



CONFRONTING RECENT NEUTRINO OSCILLATION DATA WITH STERILE NEUTRINOS

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DPF 2011 – BROWN UNIVERSITY



OUTLINE

Sterile neutrino oscillation formalism

Experimental hints

Review of global fit results

Beyond just sterile neutrinos:
e.g., sterile neutrinos + non-standard matter-like
effects

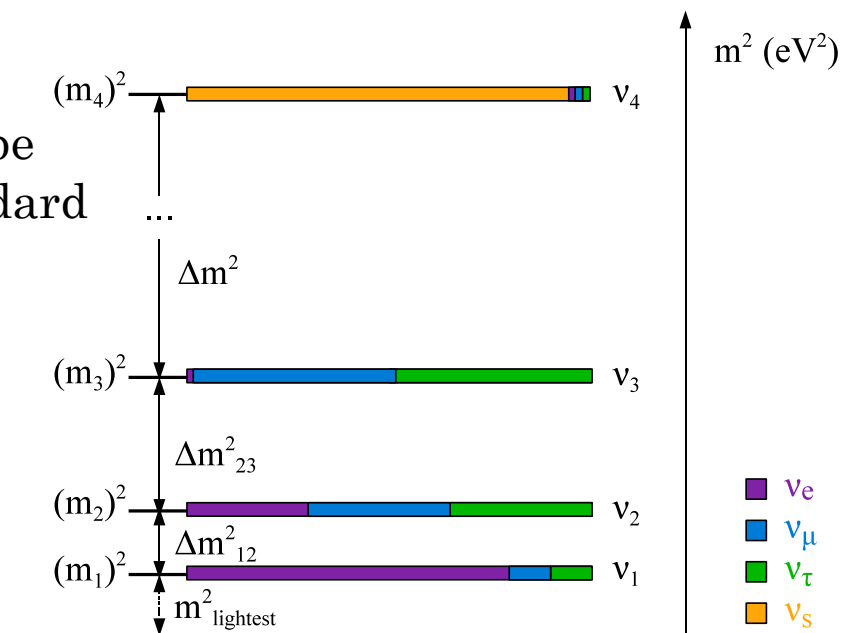
Fit results

STERILE NEUTRINO OSCILLATION FORMALISM

Additional neutrino “flavor” (and mass) states which have no weak interactions (through the standard W/Z bosons)

Additional mass states are assumed to be produced through mixing with the standard model neutrinos

→ Can affect neutrino oscillations through mixing



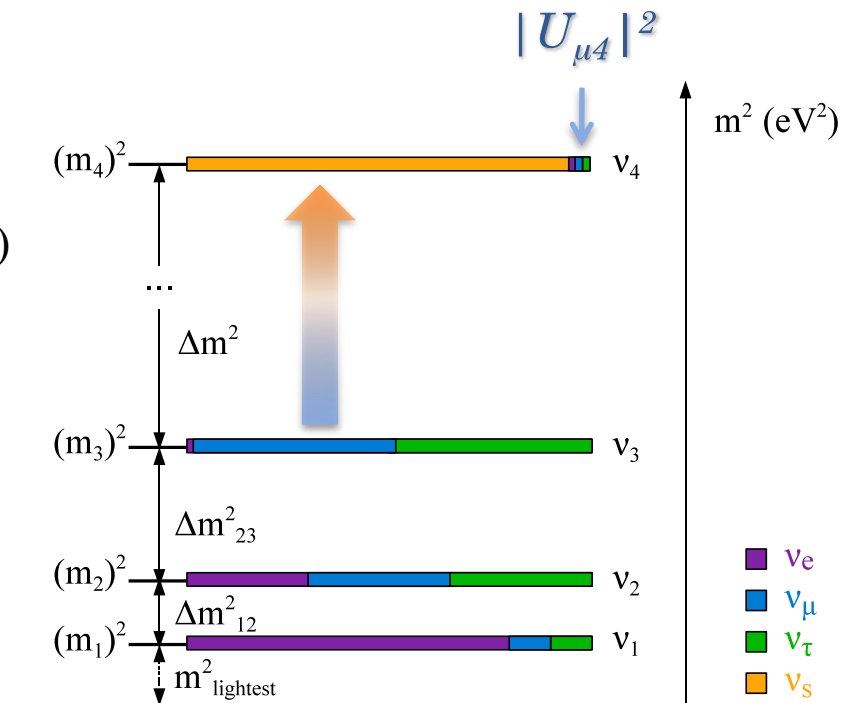
STERILE NEUTRINO OSCILLATION FORMALISM

Oscillation effects:

ν_μ disappearance*:

$$P(\nu_\mu \rightarrow \nu_\mu) = 1 - \sin^2 2\theta_{\mu\mu} \sin^2(1.27 \Delta m^2 L / E)$$

$$\rightarrow 4|U_{\mu 4}|^2(1 - |U_{\mu 4}|^2)$$



(3+1)

* $m_1, m_2, m_3 = 0$

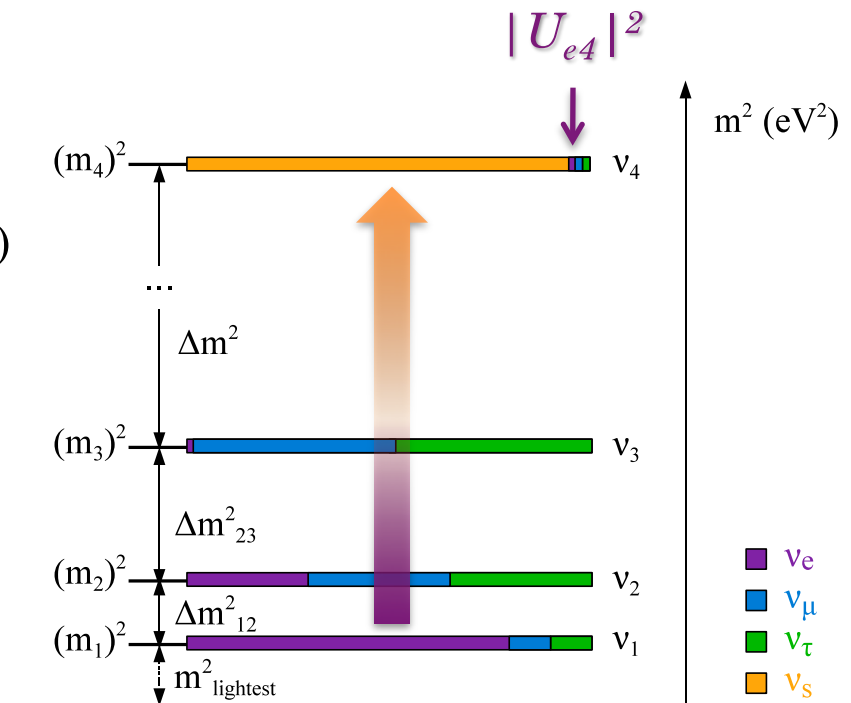
STERILE NEUTRINO OSCILLATION FORMALISM

Oscillation effects:

ν_e disappearance*:

$$P(\nu_e \rightarrow \nu_e) = 1 - \sin^2 2\vartheta_{ee} \sin^2(1.27 \Delta m^2 L / E)$$

$$\rightarrow 4|U_{e4}|^2(1 - |U_{e4}|^2)$$



(3+1)

* $m_1, m_2, m_3 = 0$

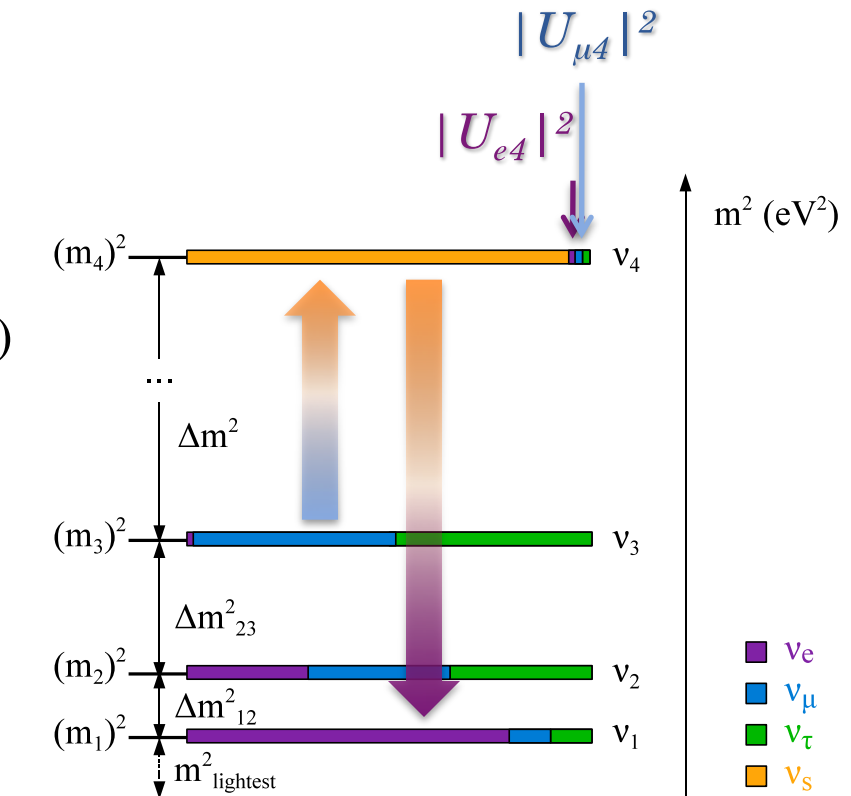
STERILE NEUTRINO OSCILLATION FORMALISM

Oscillation effects:

$\nu_\mu \rightarrow \nu_e$ appearance*:

$$P(\nu_\mu \rightarrow \nu_e) = \sin^2 2\vartheta_{\mu e} \sin^2(1.27 \Delta m^2 L / E)$$

$$\rightarrow 4|U_{e4}|^2 |U_{\mu 4}|^2$$

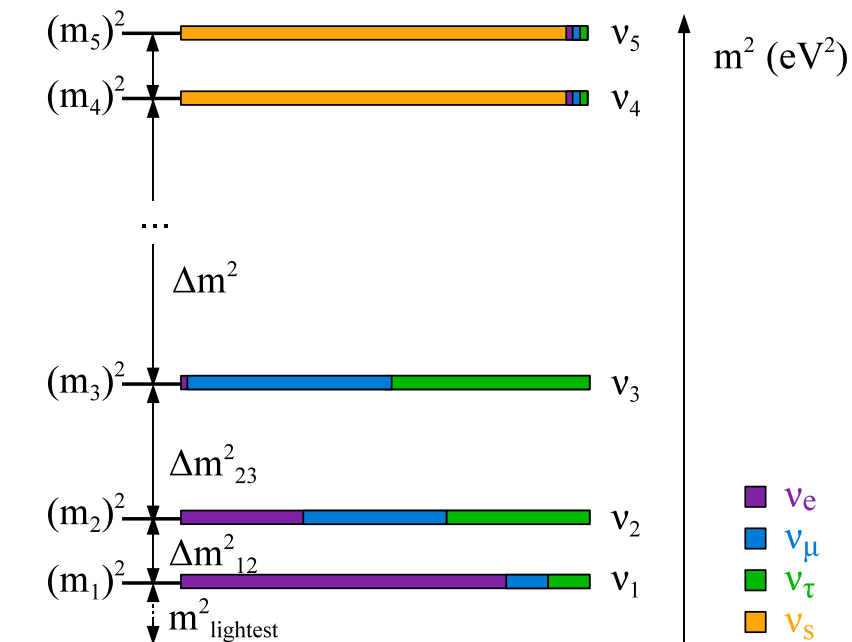


(3+1)

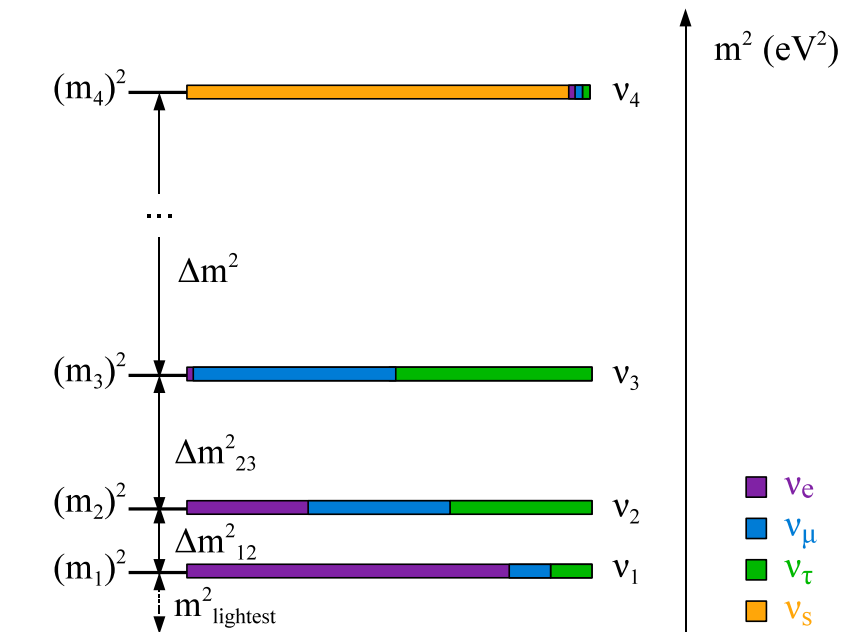
* $m_1, m_2, m_3 = 0$

STERILE NEUTRINO OSCILLATION FORMALISM

Can have **more than one new state...**



(3+2)



(3+1)

STERILE NEUTRINO OSCILLATION FORMALISM

2 \gg 1 (!)

Disappearance:

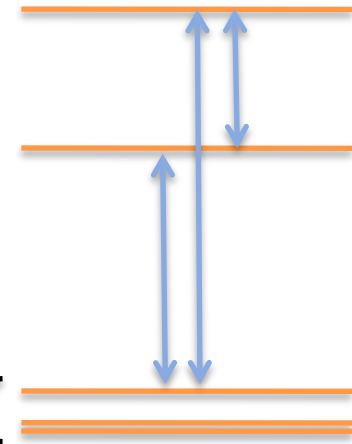
$$P(\nu_\alpha \rightarrow \nu_\alpha) = 1 - 4[(1 - |U_{\alpha 4}|^2 - |U_{\alpha 5}|^2) \cdot (|U_{\alpha 4}|^2 \sin^2 x_{41} + |U_{\alpha 5}|^2 \sin^2 x_{51}) + |U_{\alpha 4}|^2 |U_{\alpha 5}|^2 \sin^2 x_{54}]$$

Appearance:

$$P(\nu_\alpha \rightarrow \nu_{\beta \neq \alpha}) = 4|U_{\alpha 4}|^2 |U_{\beta 4}|^2 \sin^2 x_{41} + 4|U_{\alpha 5}|^2 |U_{\beta 5}|^2 \sin^2 x_{51} + 8|U_{\alpha 5}| |U_{\beta 5}| |U_{\alpha 4}| |U_{\beta 4}| \sin x_{41} \sin x_{51} \cos(x_{54} - \phi_{45})$$

$$x_{ji} \equiv 1.27 \Delta m_{ji}^2 L / E$$

assumed
degenerate {



→ 2 effective Δm^2

(3+2) is attractive because of
CP violation



OUTLINE

Sterile neutrino oscillation formalism

Experimental hints

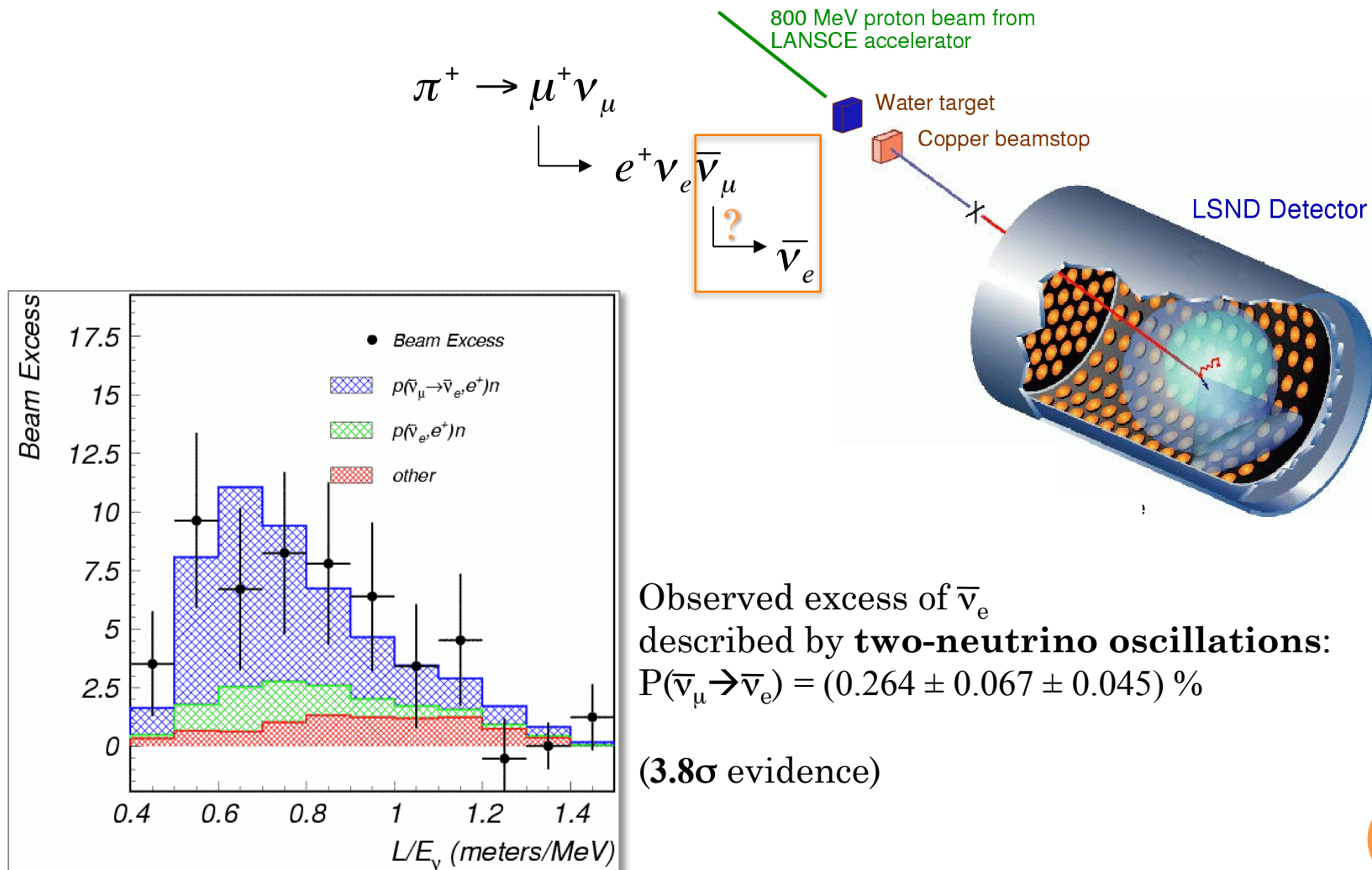
Review of global fit results

Beyond just sterile neutrinos:
e.g., sterile neutrinos + non-standard matter-like
effects

Fit results

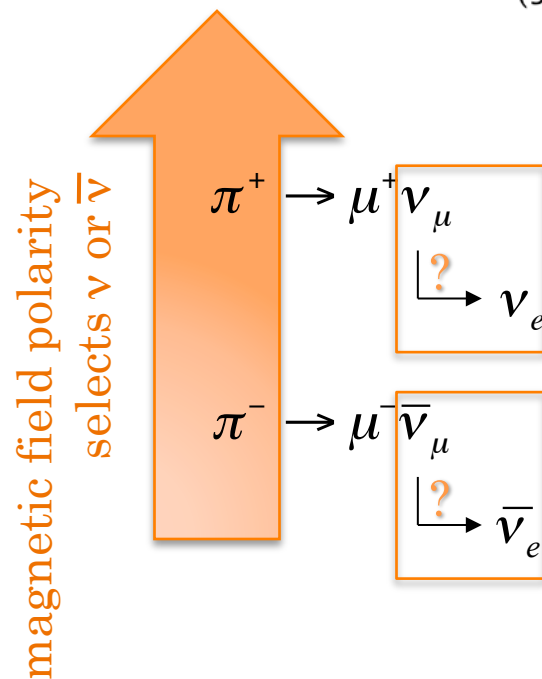
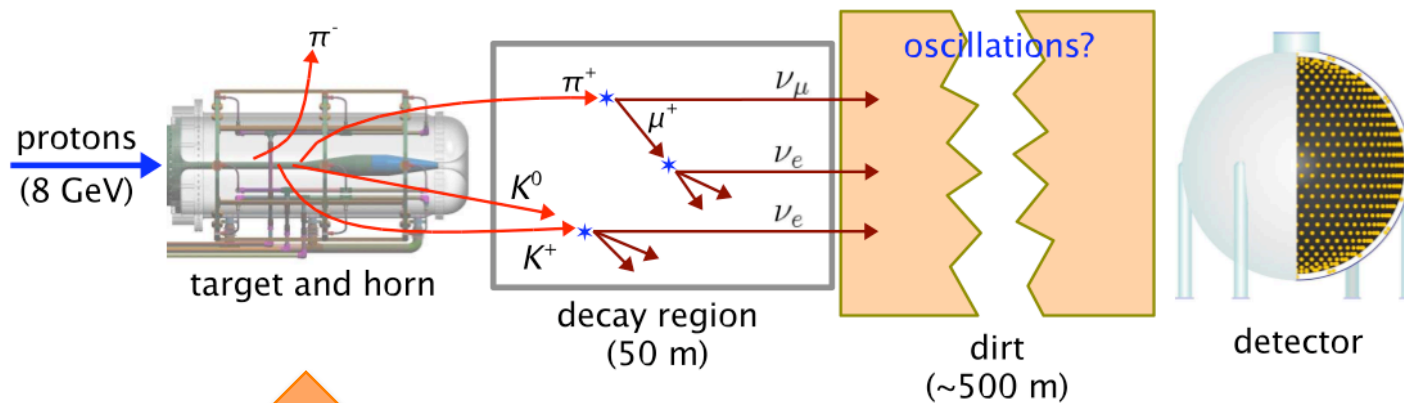
EXPERIMENTAL HINTS: THE LSND EXPERIMENT

- 1) LSND $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$



EXPERIMENTAL HINTS: THE MINIBOOONE EXPERIMENT

- 1) LSND $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$
- 2) MiniBooNE $\nu_\mu \rightarrow \nu_e$



Similar L/E as LSND

but

- Different energy, beam and detector systematics

- Different event signatures and backgrounds

EXPERIMENTAL HINTS: THE MINIBOOONE EXPERIMENT

- 1) LSND $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$
- 2) MiniBooNE $\nu_\mu \rightarrow \nu_e$

E > 475 MeV data in good agreement with background prediction.

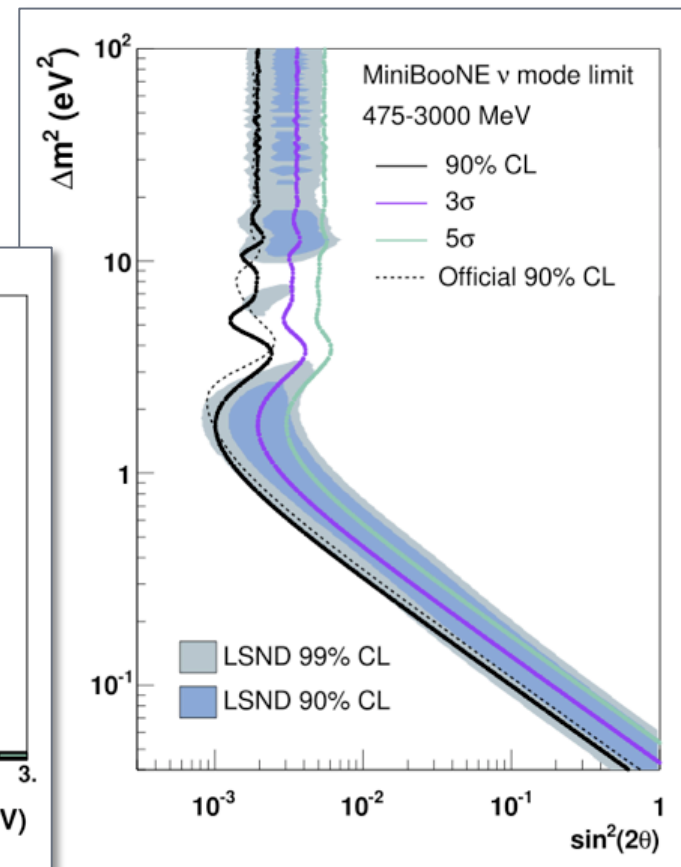
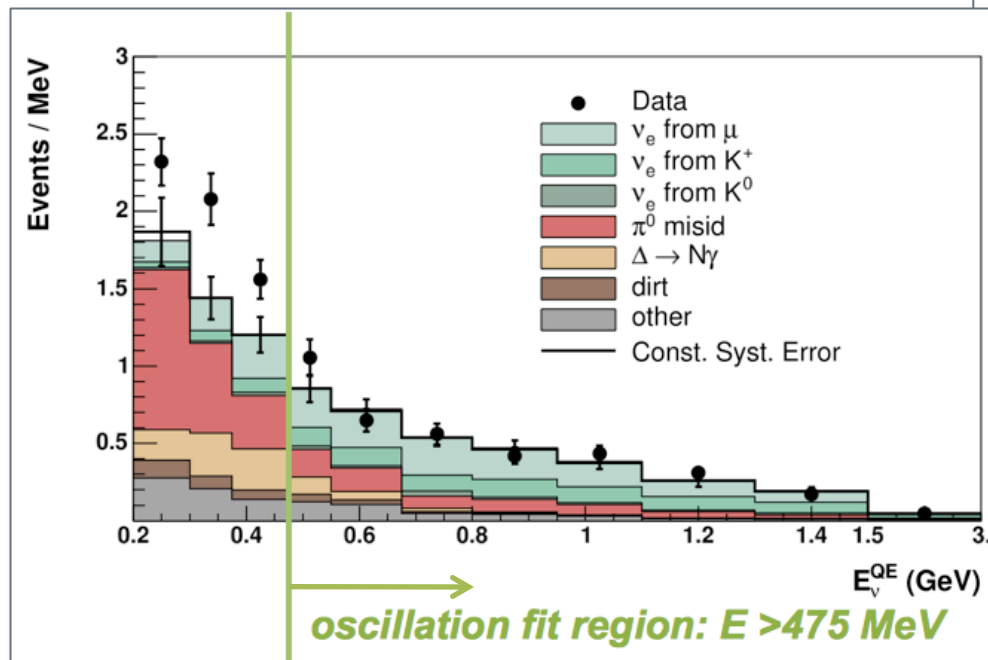
A two neutrino fit is **inconsistent with LSND** at the 90% CL assuming CP conservation.

E < 475 MeV shows a **3 σ excess at low energy**.

The total excess is consistent with magnitude of LSND signal, but inconsistent with L/E shape.

neutrino running

6.5×10^{20} POT



EXPERIMENTAL HINTS: THE MINIBOOONE EXPERIMENT

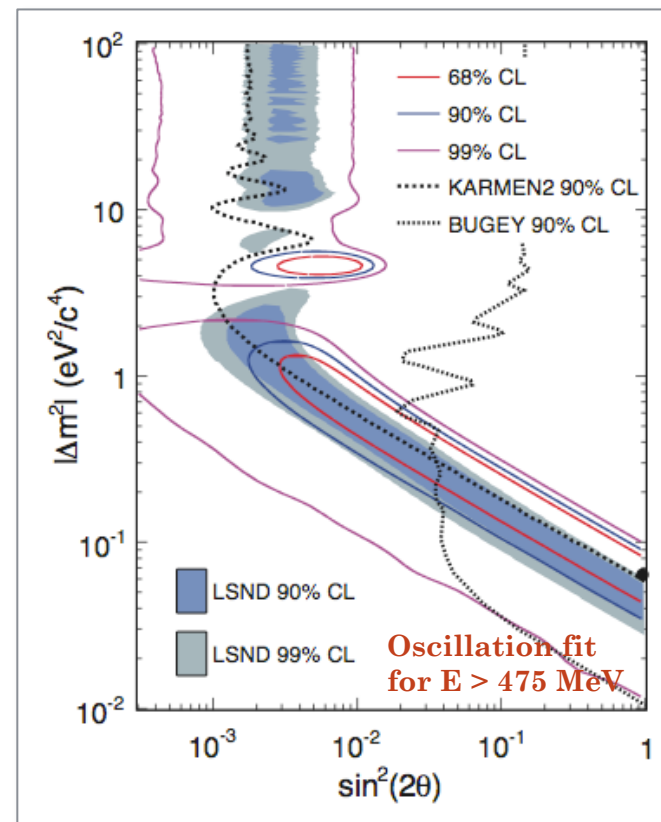
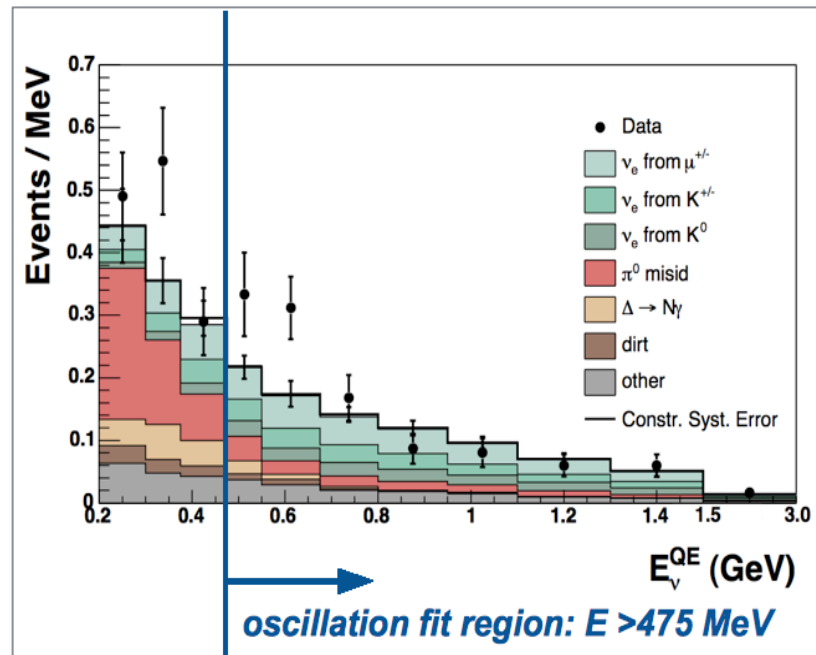
- 1) LSND $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$
- 2) MiniBooNE $\nu_\mu \rightarrow \nu_e$
- 3) MiniBooNE $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$

For $E > 475$ MeV, oscillations favored over background only (null) hypothesis at the 99.4% CL.

Low energy excess not as significant as in neutrino mode.

antineutrino running

5.7×10^{20} POT



Phys. Rev. Lett. 105, 181801 (2010)

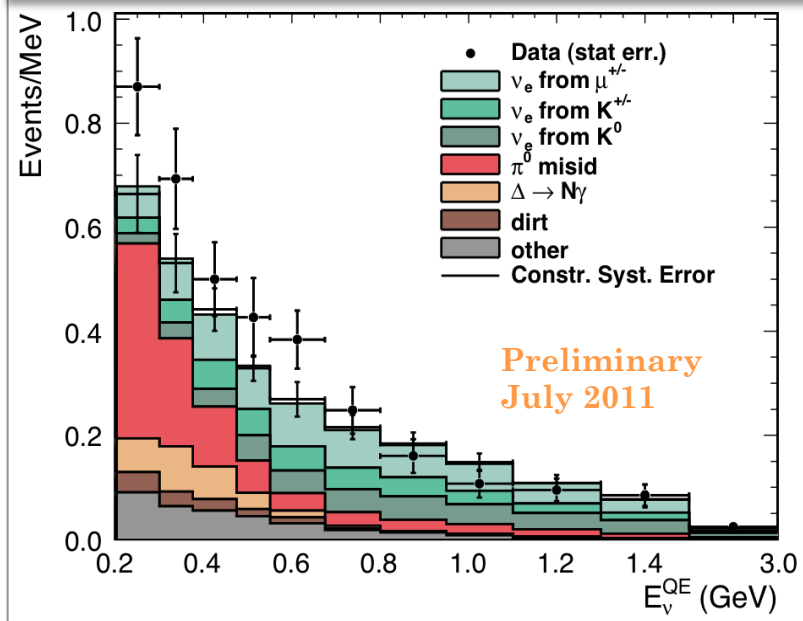
EXPERIMENTAL HINTS: THE MINIBOOONE EXPERIMENT

- 1) LSND $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$
- 2) MiniBooNE $\nu_\mu \rightarrow \nu_e$
- 3) MiniBooNE $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$

For $E > 475$ MeV, **oscillations favored** over background only (null) hypothesis **at the 91.1% CL**.

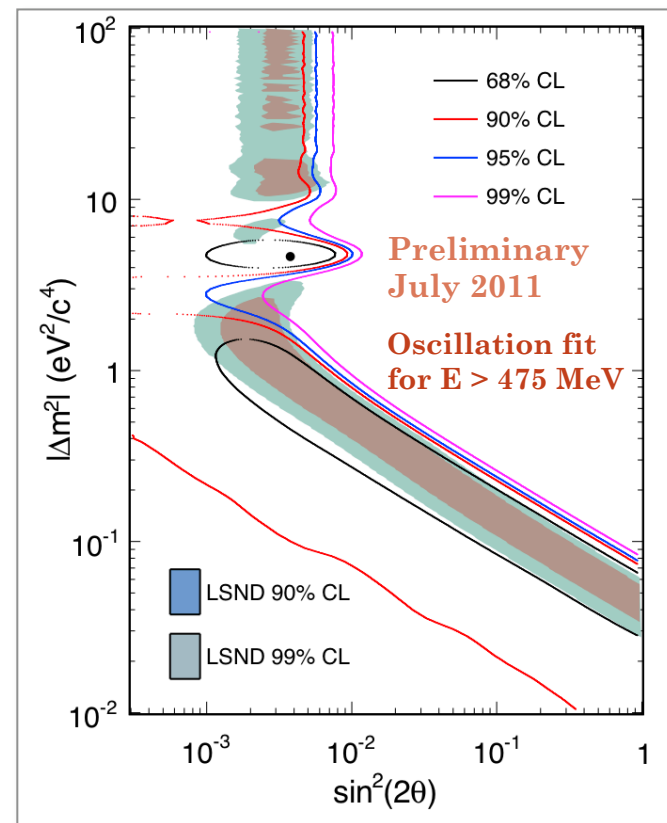
Low **energy excess** now more prominent for antineutrino running than previous result (more similar to neutrino result).

NEW! (not yet included in fits shown in this talk)



antineutrino running

8.6×10^{20} POT

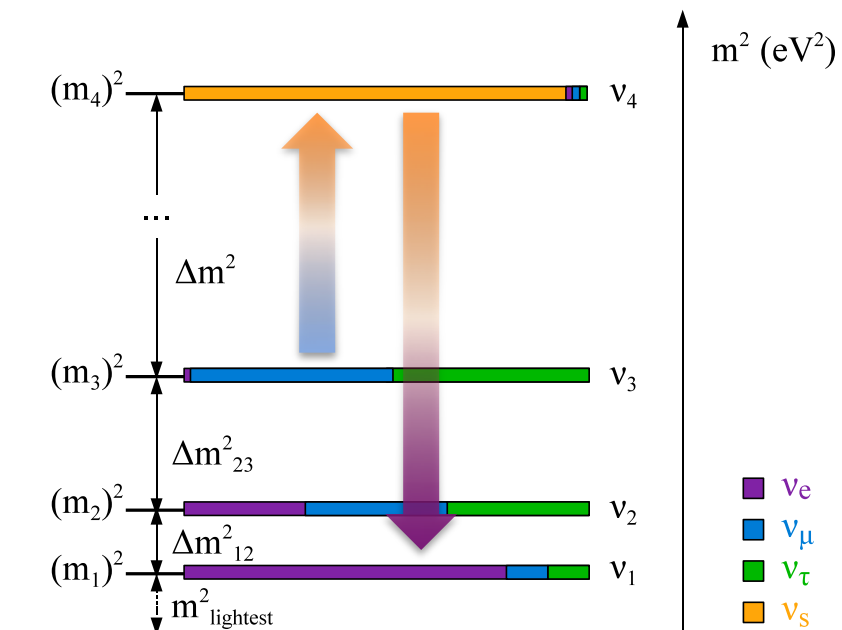


Talk by Z. Pavlovic

EXPERIMENTAL HINTS:

- 1) LSND $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$
- 2) MiniBooNE $\nu_\mu \rightarrow \nu_e$
- 3) MiniBooNE $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$

Appearance implies
disappearance...



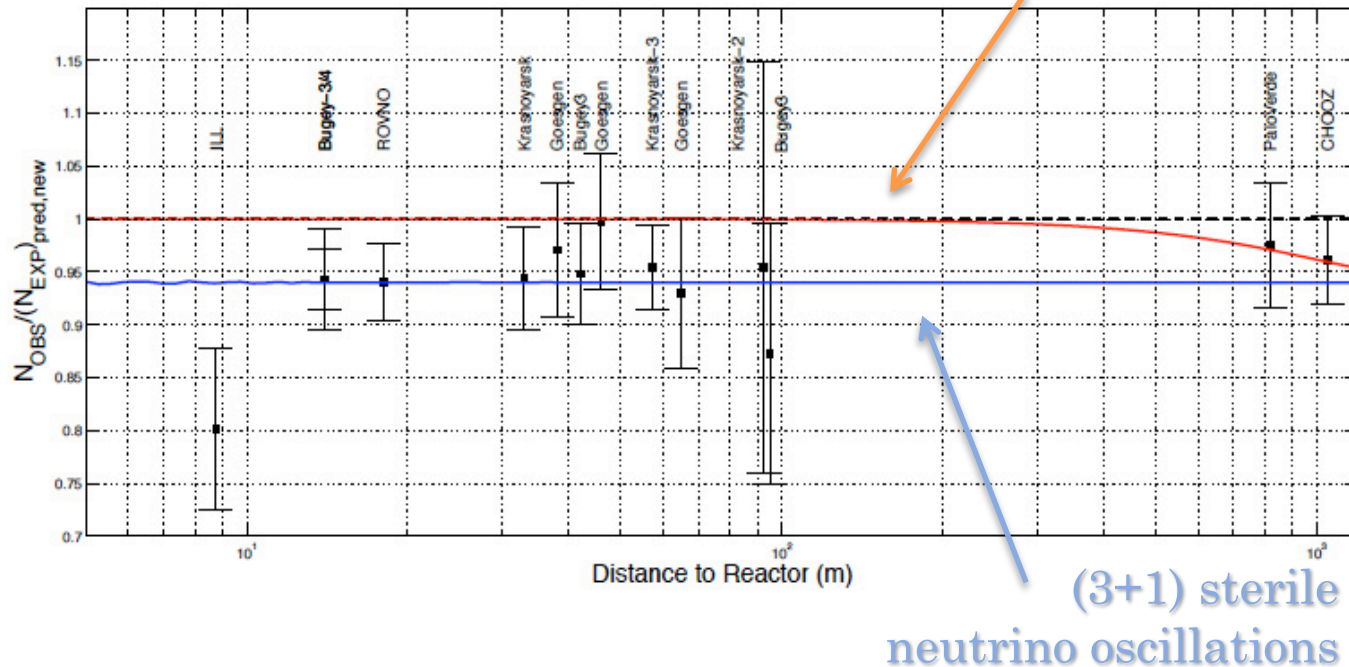
Have we seen any?

EXPERIMENTAL HINTS: THE REACTOR ANOMALY

- 1) LSND $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$
- 2) MiniBooNE $\nu_\mu \rightarrow \nu_e$
- 3) MiniBooNE $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$
- 4) Reactor $\bar{\nu}_e \rightarrow \bar{\nu}_x$

Possibly?

standard
three-neutrino oscillations



Based on **re-analysis of predicted reactor fluxes**:

- Reactor flux prediction increases by 3%.
- Re-analysis of past reactor experiments results in a deficit of electron anti-neutrinos compared to this prediction – at the 2.14σ level

→ Could be oscillations to sterile with $\Delta m^2 \sim 1\text{eV}^2$ and $\sin^2 2\theta \sim 0.1$



OUTLINE

Sterile neutrino oscillation formalism

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effects

Fit results

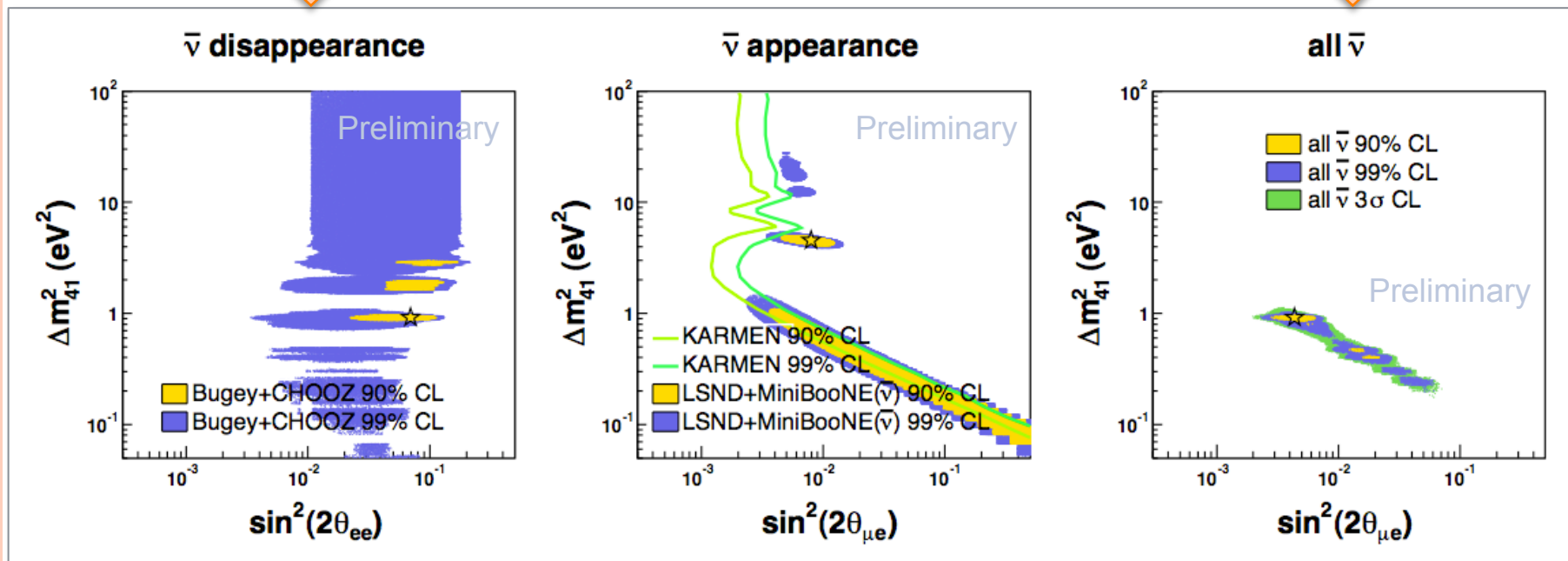
GLOBAL FITS TO SHORT-BASELINE ANTINEUTRINO: (3+1)

MiniBooNE($\bar{\nu}$) and LSND are compatible with each other
and with all other **short-baseline antineutrino results**:

Reactor anomaly:
allows oscillations at >99% CL



All antineutrino datasets:
compatibility = 22%

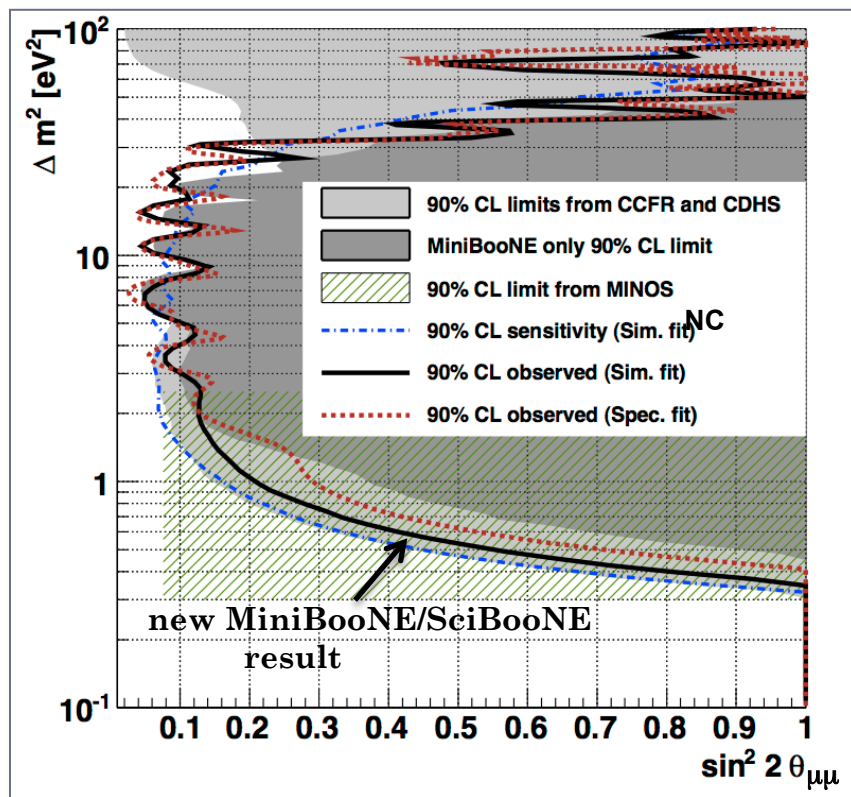


includes 2010 MiniBooNE antineutrino appearance dataset,
and new reactor flux predictions

GLOBAL FITS: (3+1)

But, strong constraints from ν_μ disappearance experiments:

Talk by M. Wascko



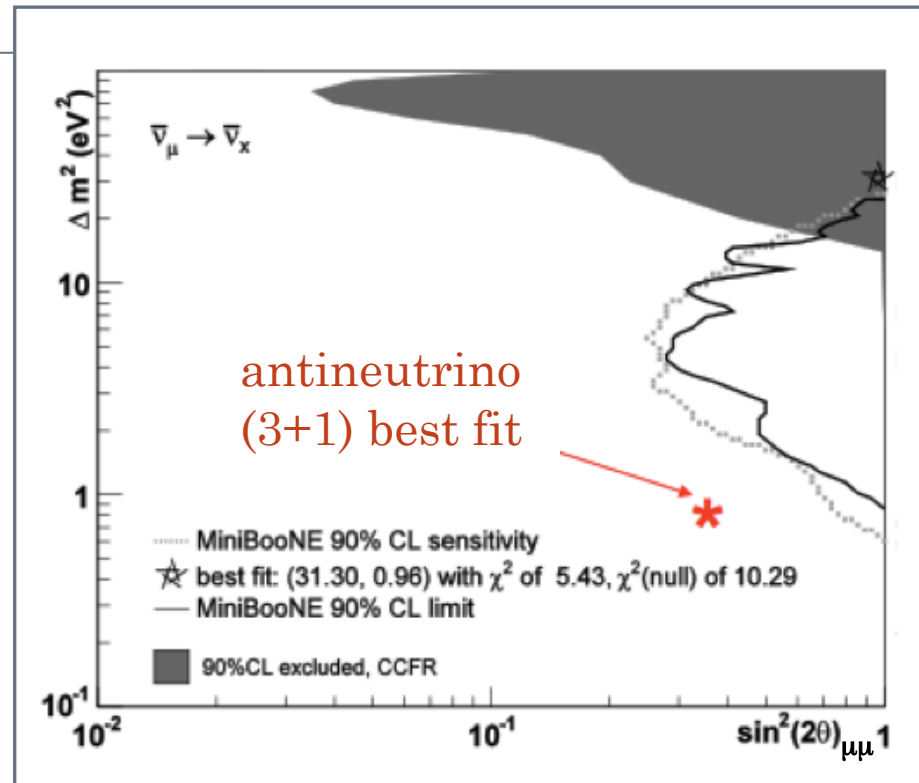
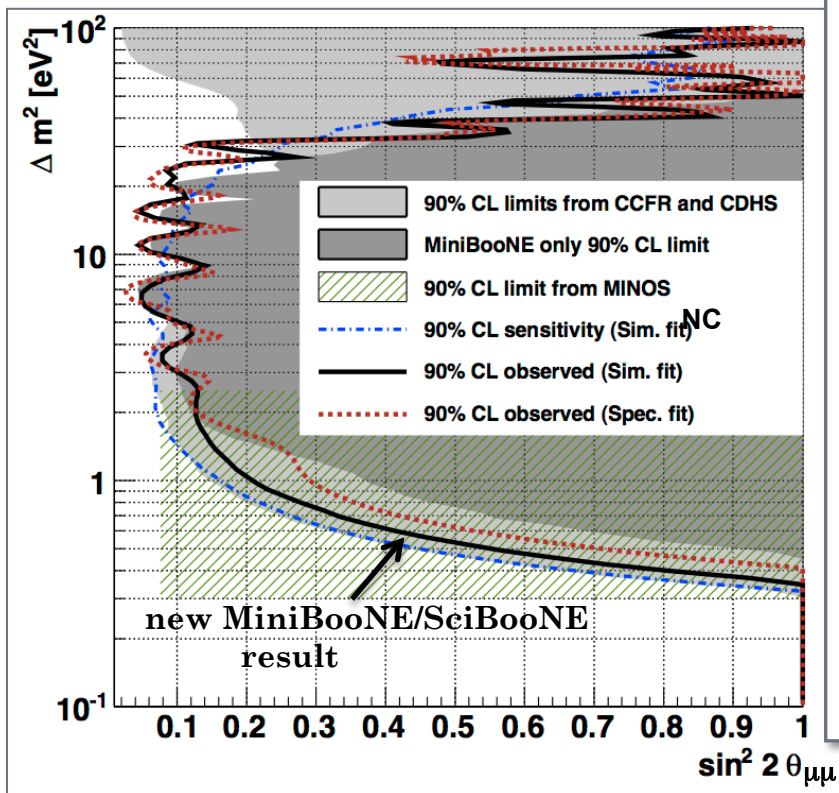
ν_μ constraint is applicable to $\bar{\nu}$ by CPT:

ν_μ and $\bar{\nu}_\mu$ disappearance in (3+1) is different ONLY if ~~CPT~~

GLOBAL FITS: (3+1)

But, strong constraints from ν_μ disappearance experiments:

Talk by M. Wascko



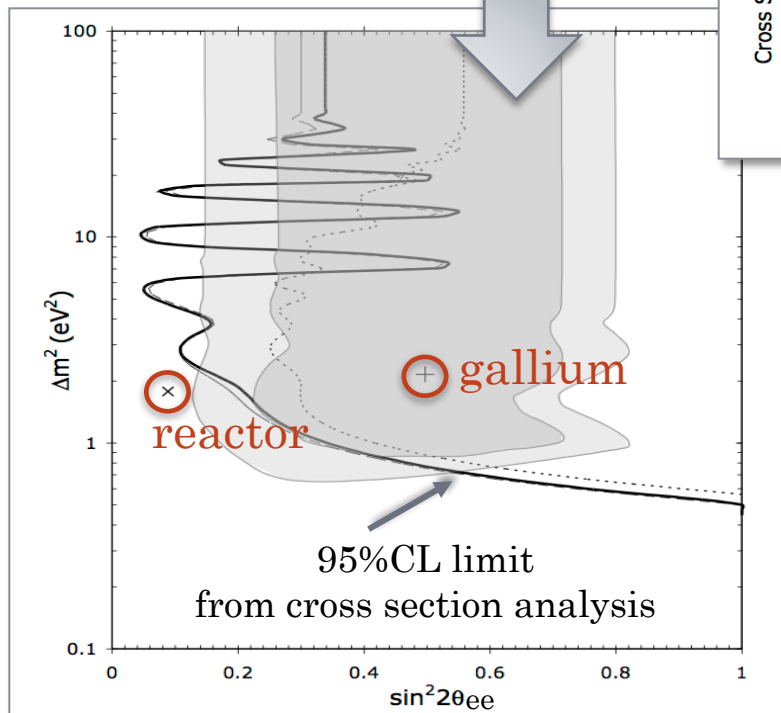
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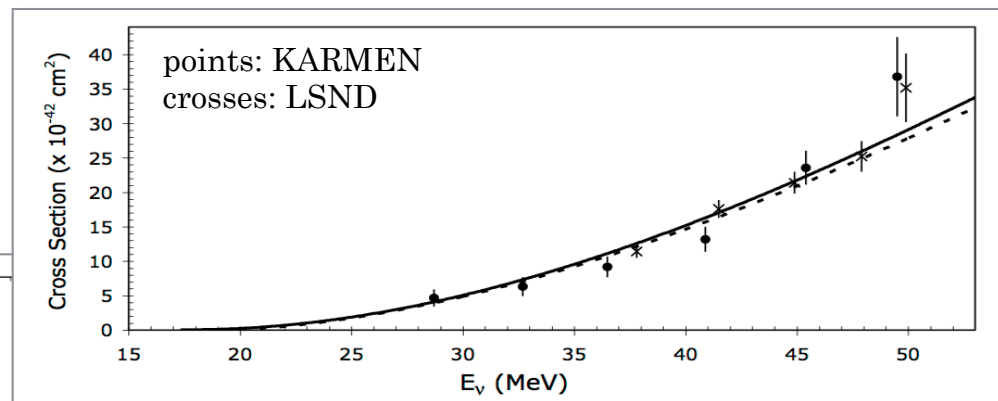
GLOBAL FITS: (3+1)

And constraints from
 ν_e disappearance experiments:

Hint from Gallium calibration
source experiments:
 ν_e disappearance



Measured cross-sections agree
with each-other (different L/E)
and with theory



Now directly excluded
by **KARMEN** and **LSND**
 ν_e cross section measurements.

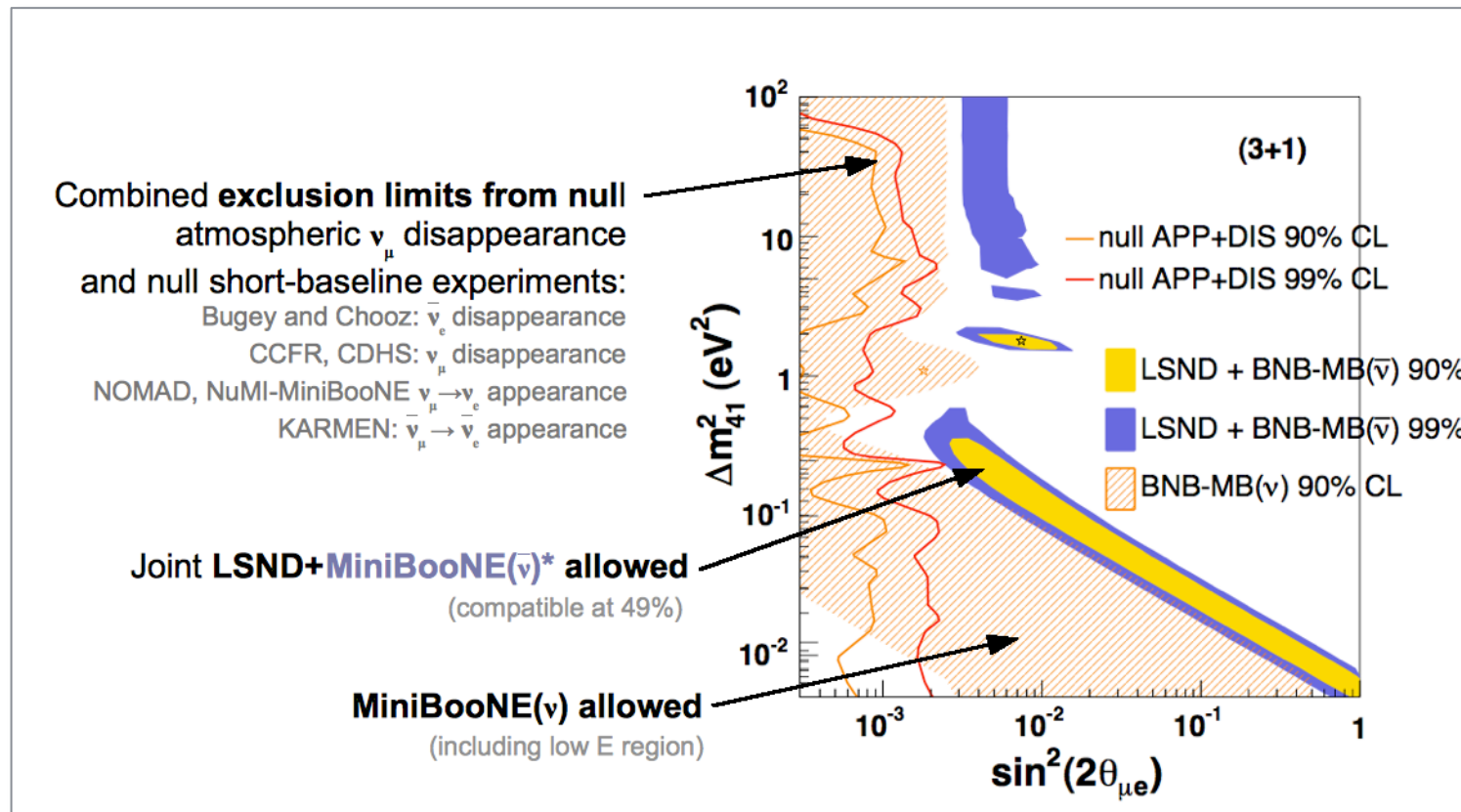
J.M.Conrad and M.H.Shaevitz,
1106.5552v2 [hep-ex]

[Reactor anomaly not excluded]

GLOBAL FITS: (3+1)

Consequently, impossible to reconcile all short-baseline results under (3+1).

Compatibility of all short-baseline datasets: 0.11%
(3+1) scenario essentially RULED OUT



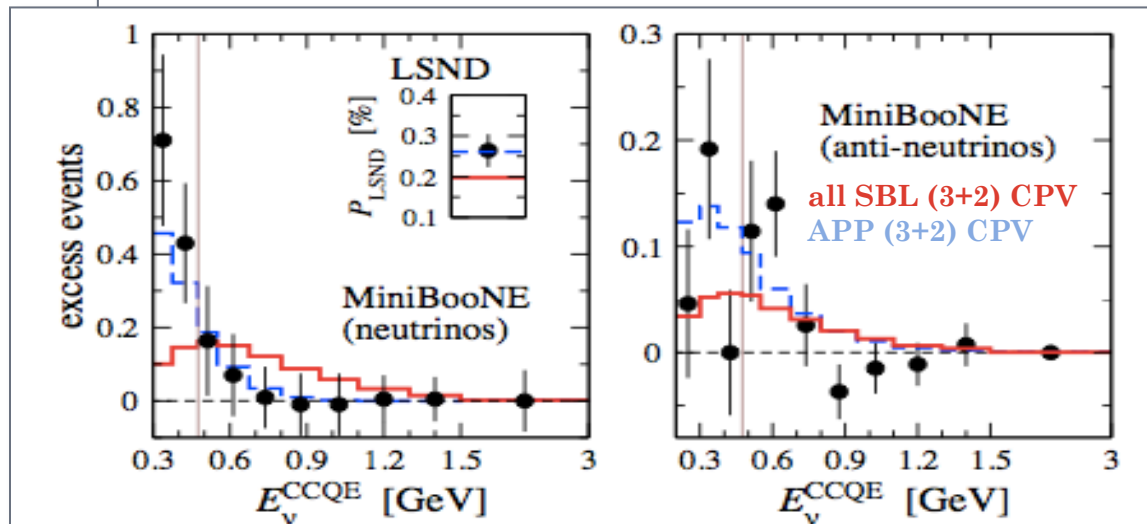
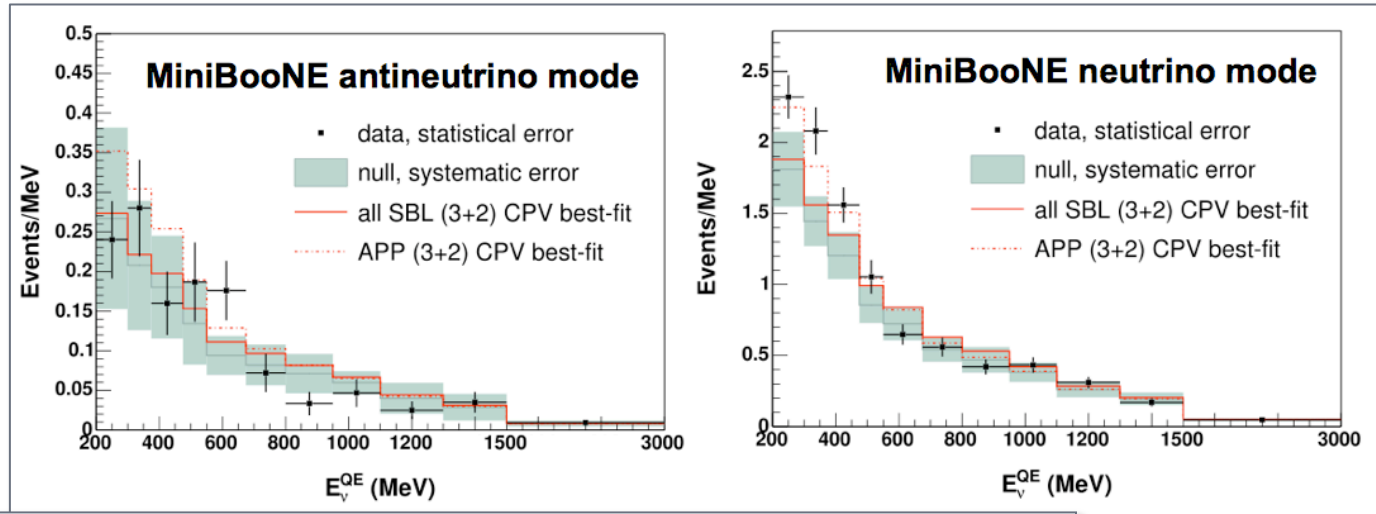
GLOBAL FITS: (3+2)



Does (3+2) + CP violation help?

GLOBAL FITS: (3+2) WITH CPV SEEMS INSUFFICIENT

	Δm_{41}^2	$ U_{e4} $	$ U_{\mu 4} $	Δm_{51}^2	$ U_{e5} $	$ U_{\mu 5} $	δ/π	χ^2/dof	
3+2	0.47	0.128	0.165	0.87	0.138	0.148	1.64	110.1/130	Kopp et al., hep-ph:1103.4570



PRD 80, 073001 (2009)
[hep-ph/0906.1997v3]

Appearance vs
disappearance and
neutrino vs antineutrino
compatibility
still not great...

GLOBAL FITS: BACK TO THE DRAWING BOARD

We know that all hints come primarily
from antineutrino experiments.

Neutrinos and antineutrinos seem to be
telling different stories,

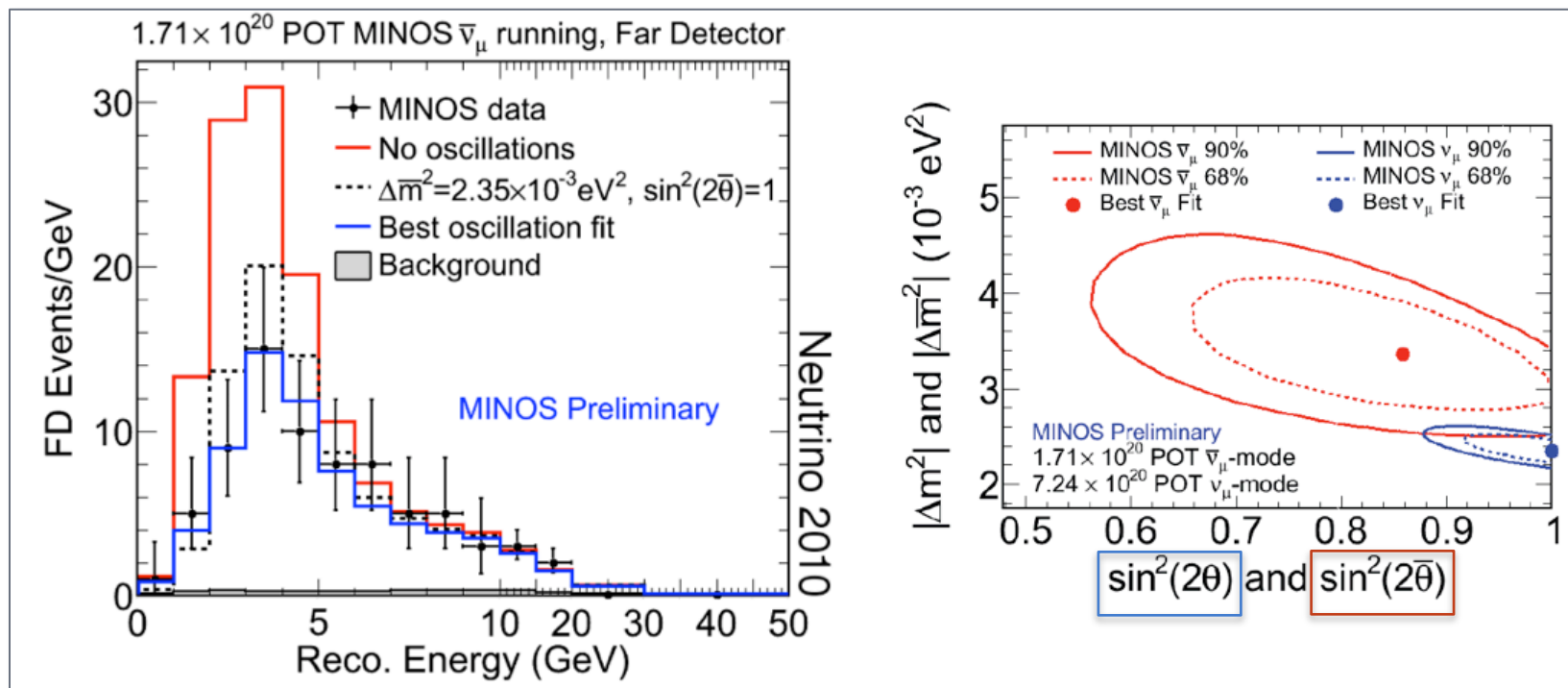
such that

sterile neutrino oscillations
with CP violation
are not enough to account for those
differences.

EXPERIMENTAL HINTS (CONTINUED): MINOS ANTINEUTRINO DISAPPEARANCE SEARCH

- 1) LSND $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$
- 2) MiniBooNE $\nu_\mu \rightarrow \nu_e$
- 3) MiniBooNE $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$
- 4) Reactor $\bar{\nu}_e \rightarrow \bar{\nu}_x$
- 5) MINOS $\bar{\nu}_\mu \rightarrow \bar{\nu}_x$

Is this effect related in any way?



See talk by X. Huang

26

ν_μ and $\bar{\nu}_\mu$ disappearance is different ONLY if ~~CPT~~

Beyond the “Reference Picture”
See talk by L. Everett

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e.g., sterile neutrinos + non-standard matter-like
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Fit results

STERILE NEUTRINO OSCILLATION FORMALISM WITH NON-STANDARD MATTER-LIKE EFFECTS

We investigate the hypothesis that the appearance signals from

MiniBooNE (ν)

MiniBooNE ($\bar{\nu}$)

LSND

are due to (3+1)

where

ν_s experience matter-like potential: $V_s = +A_s$

$\bar{\nu}_s$ experience matter-like potential: $V_s = -A_s$

Work in collaboration
with J. Conrad and M. Shaevitz

STERILE NEUTRINO OSCILLATION FORMALISM WITH NON-STANDARD MATTER-LIKE EFFECTS

Effective matter potential in neutrino flavor space:

$$V = \begin{pmatrix} V_{CC} + V_{NC} & 0 & 0 & 0 \\ 0 & V_{NC} & 0 & 0 \\ 0 & 0 & V_{NC} & 0 \\ 0 & 0 & 0 & V_s \end{pmatrix} \simeq \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & V_s \end{pmatrix} \quad V_s\text{-dominant}$$

Effective hamiltonian in matter:

$$H_m = \frac{1}{2E} U \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_4^2 \end{pmatrix} U^\dagger + V$$

m_4 -dominant

$$= \frac{\Delta m^2}{2E} \begin{pmatrix} U_{e4}U_{e4}^* & U_{e4}U_{\mu 4}^* & U_{e4}U_{\tau 4}^* & U_{e4}U_{s4}^* \\ U_{\mu 4}U_{e4}^* & U_{\mu 4}U_{\mu 4}^* & U_{\mu 4}U_{\tau 4}^* & U_{\mu 4}U_{s4}^* \\ U_{\tau 4}U_{e4}^* & U_{\tau 4}U_{\mu 4}^* & U_{\tau 4}U_{\tau 4}^* & U_{\tau 4}U_{s4}^* \\ U_{s4}U_{e4}^* & U_{s4}U_{\mu 4}^* & U_{s4}U_{\tau 4}^* & U_{s4}U_{s4}^* + 2EV_s/\Delta m^2 \end{pmatrix}$$

STERILE NEUTRINO OSCILLATION FORMALISM WITH NON-STANDARD MATTER-LIKE EFFECTS

General oscillation probability:

$$P(\nu_\mu \rightarrow \nu_e) = 4|U_{e4}^M|^2|U_{\mu4}^M|^2 \sin^2(1.27 \Delta m_M^2 L/E)$$

$$(\sin^2 2\theta_M = 4|U_{e4}^M|^2|U_{\mu4}^M|^2)$$



Gives effective mixing parameters in matter:
(as functions of Δm^2 , $|U_{e4}|$, $|U_{\mu4}|$, and E)

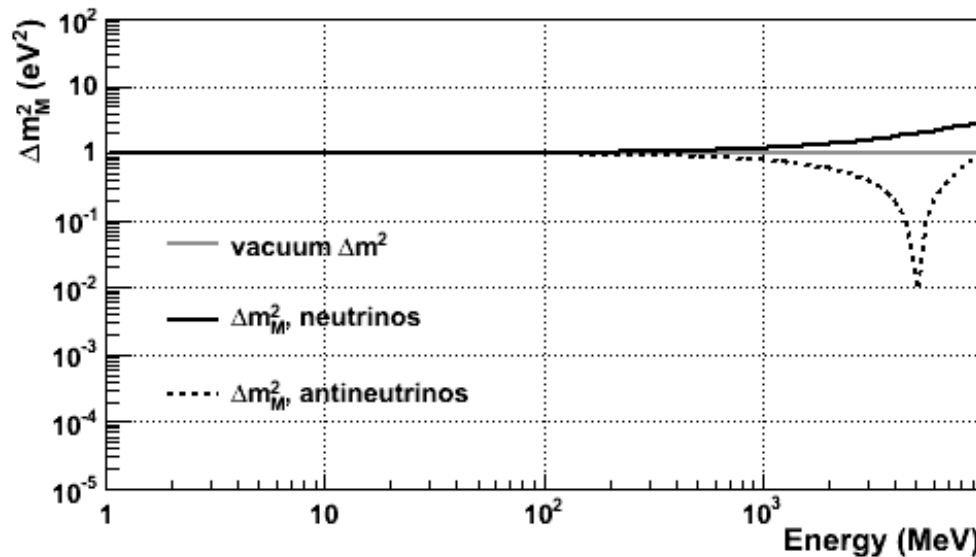
$$\Delta m_M^2 = \Delta m^2 + 2EV_s$$

$$\sin^2 2\theta_M = \frac{16(\Delta m^2)^4|U_{e4}|^2|U_{\mu4}|^2|U_{s4}|^4}{((\Delta m^2 - 2EV_s)^2 + 4(2EV_s\Delta m^2|U_{s4}|^2))(2EV_s - \Delta m^2(1 - 2|U_{s4}|^2) + \sqrt{(\Delta m^2 - 2EV_s)^2 + 4(2EV_s\Delta m^2|U_{s4}|^2)})^2}$$

Both E - and $\sqrt{\nu}$ -dependent!

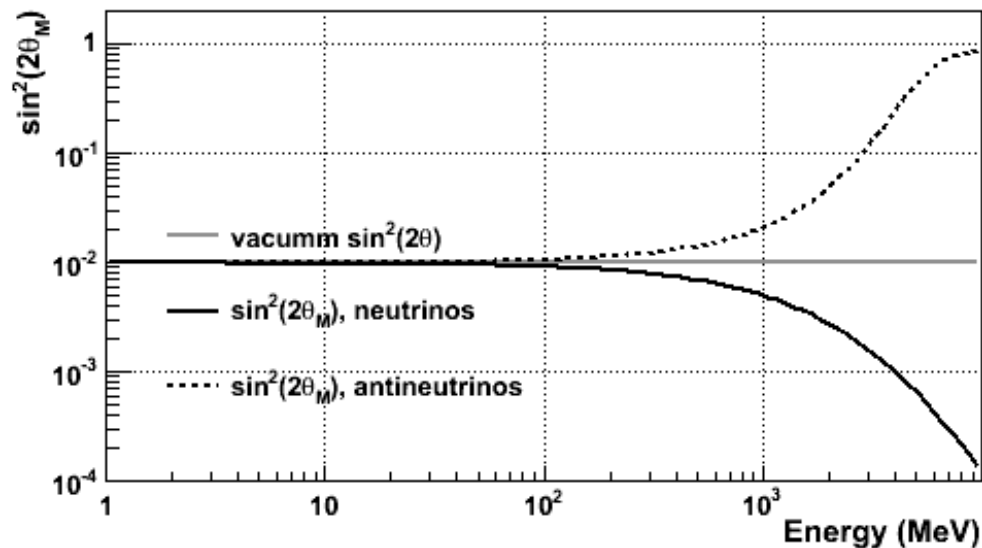
*assuming $|U_{s4}|^2 = 1 - |U_{e4}|^2 - |U_{\mu4}|^2 \approx 1$

NON-STANDARD MATTER EFFECTS CAN GIVE SIZABLE NEUTRINO – ANTINEUTRINO DIFFERENCES



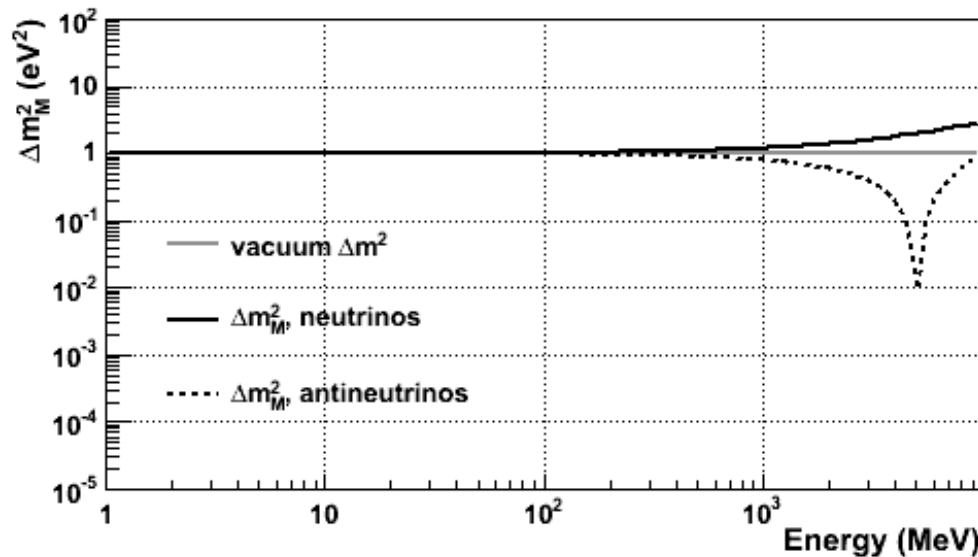
← vacuum case

Effective mixing parameters for neutrinos and antineutrinos are different!



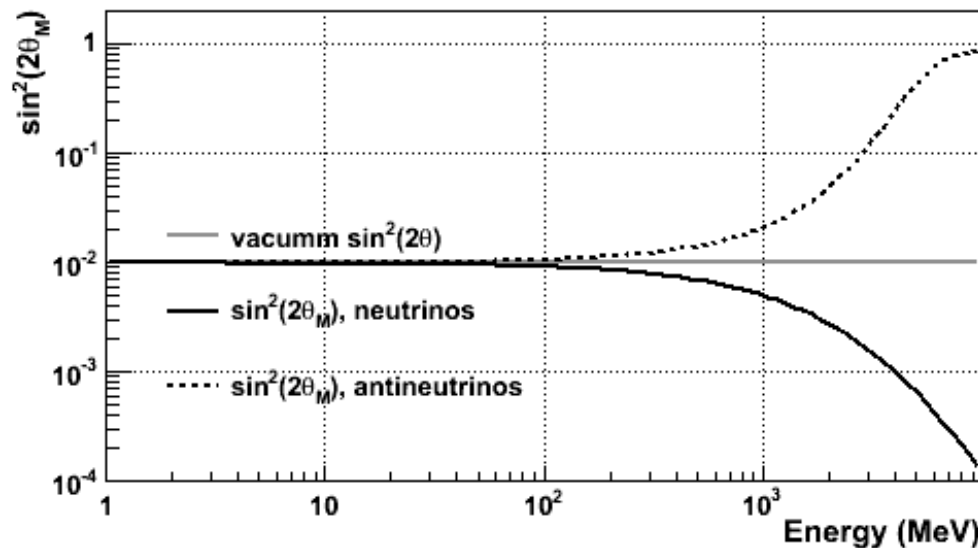
← vacuum case

NON-STANDARD MATTER EFFECTS CAN GIVE SIZABLE NEUTRINO – ANTINEUTRINO DIFFERENCES



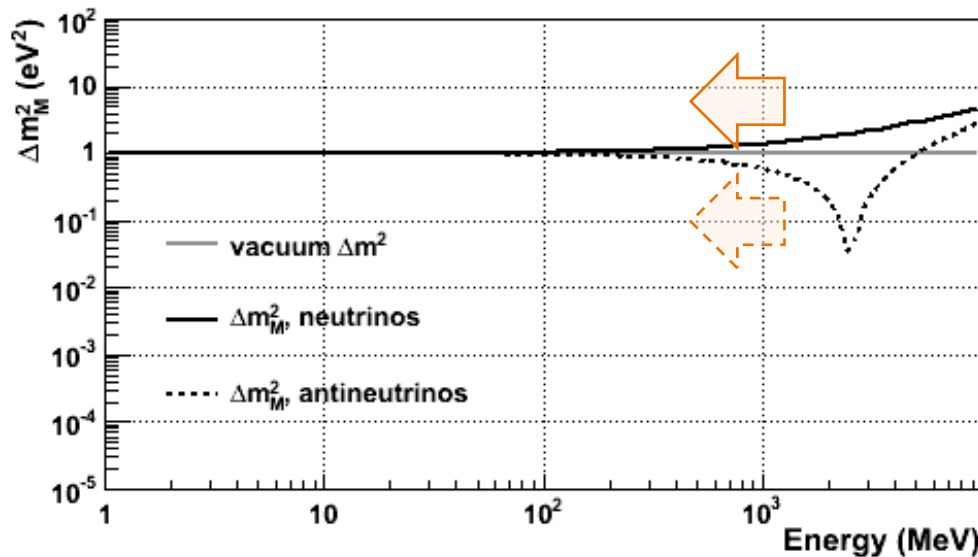
← vacuum case

For a set of vacuum parameters, **increasing A_s** leads to larger deviations from vacuum case and shifts the resonance to lower antineutrino energies.



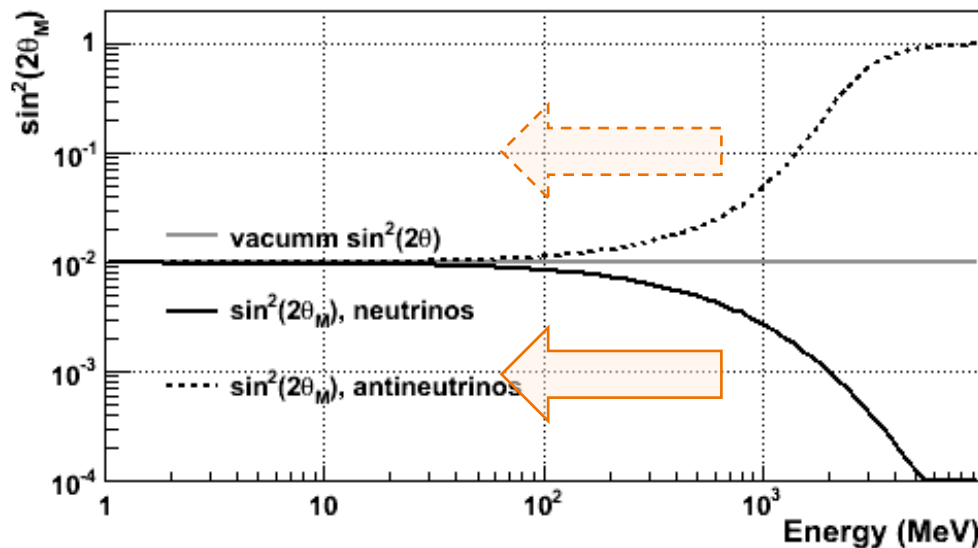
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NON-STANDARD MATTER EFFECTS CAN GIVE SIZABLE NEUTRINO – ANTINEUTRINO DIFFERENCES



← vacuum case

For a set of vacuum parameters, **increasing A_s** leads to larger deviations from vacuum case and shifts the resonance to lower antineutrino energies.



← vacuum case

STERILE NEUTRINO + NON-STANDARD MATTER-LIKE EFFECTS FIT

General oscillation probability:

$$P(\nu_\mu \rightarrow \nu_e) = 4|U_{e4}^M|^2|U_{\mu4}^M|^2 \sin^2(1.27\Delta m_M^2 L/E)$$

$$\sin^2 2\theta_M = 4|U_{e4}^M|^2|U_{\mu4}^M|^2$$

“(3+1)+M” fit to MiniBooNE and LSND:

○ Vary:

vacuum $|U_{e4}|^2, |U_{\mu4}|^2$ within 0-5% each

vacuum Δm^2 within 0.01-100 eV²

A_s within 10^{-13} - 10^{-9} eV ← short-baseline data driven

$$\sin^2 2\theta_M = \frac{16(\Delta m^2)^4 |U_{e4}|^2 |U_{\mu4}|^2 |U_{s4}|^4}{((\Delta m^2 - 2EV_s)^2 + 4(2EV_s \Delta m^2 |U_{s4}|^2))(2EV_s - \Delta m^2(1 - 2|U_{s4}|^2) + \sqrt{(\Delta m^2 - 2EV_s)^2 + 4(2EV_s \Delta m^2 |U_{s4}|^2)})^2}$$

$$\Delta m_M^2 = \Delta m^2 + 2EV_s$$



OUTLINE

Sterile neutrino oscillation formalism

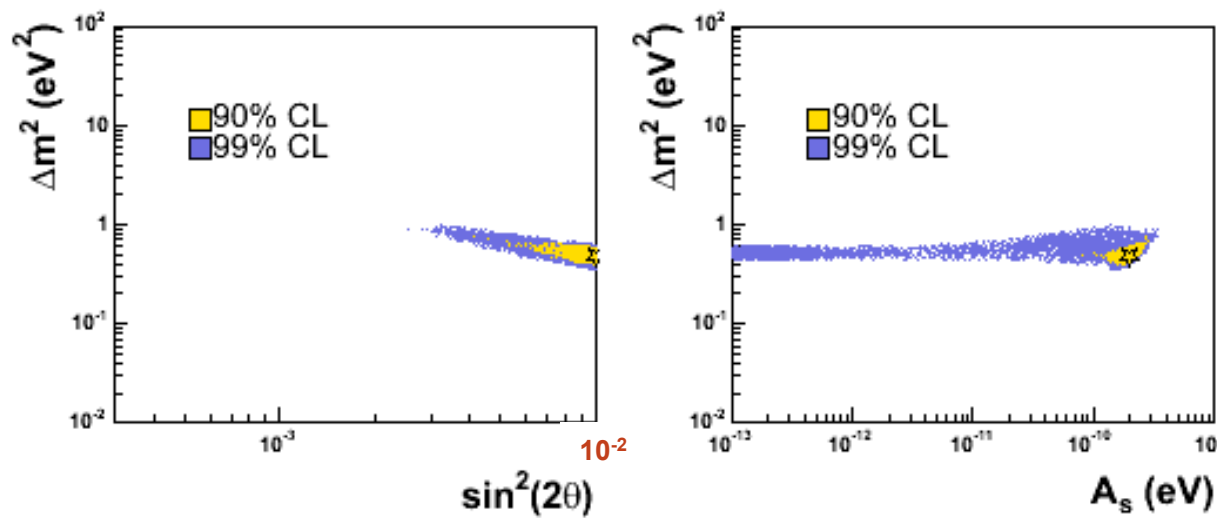
Experimental hints

Review of global fit results

Extending the sterile neutrino oscillation formalism to include non-standard matter-like effects

Fit results

FIT RESULTS: ALLOWED PARAMETERS



Fit prefers a **large** $A_s \sim 2.0 \times 10^{-10}$ eV

Best-fit vacuum oscillation parameters:

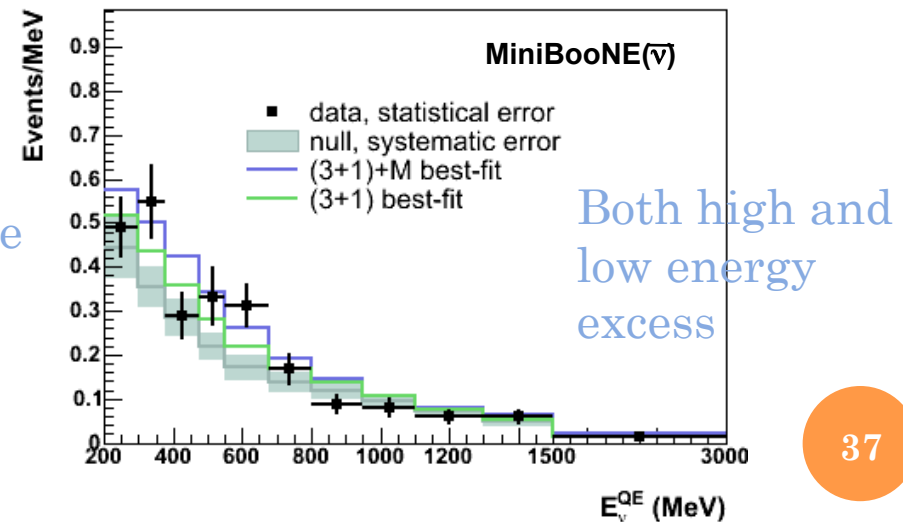
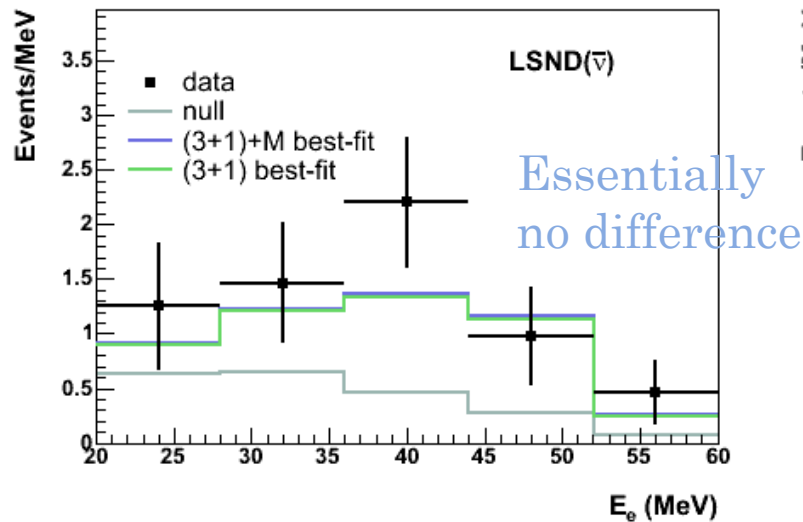
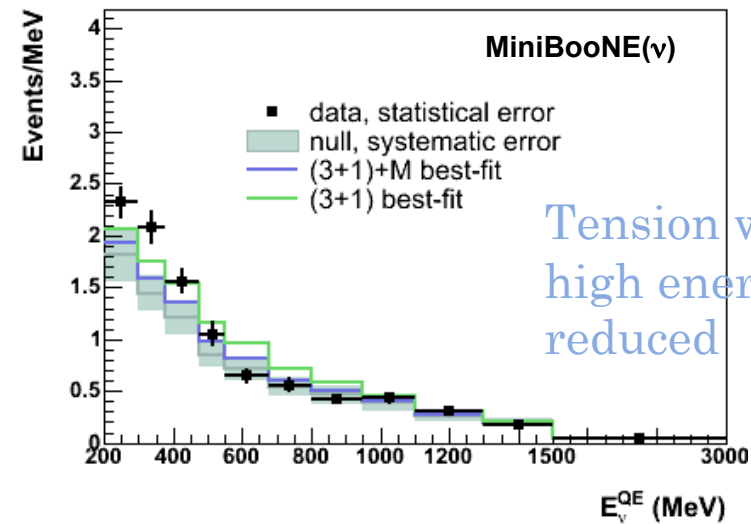
$$\Delta m^2 = 0.47 \text{ eV}^2, \quad \sin^2 2\theta = 0.0099$$

(Note: for standard matter effects, $A_e = \sqrt{2}G_F n_e \sim 10^{-13}$ eV)

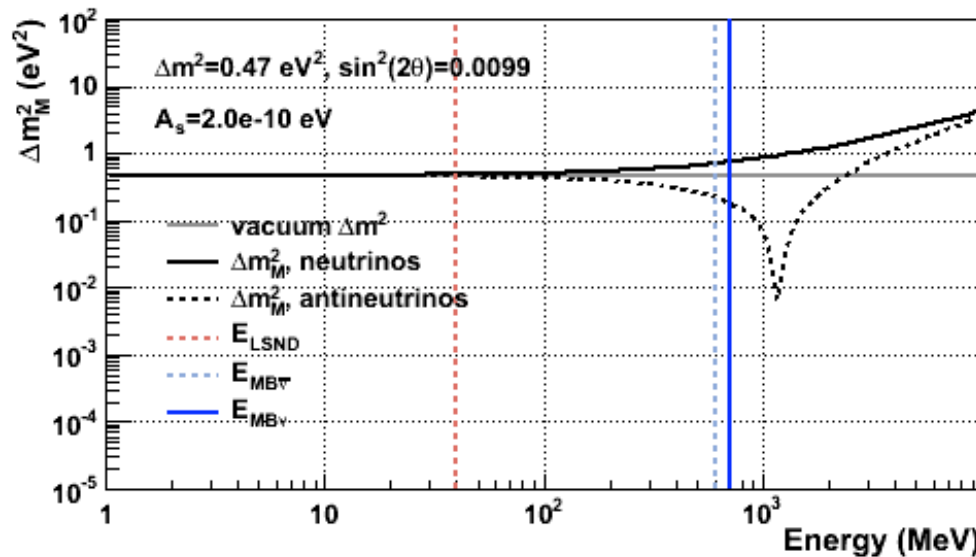
FIT RESULTS: BEST-FIT DISTRIBUTIONS

$$\begin{aligned}\chi^2 (3+1)+M &= 44.5/38 \text{ (22\%)} \\ \chi^2 (3+1) &= 52.9/39 \text{ (7\%)} \\ \Delta\chi^2/\text{dof} &= 8.5/1 \text{ fit param.}\end{aligned}$$

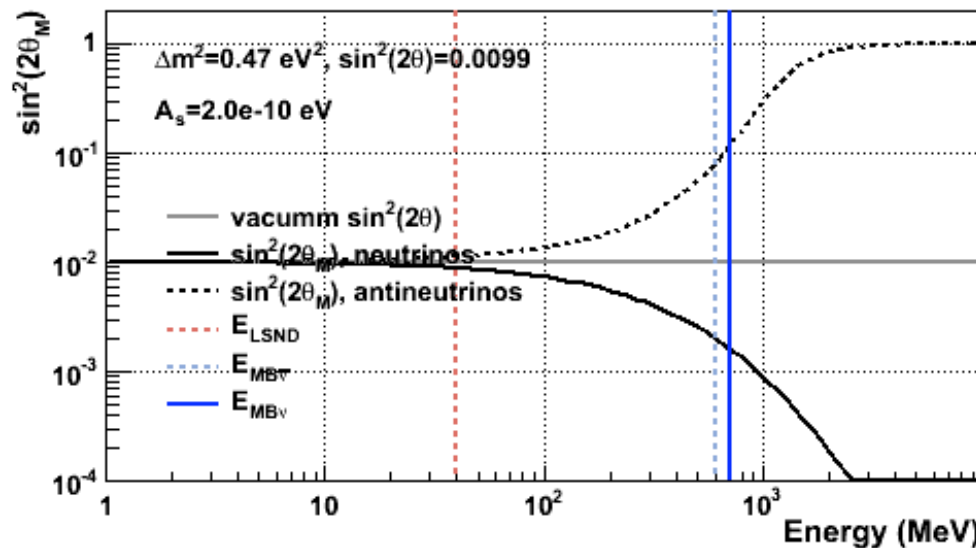
Compatibility increases
from 2.3% to 17.4%.



FIT RESULTS: EFFECTIVE MIXING PARAMETERS VS. ENERGY

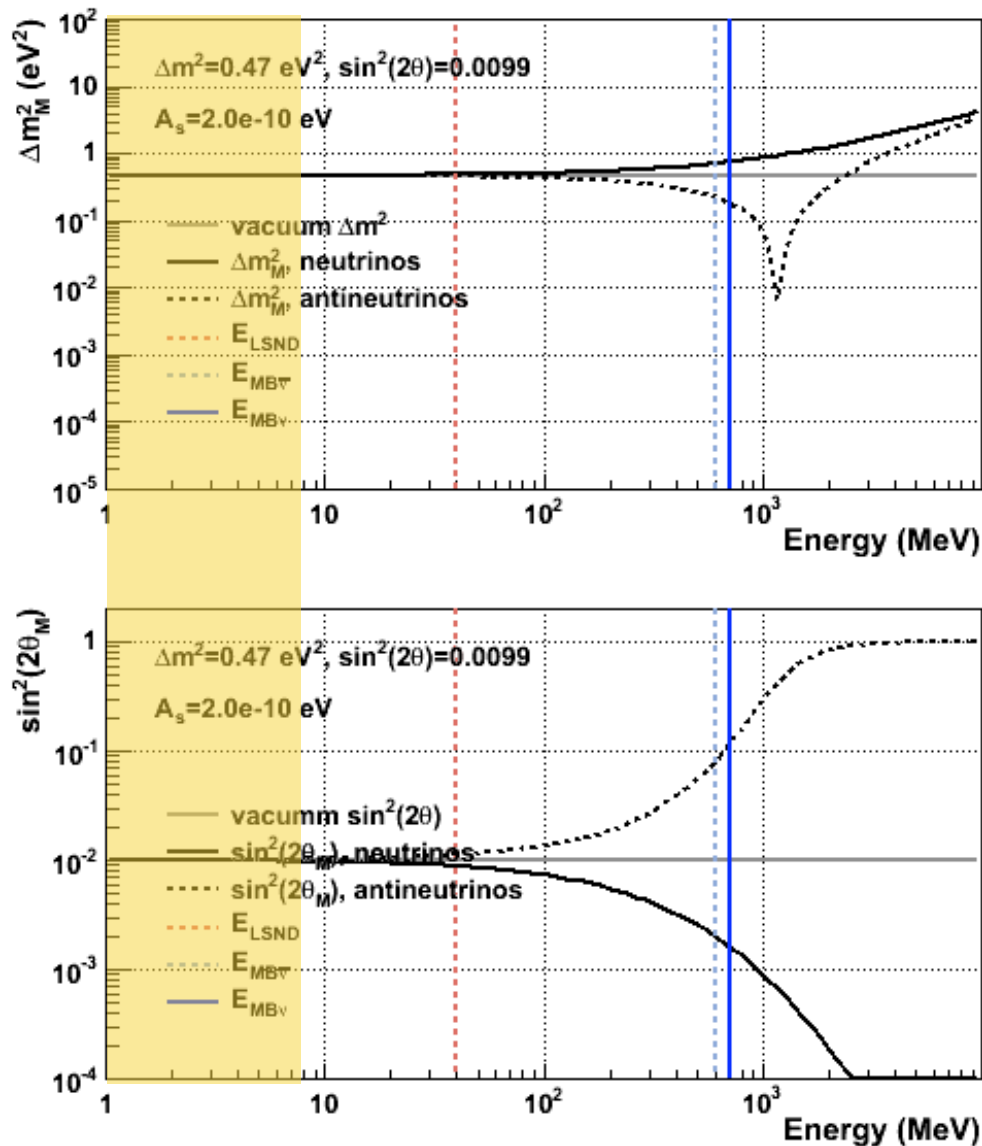


← Best-fit Δm^2 (vacuum)



← Best-fit $\sin^2 2\theta$ (vacuum)

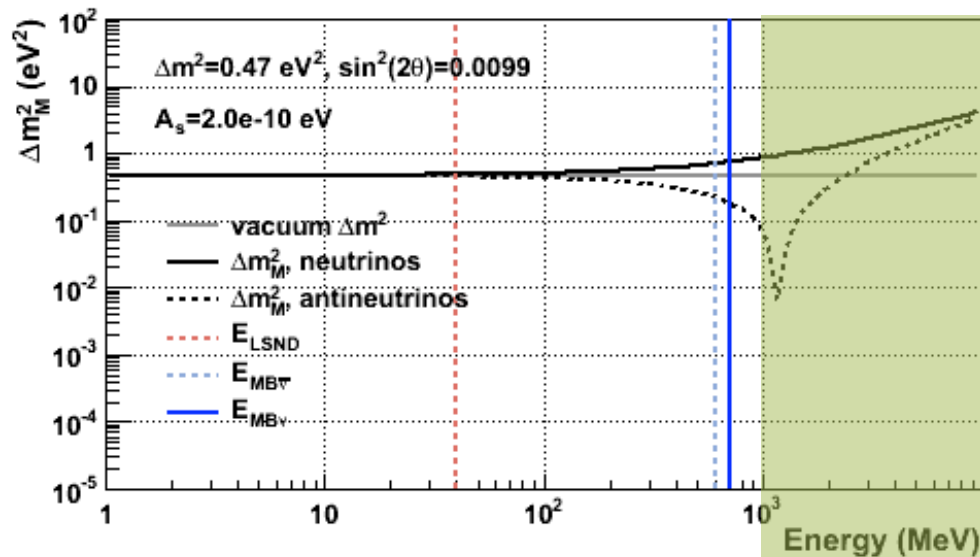
FIT RESULTS: EFFECTIVE MIXING PARAMETERS VS. ENERGY



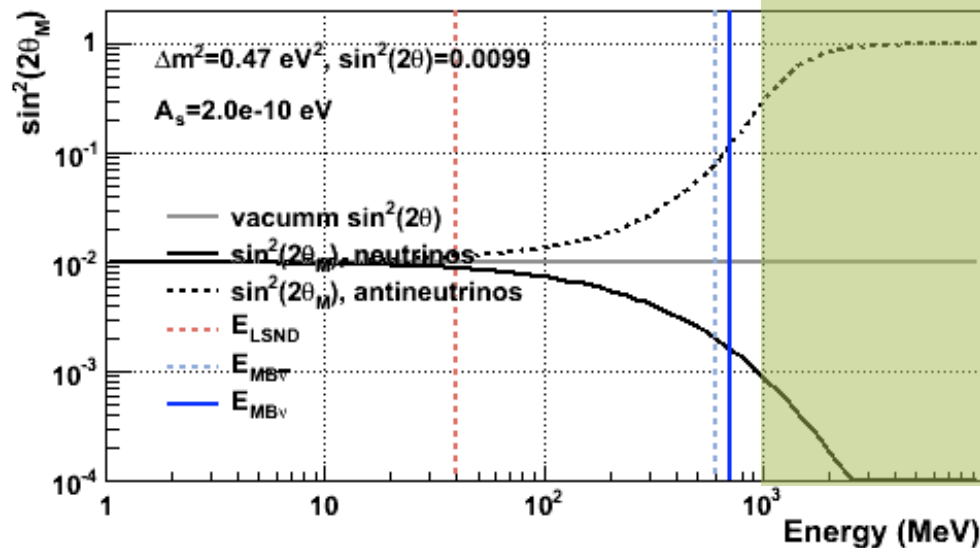
poses no threat to
reactor, solar(?) dataset
interpretations;
looks like (3+1) at low energies,
so it can accommodate reactor
anomaly

...and/but...

FIT RESULTS: EFFECTIVE MIXING PARAMETERS VS. ENERGY



suggests observable effects
in MINOS $\bar{\nu}$
and other accelerator &
atmospheric $\bar{\nu}$ measurements !



Including those datasets
is essential (work in progress...)

CONCLUSIONS

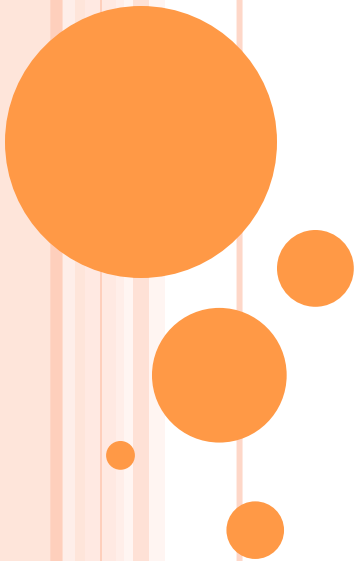
- Several **hints for sterile neutrinos**, all compatible with (3+1) oscillations at $\Delta m^2 \sim 1 \text{eV}^2$ and small mixings.
- However, there are **apparent differences between neutrino and antineutrino disappearance** which make it hard to accommodate all results in either (3+1) or (3+2).
- **Non-standard interactions/matter-like effects can lead to differences** in neutrino and antineutrino disappearance (ant appearance) probabilities with and without sterile neutrinos.
- We have presented a model which seems to accommodate MiniBooNE neutrino and antineutrino (including low energy excess), and LSND antineutrino data simultaneously:

(3+1) model with “matter-like” potential $V_s = \pm A_s$ experienced only by ($\nu/\bar{\nu}$) sterile neutrinos.

A_s is large and invites interpretations.

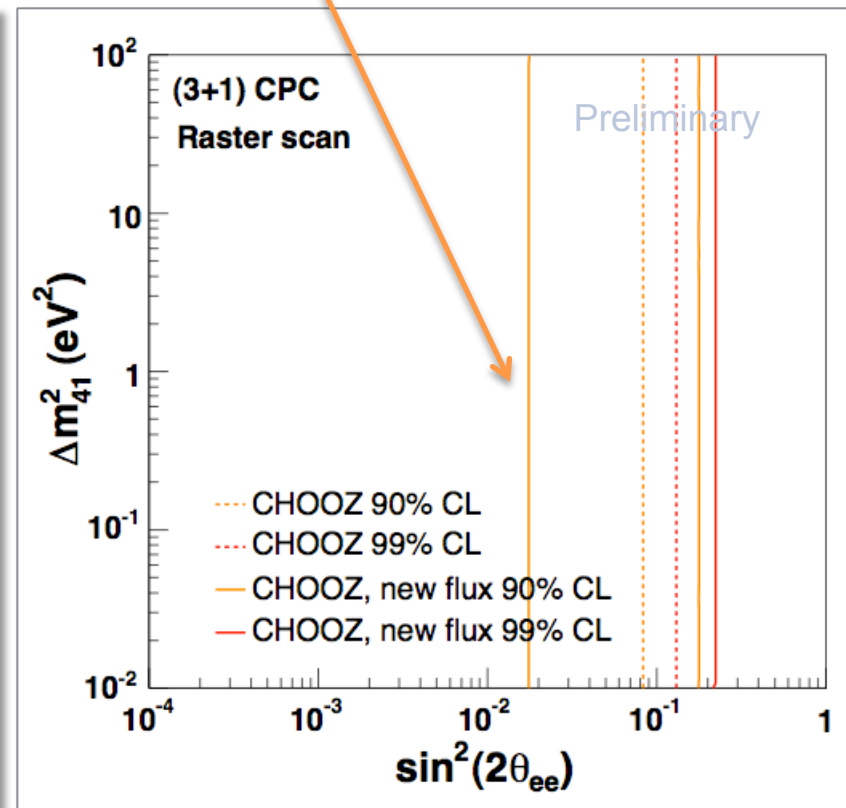
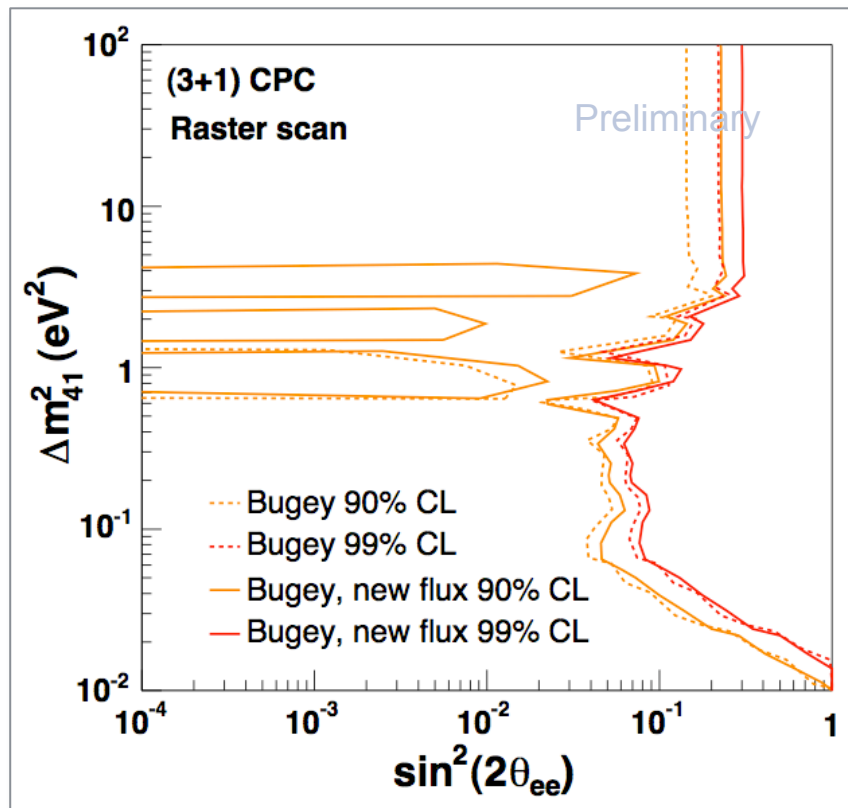
- Implications for other antineutrino datasets with $E > 100 \text{ MeV}$ are being investigated, along with other sources of external constraints.

THANK YOU.



EFFECT OF NEW FLUX ON CONSTRAINTS FROM REACTORS

No closed contours when doing global scan, but lower limit for raster scan



GLOBAL FITS: COMPATIBILITY TESTS

$$\chi_{PG}^2 = \chi_{min,all}^2 - \sum_i \chi_{min,i}^2, \quad PG = prob(\chi_{PG}^2, ndf_{PG}).$$

A measure of how much parameter regions preferred
by different subsets of data overlap.

Gives sensible results even in cases where:

- the errors are estimated very conservatively
- the total number of data points is very large

Avoids the problem that a possible disagreement between data sets becomes diluted by data points which are insensitive to the problem in the fit

Can also be very useful when a set consisting of a rather small number of data points is combined with a very large data sample

“Testing the statistical compatibility of independent data sets”,
Maltoni & Schwetz, Phys.Rev. D68 (2003) 033020

CPT CONSERVATION AND DISAPPEARANCE PROBABILITIES

$$\mathbf{C}[\nu_{L\alpha} \rightarrow \nu_{L\alpha}] = \bar{\nu}_{L\alpha} \rightarrow \bar{\nu}_{L\alpha}$$



$$\mathbf{P}[\bar{\nu}_{L\alpha} \rightarrow \bar{\nu}_{L\alpha}] = \bar{\nu}_{R\alpha} \rightarrow \bar{\nu}_{R\alpha}$$



$$\mathbf{T}[\bar{\nu}_{R\alpha} \rightarrow \bar{\nu}_{R\alpha}] = \bar{\nu}_{R\alpha} \rightarrow \bar{\nu}_{R\alpha}$$

$$\mathbf{CPT}[\nu_{\alpha} \rightarrow \nu_{\alpha}] = \bar{\nu}_{\alpha} \rightarrow \bar{\nu}_{\alpha}$$

GLOBAL FITS: (3+2)

PRELIMINARY

Dataset	CP	χ^2 (ndf)	gof	Δm_{41}^2	Δm_{51}^2	$ U_{e4} $	$ U_{\mu 4} $	$ U_{e5} $	$ U_{\mu 5} $	ϕ_{45}
all SBL+ atm	CPC	186.1 (193)	62%	0.92	23.8	0.13	0.13	0.083	0.14	0
	CPV	182.6 (192)	67%	0.92	26.6	0.14	0.14	0.077	0.15	1.7π

includes 2010 MiniBooNE antineutrino appearance dataset, and new reactor flux predictions

Change in χ^2 for CPC \rightarrow CPV: 3.5/1 dof
 Compatibility among all experiments still low: 6%

	Δm_{41}^2	$ U_{e4} $	$ U_{\mu 4} $	Δm_{51}^2	$ U_{e5} $	$ U_{\mu 5} $	δ/π	χ^2/dof
3+2	0.47	0.128	0.165	0.87	0.138	0.148	1.64	110.1/130

(Kopp et al. - hep-ph:1103.4570)

Compatibility between data sets better
 but still not very good

LSND+MB ($\bar{\nu}$) vs Rest = 0.13%

Appearance vs Disappearance = 0.53%

ON THE $\sin^2 2\theta$ CUTOFF

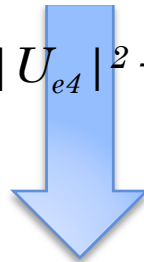
$$\lambda_1 = 0$$

$$\lambda_2 = 0$$

$$\lambda_3 = \frac{1}{4E}(2EV_s + \Delta m^2 - \sqrt{(2EV_s + \Delta m^2)^2 - 4(2EV_s \Delta m^2(1 - |U_{s4}|^2))})$$

$$\lambda_4 = \frac{1}{4E}(2EV_s + \Delta m^2 + \sqrt{(2EV_s + \Delta m^2)^2 - 4(2EV_s \Delta m^2(1 - |U_{s4}|^2))})$$

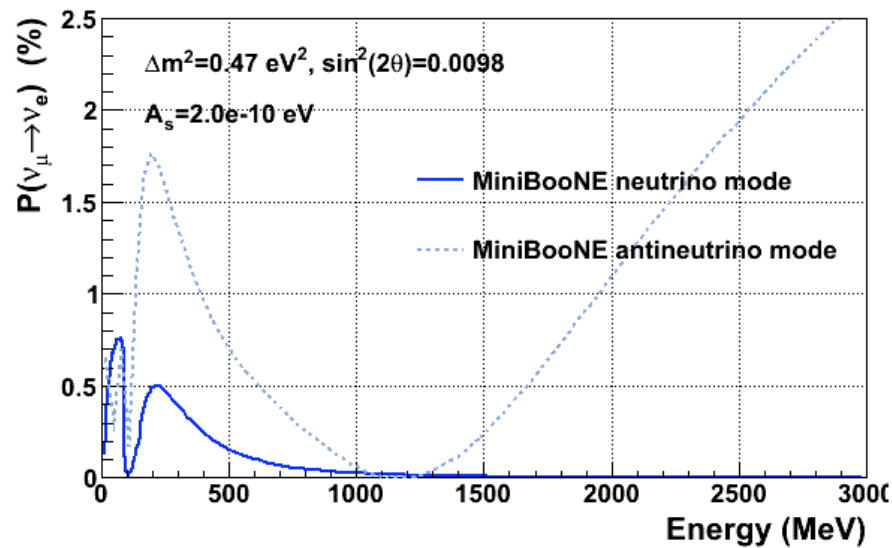
$$|U_{s4}|^2 = 1 - |U_{e4}|^2 - |U_{\mu 4}|^2 \approx 1$$



single- Δm^2 case

$$(\lambda_3=0)$$

(3+1)+M:



Predicted oscillation probabilities at MiniBooNE and LSND given best-fit parameters.

