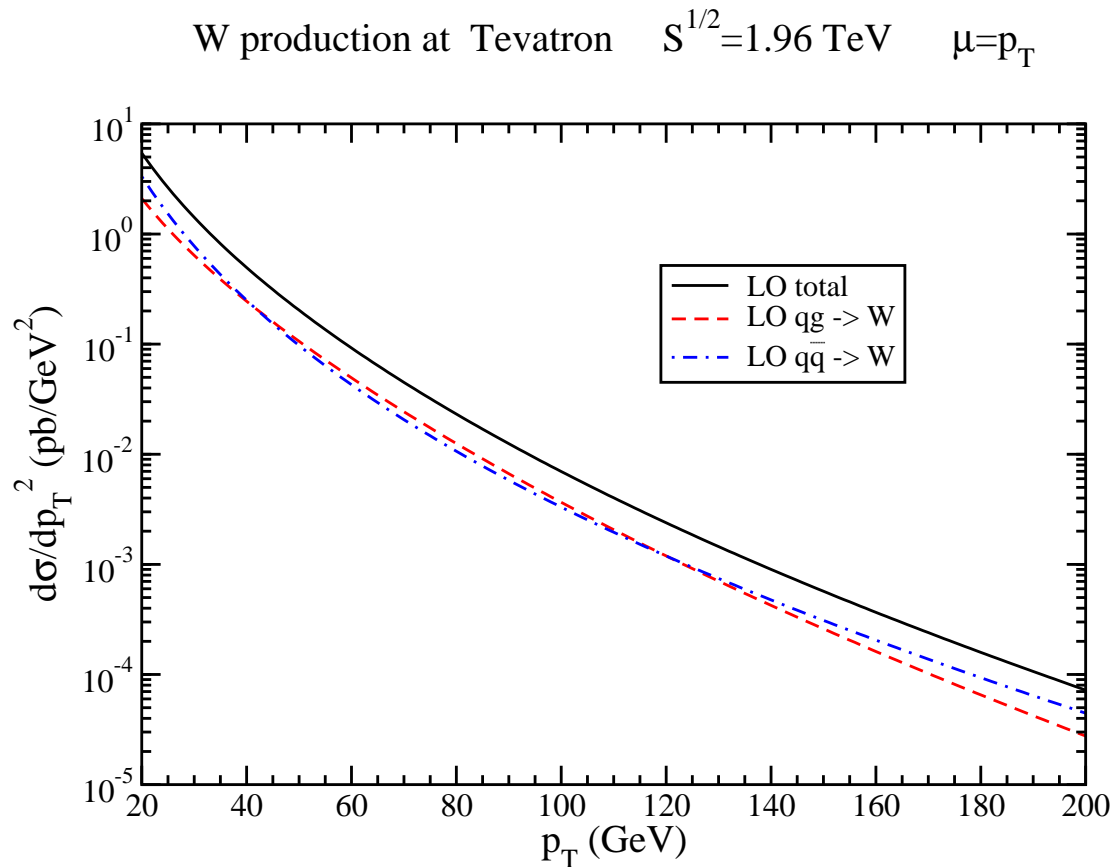
Define  $s = (p_a + p_b)^2$ ,  $t = (p_a - Q)^2$ ,  $u = (p_b - Q)^2$  and  $s_4 = s + t + u - Q^2$

At threshold  $s_4 \rightarrow 0$

**Soft corrections**  $\left[ \frac{\ln^l(s_4/p_T^2)}{s_4} \right]_+$

**Virtual corrections**  $\delta(s_4)$

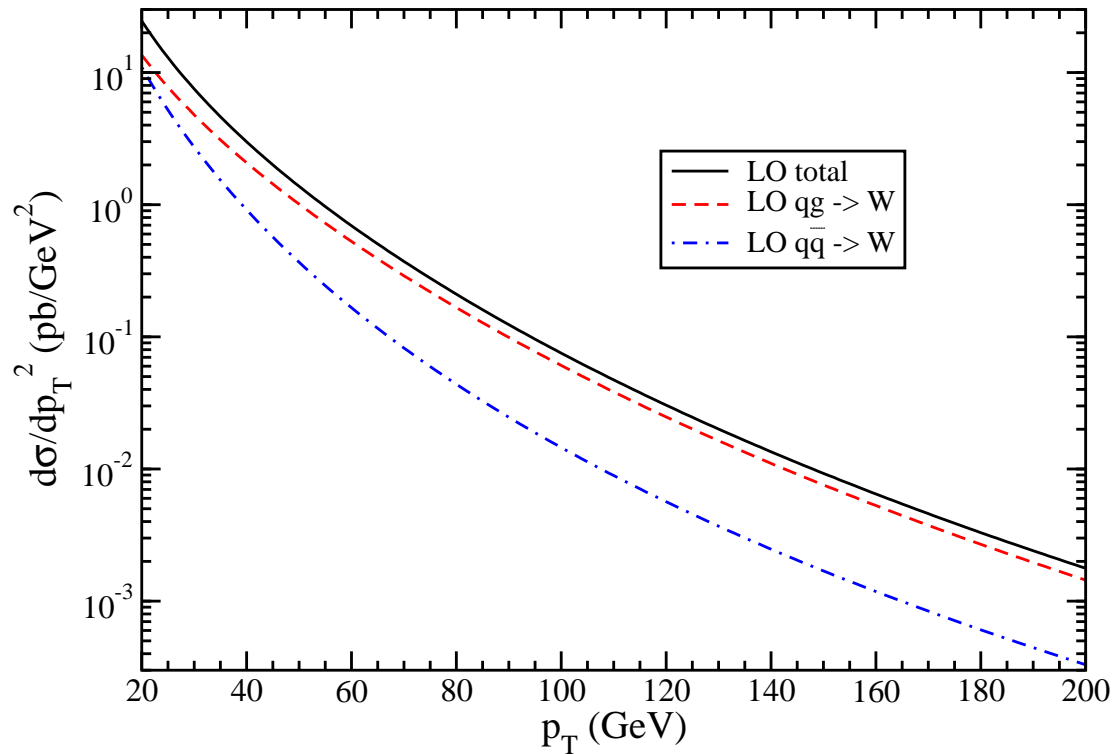
# Leading-order results for $W$ production at the Tevatron



$qg$  and  $q\bar{q}$  channel equally important

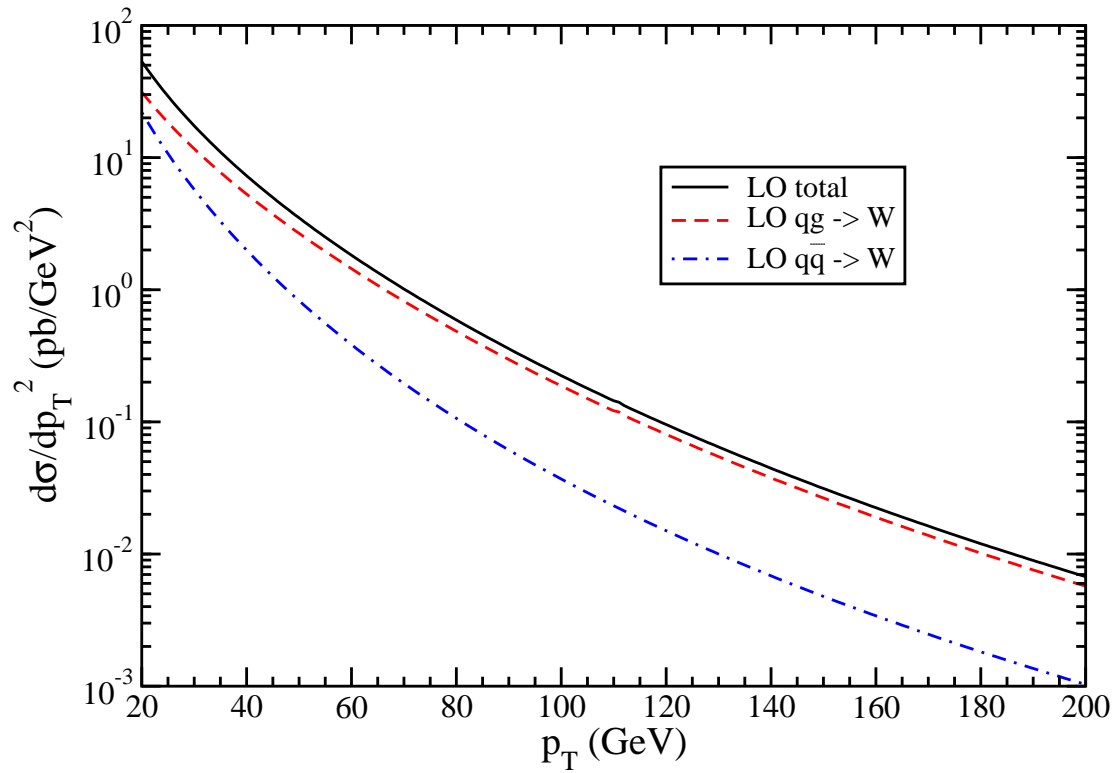
# Leading-order results for $W$ production at the LHC

W production at LHC  $S^{1/2}=7$  TeV  $\mu=p_T$



$qg$  channel is numerically dominant

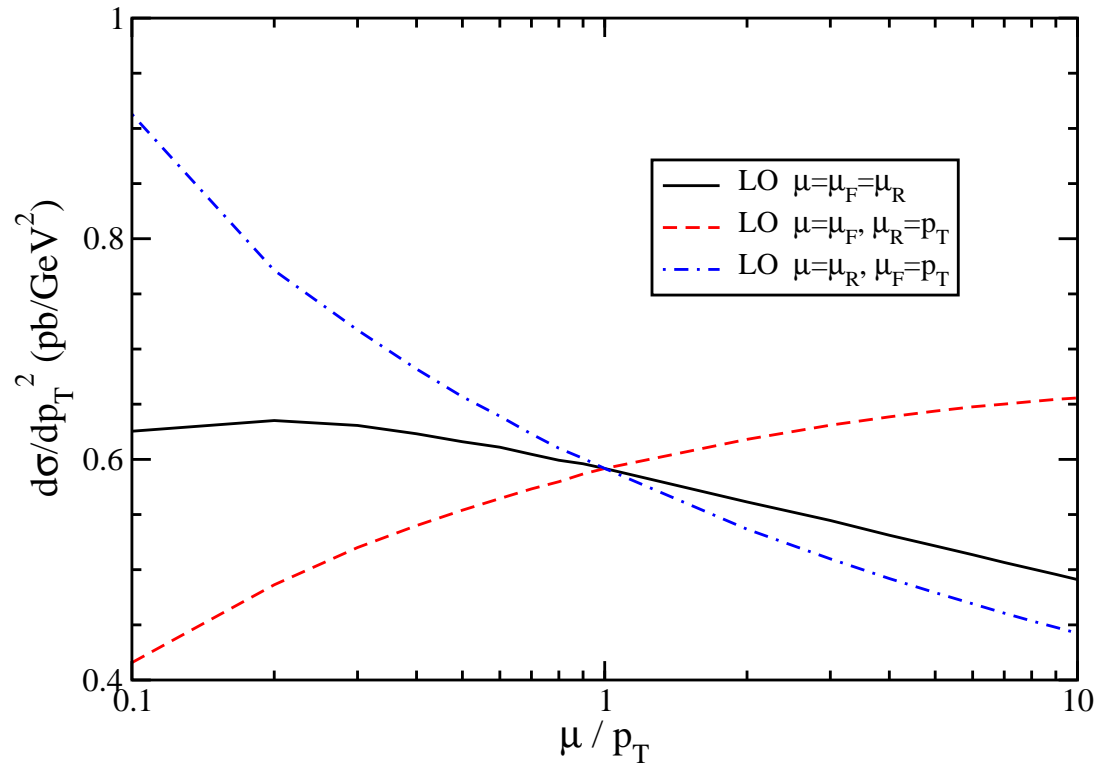
W production at LHC  $S^{1/2}=14$  TeV  $\mu=p_T$



**$qg$  channel is numerically dominant**

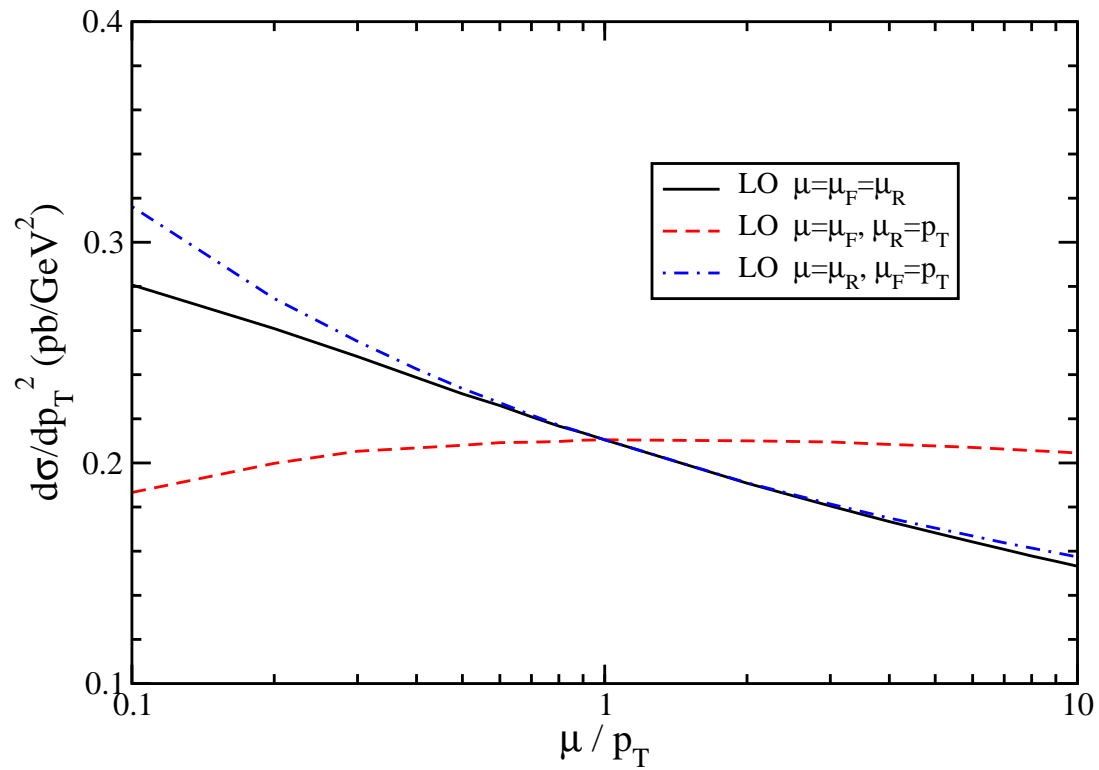
# LO scale dependence for $W$ production at the LHC

W production at LHC  $S^{1/2}=14$  TeV  $p_T=80$  GeV



At LO  $\mu_F$  and  $\mu_R$  dependence largely cancel each other

W production at LHC  $S^{1/2}=7$  TeV  $p_T=80$  GeV



somewhat different  $\mu$  dependence at 7 and 14 TeV

## NLO corrections

The NLO cross section can be written as

$$E_Q \frac{d\hat{\sigma}_{f_a f_b \rightarrow W(Q)+X}}{d^3Q} = \delta(s_4) \alpha_s(\mu_R^2) [A(s, t, u) + \alpha_s(\mu_R^2) B(s, t, u, \mu_R)] + \alpha_s^2(\mu_R^2) C(s, t, u, s_4, \mu_F)$$

The coefficient functions  $A$ ,  $B$ , and  $C$  depend on the parton flavors

The coefficient  $A(s, t, u)$  arises from the LO processes

$B(s, t, u, \mu_R)$  is the sum of virtual corrections and of singular terms  $\sim \delta(s_4)$  in the real radiative corrections

$C(s, t, u, s_4, \mu_F)$  is from real emission processes away from  $s_4 = 0$

P.B. Arnold and M.H. Reno, Nucl. Phys. B 319, 37 (1989); (E) B 330, 284 (1990)

R.J. Gonsalves, J. Pawlowski, C.-F. Wai, Phys. Rev. D 40, 2245 (1989);

Phys. Lett. B 252, 663 (1990)



## Soft-gluon corrections

$$\mathcal{D}_l(s_4) \equiv \left[ \frac{\ln^l(s_4/p_T^2)}{s_4} \right]_+$$

For the order  $\alpha_s^n$  corrections  $l \leq 2n - 1$

At NLO,  $\mathcal{D}_1(s_4)$  and  $\mathcal{D}_0(s_4)$  terms

At NNLO,  $\mathcal{D}_3(s_4)$ ,  $\mathcal{D}_2(s_4)$ ,  $\mathcal{D}_1(s_4)$ , and  $\mathcal{D}_0(s_4)$  terms

We can formally resum these logarithms for  $W$  and  $Z$  production at large  $p_T$  to all orders in  $\alpha_s$  Phys. Lett. B 480, 87 (2000)

Applied to  $W$  production at the Tevatron: JHEP 02, 027 (2004)  
and at the LHC: Phys. Rev. Lett. 95, 222001 (2005)

New two-loop results:  $\mathcal{D}_0(s_4)$  terms now fully determined

## Soft-Gluon Resummation

Resummation follows from factorization properties of the cross section  
- performed in moment space

### Resummed cross section

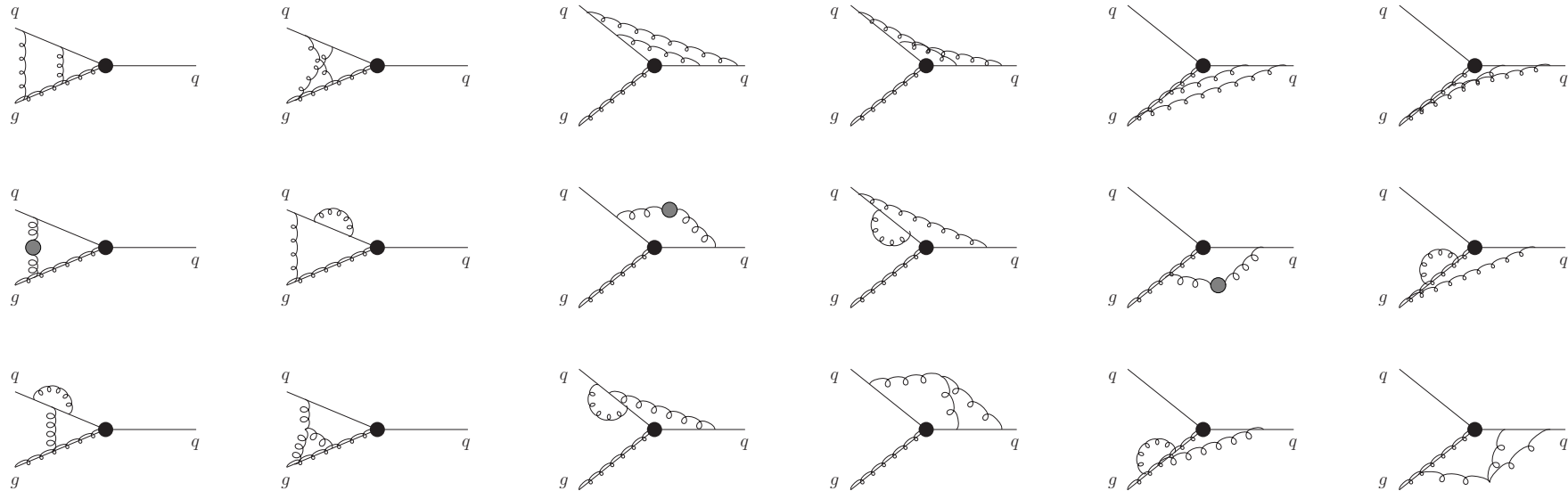
$$\hat{\sigma}^{res}(N) = \exp \left[ \sum_i E_i(N_i) \right] \exp [E'_j(N')] \exp \left[ \sum_{i=1,2} 2 \int_{\mu_F}^{\sqrt{s}} \frac{d\mu}{\mu} \gamma_{i/i}(\tilde{N}_i, \alpha_s(\mu)) \right] \\ \times H(\alpha_s) S \left( \alpha_s \left( \frac{\sqrt{s}}{\tilde{N}'} \right) \right) \exp \left[ \int_{\sqrt{s}}^{\sqrt{s}/\tilde{N}'} \frac{d\mu}{\mu} 2 \text{Re} \Gamma_S(\alpha_s(\mu)) \right]$$

$\Gamma_S$  is the soft anomalous dimension

$$\Gamma_S = \frac{\alpha_s}{\pi} \Gamma_S^{(1)} + \frac{\alpha_s^2}{\pi^2} \Gamma_S^{(2)} + \dots$$

# Two-loop soft anomalous dimension

## Two-loop eikonal diagrams for $qg \rightarrow Wq$



Determine  $\Gamma_S^{(2)}$  from UV poles of two-loop dimensionally regularized integrals

## Two-loop soft anomalous dimension

For  $qg \rightarrow Wq$  or  $qg \rightarrow Zq$

$$\Gamma_{S, qg \rightarrow Wq}^{(1)} = C_F \ln \left( \frac{-u}{s} \right) + \frac{C_A}{2} \ln \left( \frac{t}{u} \right)$$

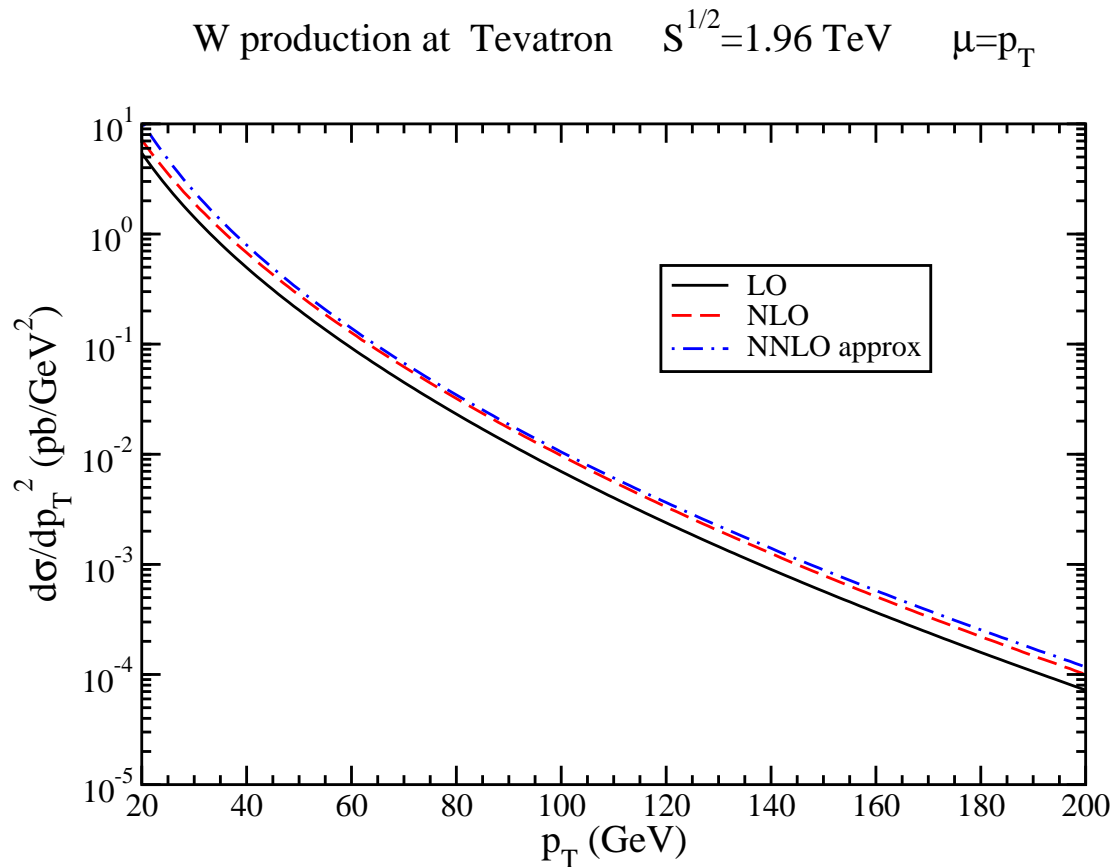
$$\Gamma_{S, qg \rightarrow Wq}^{(2)} = \frac{K}{2} \Gamma_{S, qg \rightarrow Wq}^{(1)}$$

For  $q\bar{q} \rightarrow Wg$  or  $q\bar{q} \rightarrow Zg$

$$\Gamma_{S, q\bar{q} \rightarrow Wg}^{(1)} = \frac{C_A}{2} \ln \left( \frac{tu}{s^2} \right)$$

$$\Gamma_{S, q\bar{q} \rightarrow Wg}^{(2)} = \frac{K}{2} \Gamma_{S, q\bar{q} \rightarrow Wg}^{(1)}$$

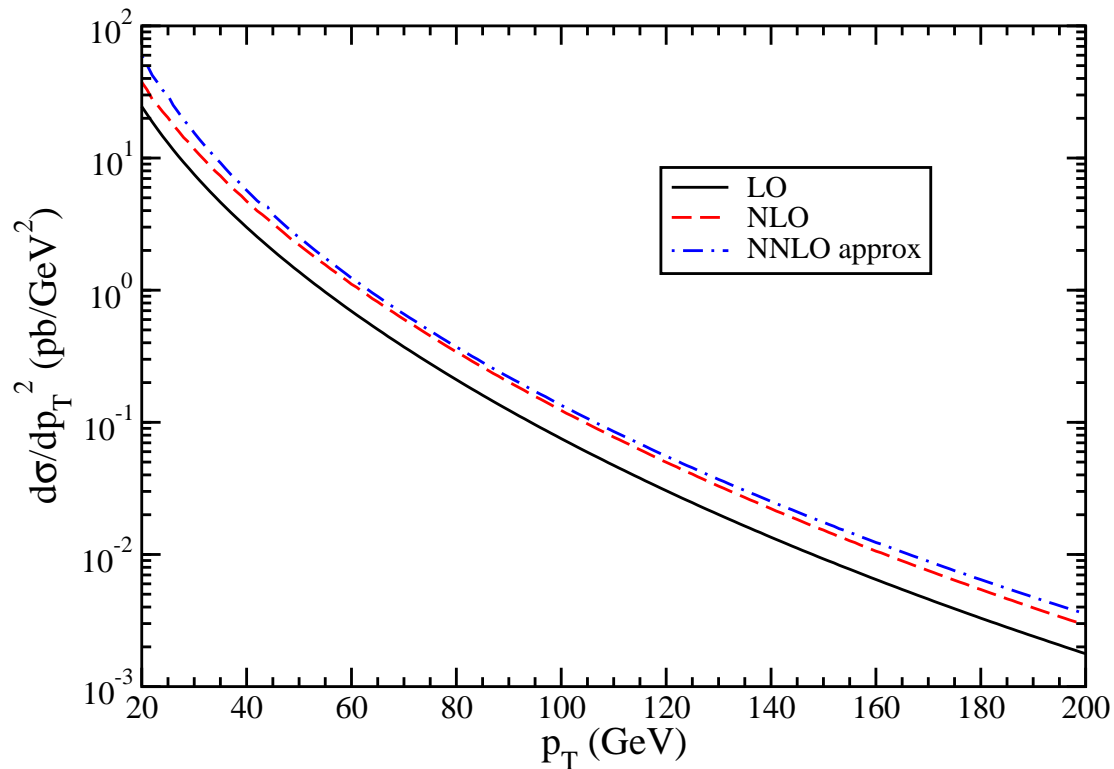
# NLO and NNLO approx for $W$ production at the Tevatron



NLO corrections and NNLO approximate corrections are significant

# NLO and NNLO approx for $W$ production at the LHC

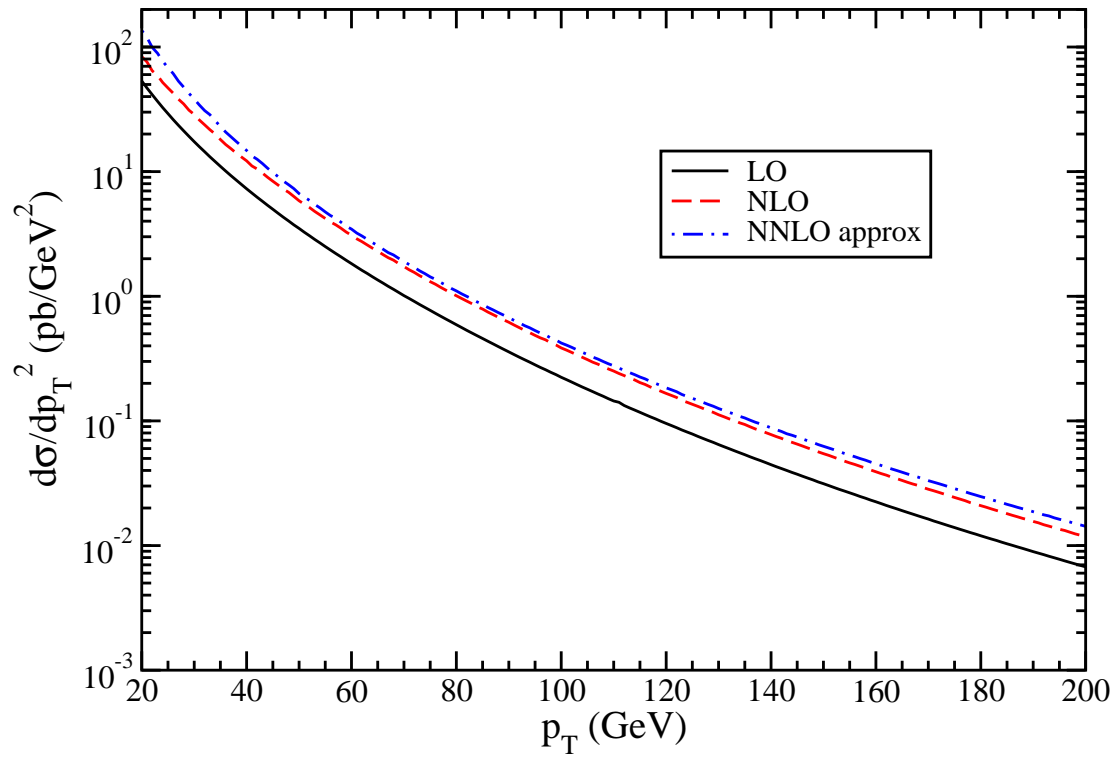
W production at LHC  $S^{1/2}=7$  TeV  $\mu=p_T$



large NLO corrections

significant NNLO approximate corrections

W production at LHC  $S^{1/2}=14$  TeV  $\mu=p_T$



large NLO corrections

significant NNLO approximate corrections

## Summary

- $W$  and  $Z$  production at large  $p_T$
- LO and NLO results
- Soft-gluon threshold corrections
- Two-loop resummation
- NNLO threshold corrections have been calculated
- Important for greater theoretical accuracy
- $W$  production at Tevatron and LHC
- More work is under way