QCD critical point and fluctuations

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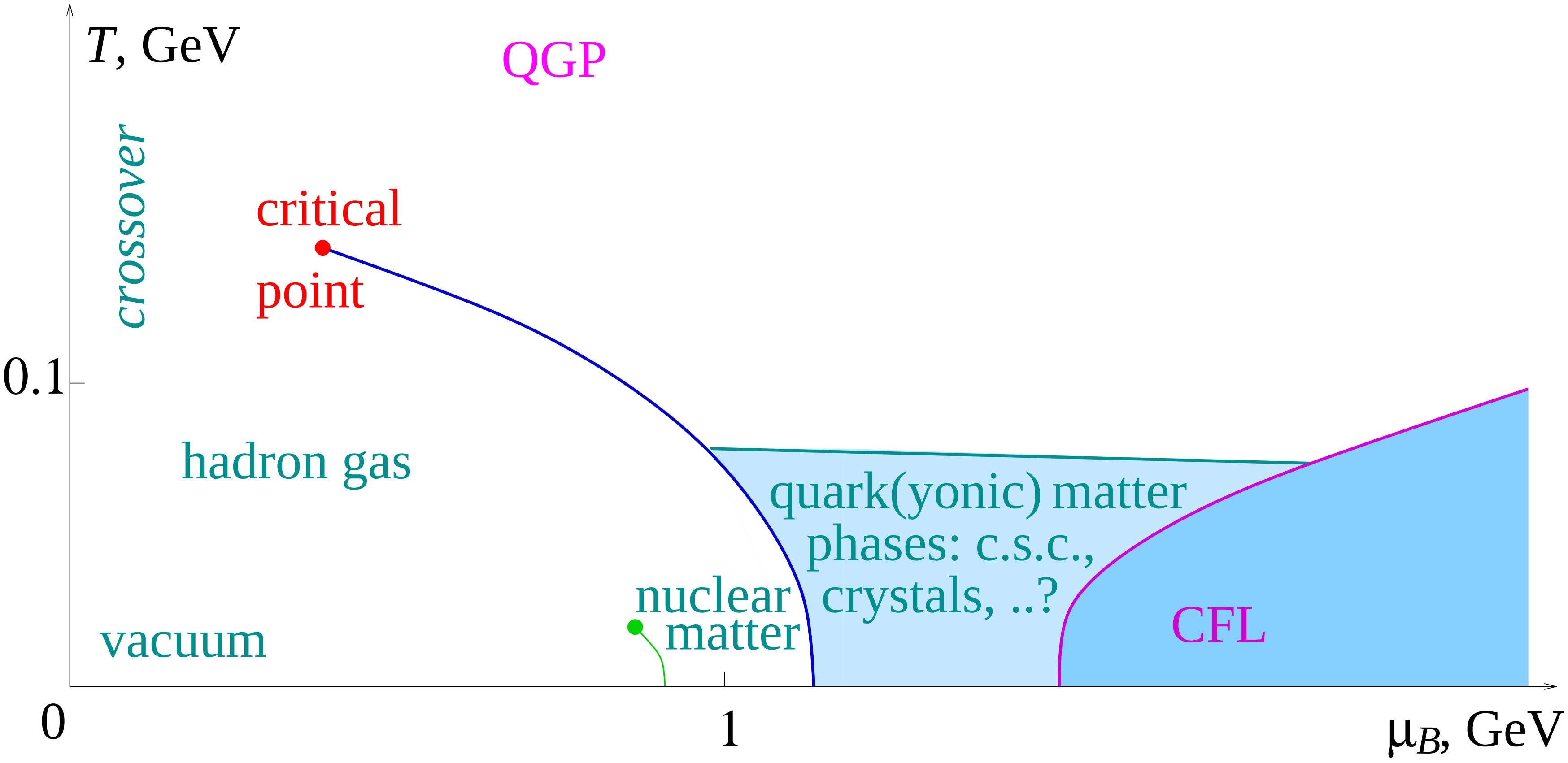
QCD phase diagram (a sketch)

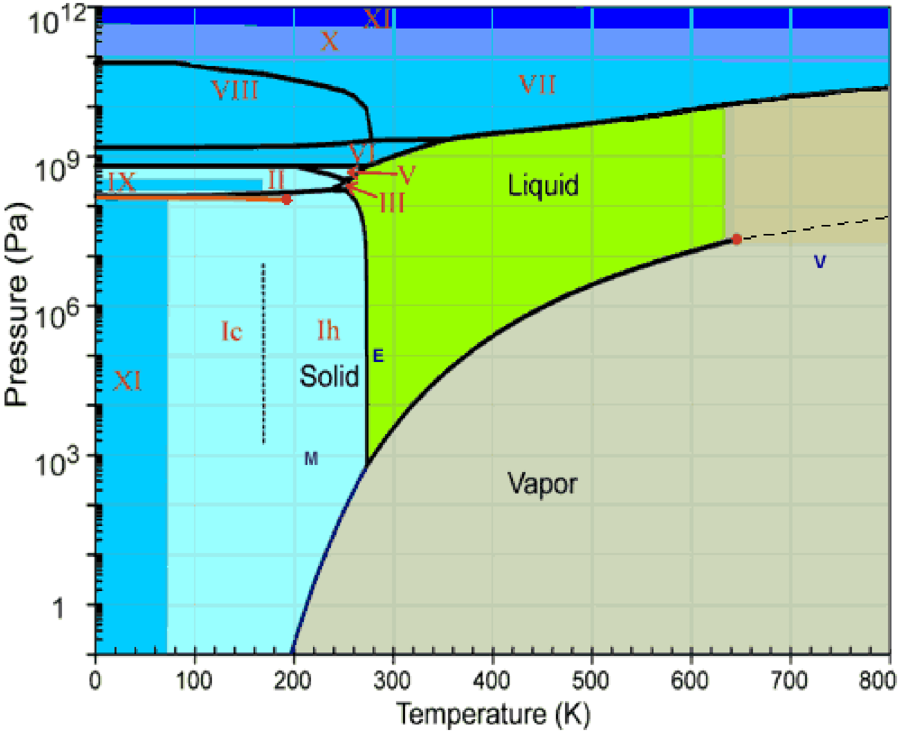


- Models (and lattice) suggest the transition becomes 1st order at some μ_B.
- Can we observe the critical point in heavy ion collisions, and how?

Critical point(s) in known liquids

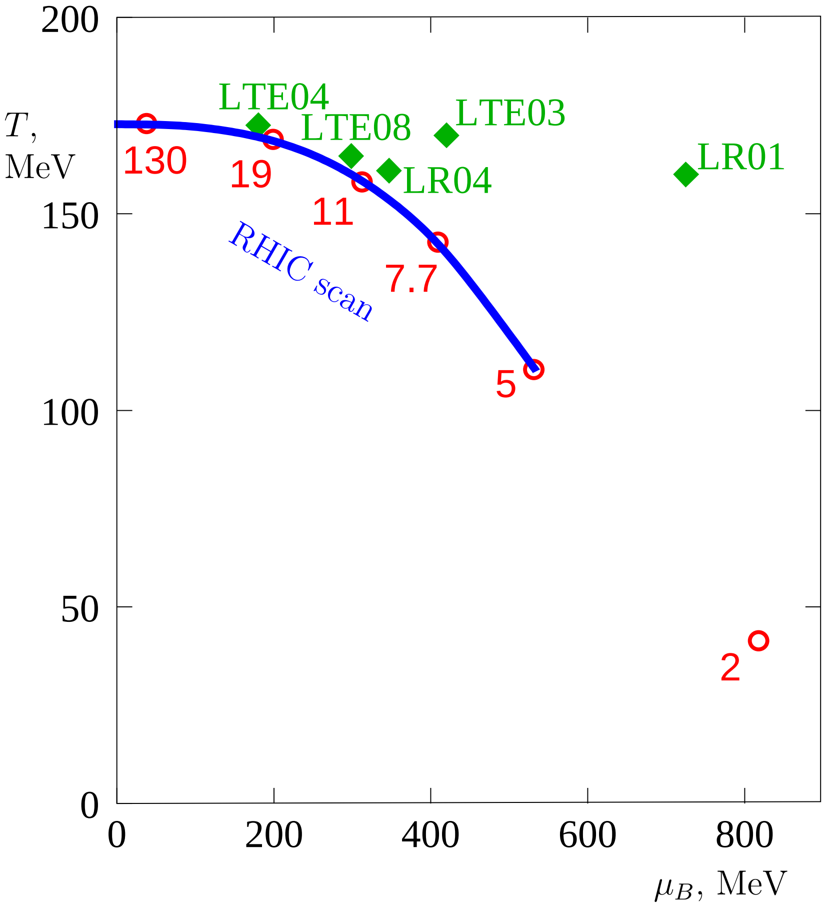
Most liquids have a critical point (seen, e.g., by critical opalescence).

Water:



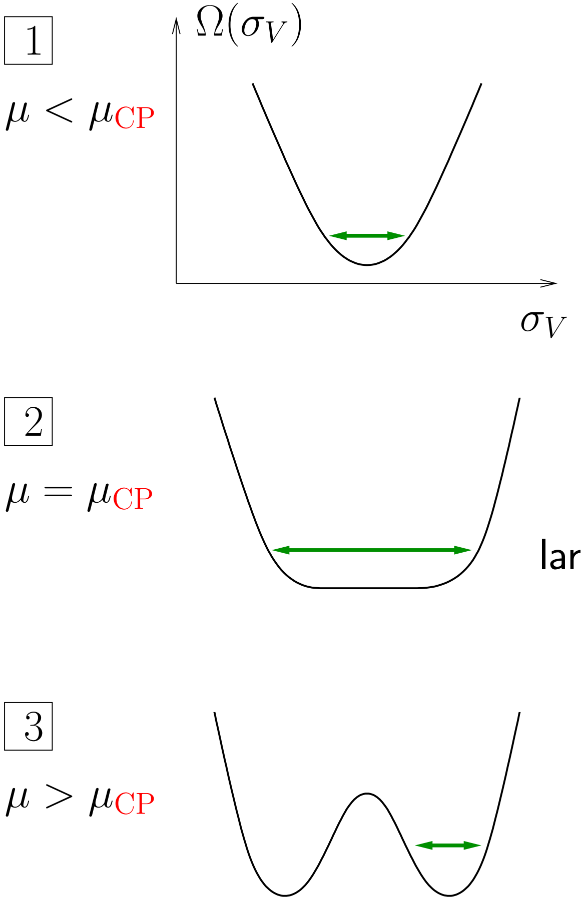
Does QCD “perfect liquid” have one?

What do we need to discover the critical point?



- Experiments: RHIC, NA61/SPS, FAIR/GSI, NICA.
- Better lattice predictions, with controllable systematics.
- Sensitive experimental signatures.

Critical fluctuations: theory

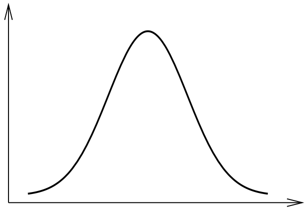


Consider an observable such as,
e.g., $\sigma_V = \int_V \sigma$, where $\sigma \sim \bar{\psi}\psi$.

$\langle \sigma_V^2 \rangle \sim (\Omega'')^{-1}$

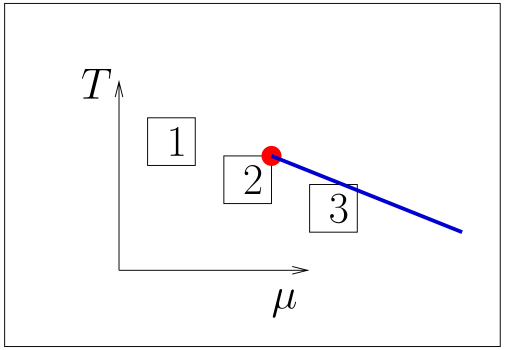
Einstein, 1910:

$P(\sigma_V) \sim$ number
of states with that σ_V
i.e., e^S, or $e^{-\Omega/T}$



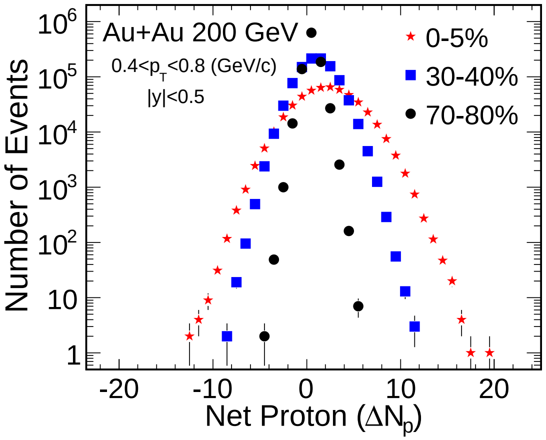
$(\Omega'')^{-1} \to \infty$

large equilibrium fluctuations



- Why does CP defy the central limit theorem?
  Because, correlation length $\xi \to \infty$. This is a collective phenomenon.
  The magnitude of fluctuations $\langle \sigma_V^2 \rangle \sim \xi^2$.

Fluctuation signatures



- Experiments measure multiplicities N_π, N_p, $\ldots$, mean p_T, etc.

  These quantities fluctuate event-by-event.

  - Fluctuation magnitude is quantified by e.g., $\langle(\delta N)^2\rangle$,$\langle(\delta p_T)^2\rangle$.

- What is the magnitude of these fluctuations near the QCD C.P.? (Rajagopal-Shuryak-MS, 1998)

- Universality tells us how it grows at the critical point: $\langle(\delta N)^2\rangle \sim \xi^2$.

- Magnitude of ξ is limited $< \mathcal{O}($2–3 fm$)$ (Berdnikov-Rajagopal).

- "Shape" of the fluctuations can be measured: non-Gaussian moments. As $\xi \to \infty$ fluctuations become less Gaussian.

- Higher cumulants show even stronger dependence on ξ (PRL 102:032301,2009):

$$\langle(\delta N)^3\rangle \sim \xi^{4.5}, \qquad \langle(\delta N)^4\rangle - 3\langle(\delta N)^2\rangle^2 \sim \xi^{7}$$

  which makes them more sensitive signatures of the critical point.

Higher moments (cumulants) and ξ

- Consider probability distribution for the order-parameter field:

$$P[\sigma] \sim \exp\{-\Omega[\sigma]/T\},$$

$$\Omega = \int d^3x \left[\frac{1}{2}(\boldsymbol{\nabla}\sigma)^2 + \frac{m_\sigma^2}{2}\sigma^2 + \frac{\lambda_3}{3}\sigma^3 + \frac{\lambda_4}{4}\sigma^4 + \dots\right]. \qquad \Rightarrow \qquad \xi = m_\sigma^{-1}$$

- Moments (connected) of $\boldsymbol{q} = 0$ mode $\sigma_V \equiv \int d^3x\,\sigma(x)$:

$$\kappa_2 = \langle\sigma_V^2\rangle = VT\,\xi^2; \qquad \kappa_3 = \langle\sigma_V^3\rangle = 2VT^2\,\lambda_3\,\xi^6;$$

$$\kappa_4 = \langle\sigma_V^4\rangle_c \equiv \langle\sigma_V^4\rangle - 3\langle\sigma_V^2\rangle^2 = 6VT^3\,[\,2(\lambda_3\xi)^2 - \lambda_4\,]\,\xi^8\,.$$

- Tree graphs. Each propagator gives ξ^2.

- Scaling requires “running”: $\lambda_3 = \tilde\lambda_3 T(T\xi)^{-3/2}$ and $\lambda_4 = \tilde\lambda_4(T\xi)^{-1}$, i.e.,

$$\kappa_3 = \langle\sigma_V^3\rangle = 2VT^{3/2}\,\tilde\lambda_3\,\xi^{4.5}; \qquad \kappa_4 = 6VT^2\,[\,2(\tilde\lambda_3)^2 - \tilde\lambda_4\,]\,\xi^7\,.$$

Moments of observables

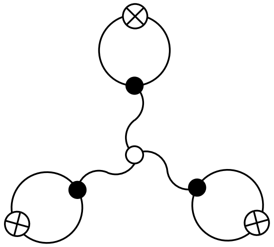
- Example: Fluctuation of multiplicity is the fluctuation of occup. numbers,

$$\delta N = \sum\nolimits_{\boldsymbol{p}} \delta n_{\boldsymbol{p}} .$$

Any moment of the multiplicity distribution is related to a correlator of $\delta n_{\boldsymbol{p}}$:

$$\kappa_{3\pi} = \langle (\delta N)^3 \rangle = \sum\nolimits_{\boldsymbol{p}_1} \sum\nolimits_{\boldsymbol{p}_2} \sum\nolimits_{\boldsymbol{p}_3} \langle \delta n_{\boldsymbol{p}_1} \delta n_{\boldsymbol{p}_2} \delta n_{\boldsymbol{p}_3} \rangle \,, \qquad \text{where } \sum\nolimits_{\boldsymbol{p}} = V \int \frac{d^3 \boldsymbol{p}}{(2\pi)^3} .$$

- $n_{\boldsymbol{p}}$ fluctuates around $\bar{n}_{\boldsymbol{p}}(m)$,
  which also fluctuates: $\delta m = g \delta\sigma$, i.e.,



$$\delta n_{\boldsymbol{p}} = \delta n^0_{\boldsymbol{p}} + \frac{\partial \bar{n}_{\boldsymbol{p}}}{\partial m} \, g \, \delta\sigma \, .$$

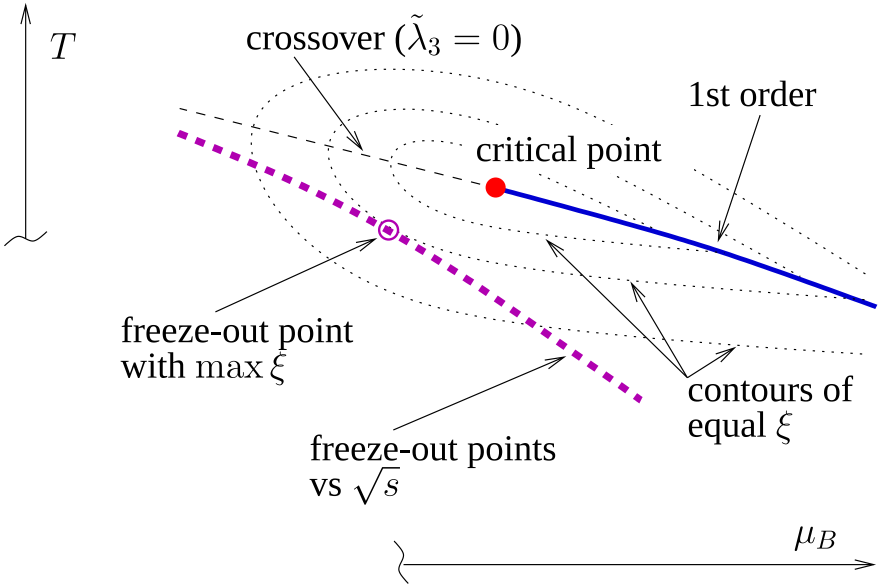
$$\langle \delta n_{\boldsymbol{p}_1} \delta n_{\boldsymbol{p}_2} \delta n_{\boldsymbol{p}_3} \rangle_\sigma = \frac{2\lambda_3}{V^2 T} \left(\frac{g}{m_\sigma^2} \right)^3 \frac{v^2_{\boldsymbol{p}_1}}{\gamma_{\boldsymbol{p}_1}} \frac{v^2_{\boldsymbol{p}_2}}{\gamma_{\boldsymbol{p}_2}} \frac{v^2_{\boldsymbol{p}_3}}{\gamma_{\boldsymbol{p}_3}}$$

$$v^2_{\boldsymbol{p}} = \bar{n}_{\boldsymbol{p}} (1 \pm \bar{n}_{\boldsymbol{p}}), \quad \gamma_{\boldsymbol{p}} = (dE_{\boldsymbol{p}} / dm)^{-1}$$

  Similarly for $\langle (\delta N)^4 \rangle_c$.

- Since $\langle (\delta N)^3 \rangle$ scales as V^1 we suggest $\omega_3(N) \equiv \dfrac{\langle (\delta N)^3 \rangle}{\bar{N}}$ which is V^0.

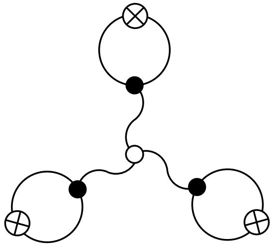
Energy scan and fluctuation signatures: notes



- Higher moments provide more sensitive signatures.
- As usual, value comes at a price:
  - Harder to predict – more theoretical uncertainties.
  - Signal/noise is worse for higher moments.
- But one can, e.g., combine various higher moments to optimize or eliminate uncertainties.

Using ratios and mixed moments

Athanasiou, Rajagopal, MS (2010)



- The dominant dependence on μ_B (i.e., on $\sqrt{s}$) is from two sources ξ and n_p, e.g., $\kappa_{3p} \sim \tilde{\lambda}_3\, g_p^3\, \xi^{4.5} n_p^3$.
  - $\xi(\mu_B)$ has a peak at $\mu_B = \mu_B^{\mathrm{critical}}$;
  - $n_B \sim e^{\mu_B^{\mathrm{critical}}/T}$ determines the height of the peak;
  - other factors: g_p^3 and $\tilde{\lambda}_3$ depend on μ_B weaker.
- Leading dependence on $\mu_B^{\mathrm{critical}}$ can be cancelled in ratios. E.g.,

$$\frac{\kappa_{3p}}{N_p}\left(\frac{N_\pi}{N_p}\right)^2 \sim \tilde{\lambda}_3\, g_p^3\, \xi^{4.5}$$

- Unknown/poorly known coupling parameters g_p or g_π can be also cancelled in ratios. E.g., no uncertainties in these ratios

$$\frac{\kappa_{4p}}{\kappa_{2p}^2}\frac{\kappa_{2\pi}^2}{\kappa_{4\pi}}, \qquad \text{or} \qquad \frac{\kappa_{4p}^3}{\kappa_{3p}^4}\frac{\kappa_{3\pi}^4}{\kappa_{4\pi}^3}.$$

  when critical fluctuations dominate. They are 1.
- Mixed moments allow more possibilities. E.g.,

$$\frac{\kappa_{2p2\pi}^2}{\kappa_{4p}\kappa_{4\pi}}.$$

  Mixed moments have no trivial Poisson contribution.

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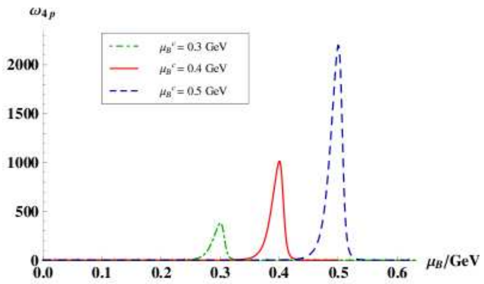

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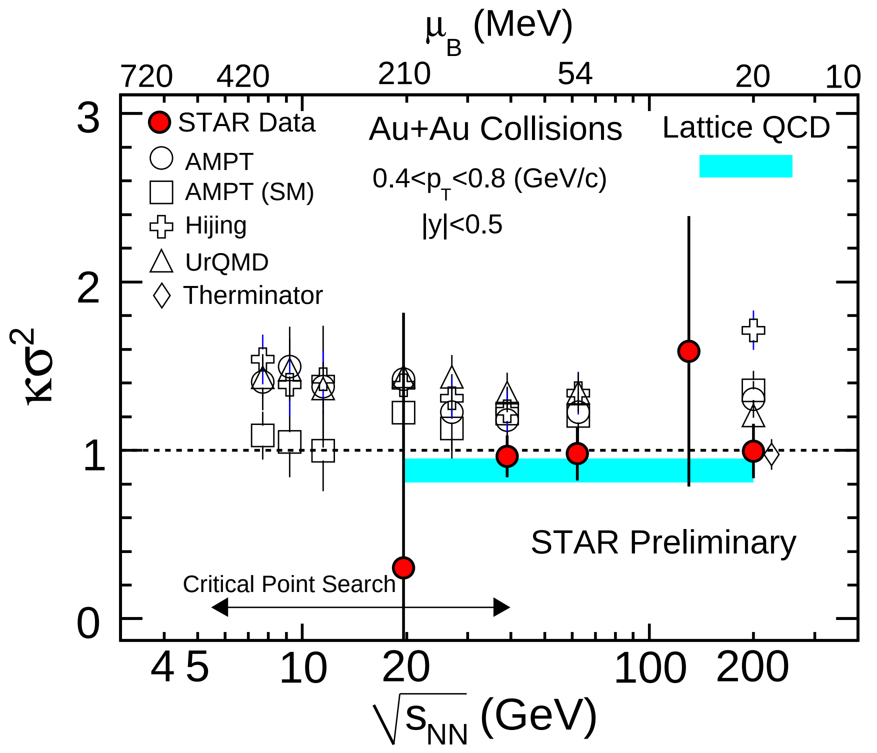
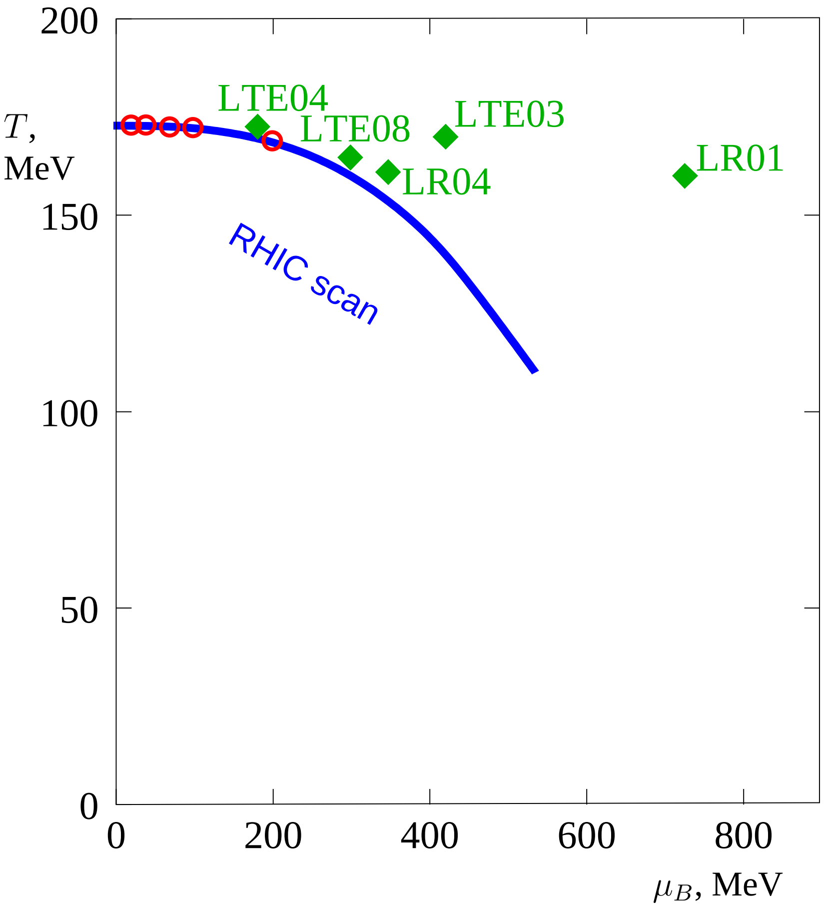
Experimental data (pre-QM)





($\kappa\sigma^2 = \kappa_4/\kappa_2 \approx \omega_4$ if $\kappa_2 \approx N$).

No critical signatures seen at those values of μ_B.

Consistent with expectations that $\mu_B^{\rm critical} > 200$ MeV.

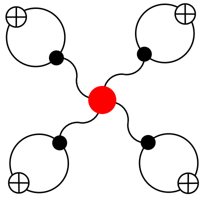
What is happening at $\sqrt{s} = 19.6$ GeV? Low statistics.

Large positive contribution to Poisson is excluded, but large negative — is not.

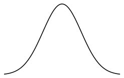
Negative kurtosis?

- Could the critical contribution to kurtosis be negative? (MS, arxiv:1104.1627)

$$\langle(\delta N)^4\rangle_c = \langle N\rangle + \langle\sigma_V^4\rangle_c \left(\frac{g}{T}\int_{\boldsymbol{p}}\frac{v_{\boldsymbol{p}}^2}{\gamma_{\boldsymbol{p}}}\right)^4 + \dots,$$

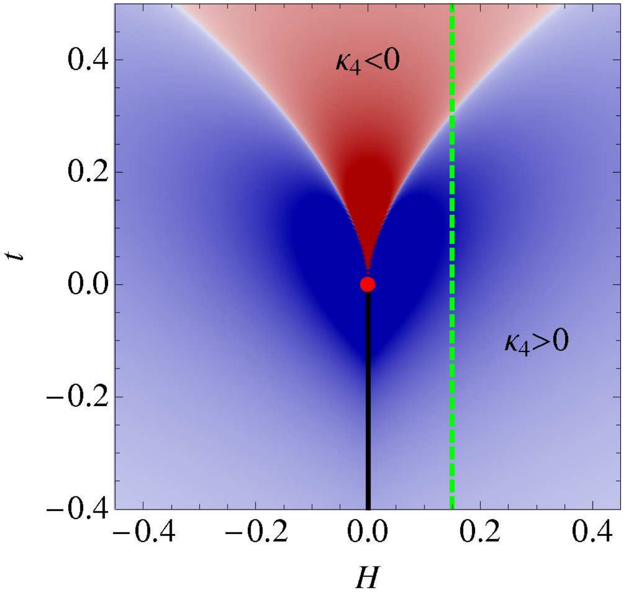


$$\langle\sigma_V^4\rangle_c = 6VT^2\,[\,2\tilde\lambda_3^2 - \tilde\lambda_4\,]\,\xi^7\,.$$

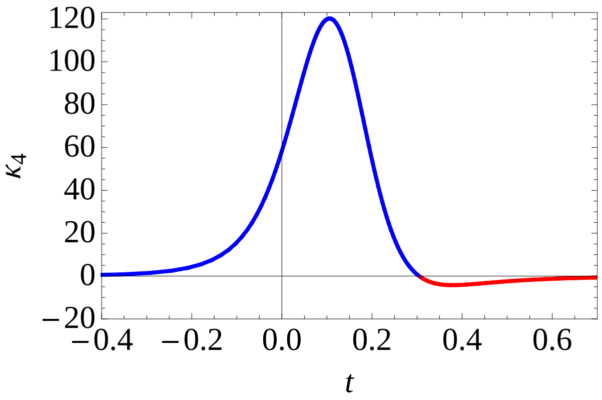
  - On the crossover line $\tilde\lambda_3 = 0$ by symmetry, while $\tilde\lambda_4 \approx 4. > 0$.
  - $P(\sigma_V)$:  $\rightarrow$ 

  Thus $\langle\sigma_V^4\rangle_c < 0$ and $\omega_4(N) < 1$ on the crossover line. And around it.

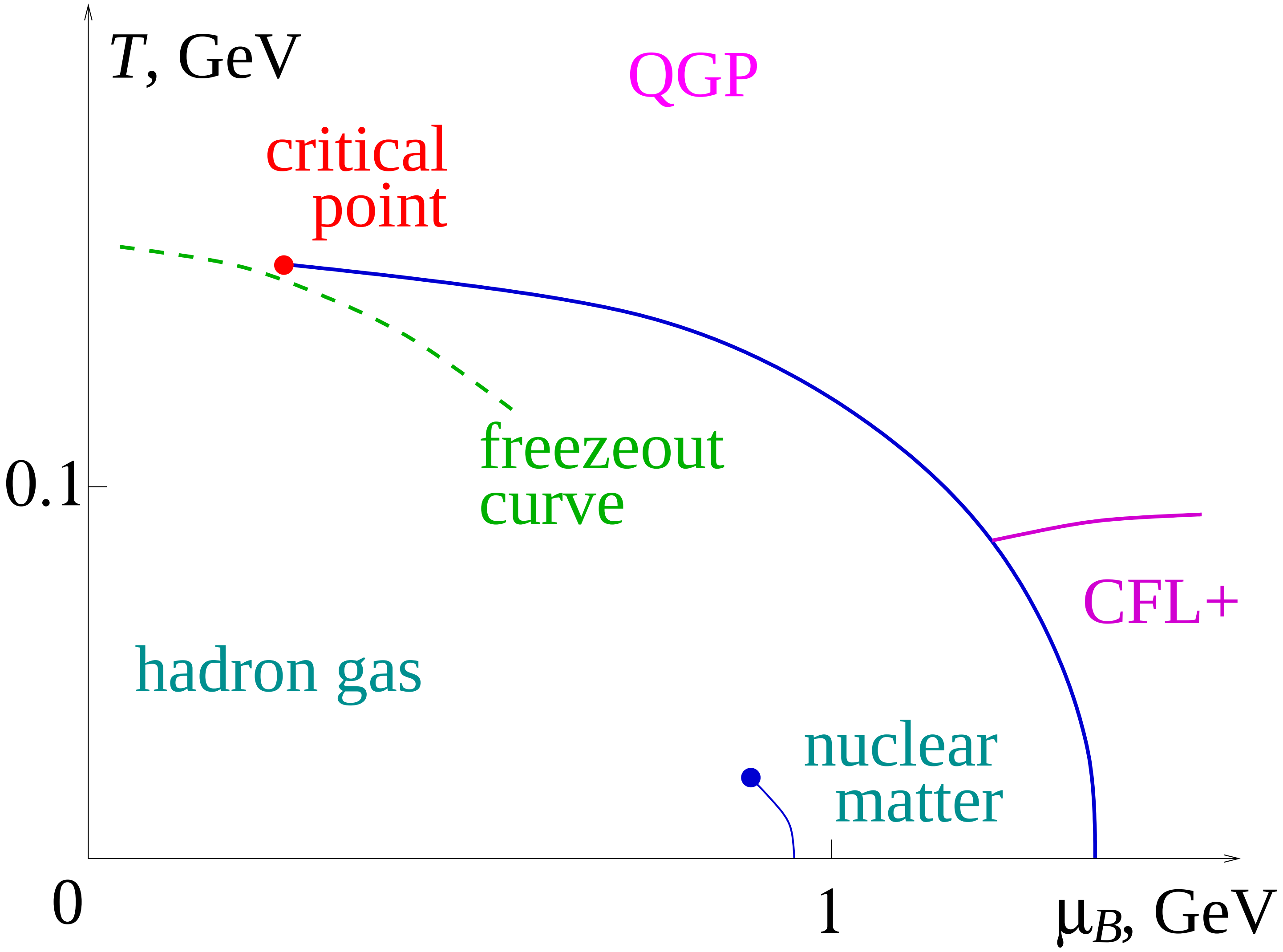
- Universal Ising eq. of state: $M = R^\beta\theta$, $t = R(1-\theta^2)$, $H = R^{\beta\delta}h(\theta)$



here κ_4 is $\kappa_4(M) \equiv \langle M^4\rangle_c$



Implications for the energy scan



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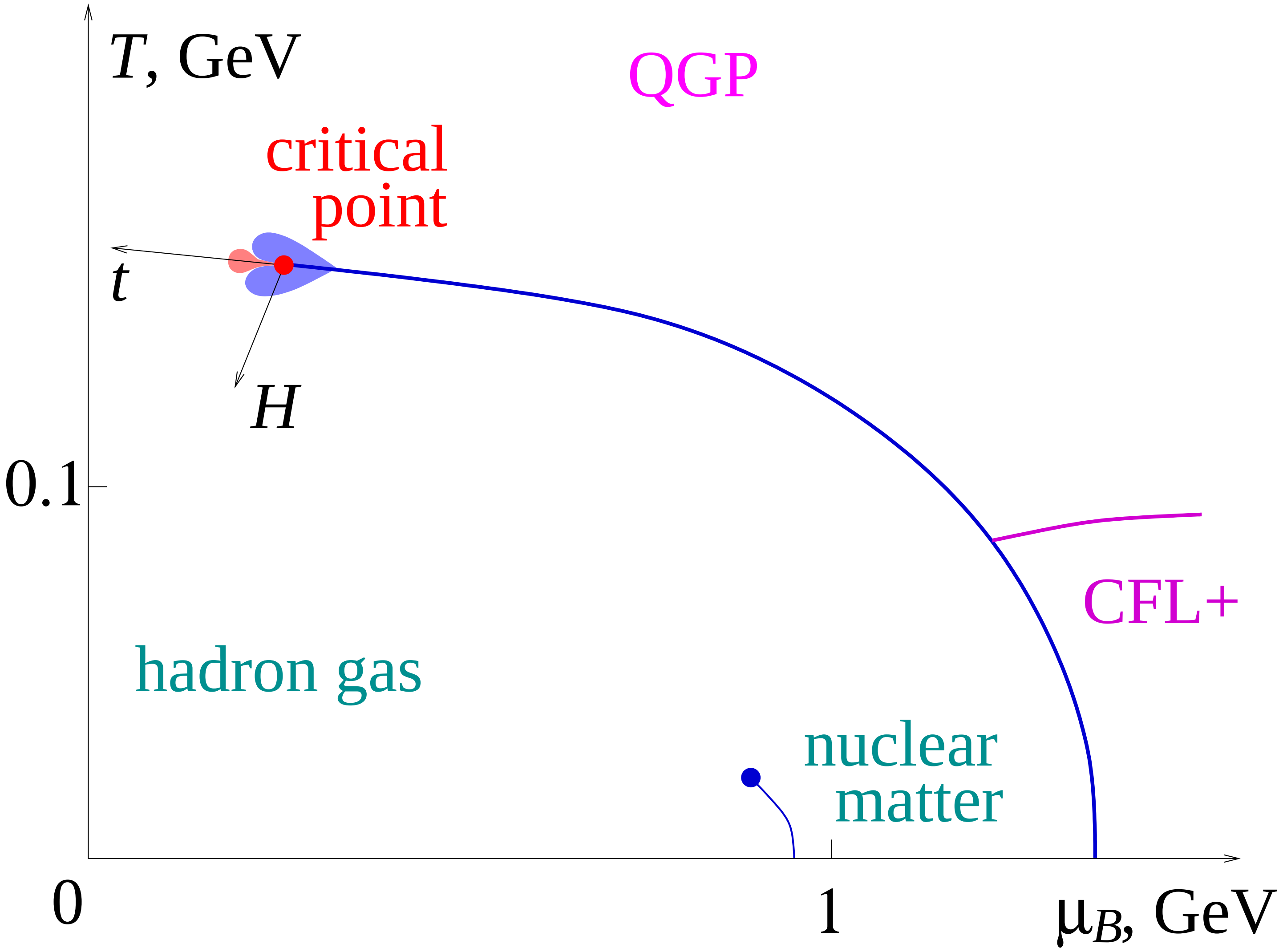

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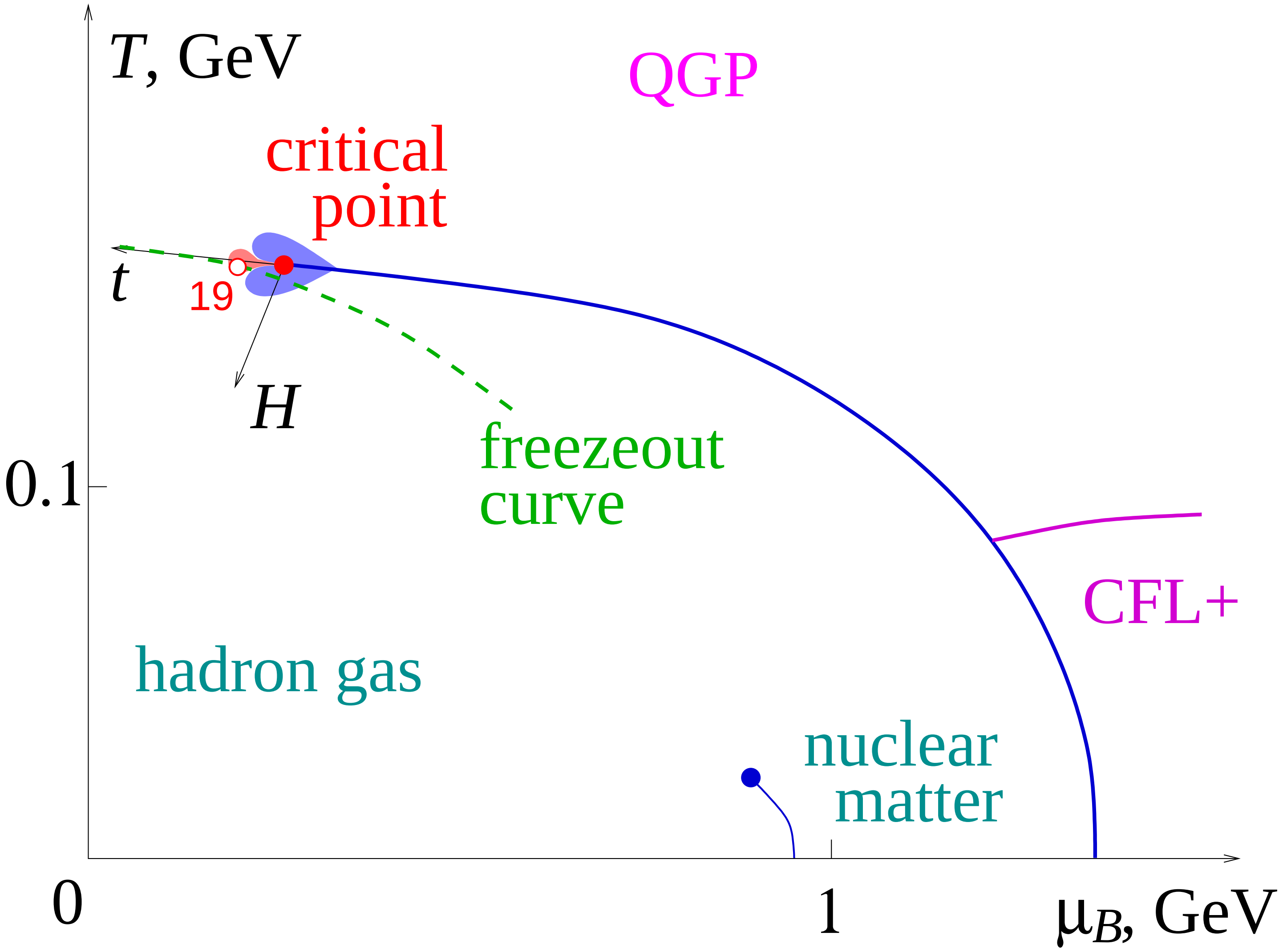
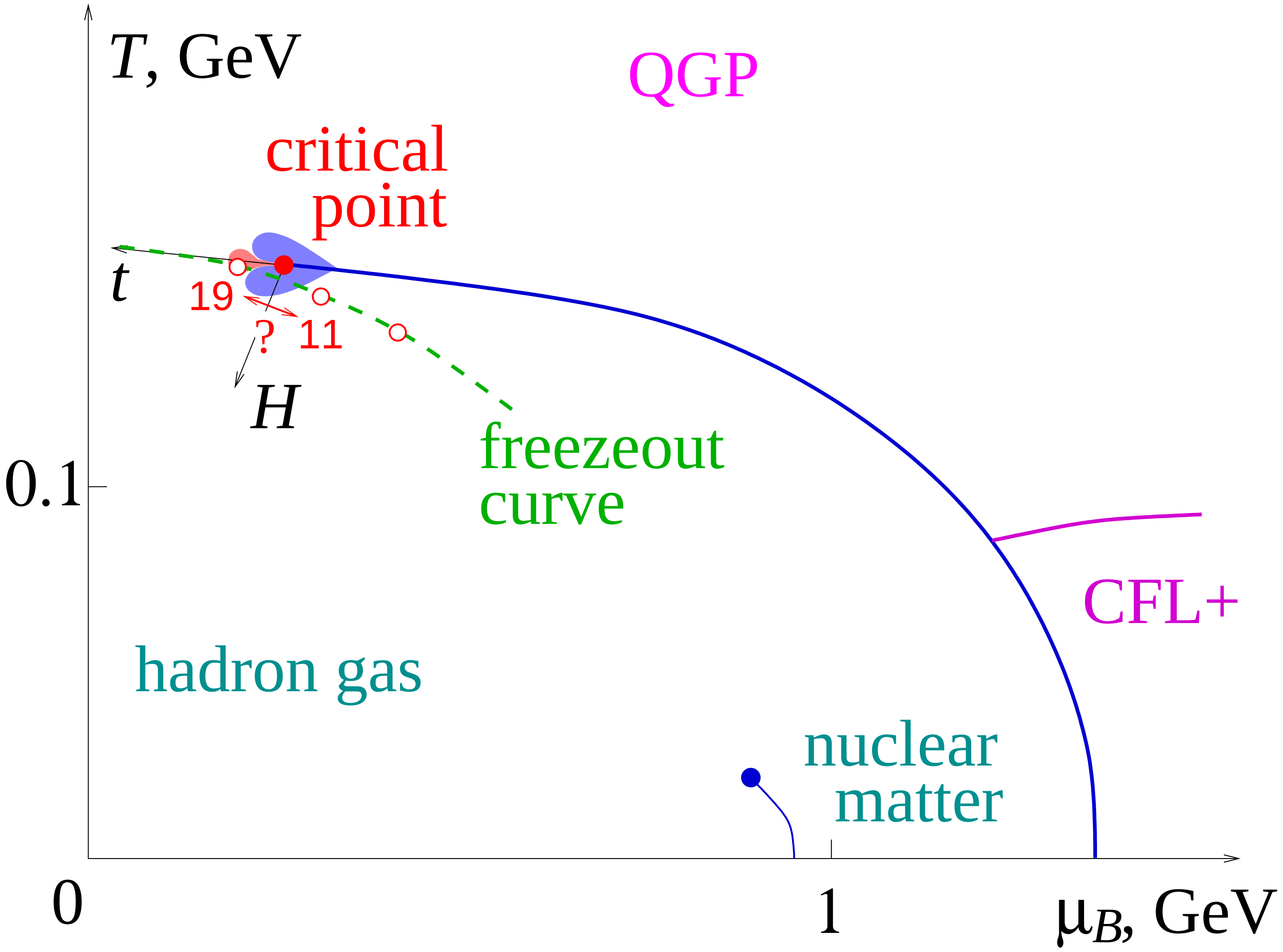


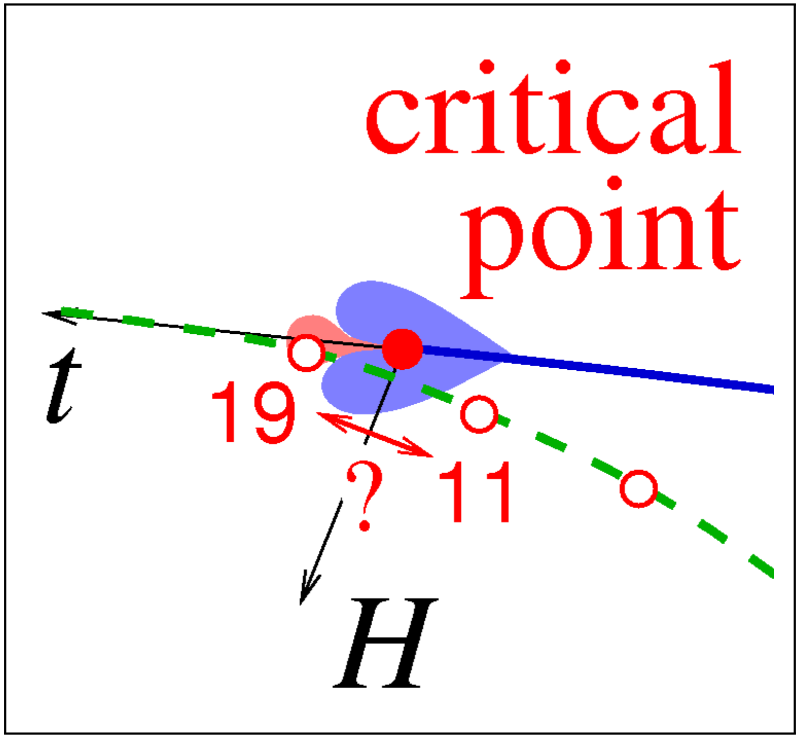
- On the crossover side, for $\sqrt{s} = 19$ GeV: $\omega_{4p} - 1 \approx -\mathcal{O}(1)$ at $\xi \approx 1.5$ fm.

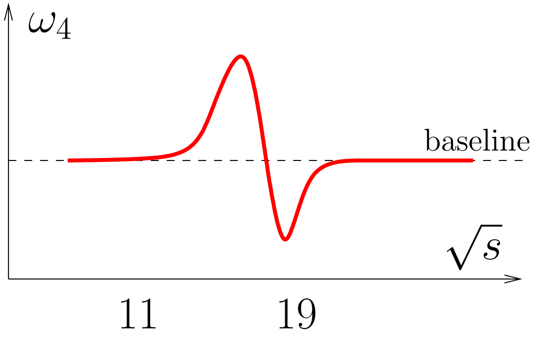
Implications for the energy scan



- If the kurtosis stays significantly below Poisson value in 19 GeV data, the logical place to take a closer look is between 19 and 11 GeV.

Implications for the energy scan





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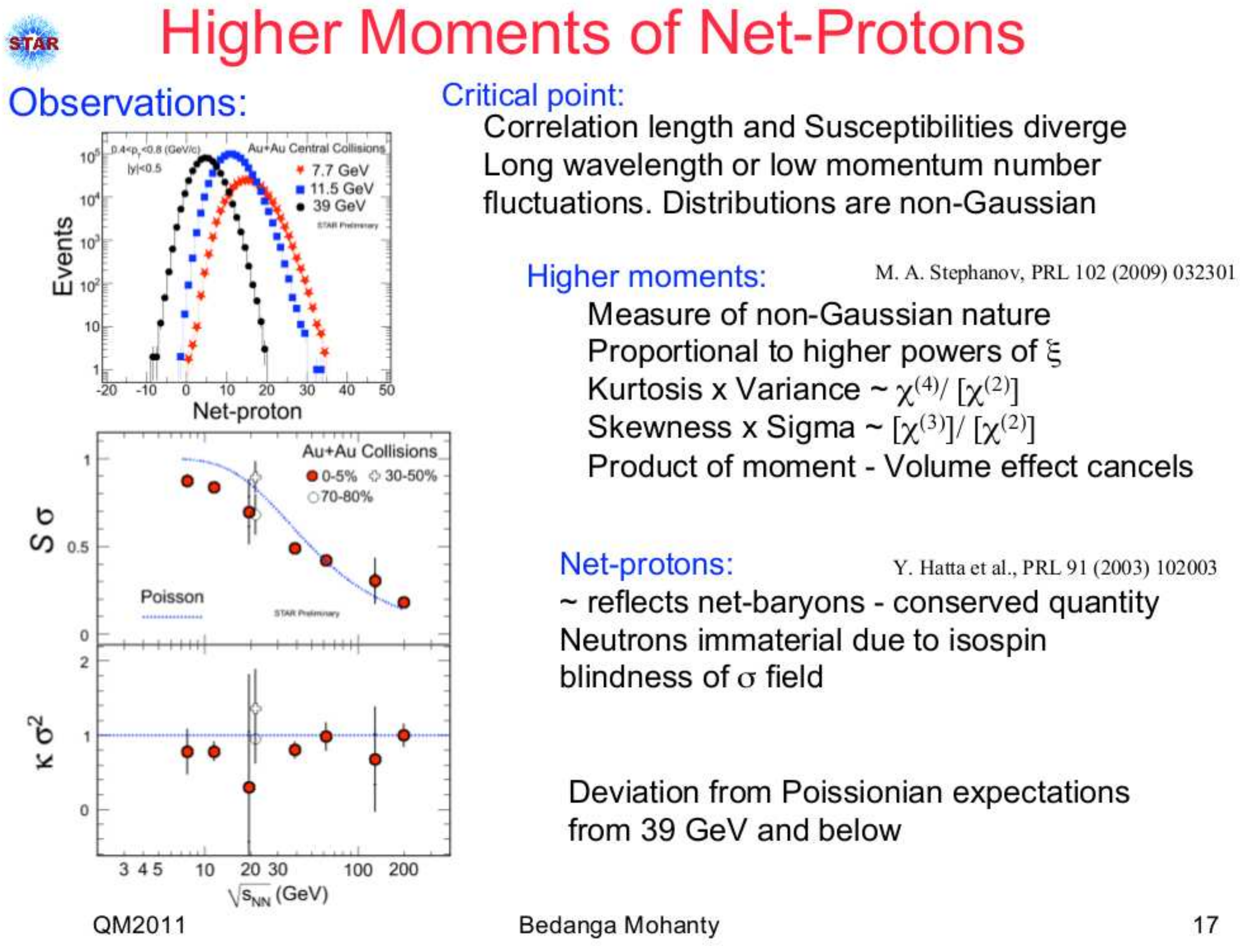
Experimental data at QM



Higher Moments of Net-Protons

Observations:

Critical point:

Correlation length and Susceptibilities diverge
Long wavelength or low momentum number fluctuations. Distributions are non-Gaussian

Higher moments: M. A. Stephanov, PRL 102 (2009) 032301

Measure of non-Gaussian nature
Proportional to higher powers of ξ
Kurtosis x Variance ~ $\chi^{(4)}/\,[\chi^{(2)}]$
Skewness x Sigma ~ $[\chi^{(3)}]/\,[\chi^{(2)}]$
Product of moment - Volume effect cancels

Net-protons: Y. Hatta et al., PRL 91 (2003) 102003

~ reflects net-baryons - conserved quantity
Neutrons immaterial due to isospin blindness of σ field

Deviation from Poissionian expectations from 39 GeV and below

QM2011 Bedanga Mohanty 17

- Potential sources of baseline shift (from Poisson) at high baryon density:
  - Fermi statistics: $\omega_4 \approx 1 - 7\langle n_{\boldsymbol{p}}\rangle_{\boldsymbol{p}}$ (small effect, but grows with μ_B).
  - O(4) critical line (Friman-Karsch-Redlich-Skokov).
  - Baryon number conservation?

Concluding remarks

- Critical point is a special singular point on the phase diagram, with unique signatures. This makes its experimental discovery possible.

- Locating the point is still a challenge for theory.

- The search for the critical point is on. New RHIC results for 2 points with $\mu_B > 200$ MeV ($\sqrt{s} = 11$ and 7.7 GeV) were presented at QM.

- If kurtosis stays significantly below Poisson value at $\sqrt{s} = 19$ GeV, then the critical point could be close, to the right, on the phase diagram.

  Then: $\sqrt{s} = 15$ GeV?