Two-loop corrections to $W$ and $Z$ boson production at high $p_T$

Nikolaos Kidonakis
(Kennesaw State University)

- Partonic production channels at LO
- NLO corrections
- Soft-gluon corrections
- Two-loop calculations and NNLL resummation
- Approximate NNLO $p_T$ distribution at LHC and Tevatron

In collaboration with R. J. Gonsalves
W and Z production at large \( p_T \) - parton processes

W and Z hadroproduction useful in testing the SM and in estimates of backgrounds to Higgs production and new physics (new gauge bosons)

\( p_T \) distribution falls rapidly as \( p_T \) increases

Partonic channels at LO

\[ q(p_a) + g(p_b) \rightarrow W(Q) + q(p_c) \]

\[ q(p_a) + \bar{q}(p_b) \rightarrow W(Q) + g(p_c) \]

Define \( s = (p_a + p_b)^2 \), \( t = (p_a - Q)^2 \), \( u = (p_b - Q)^2 \) and \( s_4 = s + t + u - Q^2 \)

At threshold \( s_4 \rightarrow 0 \)

Soft corrections \( \left[ \frac{\ln^t(s_4/p_T^2)}{s_4} \right]_+ \)

Virtual corrections \( \delta(s_4) \)
Leading-order results for $W$ production at the Tevatron

$qg$ and $q\bar{q}$ channel equally important
Leading-order results for $W$ production at the LHC

$W$ production at LHC $S^{1/2}=7 \text{ TeV}$ $\mu=p_T$

$qg$ channel is numerically dominant
W production at LHC \( S^{1/2} = 14 \) TeV \( \mu = p_T \)

\[ \frac{d\sigma}{dp_T^2} \text{(pb/GeV}^2) \]

- **LO total**
- **LO qg -> W**
- **LO q\bar{q} -> W**

\textit{qg channel is numerically dominant}
LO scale dependence for $W$ production at the LHC

$W$ production at LHC  $S^{1/2}=14$ TeV  $p_T=80$ GeV

At LO $\mu_F$ and $\mu_R$ dependence largely cancel each other

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W production at LHC \( S^{1/2} = 7 \text{ TeV} \) \( p_T = 80 \text{ GeV} \)

somewhat different \( \mu \) dependence at 7 and 14 TeV
NLO corrections

The NLO cross section can be written as

\[
E_Q \frac{d\hat{\sigma}_{f_a f_b \rightarrow W(Q)+X}}{d^3Q} = \delta(s_4)\alpha_s(\mu_R^2) [A(s, t, u) + \alpha_s(\mu_R^2) B(s, t, u, \mu_R)] + \alpha_s^2(\mu_R^2) C(s, t, u, s_4, \mu_F)
\]

The coefficient functions \(A, B,\) and \(C\) depend on the parton flavors.

The coefficient \(A(s, t, u)\) arises from the LO processes.

\(B(s, t, u, \mu_R)\) is the sum of virtual corrections and of singular terms \(\sim \delta(s_4)\) in the real radiative corrections.

\(C(s, t, u, s_4, \mu_F)\) is from real emission processes away from \(s_4 = 0\)

Soft-gluon corrections

\[ D_l(s_4) \equiv \left[ \ln^l \left( \frac{s_4}{p_T^2} \right) \right]_+ \]

For the order \( \alpha_s^n \) corrections \( l \leq 2n - 1 \)

At NLO, \( D_1(s_4) \) and \( D_0(s_4) \) terms

At NNLO, \( D_3(s_4), D_2(s_4), D_1(s_4), \) and \( D_0(s_4) \) terms

We can formally resum these logarithms for \( W \) and \( Z \) production at large \( p_T \) to all orders in \( \alpha_s \)


Applied to \( W \) production at the Tevatron: JHEP 02, 027 (2004)


New two-loop results: \( D_0(s_4) \) terms now fully determined
Soft-Gluon Resummation

Resummation follows from factorization properties of the cross section - performed in moment space

Resummed cross section

\[ \hat{\sigma}^{res}(N) = \exp \left[ \sum_i E_i(N_i) \right] \exp \left[ E_j'(N') \right] \exp \left[ \sum_{i=1,2} 2 \int_{\mu_F}^{\sqrt{s}} \frac{d\mu}{\mu} \gamma_{i/i} \left( \tilde{N}_i, \alpha_s(\mu) \right) \right] \times H(\alpha_s) S \left( \alpha_s \left( \frac{\sqrt{s}}{\tilde{N}'} \right) \right) \exp \left[ \int_{\sqrt{s}}^{\sqrt{s}/\tilde{N}'} \frac{d\mu}{\mu} 2 \text{Re} \Gamma_S(\alpha_s(\mu)) \right] \]

\[ \Gamma_S \text{ is the soft anomalous dimension} \]

\[ \Gamma_S = \frac{\alpha_s}{\pi} \Gamma_S^{(1)} + \frac{\alpha_s^2}{\pi^2} \Gamma_S^{(2)} + \cdots \]

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Two-loop soft anomalous dimension

Two-loop eikonal diagrams for $qg \rightarrow Wq$

Determine $\Gamma^{(2)}_S$ from UV poles of two-loop dimensionally regularized integrals
Two-loop soft anomalous dimension

For $qg \to Wq$ or $qg \to Zq$

$$\Gamma^{(1)}_{S, qg\to Wq} = C_F \ln \left( \frac{-u}{s} \right) + \frac{C_A}{2} \ln \left( \frac{t}{u} \right)$$

$$\Gamma^{(2)}_{S, qg\to Wq} = \frac{K}{2} \Gamma^{(1)}_{S, qg\to Wq}$$

For $q\bar{q} \to Wg$ or $q\bar{q} \to Zg$

$$\Gamma^{(1)}_{S, q\bar{q}\to Wg} = \frac{C_A}{2} \ln \left( \frac{tu}{s^2} \right)$$

$$\Gamma^{(2)}_{S, q\bar{q}\to Wg} = \frac{K}{2} \Gamma^{(1)}_{S, q\bar{q}\to Wg}$$
NLO and NNLO approx for $W$ production at the Tevatron

NLO corrections and NNLO approximate corrections are significant

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large NLO corrections
significant NNLO approximate corrections
W production at LHC $S^{1/2}=14$ TeV $\mu=\mathbf{p}_T$

large NLO corrections
significant NNLO approximate corrections

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Summary

- $W$ and $Z$ production at large $p_T$
- LO and NLO results
- Soft-gluon threshold corrections
- Two-loop resummation
- NNLO threshold corrections have been calculated
- Important for greater theoretical accuracy
- $W$ production at Tevatron and LHC
- More work is under way