COFI Advanced Instrumentation and Analysis Techniques School

December 9-17, 2023

Principles (and future) of **Quantum Computing**

Jens Koch | Northwestern University

logistics

Roadmap

Quantum Simulation for High-Energy Physics

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HHL algorithm & ML relevance

Harrow, Hassadim & Lloyd 2008

$$A\vec{x} = \vec{b}$$

$$\vec{x} = A^{-1}\vec{b}$$

Use of HHL in training of deep neural networks offers exponential speedup over classical training

quantum computing
history

1980s

Foundations laid by Paul Benioff, Richard Feynman, and Yuri Manin

1990s

Invention of first quantum algorithms

Invention of quantum error correction

2000s

Development of basic quantum gates and first small-scale “quantum processors”

2010s

Significant progress in quantum hardware & implementation of algorithms.

“NISQ” (2016)

Google claims “quantum supremacy” (2019)

2020s

Ongoing research focuses on scaling, QEC, NISQ-based apps

computer

computer

computer


INPUT
011010100...101101010
n binary digits
OUTPUT
0101010110...1001
m binary digits

computer

Notation

$\mathbb{B} = \{0, 1\}, \quad n, m \in \mathbb{N}$

computer

INPUT

$\vec{x} \in \mathbb{B}^n$

$$f : \mathbb{B}^n \to \mathbb{B}^m$$

OUTPUT

$\vec{y} \in \mathbb{B}^m$

Notation

$\mathbb{B} = \{0, 1\}, \quad n, m \in \mathbb{N}$

computer

INPUT

$\vec{x} \in \mathbb{B}^n$

$$f : \mathbb{B}^n \times \Omega \to \mathbb{B}^m$$

OUTPUT

$\vec{y} \in \mathbb{B}^m$

Notation

$\mathbb{B} = \{0, 1\}, \quad n, m \in \mathbb{N}$

Ω sample space
for random variable(s)

computer

a black box* implementing map *f*

$$f : \mathbb{B}^n \times \Omega \to \mathbb{B}^m$$

Examples

*a “device” obeying the laws of physics

computer

a black box* implementing map *f*

$$f : \mathbb{B}^n \times \Omega \to \mathbb{B}^m$$

Examples

*a "device" obeying the laws of physics

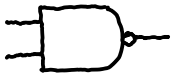
computer

a black box* implementing map *f*

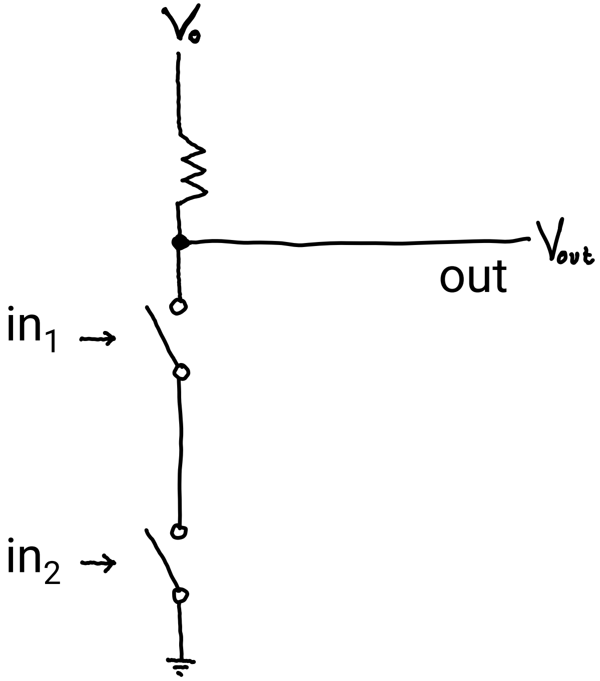
$$f : \mathbb{B}^n \times \Omega \to \mathbb{B}^m$$

Examples

NAND gate



in_1	in_2	out
0	0	1
0	1	0
1	0	0
1	1	0



*a “device” obeying the laws of physics

computer

a black box* implementing map *f*

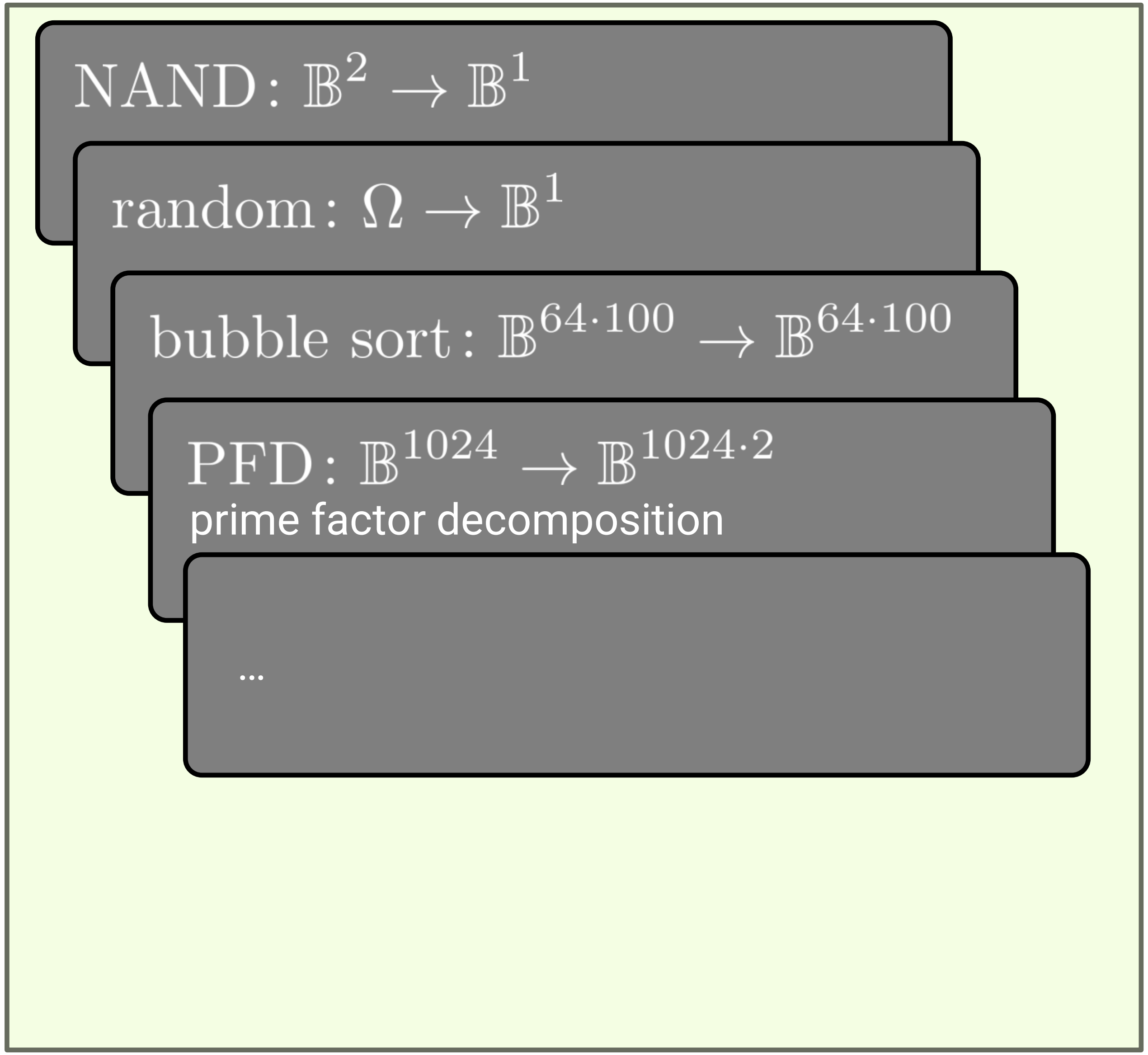
Examples

$$f : \mathbb{B}^n \times \Omega \to \mathbb{B}^m$$

*a "device" obeying the laws of physics

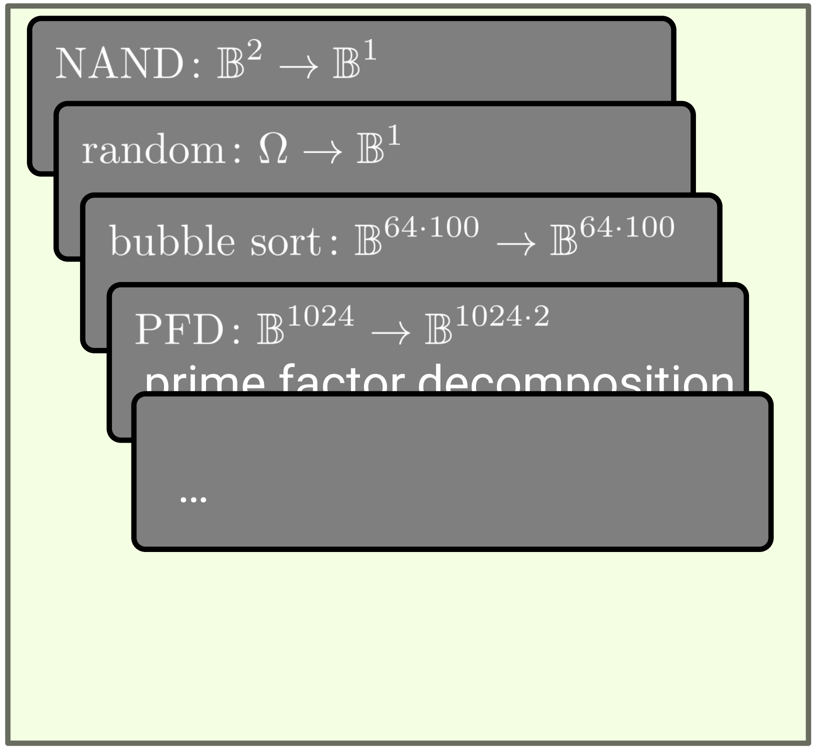
programmable computer

a black box* that can implement** any*** given *f*



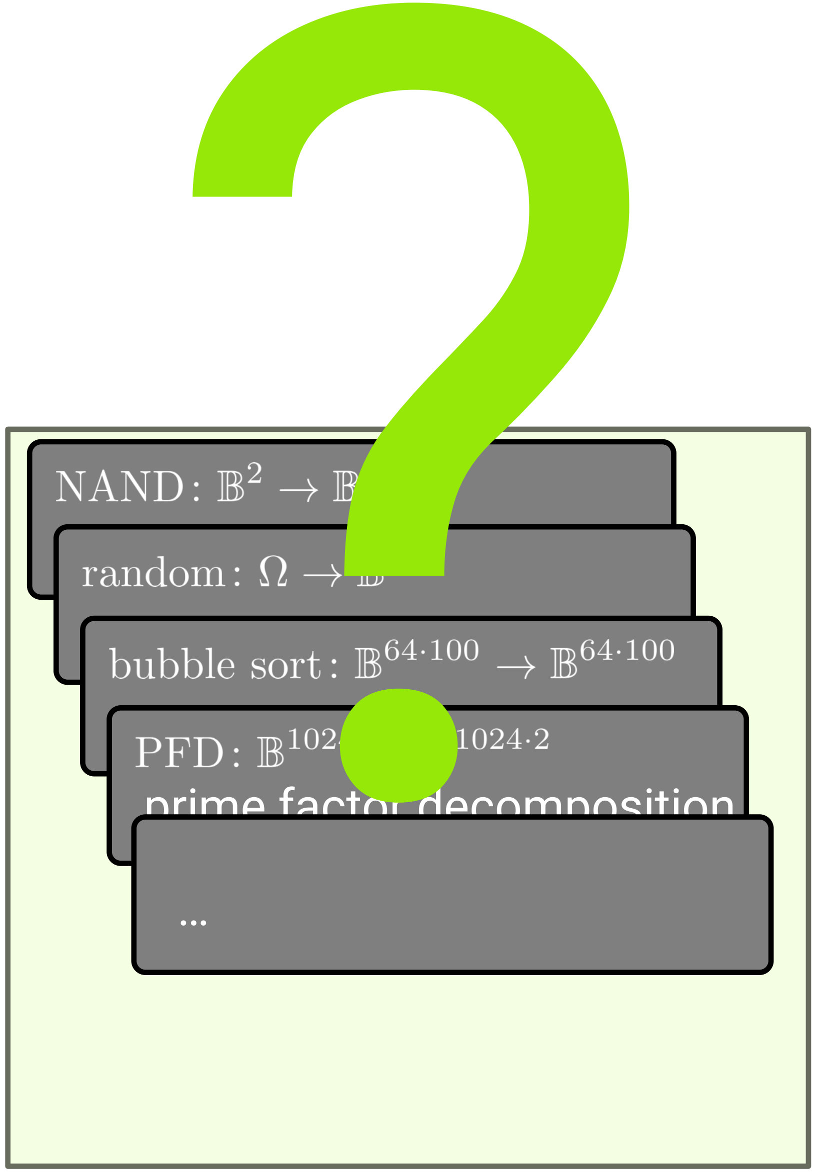
* a “device” obeying the laws of physics
** with input & output sizes $n, m \leq N_0$
*** output is provided in finite time

classical vs. quantum computer



classical

"uses purely classical physics paradigms for computing"



quantum

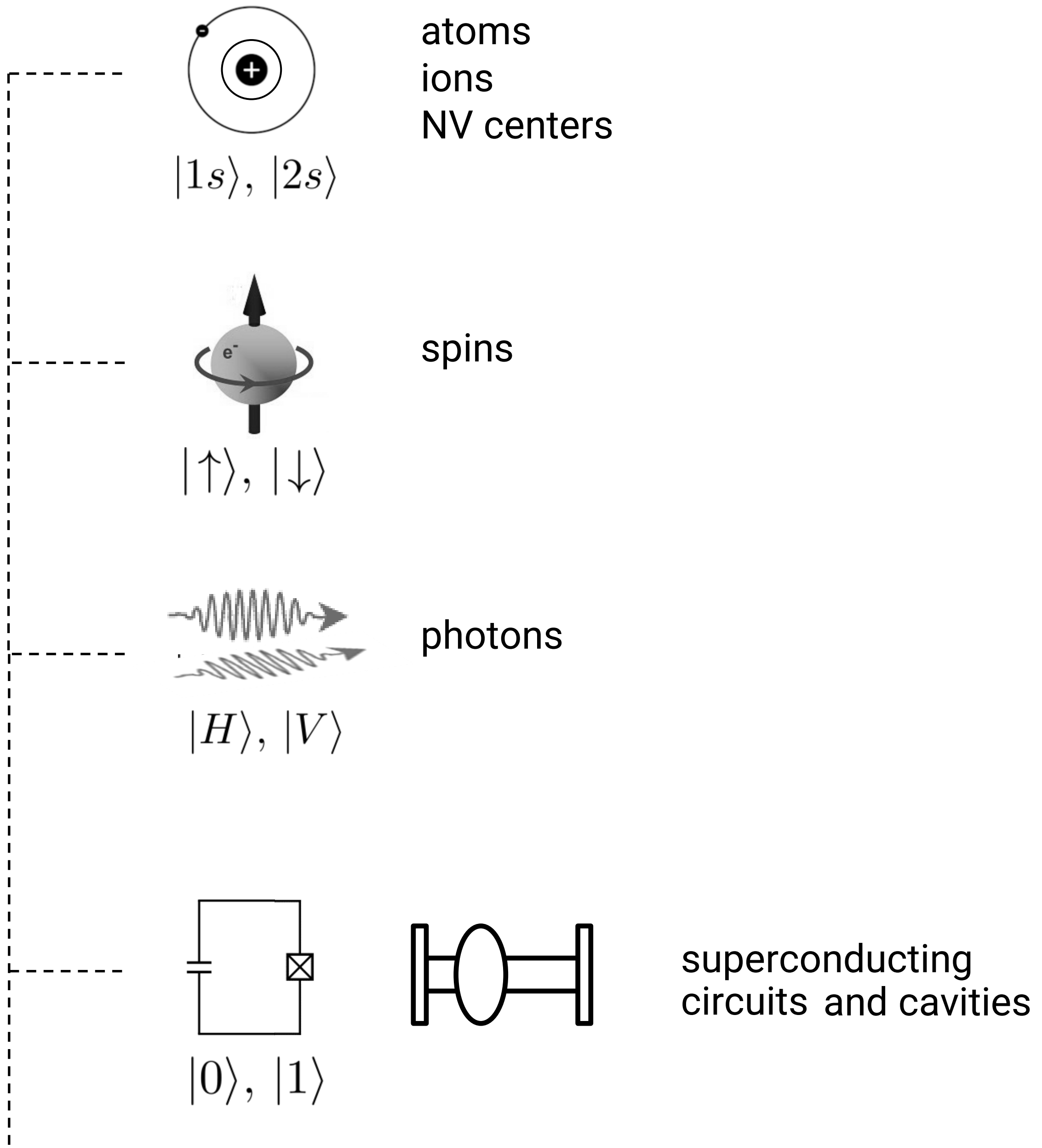
"uses concepts specific to quantum physics"

Jozsa & Linden (2003)

For any quantum algorithm operating on pure states,
we prove that the presence of **multi-partite entanglement**,
with a number of parties that **increases unboundedly with input size**, is **necessary** if the quantum algorithm is to offer an **exponential speed-up over classical computation**.

entanglement

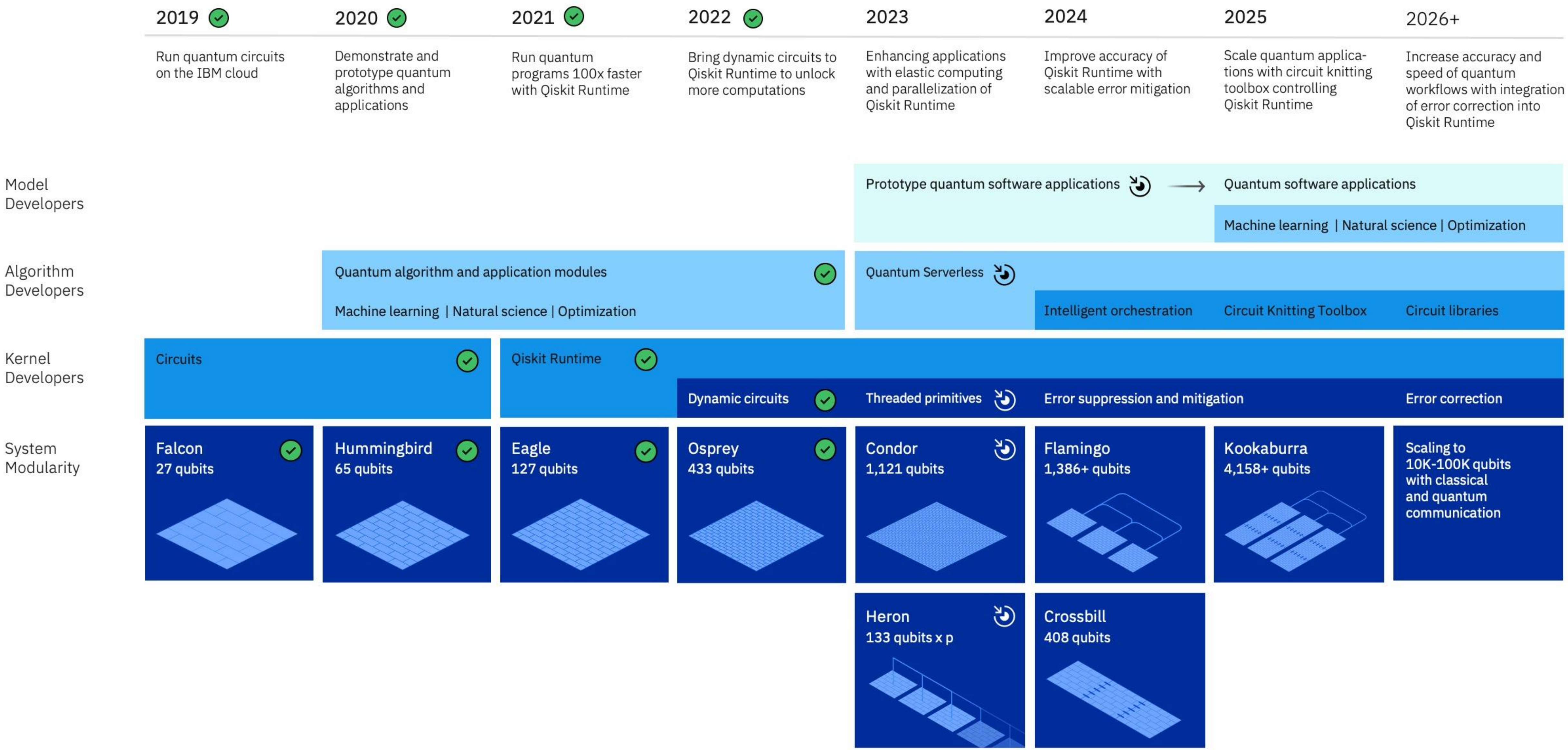
quantum computing hardware



Development Roadmap

Executed by IBM
On target

IBM **Quantum**



Quantum Computing Market Map

THE QUANTUM INSIDER

Non exhaustive and in no particular order. Excludes details on control systems, assembly languages, circuit design, etc.

Users
Select examples

Material Science
Merck, OTI, Airbus, BMW

Finance
Goldman Sachs, Bank of America, J.P.Morgan, Wells Fargo

Life Sciences
Odyssey Therapeutics, Boehringer Ingelheim, AstraZeneca, gsk

Other
VW, Johnson Electric, Denso

Applications
Not mapped to verticals

Software offerings
Includes control software

Not strictly categorized given diversity of operations[1]
Multiverse Computing, QC Ware, NQCG, 1QBit, Aliro Quantum, Horizon Quantum Computing, HQS Quantum Simulations, Zapata, QCi Ready-to-run Quantum Software, Entropica Labs, river lane, Strange Works, QuantFi, Phasecraft, aq, QunaSys, Terra Quantum, ⟨b| e^{iθ}, ProteinQure, Keysight Technologies, Qubit pharmaceuticals, Q-CTRL, Classiq, Quantum-South, menten.AI, ParityQC

Cloud access to QPUs
aws, Google Cloud, Azure

Simulators / q-inspired / etc
Atos, Fujitsu, NVIDIA, Huawei Cloud

QPUs[2]

Superconducting
rigetti, Google AI Quantum, OQC, QILIMANJARO Quantum Tech, IBM Q, qci, IQM, Origin Quantum, Nord Quantique, aws, seeqc, D-Wave The Quantum Computing Company, Alibaba.com

Ion Trap
Quantinuum, IONQ, Universal Quantum, AQT, oxford ionics, Qudoor, eleQtron

Neutral Atoms
atom computing, ColdQuanta, QuEra, Pasqal

Silicon
Silicon Quantum Computing, intel, Quantum Motion, equal1, photonic

Photonics
PsiQuantum, QC82, Xanadu, ORCA Computing, Quandela, Turing

Other
EeroQ – Electrons on helium; Quantum Brilliance – NV Diamond

Hardware / components
Select examples only – not representative of entire ecosystem

Cryogenics (includes testing)
Oxford Instruments, FormFactor, Lake Shore Cryotronics, Bluefors, Montana Instruments, ICE, Maybell

Lights and lasers
Vescent Photonics, NKT Photonics, M Squared, Coherent, Qant, Toptica Photonics, Ligentec

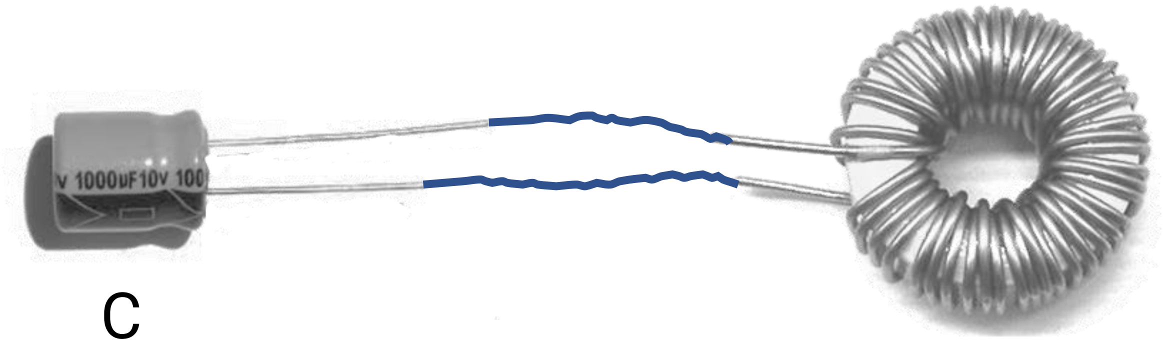
Other componentry (examples)
Delft Circuits, Keysight Technologies, Quantum Machines, Quantum Design, CryoCoax, ZNF, Qblox, QDevil, Zurich Instruments

[1] Software offerings can be further classified into SDKs, firmware / enablers, algorithms / applications, simulators etc. but many companies are offering a mixture across the stack

[2] Many QPU providers are offering full stack services (e.g. Pasqal acquired Qu&Co, Quantinuum was originally CQC prior to merger with HQS, etc.

Building quantum bits from circuits

?



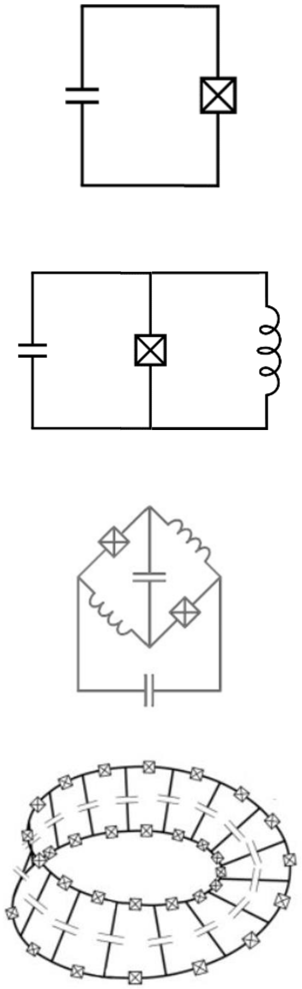
C

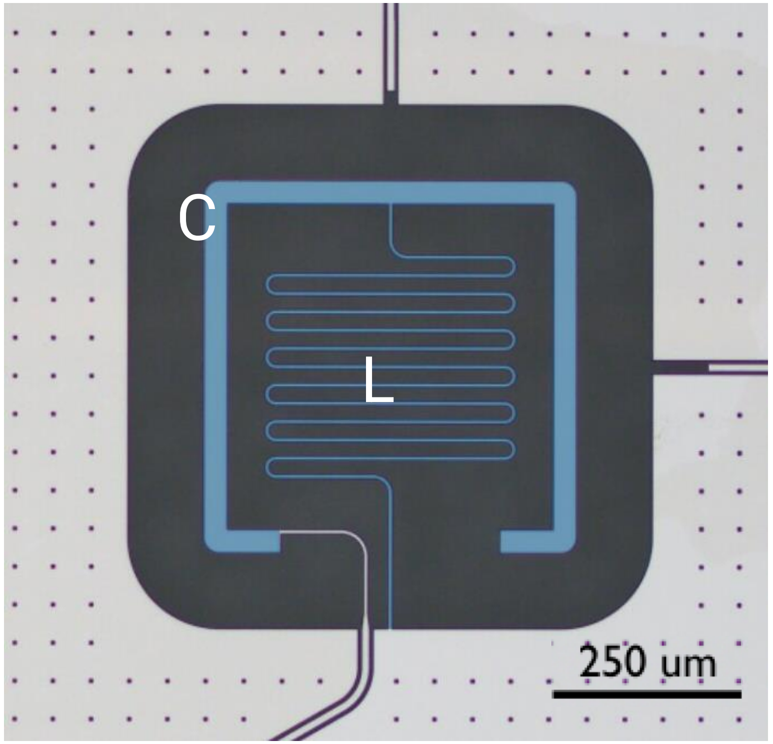
L

Ohmic losses
energy dissipation, $V = RI$

interaction with the environment **dramatically reduces quantum effects**

Building quantum bits from
superconducting circuits



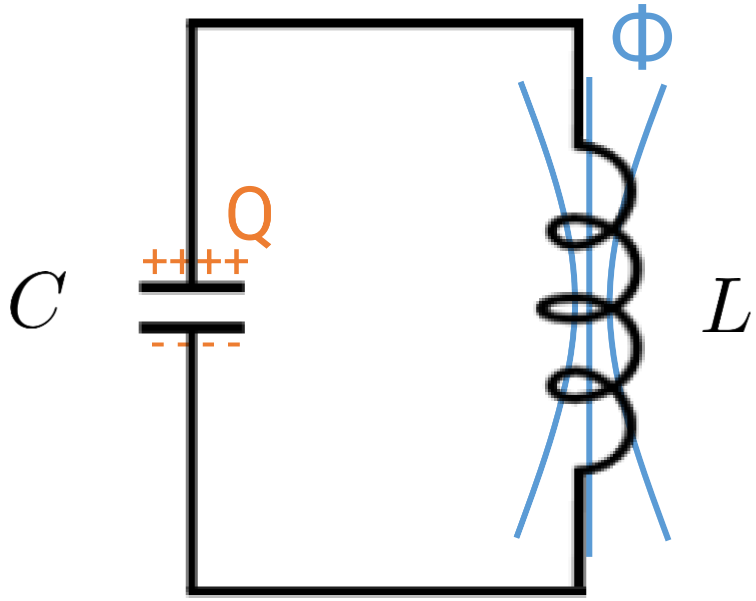


$$H = \frac{Q^2}{2C} + \frac{\Phi^2}{2L}$$

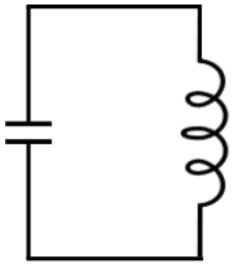
charging energy

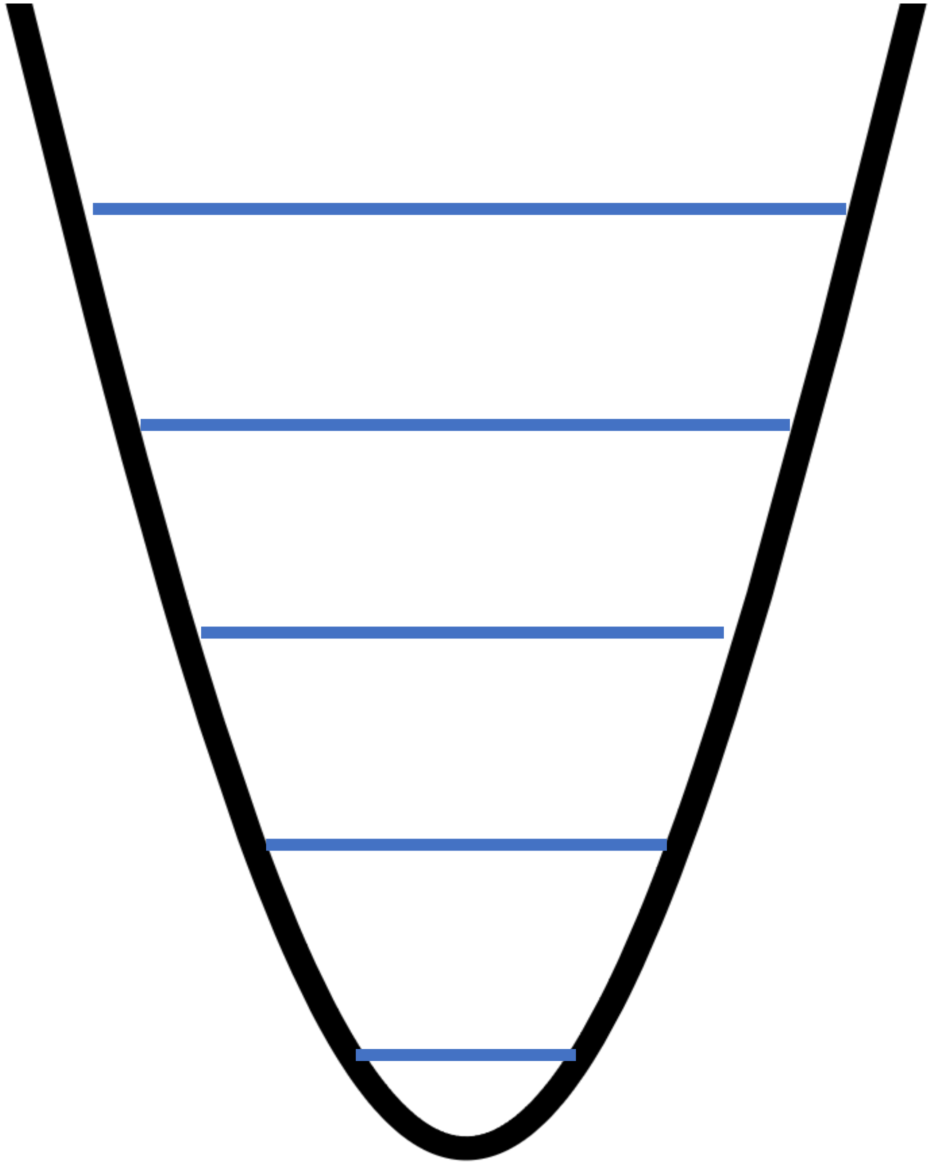
inductive energy

$$= \hbar\omega\,(a^\dagger a + \tfrac{1}{2})$$

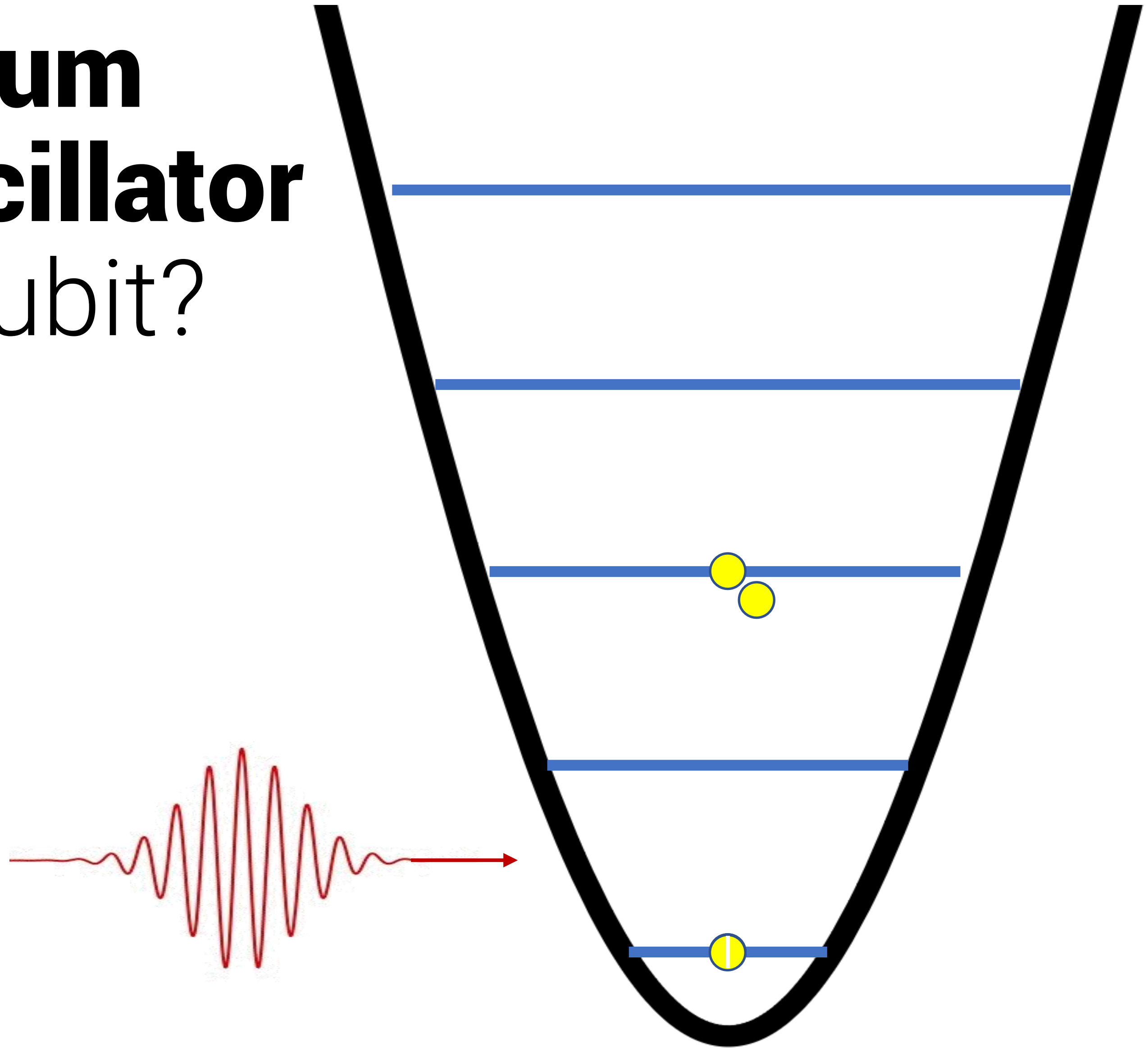


$$[\Phi, Q] = i\hbar$$



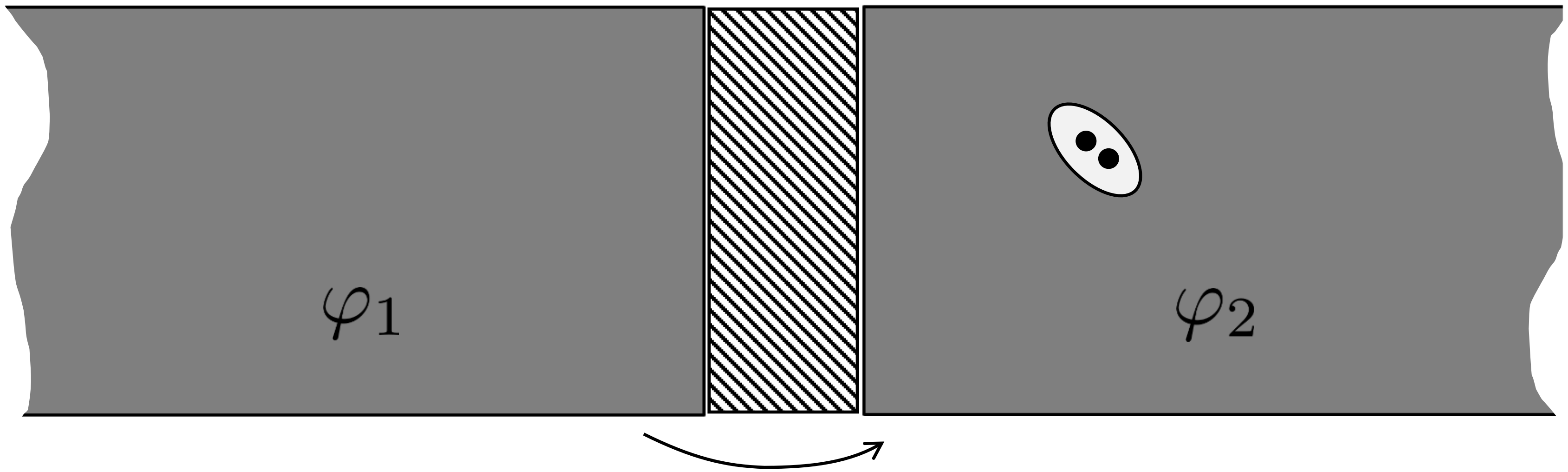


Quantum
LC oscillator
as a qubit?



not a good qubit…
need nonlinearity

From LC oscillator to transmon



$$\varphi = \varphi_1 - \varphi_2$$

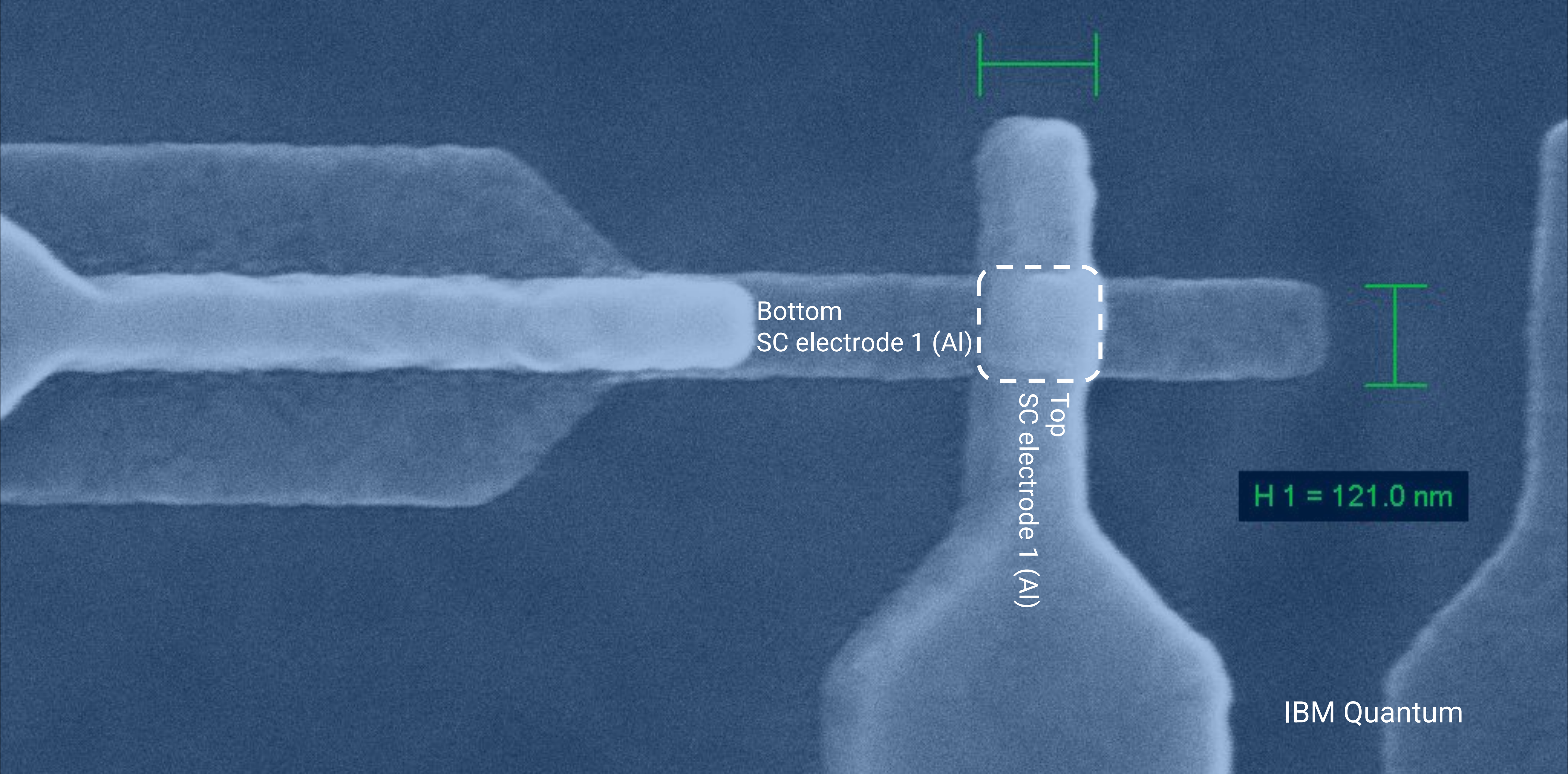
Brian Josephson
Nobel Prize 1973

Josephson effect

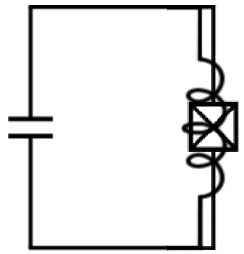
current-phase relation: $I = I_0 \sin\varphi$

phase evolution equation: $\frac{d\varphi}{dt} = V \cdot \frac{2\pi}{\Phi_0}$

Josephson junction

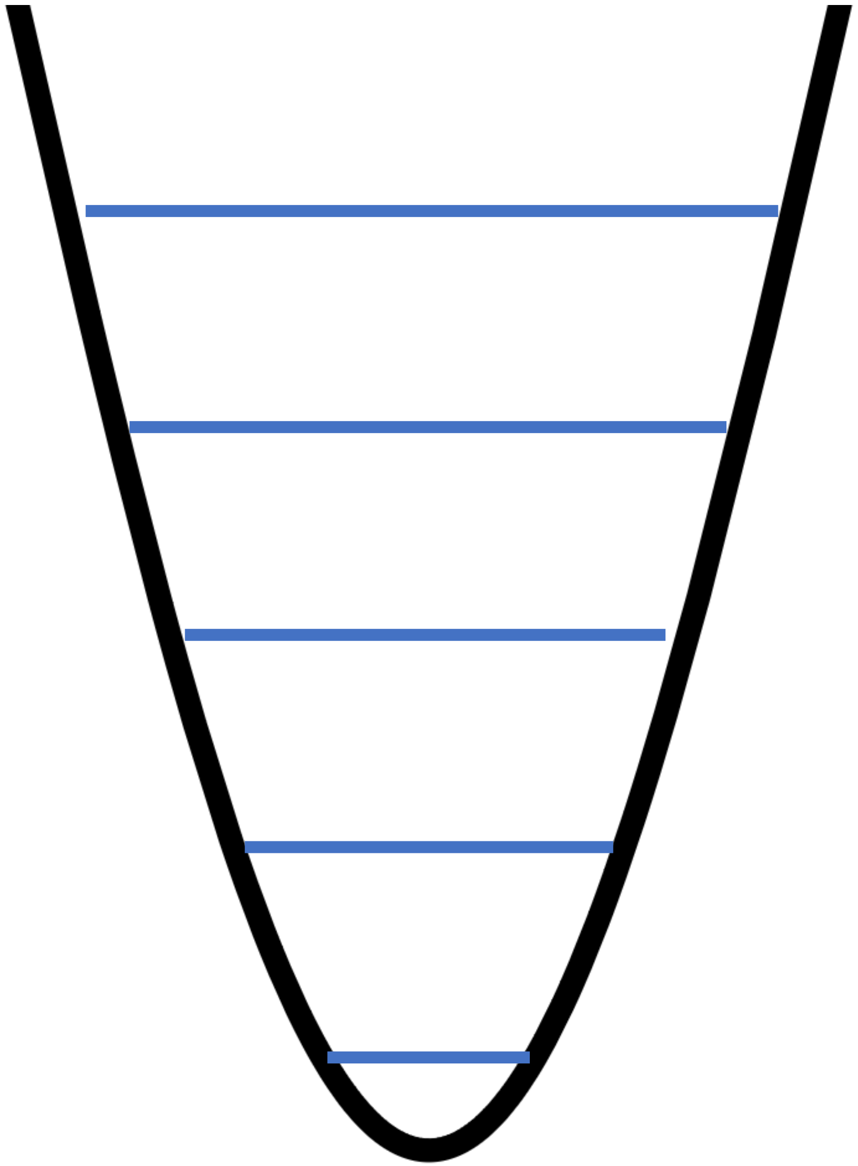


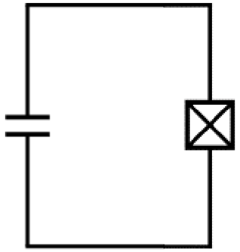
IBM Quantum



$$\frac{\Phi^2}{2L}$$

$$-E_J \cos(2\pi\Phi/\Phi_0)$$

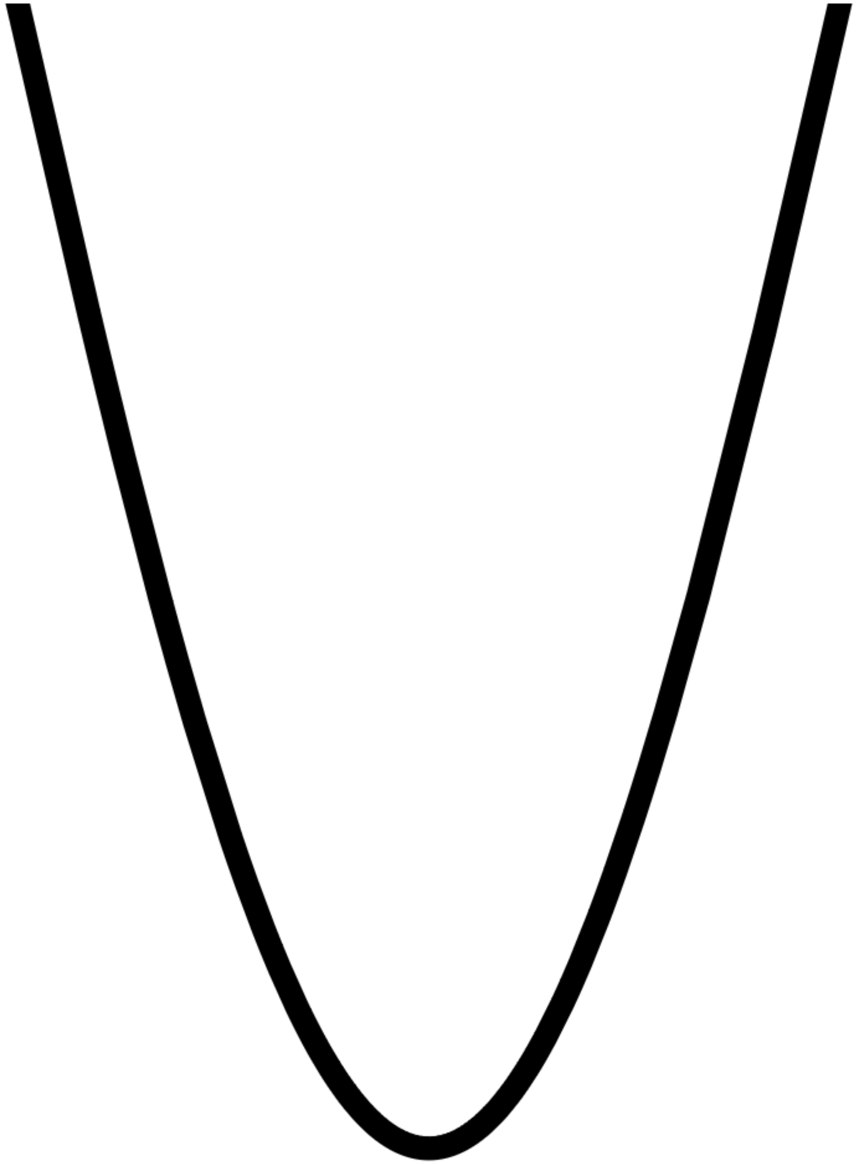


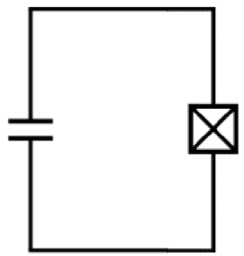


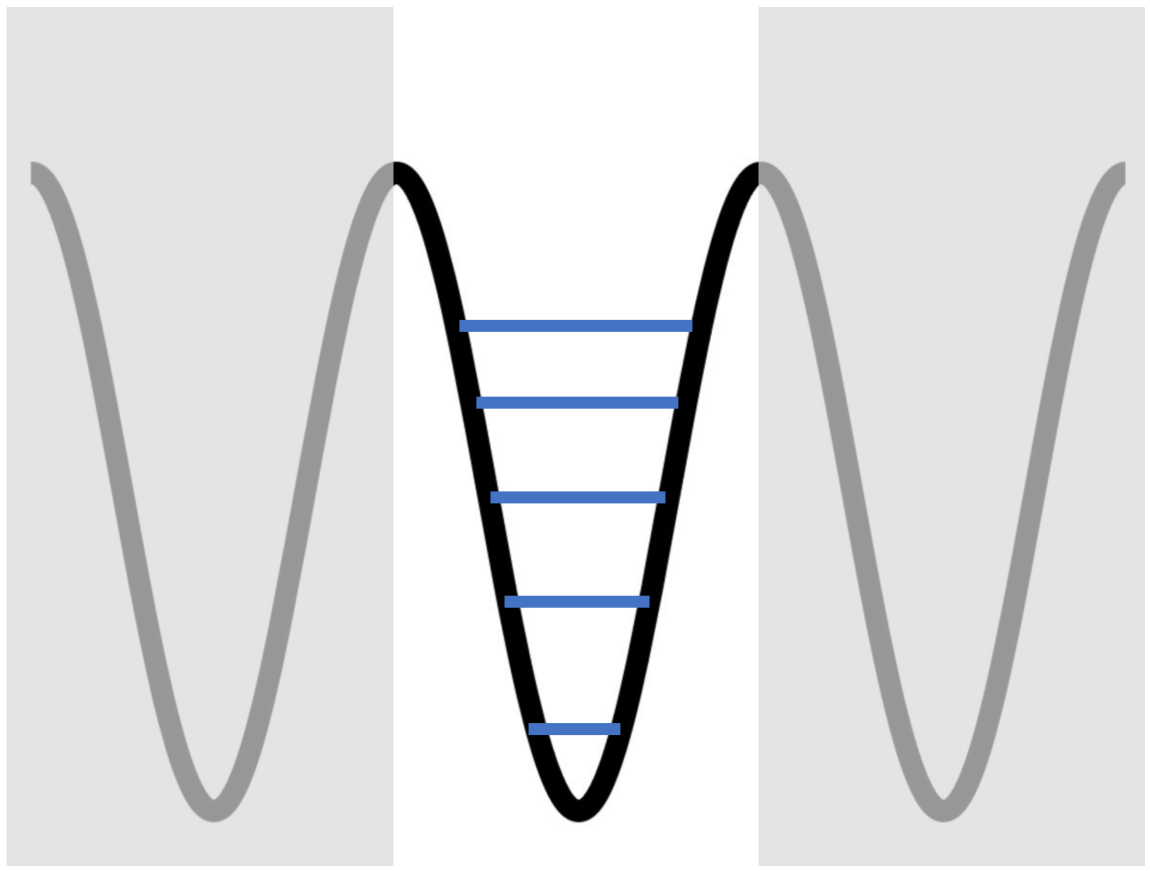
$$\frac{\Phi^2}{2L}$$

$$\downarrow$$

$$-E_J \cos(2\pi\Phi/\Phi_0)$$







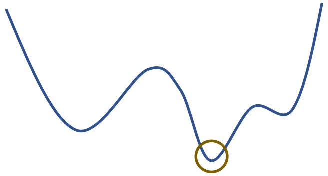
quantum computing:

use quantum parallelism
and entanglement
to beat classical
computers

Shor's algorithm

N = p q (p,q prime)
period-finding of modular functions

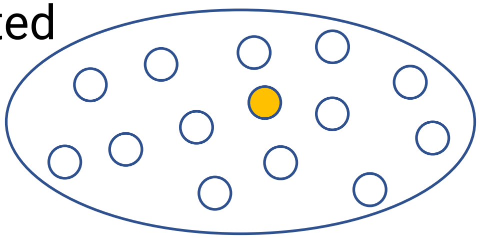
quantum adiabatic algorithm



optimization

Grover's algorithm

search unsorted database



digital quantum simulation

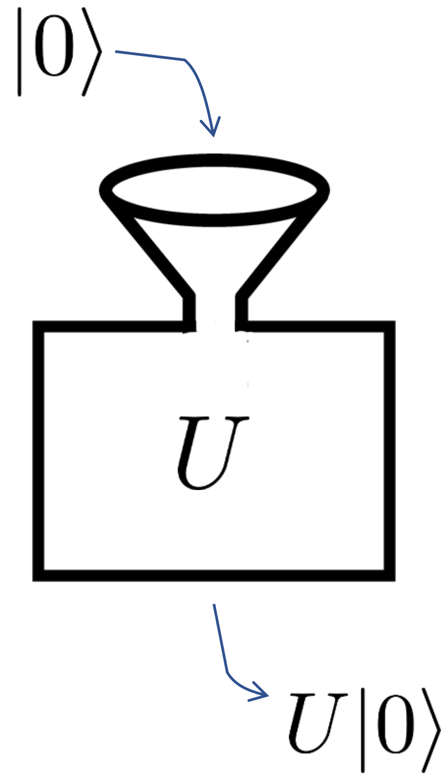
$$i\hbar \frac{d}{dt}|\Psi(t)\rangle = H|\Psi(t)\rangle$$

Harrow-Hassidim-Lloyd (HHL) algorithm

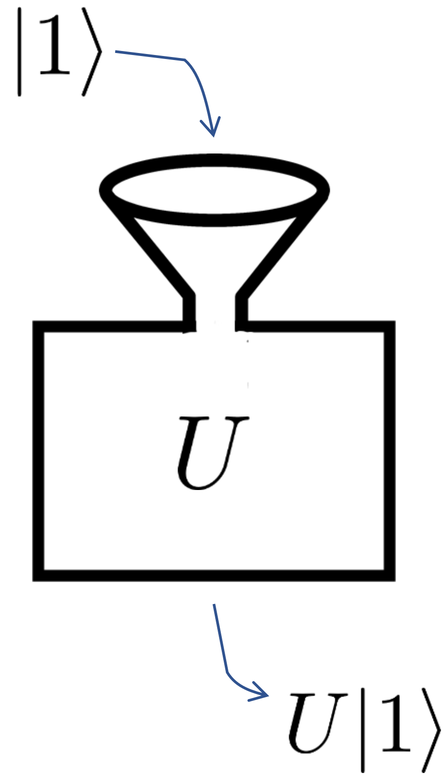
matrix inversion* $\vec{x} = \mathcal{A}^{-1}\vec{b}$

quantumalgorithmzoo.org

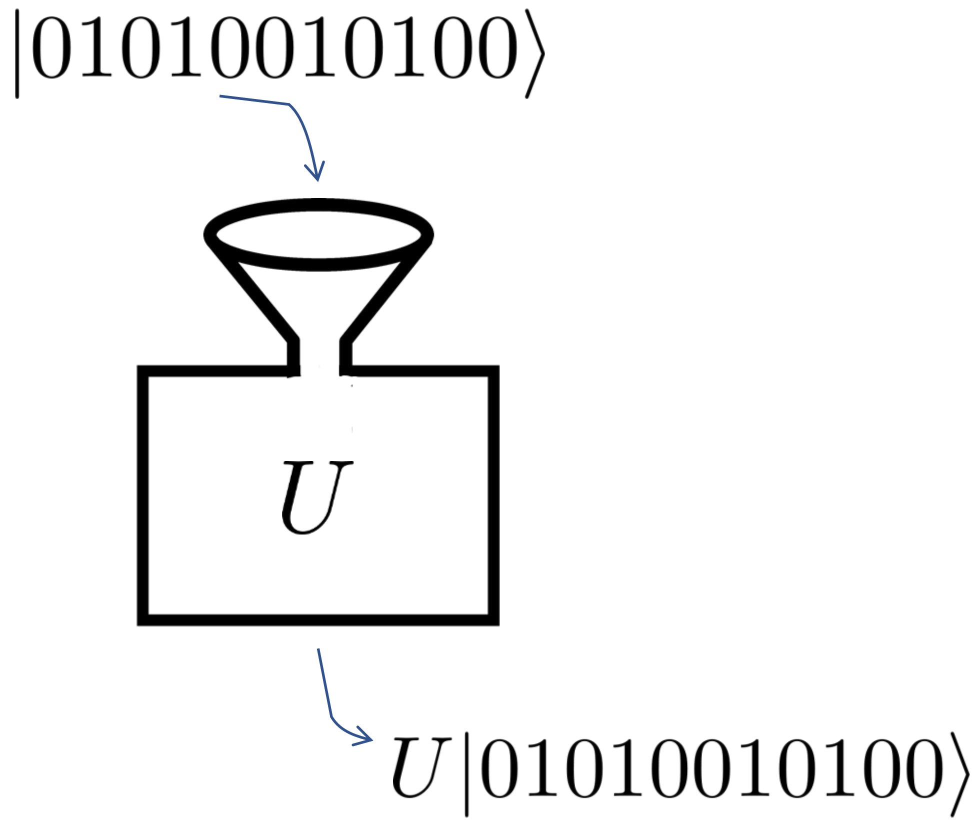
quantum parallelism



quantum parallelism

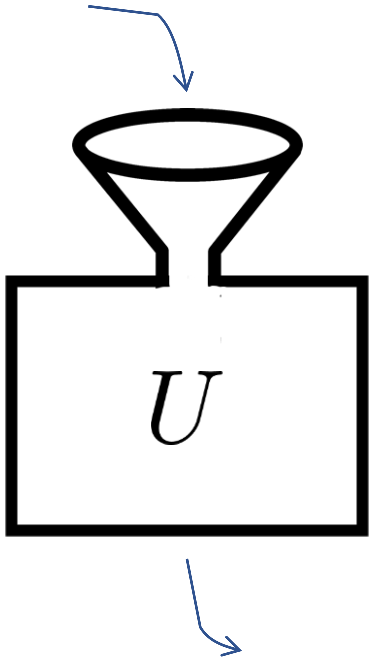


What makes a quantum computer tick?



quantum parallelism

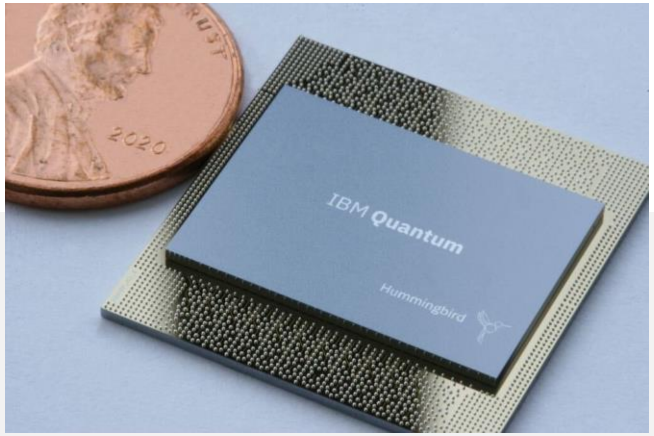
$$|00000\ldots0\rangle + |00000\ldots1\rangle + \cdots + |11111\ldots1|$$



$$U|00000\ldots0\rangle + U|00000\ldots1\rangle + \cdots + U|11111\ldots1|$$

storing quantum states

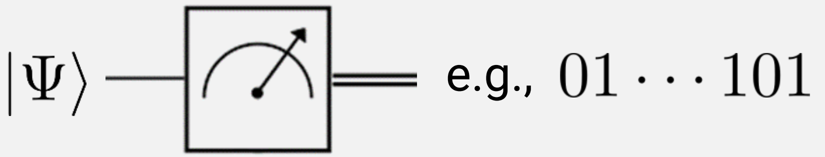
$$
\begin{aligned}
|\Psi\rangle = &\ \alpha_0|00\cdots000\rangle \\
&+ \alpha_1|00\cdots001\rangle \\
&+ \alpha_2|00\cdots010\rangle \\
&+ \alpha_3|00\cdots011\rangle \\
&\ \ \vdots \\
&+ \alpha_{2^N}|1\cdots111\rangle
\end{aligned}
$$



- 2^N complex amplitudes
- storage: 2^N x 128 bits

N= 65 qubits → memory/disk space: 590 billion GB

- **but**: output only N classical bits



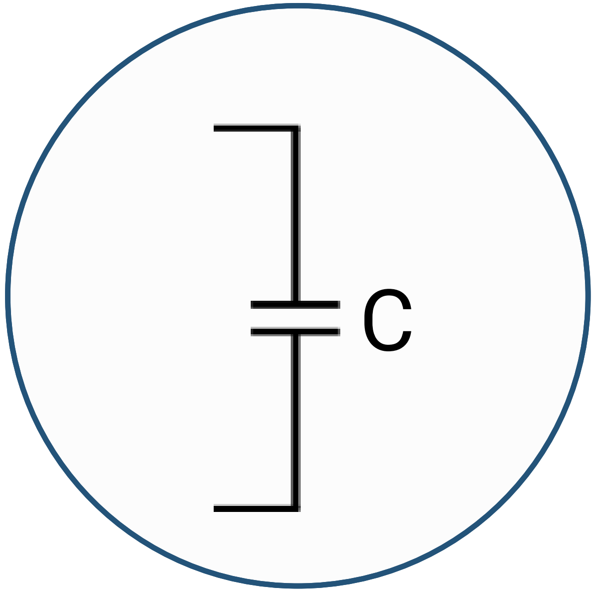
quantum parallelism

- can process all possible inputs at once
- **not sufficient** for quantum speedup

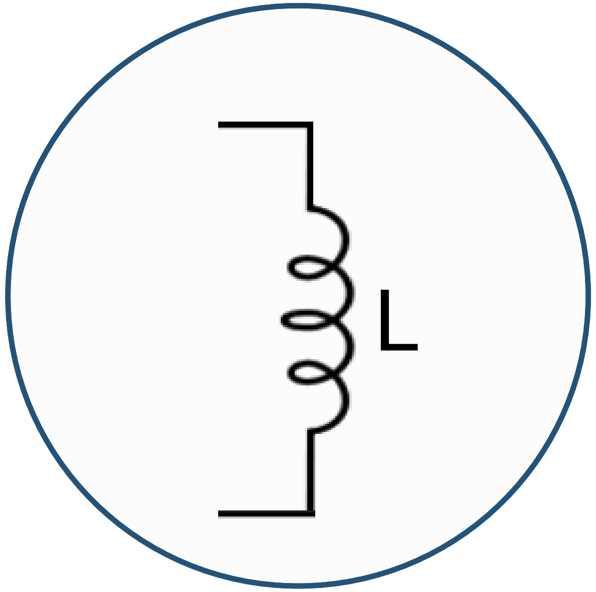
  - measurement only accesses one output state

Circuit elements

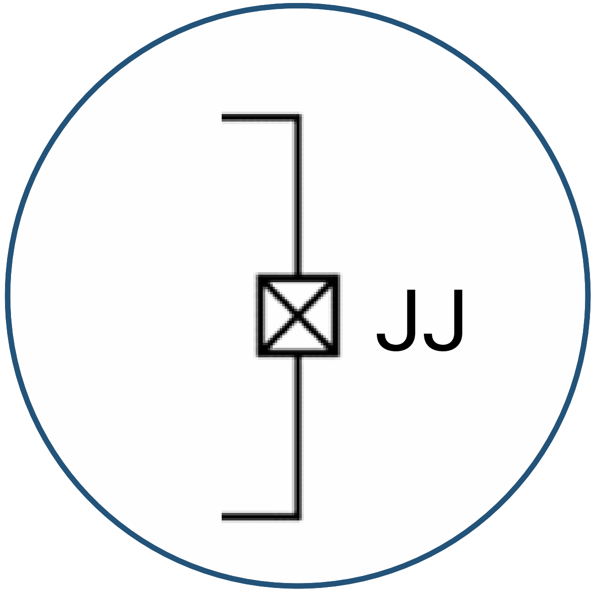
Building blocks for superconducting qubits



Capacitance

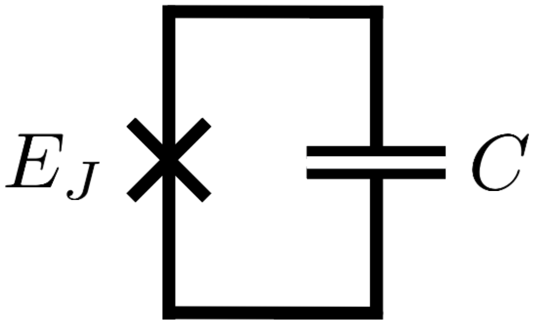


Inductance



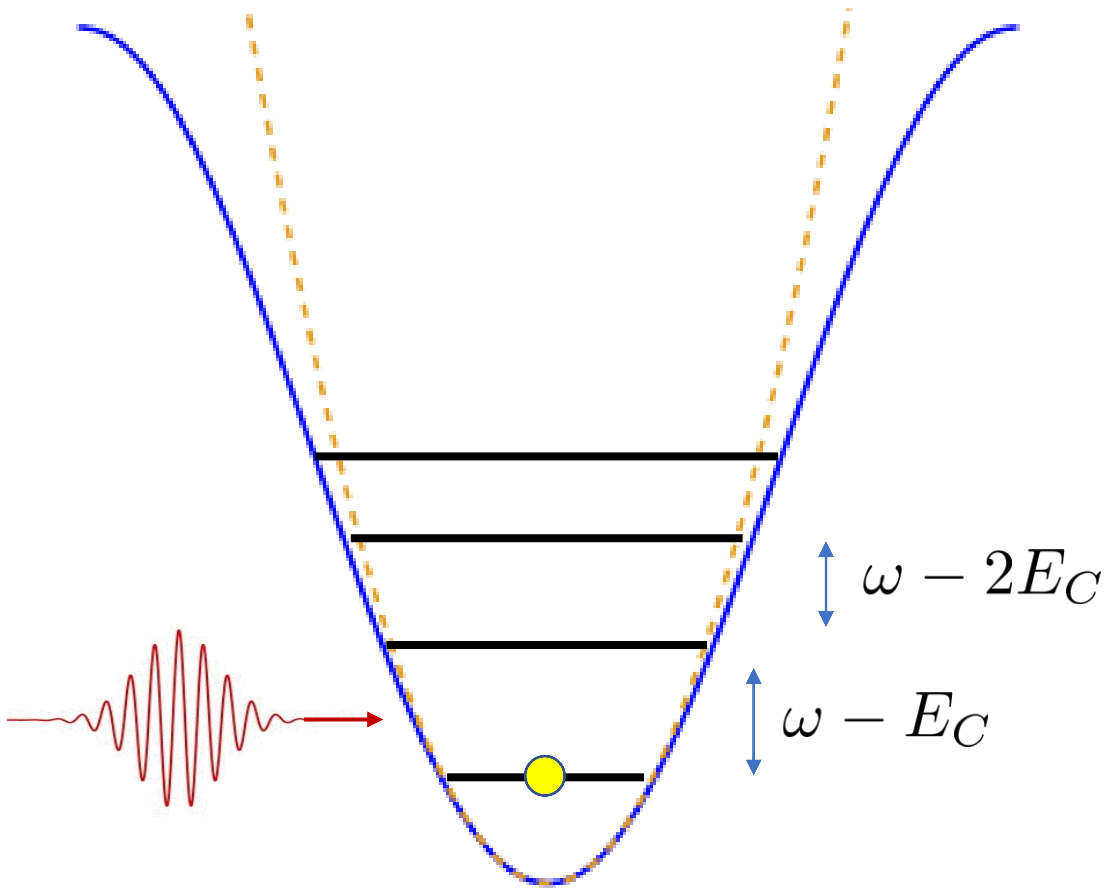
Josephson junction

transmon qubit



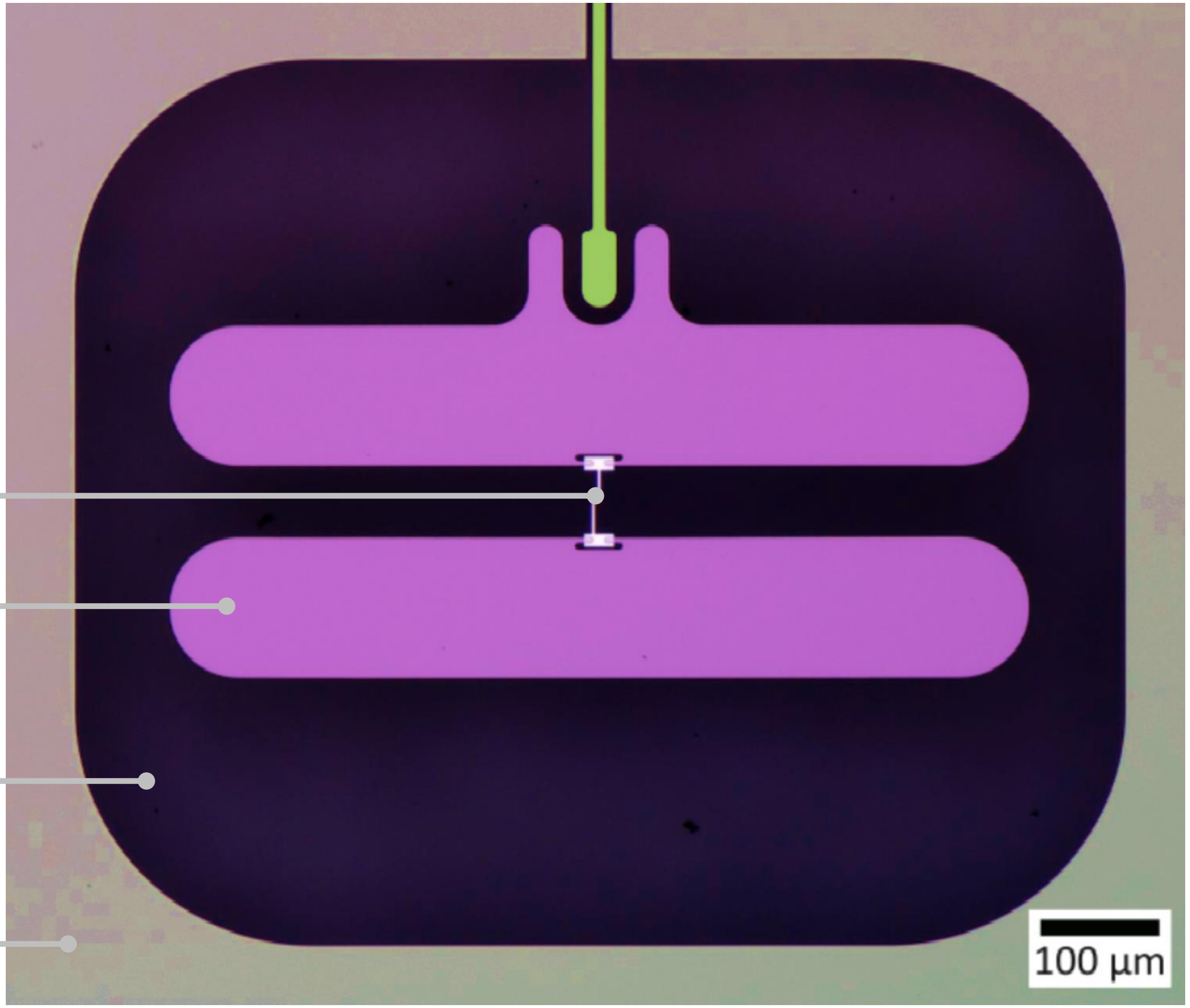
Introduce nonlinearity:
replace inductor with Josephson junction

$$H = \frac{Q^2}{2C} - E_J \cos(\underbrace{2\pi\Phi/\Phi_0}_{\varphi})$$



transmon qubit

Houck lab



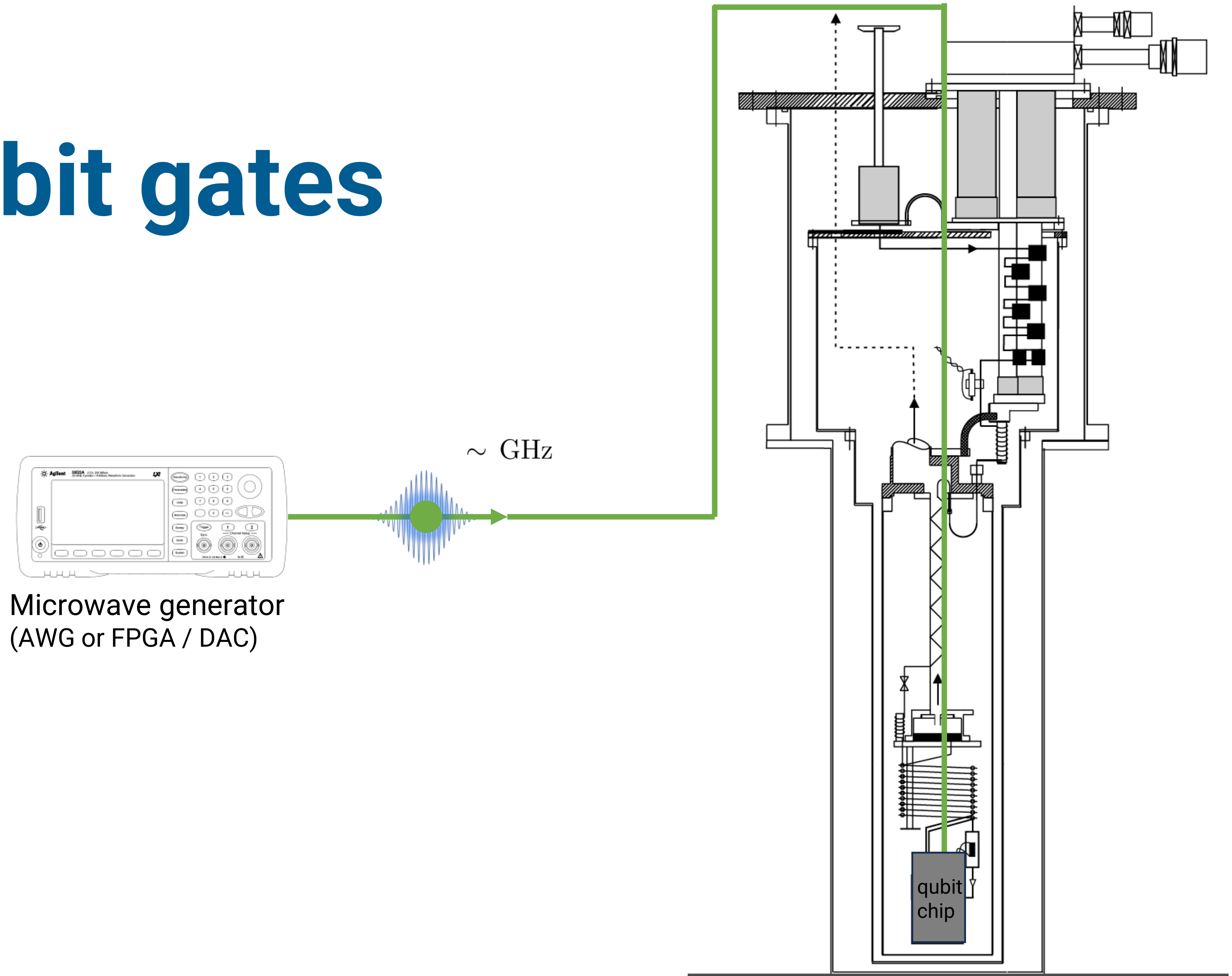
JJ
(Al-AlOx)

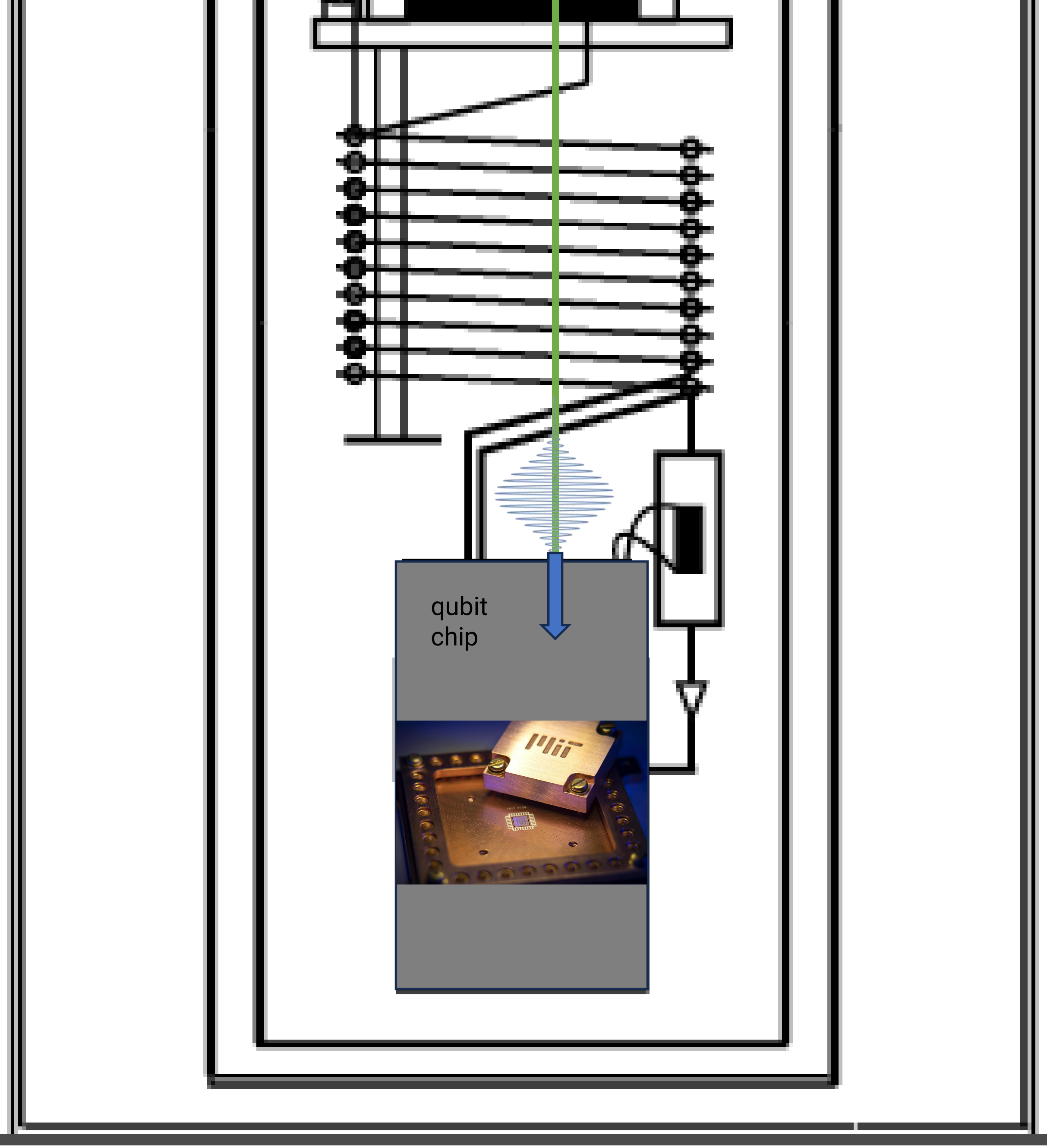
capacitor pads
(Nb, Al, or Ta)

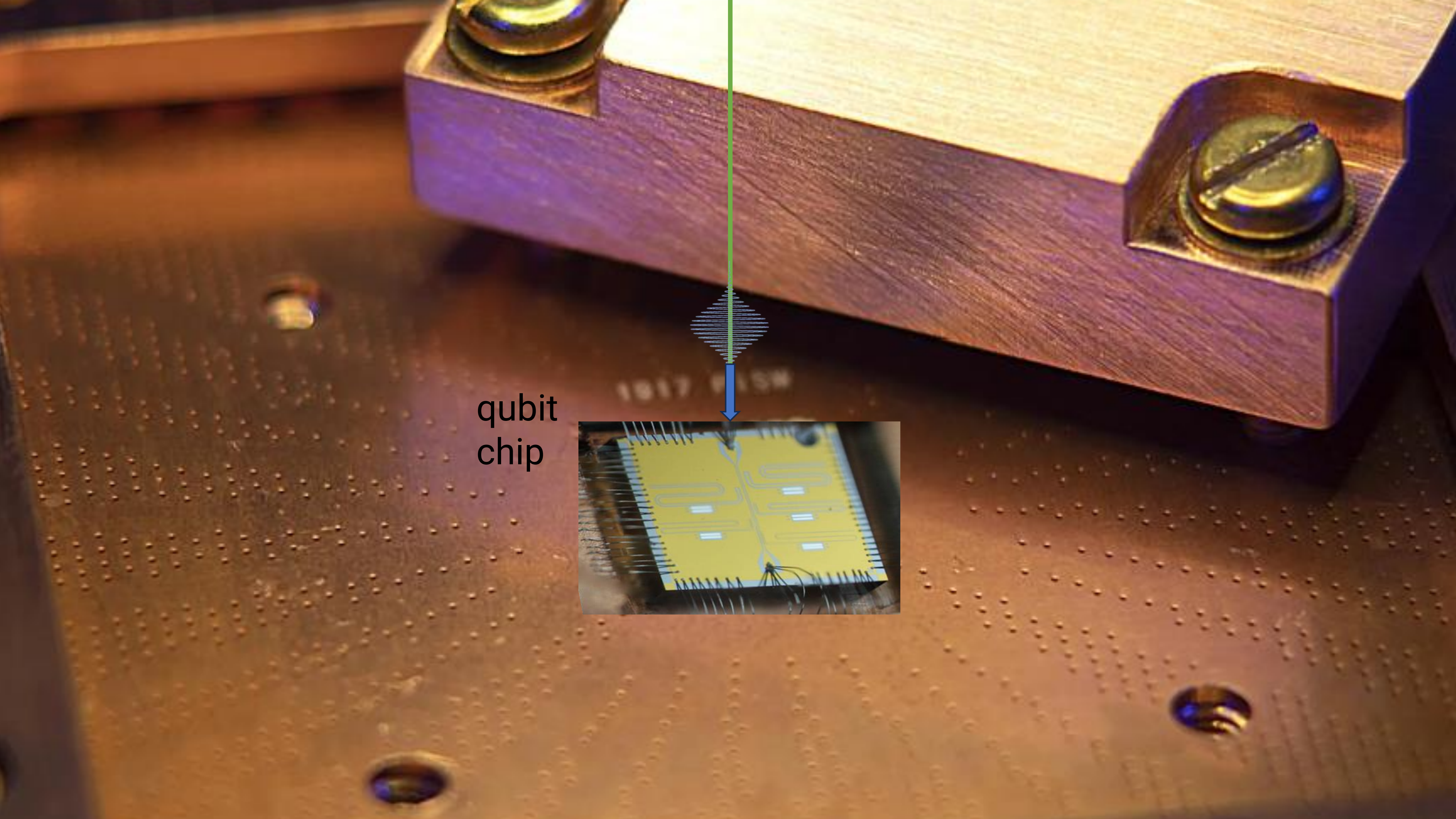
substrate
(Si or sapphire)

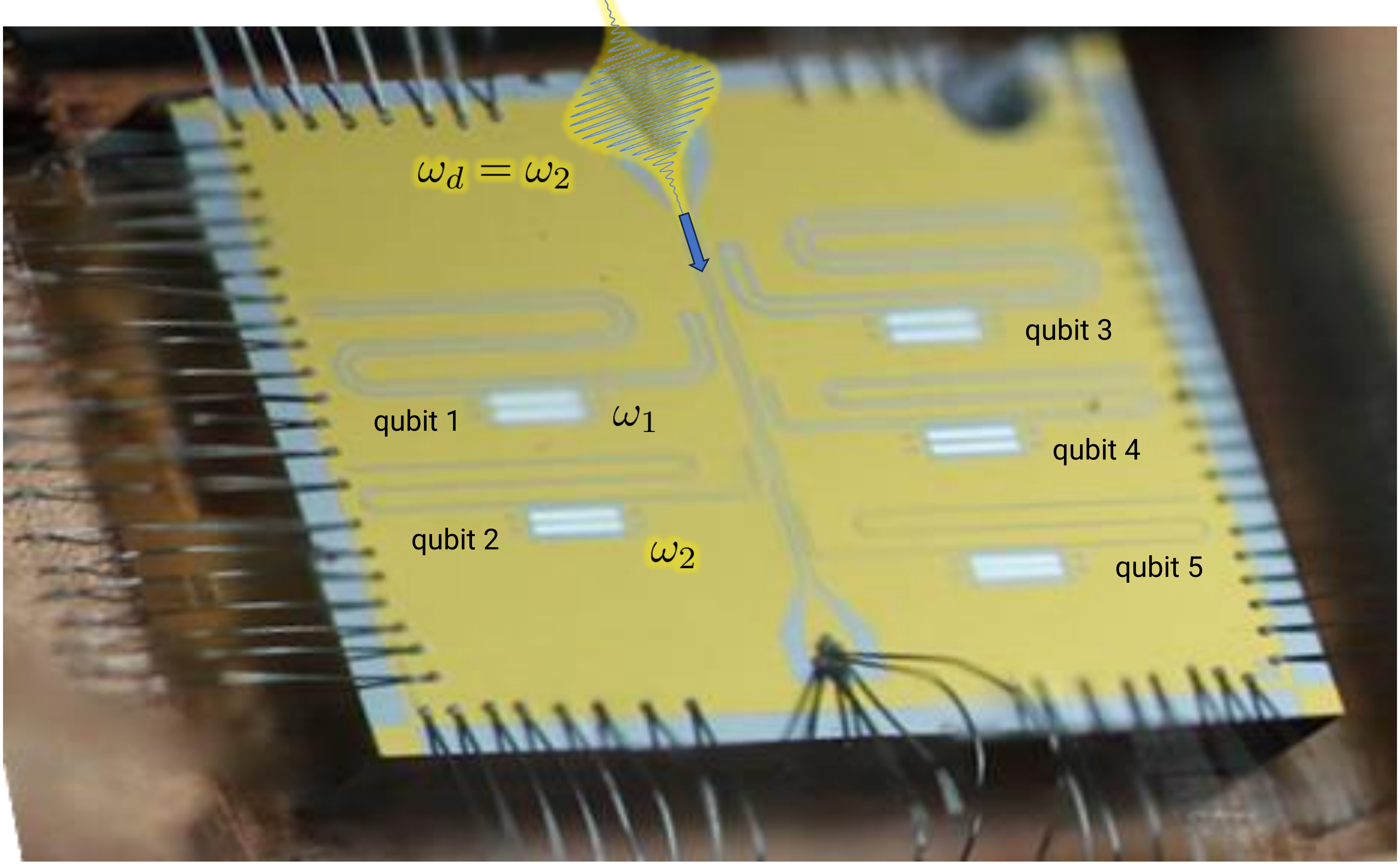
ground plane
(Nb, Al, or Ta)

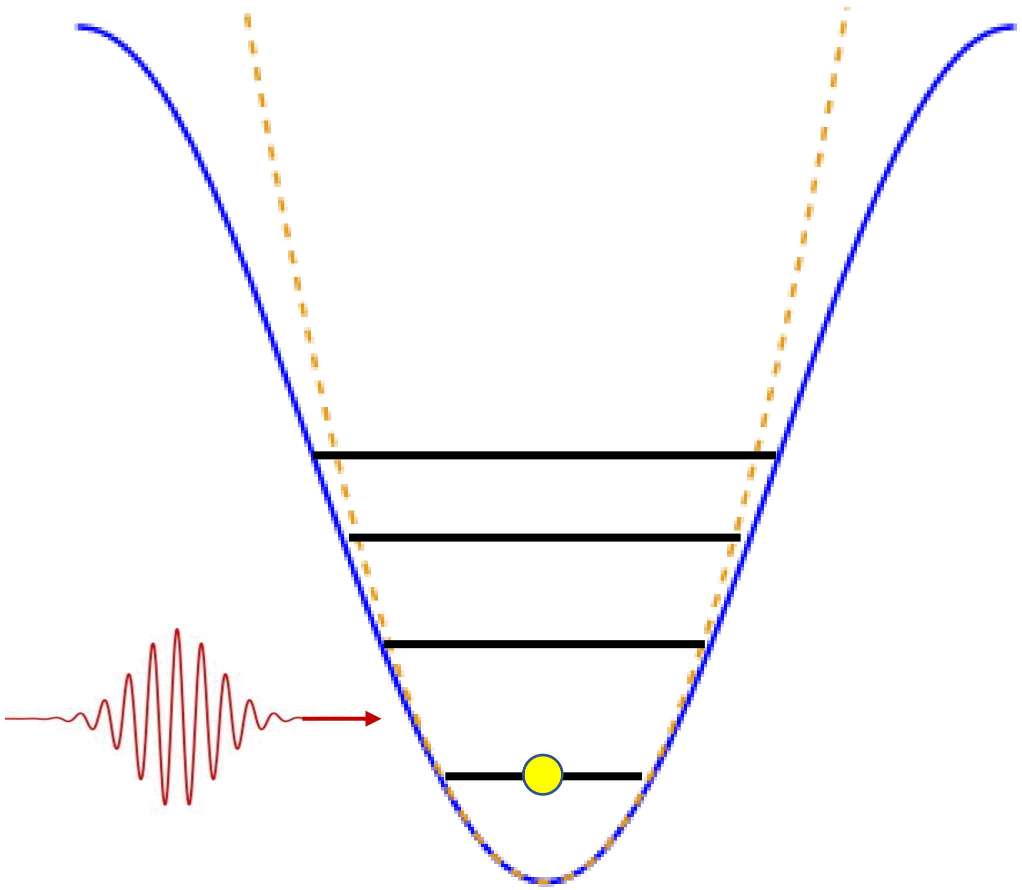
single-qubit gates

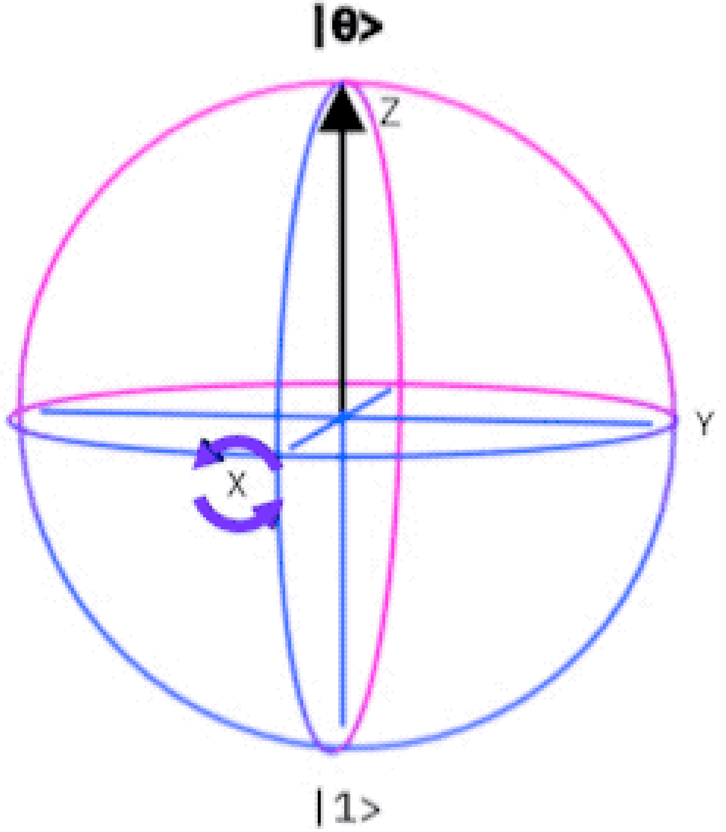



qubit
chip


qubit
chip


$\omega_d = \omega_2$
qubit 3
qubit 1
ω_1
qubit 4
qubit 2
ω_2
qubit 5



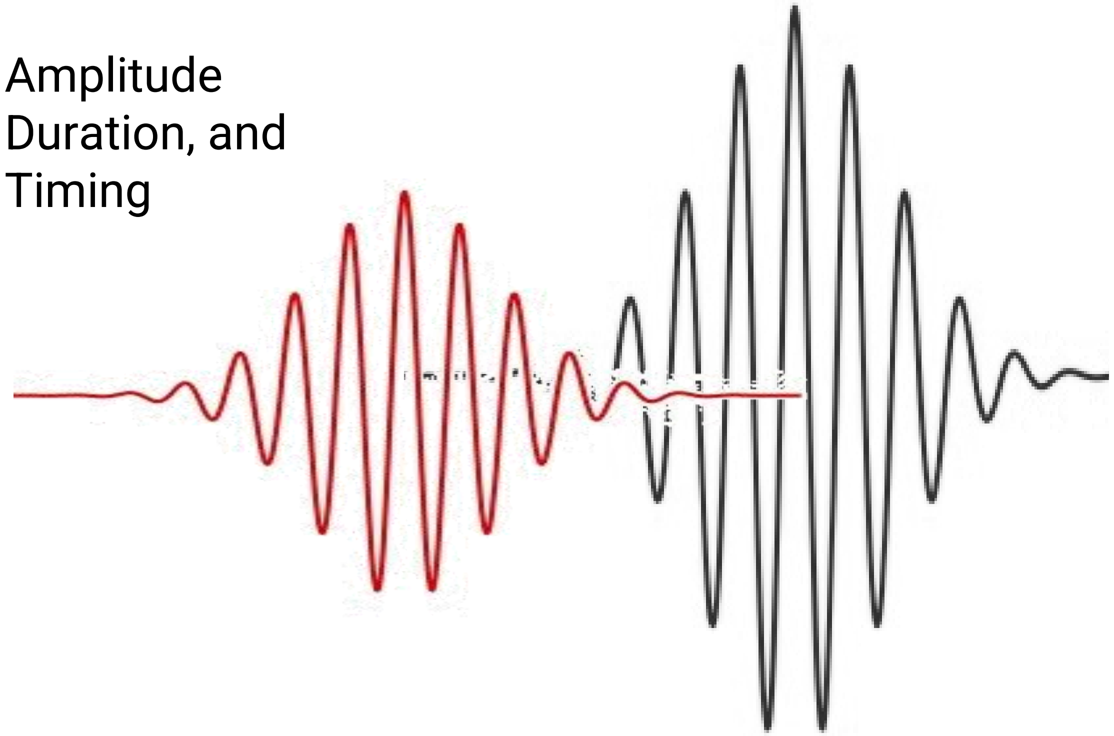


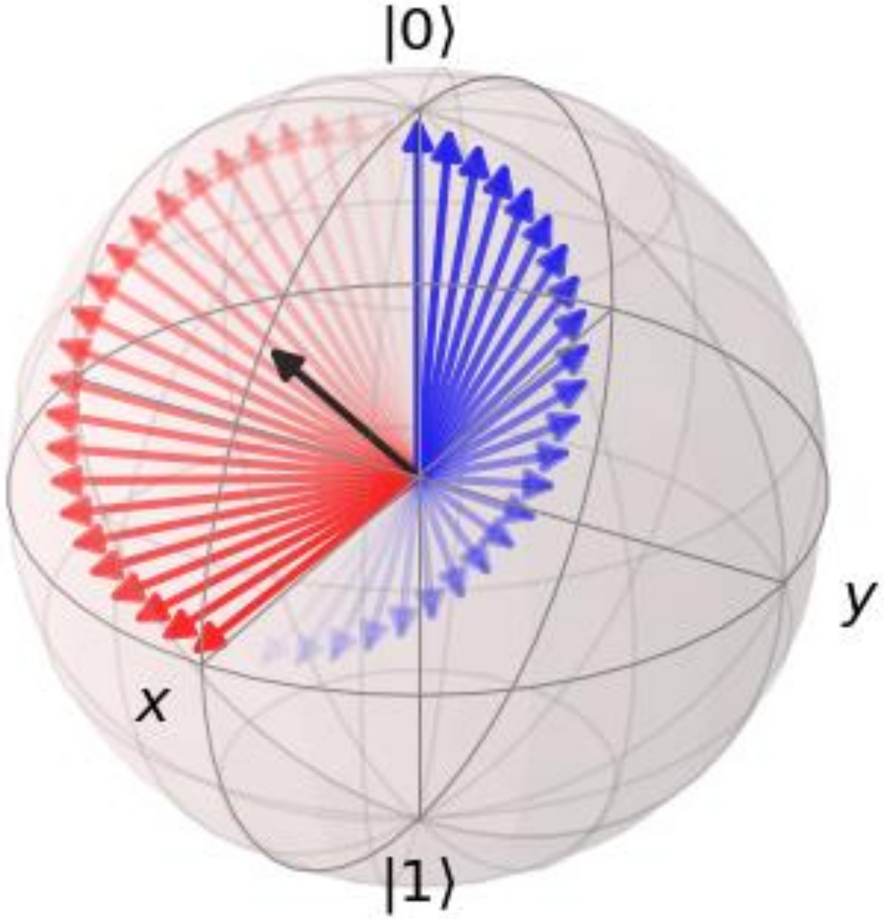
$|0\rangle \rightarrow |1\rangle$

arbitrary single-qubit gates

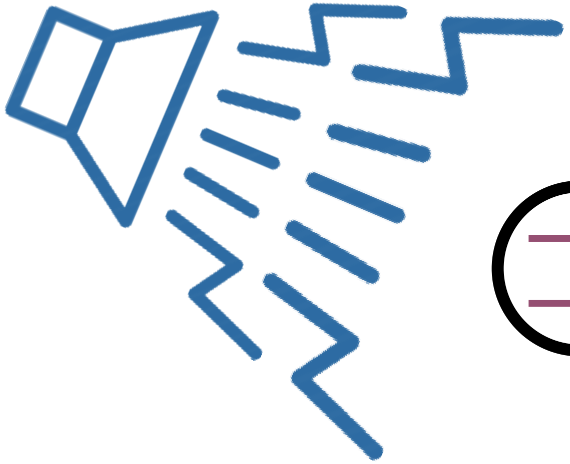
Vary pulse

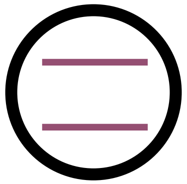
- Amplitude
- Duration, and
- Timing





noise



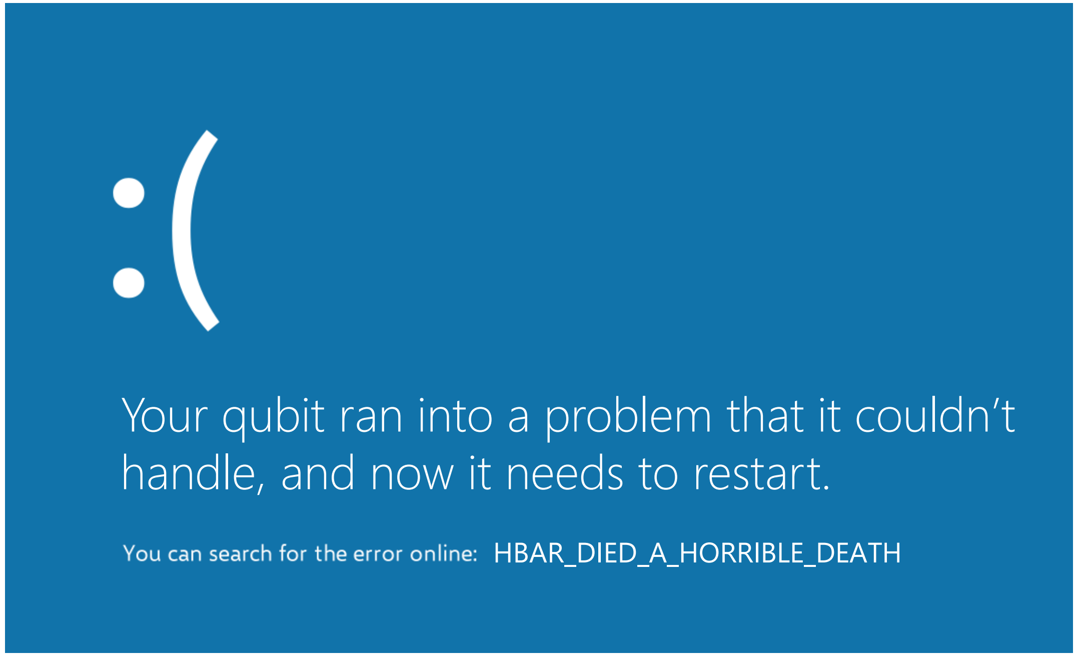


QUBIT

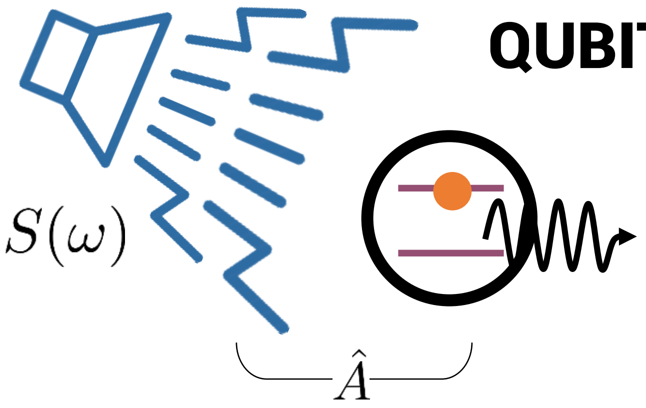
$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

1/f charge noise
1/f flux noise
critical current noise
dielectric loss
photon shot noise
Purcell decay
nonequilibrium quasiparticles
…

DECOHERENCE / ERRORS



noise



QUBIT

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$|0\rangle$$

DECOHERENCE

depolarization
(T_1)

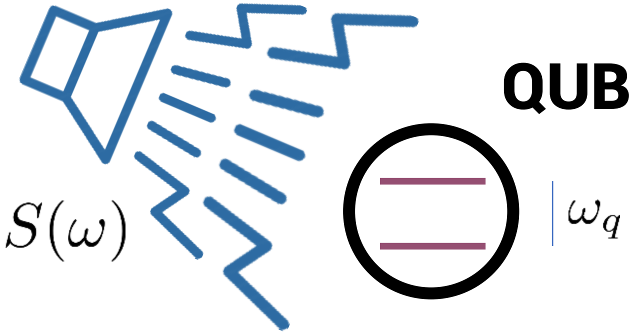
rate: (Fermi's golden rule)

$$1/T_1 \sim |\langle 0|\hat{A}\,|1\rangle|^2 S(\omega_q)$$

transition matrix element

noise spectral density

noise



$$\rho = |\psi\rangle\langle\psi| = \begin{pmatrix} |\alpha|^2 & \alpha\beta^* e^{-i\omega_q t} \\ \alpha^*\beta e^{i\omega_q t} & |\beta|^2 \end{pmatrix}$$

$$\begin{pmatrix} |\alpha|^2 & 0 \\ 0 & |\beta|^2 \end{pmatrix}$$

DECOHERENCE

depolarization (T_1)

rate: (Fermi's golden rule)

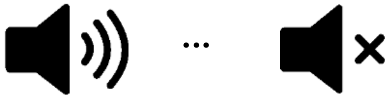
$$1/T_1 \sim |\langle 0|\hat{A}\,|1\rangle|^2 S(\omega_q)$$

pure dephasing (T_φ)

$$1/T_\varphi \sim \left(\frac{\partial\omega_q}{\partial\lambda}\right)^2 S(0)$$

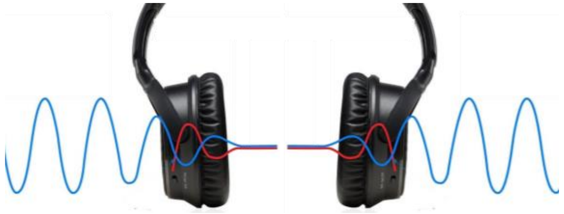
linear sensitivity
of qubit freq. to noise

error mitigation strategies



❶ reduce noise
(materials science, fabrication, microwave engineering, …)

❷ intrinsic noise protection
(develop circuits that are insensitive to noise)



❸ active quantum error correction
(monitor for errors, apply correction steps when required in feedback loop)

1/f noise

charge noise — Cooper pair box

flux noise — rf SQUID qubit, 3-junction flux qubit

Cooper pair box

letters to nature

Coherent control of macroscopic quantum states in a single-Cooper-pair box

Y. Nakamura*, Yu. A. Pashkin† & J. S. Tsai*

** NEC Fundamental Research Laboratories, Tsukuba, Ibaraki 305-8051, Japan*
† CREST, Japan Science and Technology Corporation (JST), Kawaguchi, Saitama 332-0012, Japan

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Physica Scripta. Vol. T76, 165–170, 1998

Quantum Coherence with a Single Cooper Pair

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Received October 27, 1997; revised version received January 15, 1998; accepted January 23, 1998

Cooper pair box

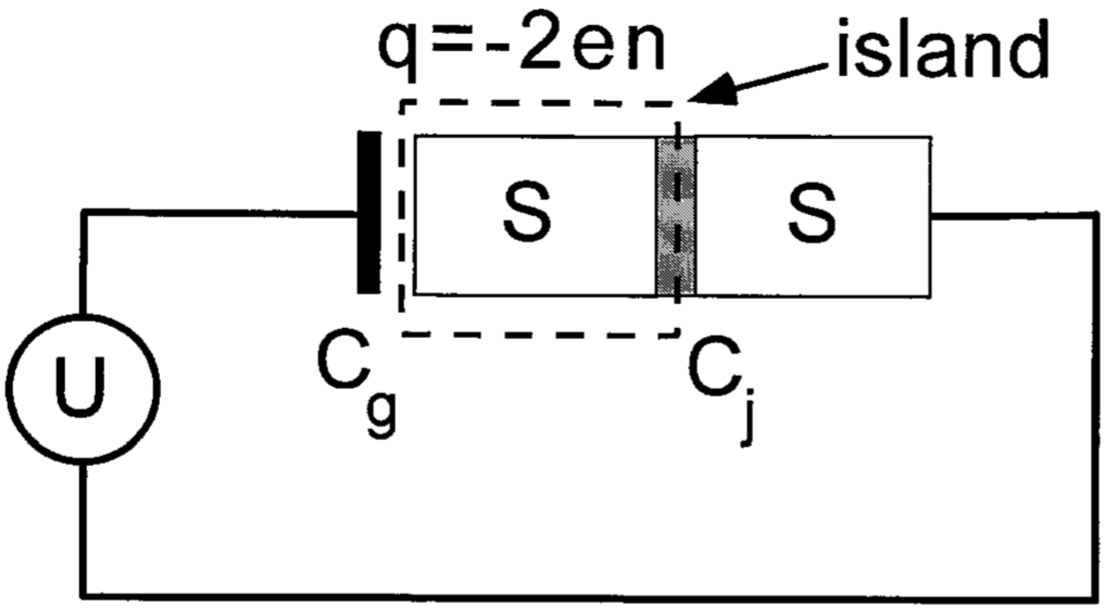
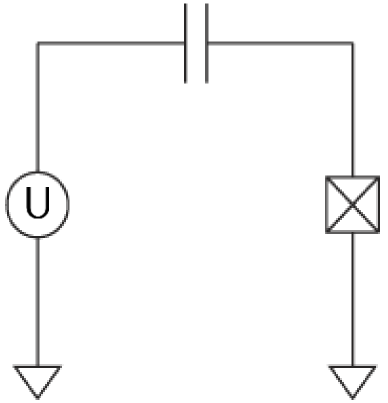


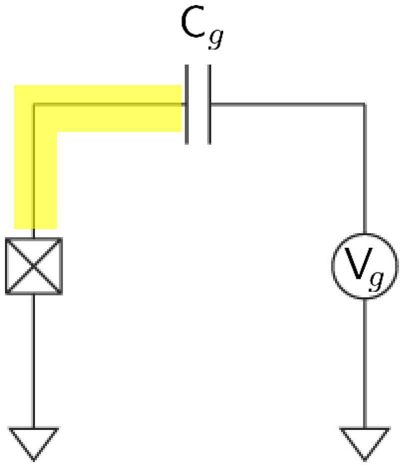
Fig. 1. Schematics of the single Cooper box: a superconducting electrode (island) is in contact with a superconducting reservoir though a tunnel junction (grey zone) with capacitance C_j. Excess Cooper pairs tunnel onto the island in response to an electric field applied by means of the gate capacitance C_g and voltage U.

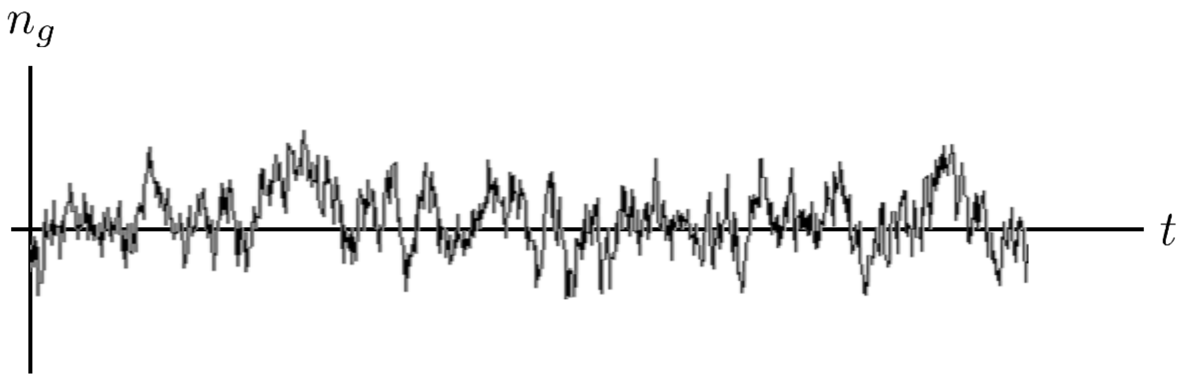


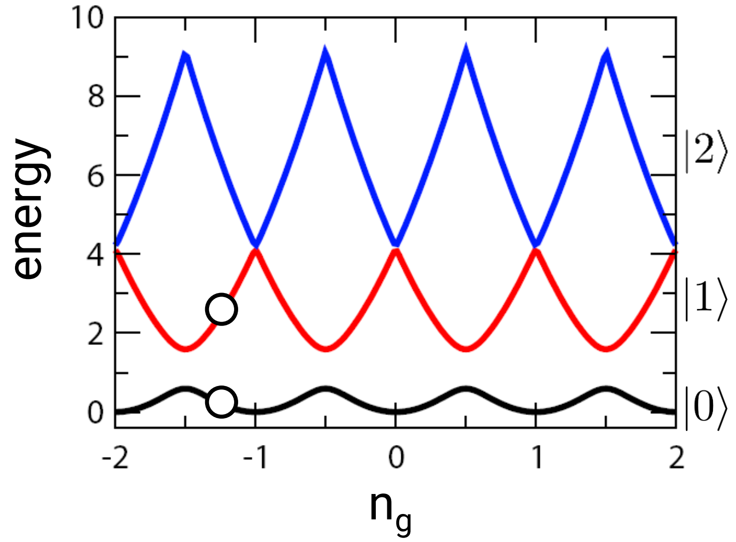
$$T_\varphi \approx 1\,\mathrm{ns}$$

V Bouchiat et al., Phys. Scr. **1998,** 165 (1998)

charge noise dephasing



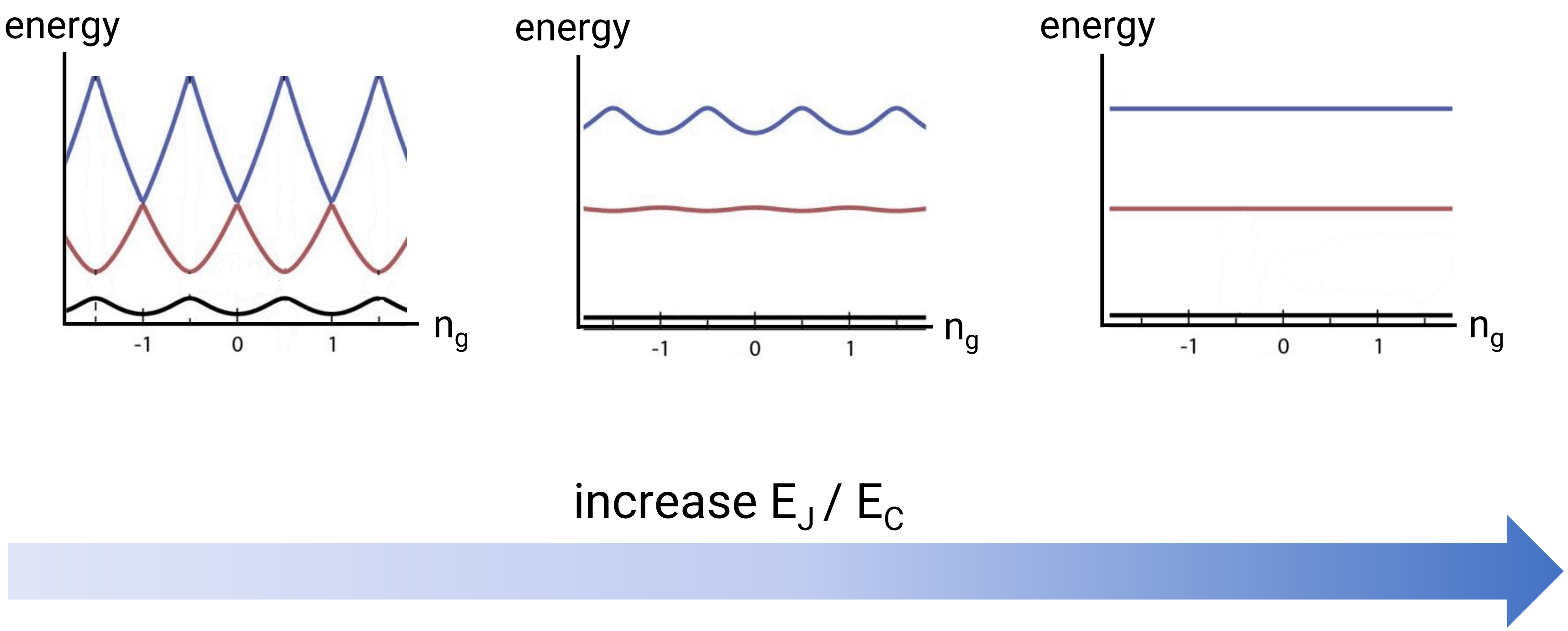




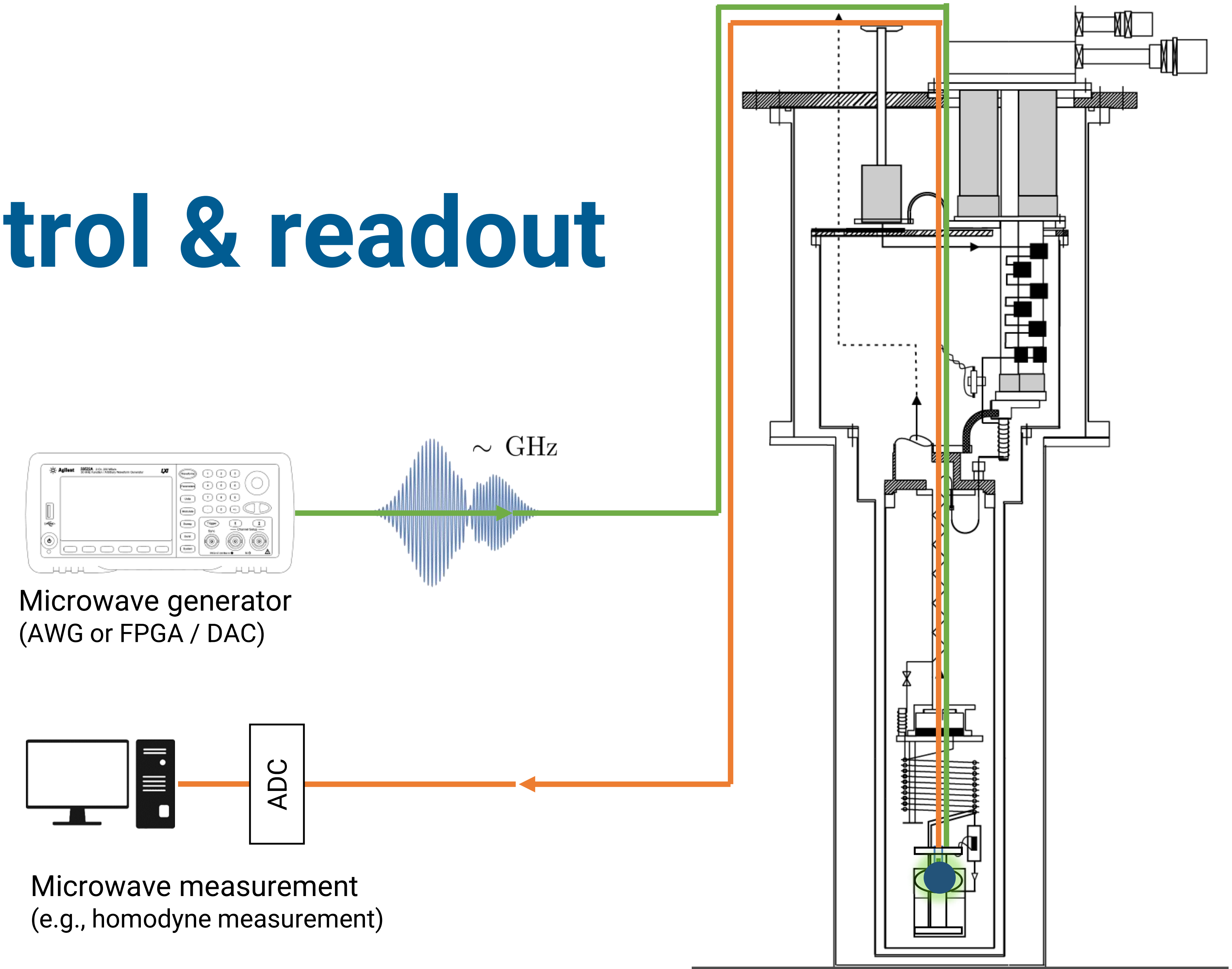
- ▶ dephasing

$$1/T_\varphi \sim \left(\frac{\partial \omega_q}{\partial \lambda}\right)^2 S(0)$$

Flattening the spectrum
for charge noise protection



basics of qubit control & readout



Qubit operations and readout

$$\hat{H}_{\mathrm{JC}} = \tfrac{1}{2}\hbar\omega_q\hat{\sigma}_z + \hbar\omega_r\hat{a}^\dagger\hat{a} + \hbar g(\hat{a}\hat{\sigma}_+ + \hat{a}^\dagger\hat{\sigma}_-)$$

Perturbation theory in $g/\Delta \ll 1$

(Schrieffer-Wolff transf. / adiabatic elimination)

Lamb shift: $\hbar\omega_q' = \hbar\omega_q + \frac{g^2}{\Delta}$

$$\hat{H}_{\mathrm{JC}}^{\mathrm{eff}} = \hbar\omega_r\hat{a}^\dagger\hat{a} + \frac{\hbar\omega_q'}{2}\hat{\sigma}_z + \hbar\chi\hat{a}^\dagger\hat{a}\hat{\sigma}_z$$

ac Stark shift: $\chi = \frac{g^2}{\Delta}$

dispersive regime

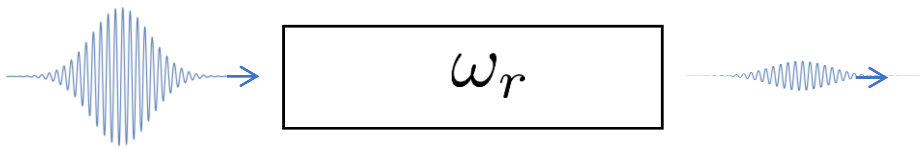
$$\hat{H}_{\mathrm{JC}}^{\mathrm{eff}} = \hbar\omega_r \hat{a}^\dagger \hat{a} + \frac{\hbar\omega_q'}{2}\hat{\sigma}_z + \hbar\chi \hat{a}^\dagger \hat{a}\hat{\sigma}_z$$

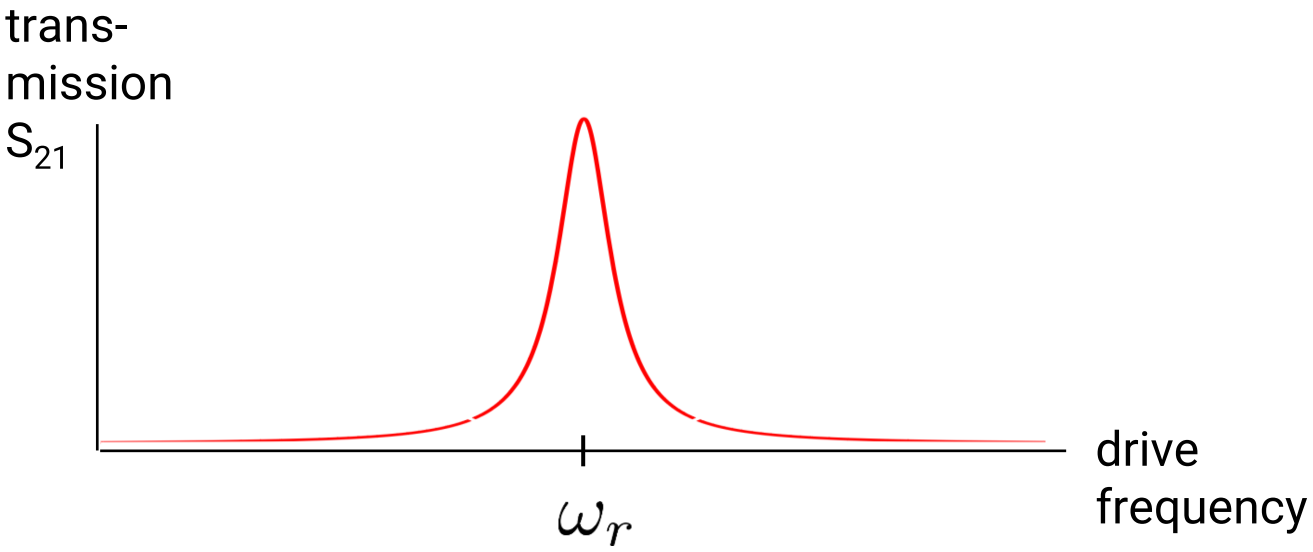
$$= \hbar(\omega_r + \chi\hat{\sigma}_z)\hat{a}^\dagger \hat{a} + \frac{\hbar\omega_q'}{2}\hat{\sigma}_z$$

Resonator frequency depends on qubit state!

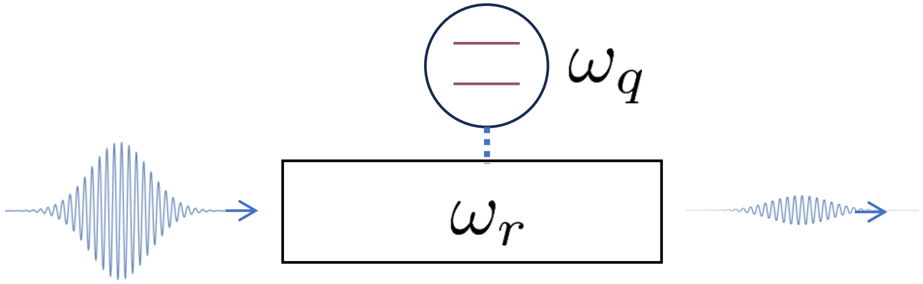
dispersive readout

$$\hat{H} = \hbar\omega_r \hat{a}^\dagger \hat{a} + \hat{H}_{\text{drive}}$$

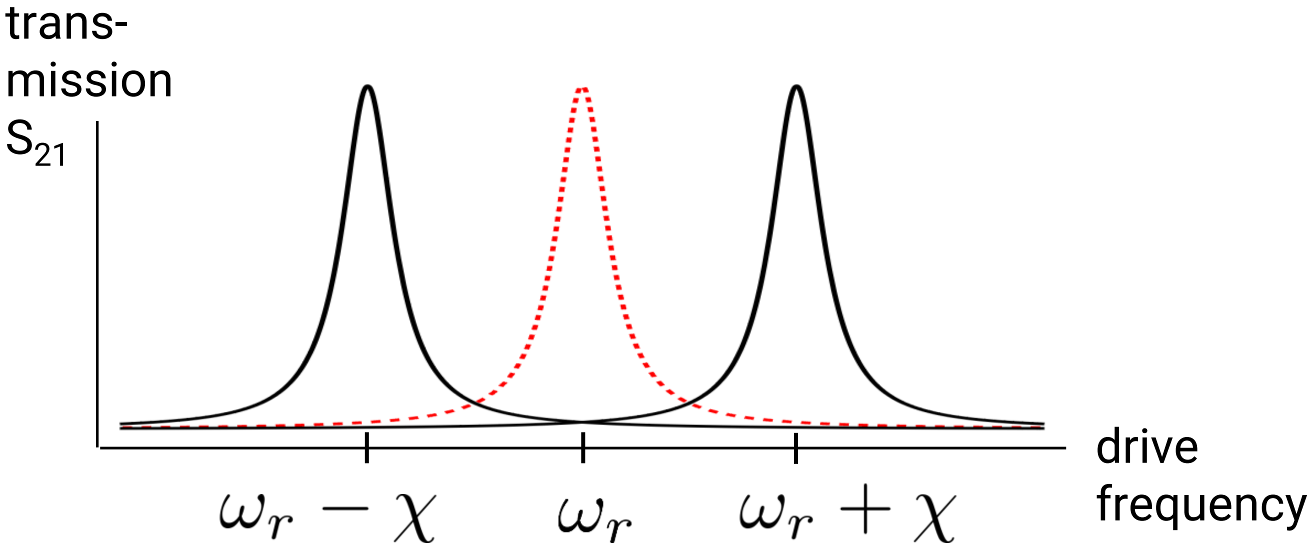




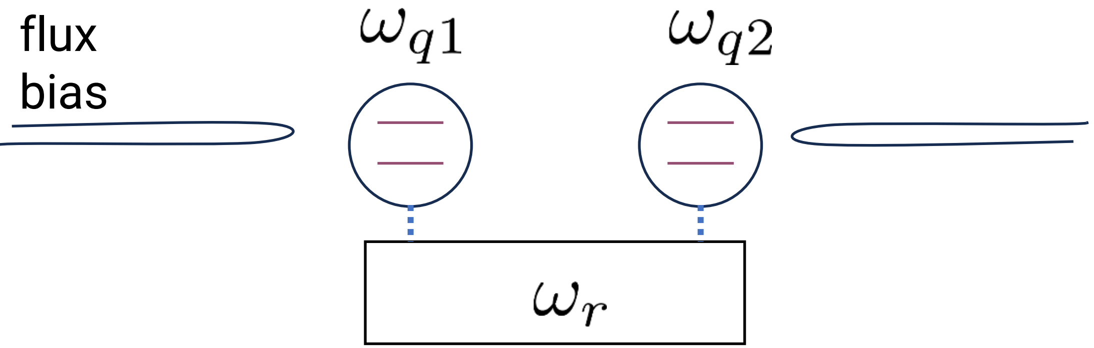
dispersive readout

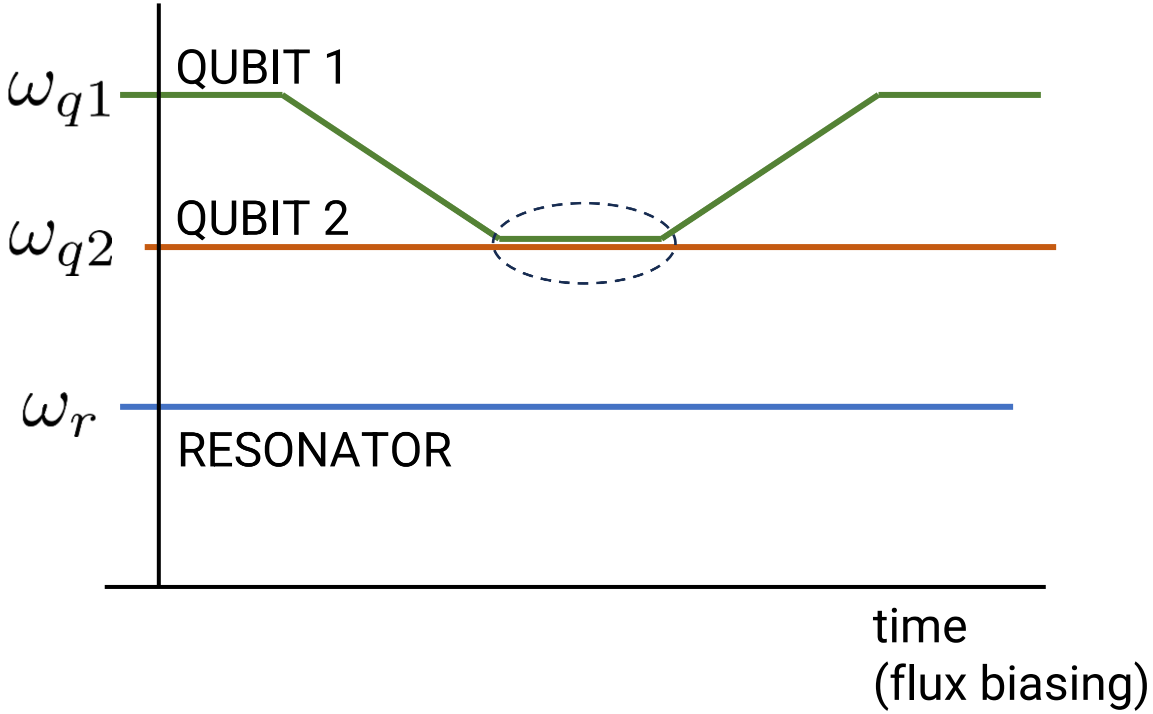


$$\hat{H}_{\mathrm{JC}}^{\mathrm{eff}} = \hbar(\omega_r + \chi\hat{\sigma}_z)\hat{a}^\dagger\hat{a} + \frac{\hbar\omega_q'}{2}\hat{\sigma}_z$$

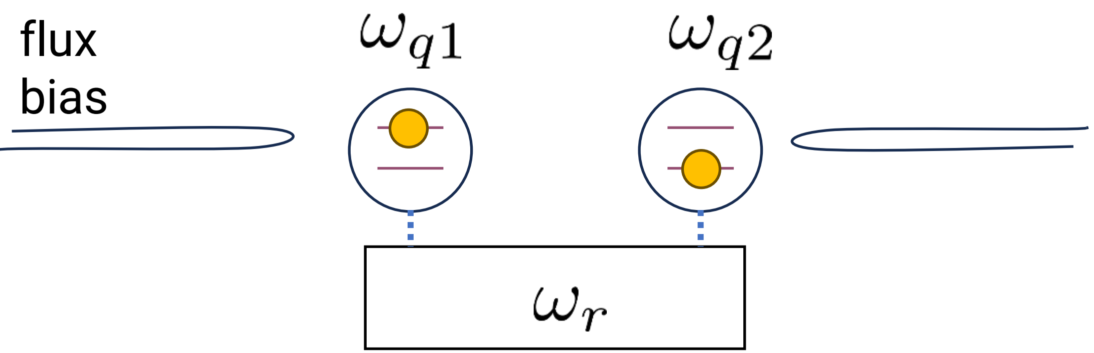


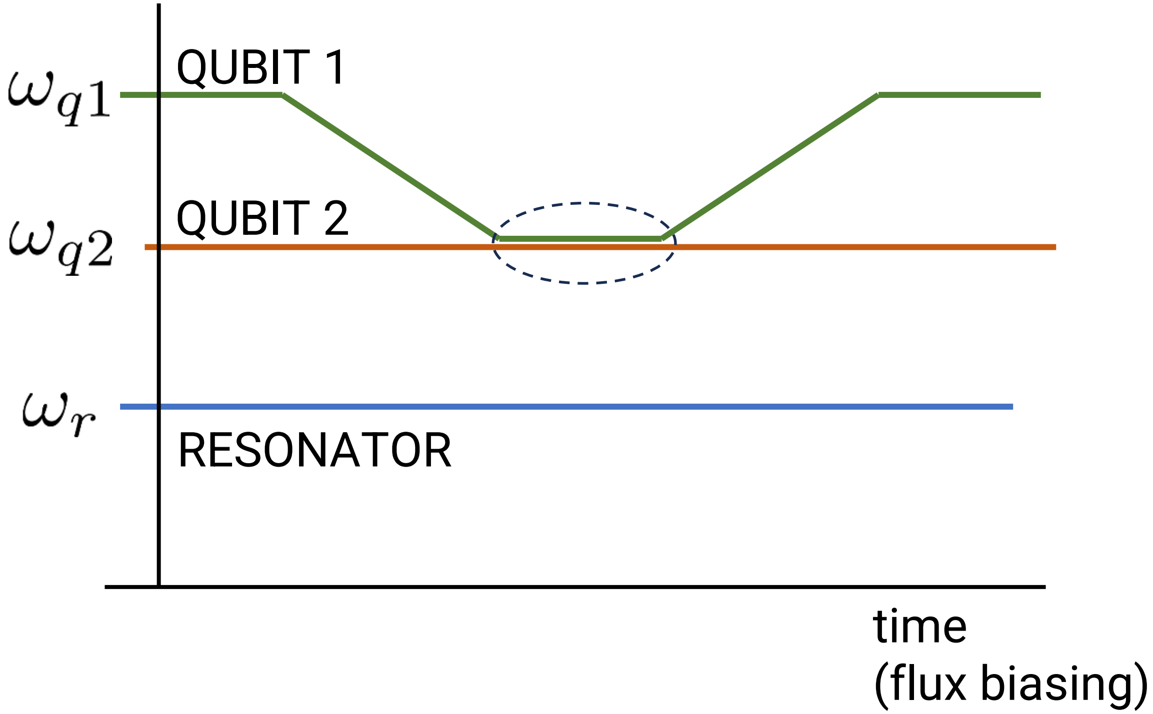
two-qubit gate
(example)



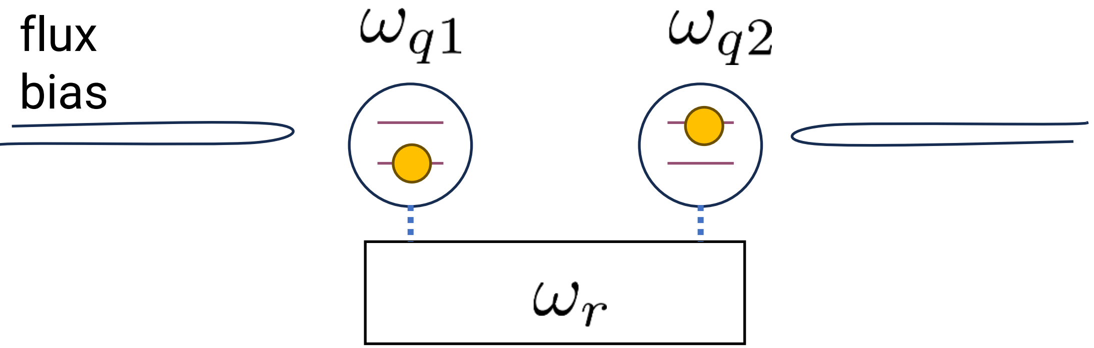


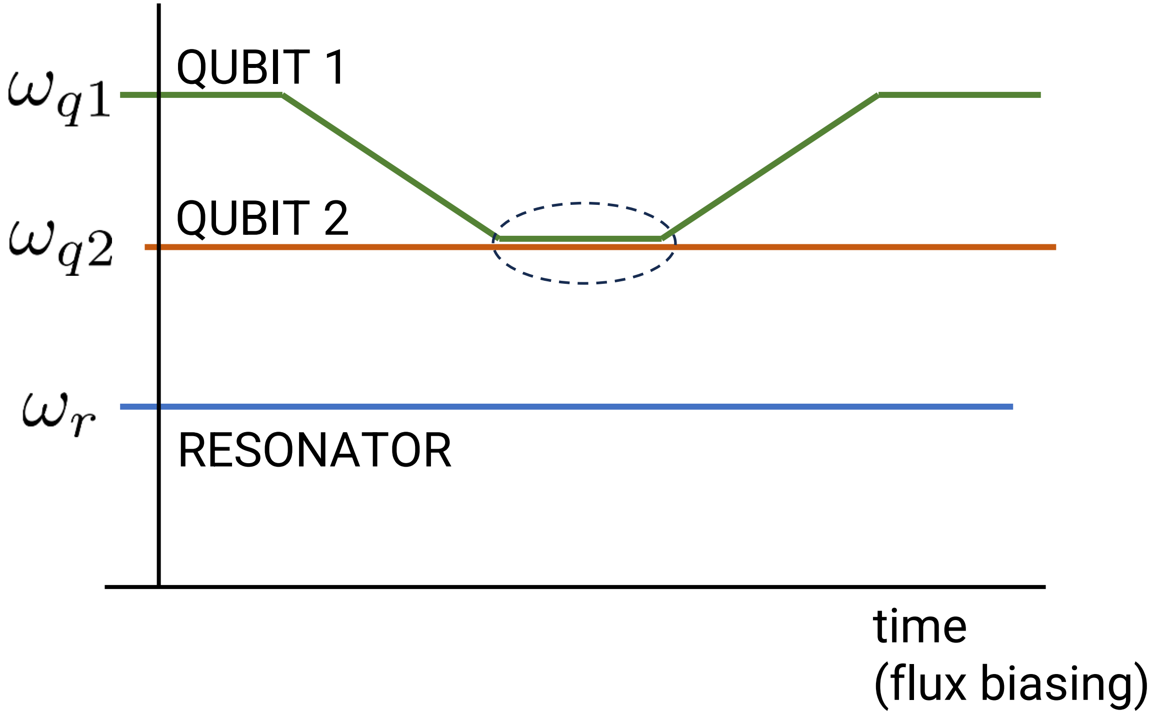
two-qubit gate
(example)



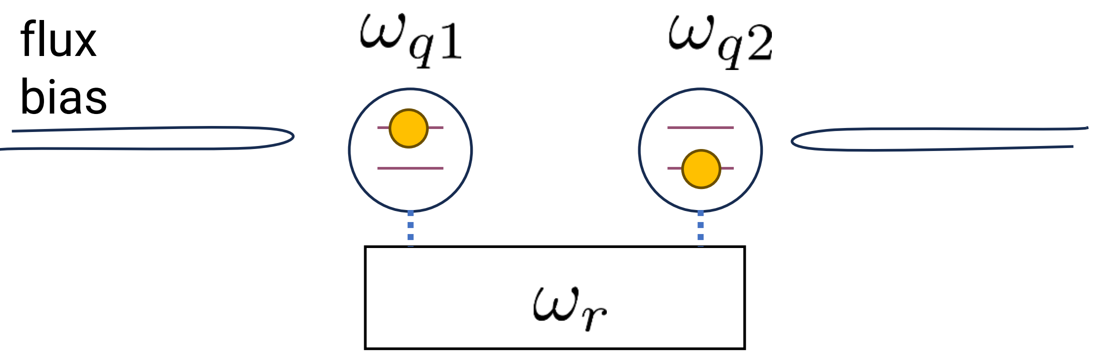


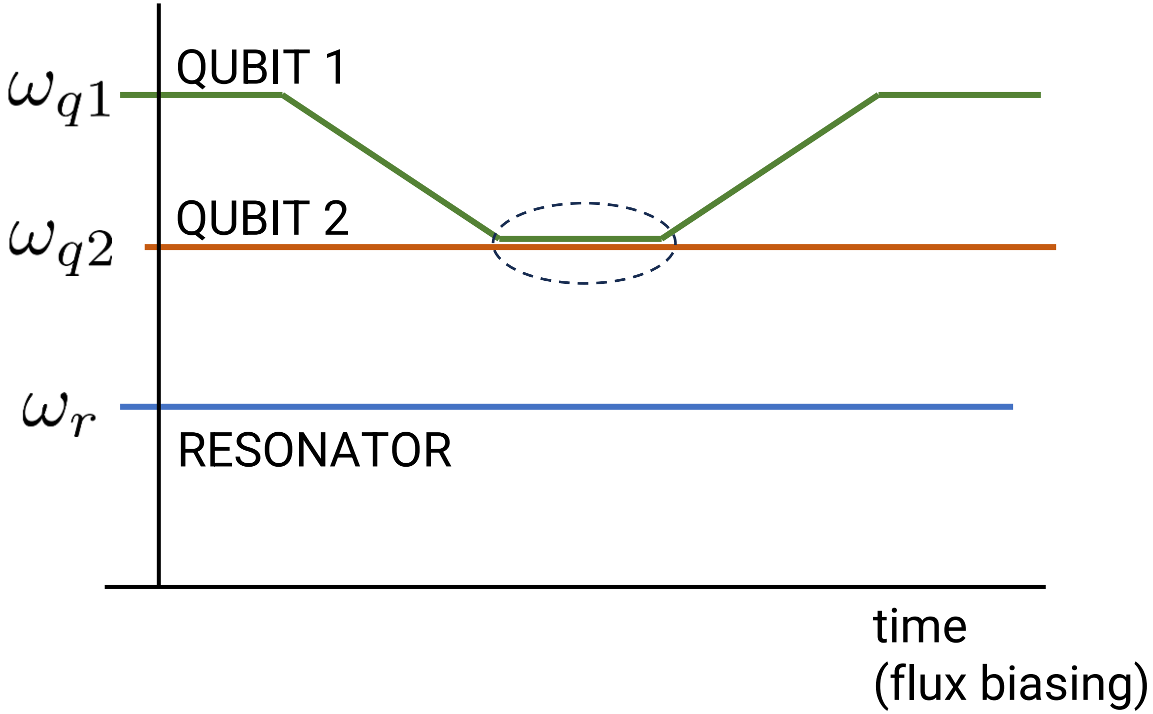
two-qubit gate
(example)





two-qubit gate
(example)





After quarter period: √iSWAP gate

task

$$\sqrt{\mathrm{iSWAP}}|01\rangle = \quad ?$$

$$\sqrt{\text{iSWAP}} = \sqrt{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 \\ 0 & \frac{i}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$|0\rangle \mapsto \begin{pmatrix} 1 \\ 0 \end{pmatrix}, |1\rangle \mapsto \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\sqrt{\text{iSWAP}} = \sqrt{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 \\ 0 & \frac{i}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$|01\rangle = |0\rangle \otimes |1\rangle \quad \mapsto \quad \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \begin{pmatrix} 0 \\ 1 \end{pmatrix} \\ 0 \begin{pmatrix} 0 \\ 1 \end{pmatrix} \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\sqrt{\text{iSWAP}} = \sqrt{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 \\ 0 & \frac{i}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\sqrt{\text{iSWAP}}|01\rangle = $$?

$$|0\rangle \mapsto \begin{pmatrix} 1 \\ 0 \end{pmatrix}, |1\rangle \mapsto \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\sqrt{\text{iSWAP}} = \sqrt{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 \\ 0 & \frac{i}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\sqrt{\text{iSWAP}}|01\rangle = \ ?$$

$$|0\rangle \mapsto \begin{pmatrix} 1 \\ 0 \end{pmatrix}, |1\rangle \mapsto \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

universal control

For gate based quantum computing:
what is a

minimal set of elementary qubit operations

sufficient to carry out an **arbitrary algorithm**?

universal control

1. arbitrary single-qubit gates
   (can approximate and decompose into sequence of H, T)

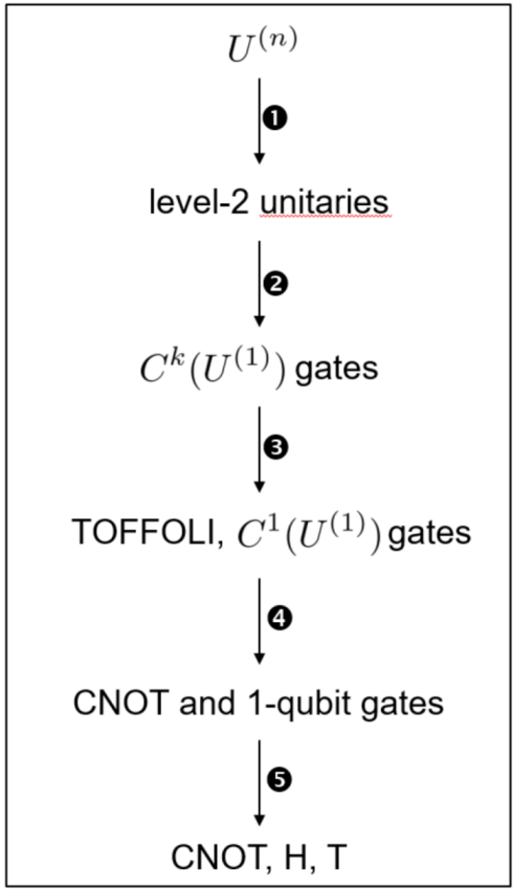
2. one 2-qubit entangling gate
   (like CNOT or √iSWAP)

universal control

$$U =$$

$2^n \times 2^n$ unitary

Summary of decomposition chain:



① Iterative reduction of general n-qubit unitary into n-qubit level-2 unitaries

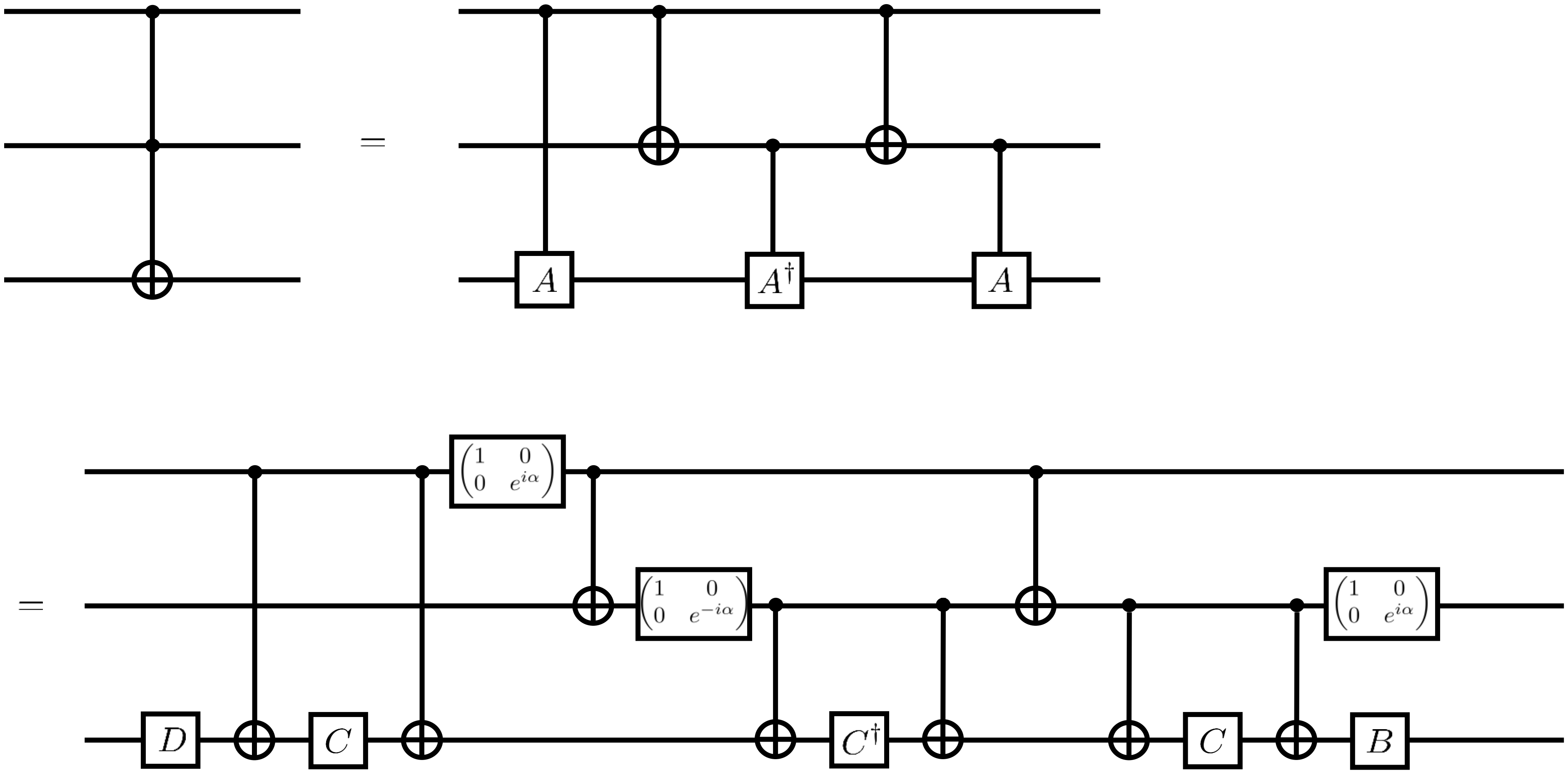
② Conversion of n-qubit level-2 unitaries into $C^k(U^{(1)})$ gates ($k=n-1$), if necessary with the help of Gray codes

③ Decompose $C^k(U^{(1)})$ gate into cascade of TOFFOLIs and one $C^1(U^{(1)})$ gate, using n-1 auxiliary qubits

④ Express TOFFOLI in terms of CA gates where $A^2 = X$. Then, use AXBXC decomposition on CA and $C^1(U^{(1)})$ to reduce to CNOTs and 1-qubit gates.

⑤ Approximate 1-qubit gates by sequences of H and T gates, using irrational-angle decomposition.

TOFFOLI gate decomposition

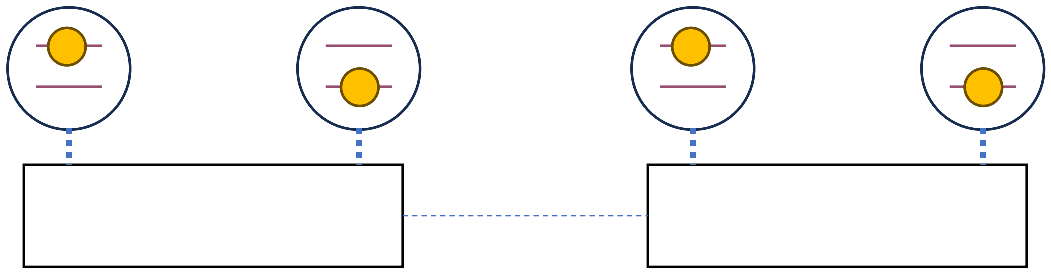


who’s the qubit?

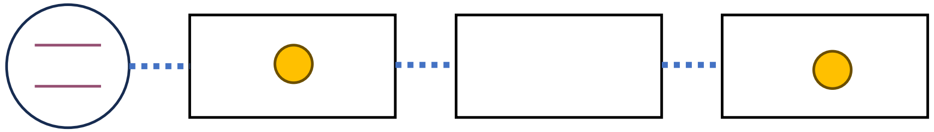
bosonic-qubit architectures

who’s the qubit?

SCHEME 1: The qubit is the qubit (duh!)
(Google, IBM, Rigetti etc.)



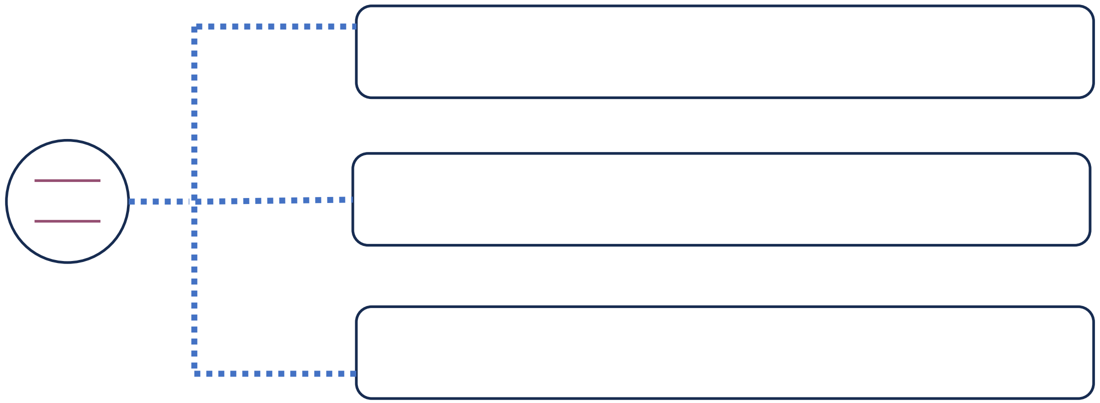
SCHEME 2: The qubit is a linear cavity mode
Yale, FNAL-SQMS, Schuster



- Fock state encoding
- bosonic codes
- cat states
- GKP states

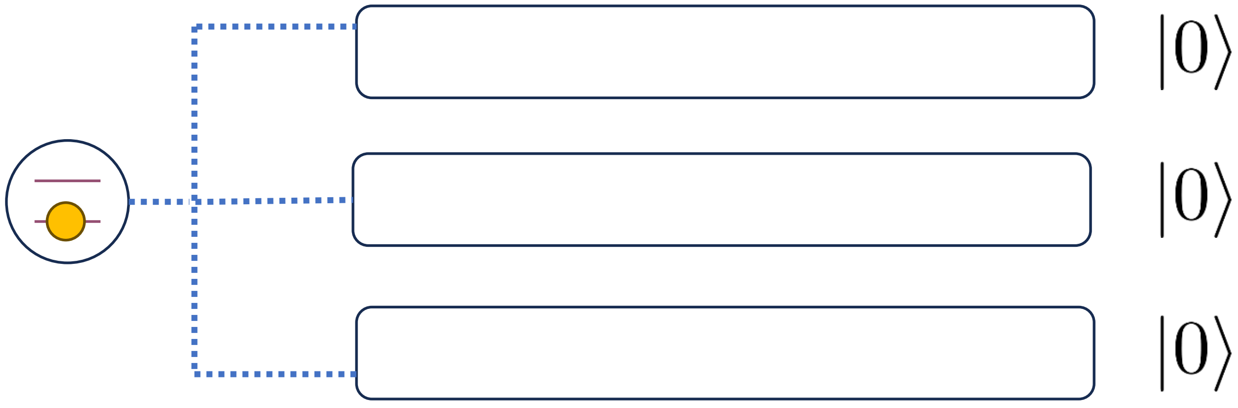
multi-mode control

$$\hat{H} = \sum_j \omega_j \hat{a}_j^\dagger \hat{a}_j + \sum_j \chi_j \hat{a}_j^\dagger \hat{a}_j \hat{\sigma}_z + \frac{\omega_q'}{2} \hat{\sigma}_z$$



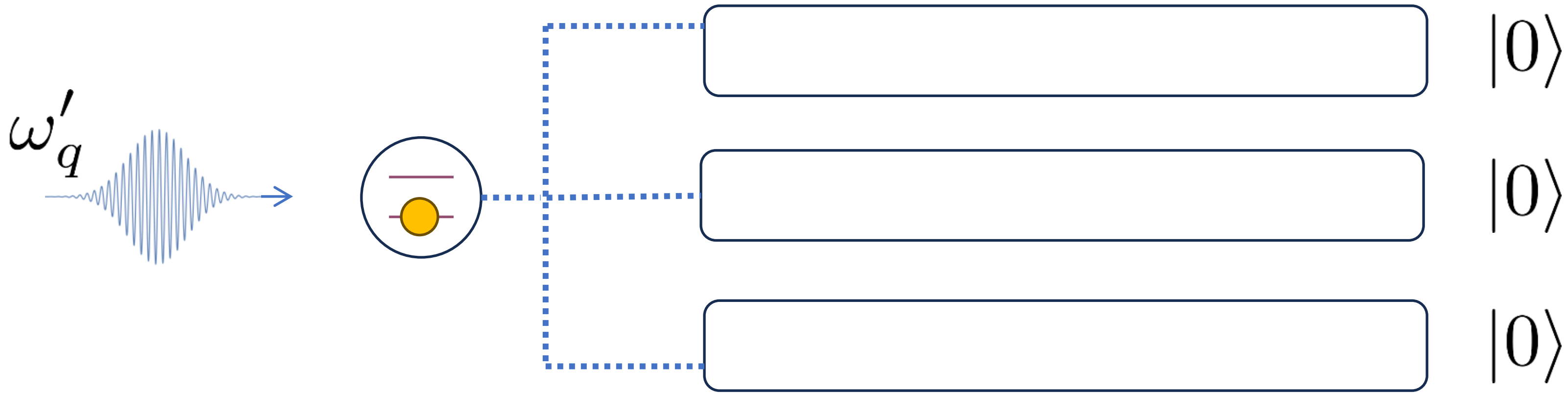
multi-mode control

$$\hat{H} = \sum_j \omega_j \hat{a}_j^\dagger \hat{a}_j + \sum_j \chi_j \hat{a}_j^\dagger \hat{a}_j \hat{\sigma}_z + \frac{\omega_q'}{2}\hat{\sigma}_z$$



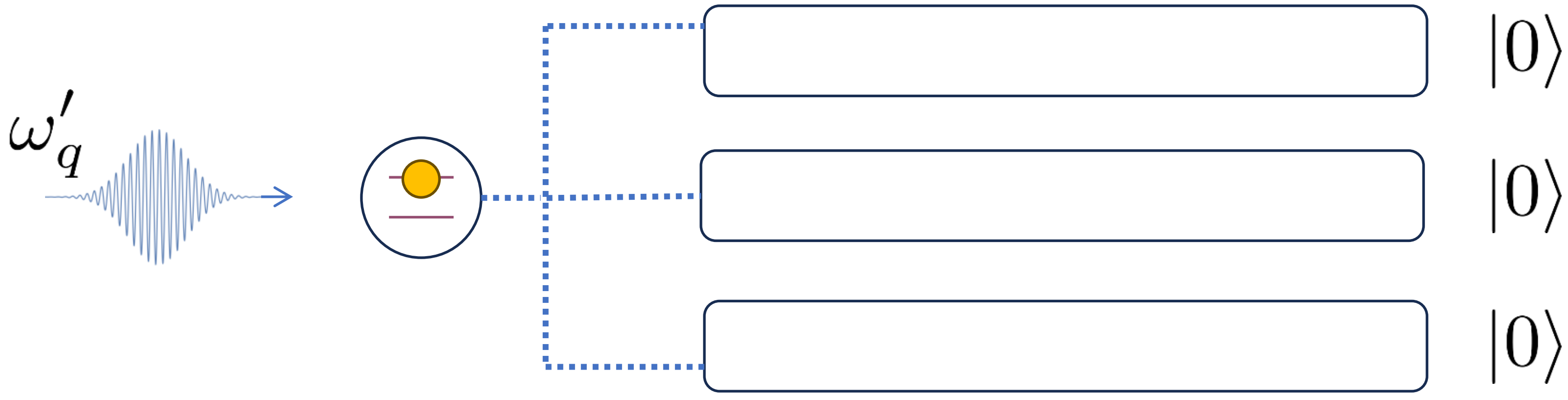
multi-mode control

$$\hat{H} = \sum_j \omega_j \hat{a}_j^\dagger \hat{a}_j + \sum_j \chi_j \hat{a}_j^\dagger \hat{a}_j \hat{\sigma}_z + \frac{\omega_q'}{2} \hat{\sigma}_z$$



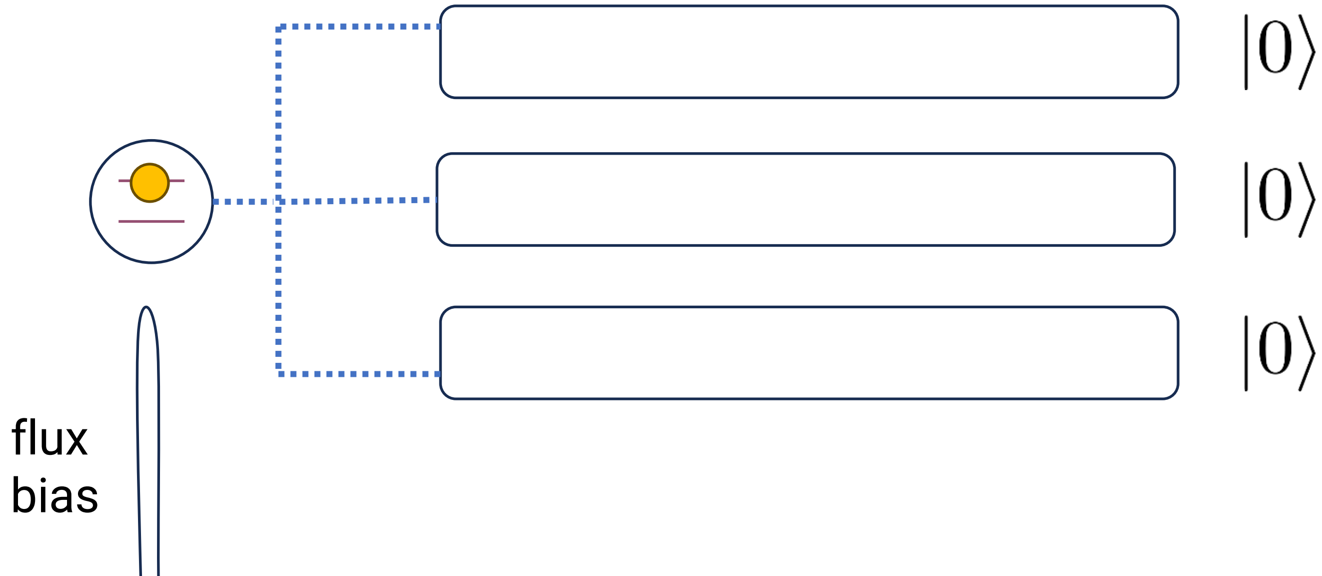
multi-mode control

$$\hat{H} = \sum_j \omega_j \hat{a}_j^\dagger \hat{a}_j + \sum_j \chi_j \hat{a}_j^\dagger \hat{a}_j \hat{\sigma}_z + \frac{\omega_q'}{2}\hat{\sigma}_z$$



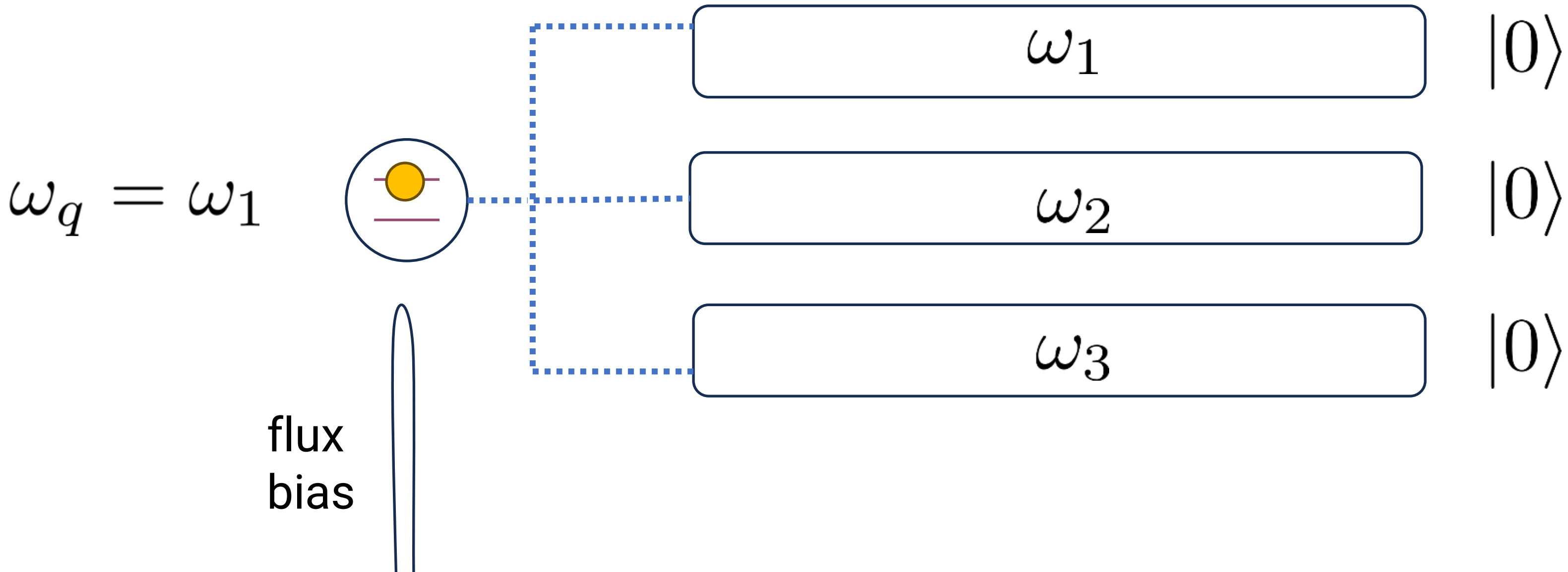
multi-mode control

$$\hat{H} = \sum_j \omega_j \hat{a}_j^\dagger \hat{a}_j + \sum_j \chi_j \hat{a}_j^\dagger \hat{a}_j \hat{\sigma}_z + \frac{\omega_q'}{2}\hat{\sigma}_z$$



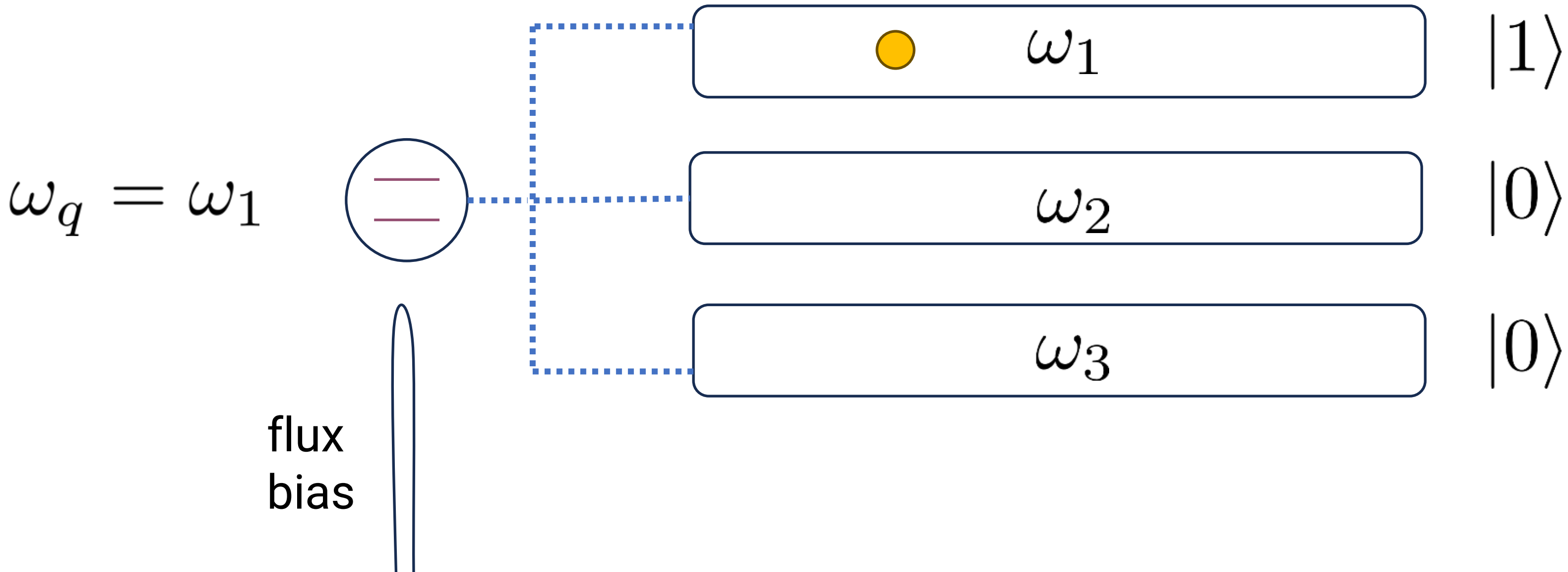
multi-mode control

$$\hat{H} = \sum_j \omega_j \hat{a}_j^\dagger \hat{a}_j + \sum_j \chi_j \hat{a}_j^\dagger \hat{a}_j \hat{\sigma}_z + \frac{\omega_q'}{2} \hat{\sigma}_z$$



multi-mode control

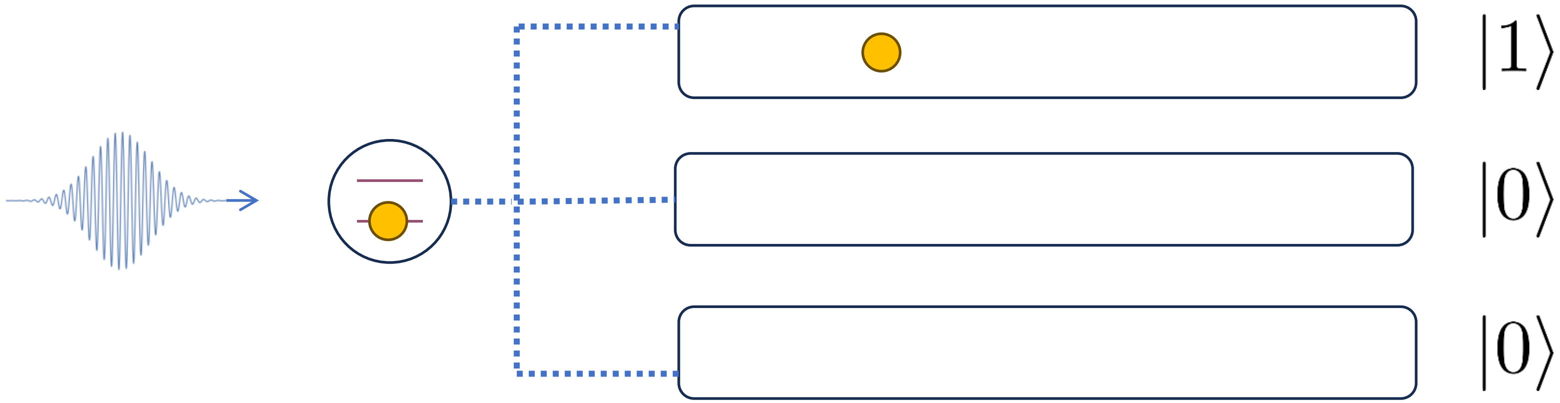
$$\hat{H} = \sum_j \omega_j \hat{a}_j^\dagger \hat{a}_j + \sum_j \chi_j \hat{a}_j^\dagger \hat{a}_j \hat{\sigma}_z + \frac{\omega_q'}{2}\hat{\sigma}_z$$



NEXT: load an excitation into mode 2
(π pulse on mode 2)

multi-mode control

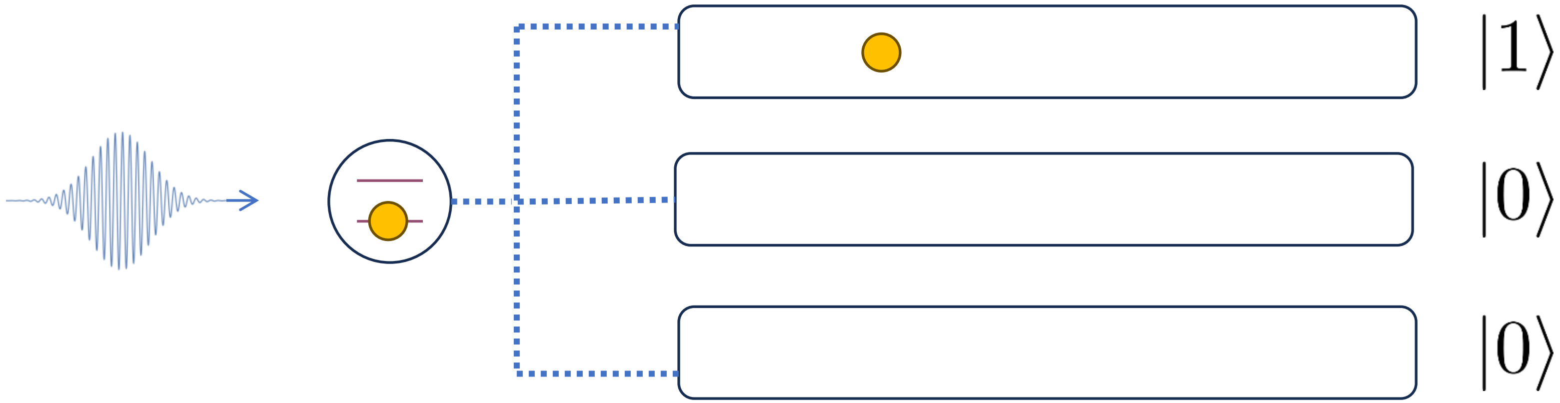
$$\hat{H} = \sum_j \omega_j \hat{a}_j^\dagger \hat{a}_j + \sum_j \chi_j \hat{a}_j^\dagger \hat{a}_j \hat{\sigma}_z + \frac{\omega_q'}{2} \hat{\sigma}_z$$



multi-mode control

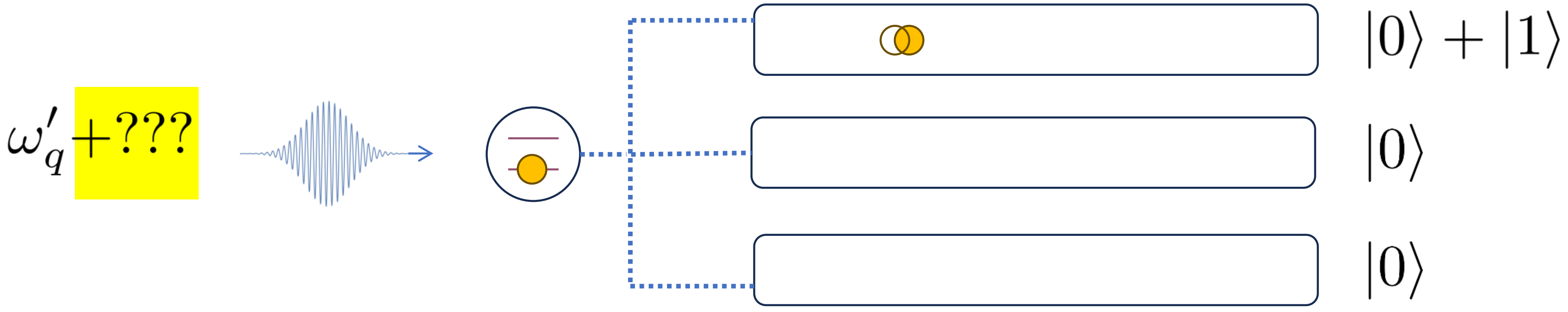
$$\hat{H} = \sum_j \omega_j \hat{a}_j^\dagger \hat{a}_j + \sum_j \chi_j \hat{a}_j^\dagger \hat{a}_j \hat{\sigma}_z + \frac{\omega_q'}{2}\hat{\sigma}_z$$

$\omega_q' + \chi_1$

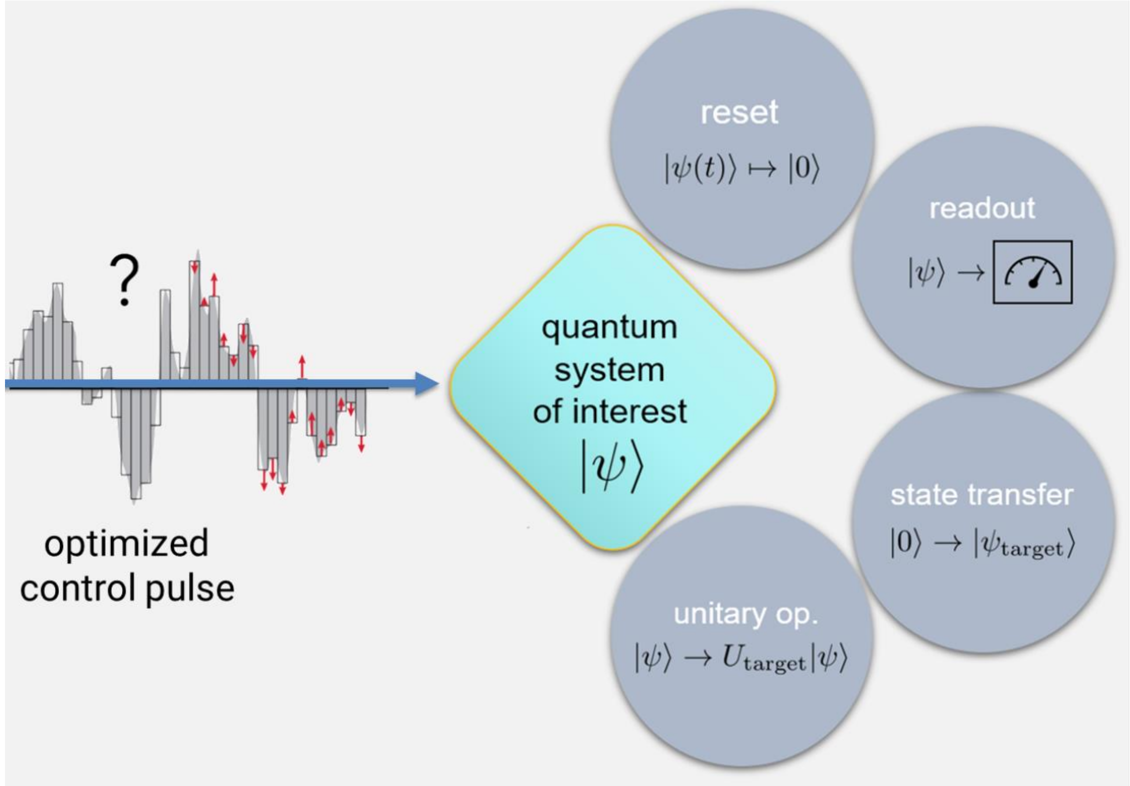


multi-mode control

$$\hat{H} = \sum_j \omega_j \hat{a}_j^\dagger \hat{a}_j + \sum_j \chi_j \hat{a}_j^\dagger \hat{a}_j \hat{\sigma}_z + \frac{\omega_q'}{2} \hat{\sigma}_z$$



quantum optimal control



N. Khaneja, …, S. J. Glaser,
Optimal Control of Coupled Spin Dynamics: Design of NMR Pulse Sequences by Gradient Ascent Algorithms

J. Magn. Reson. **172**, 296-305 (2005)

robust quantum control

Crosstalk-Robust Quantum Control in Multimode Bosonic Systems

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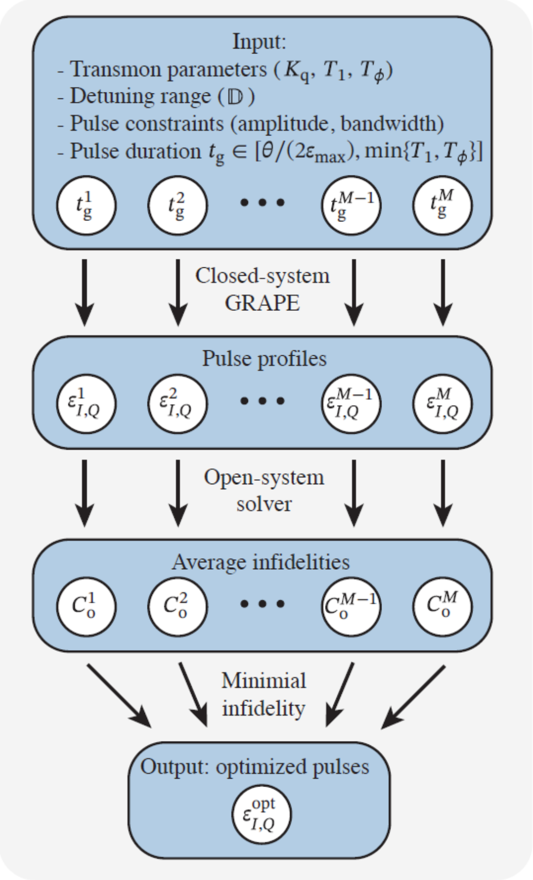
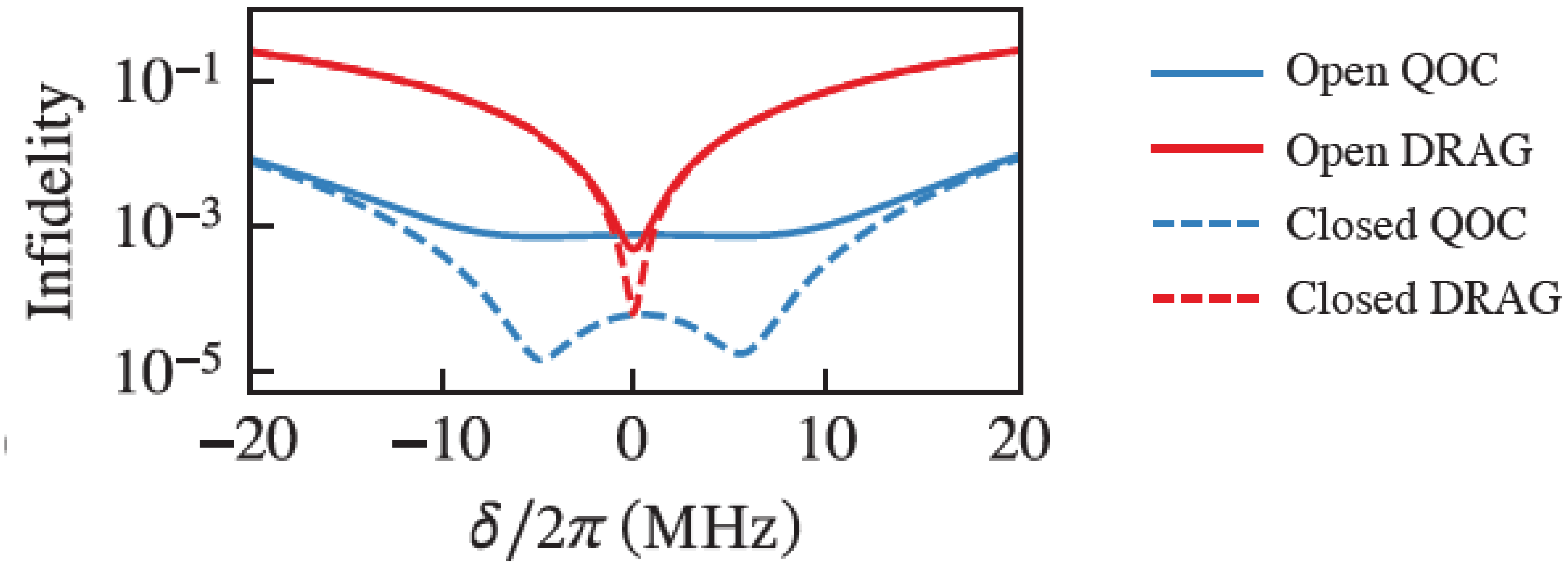
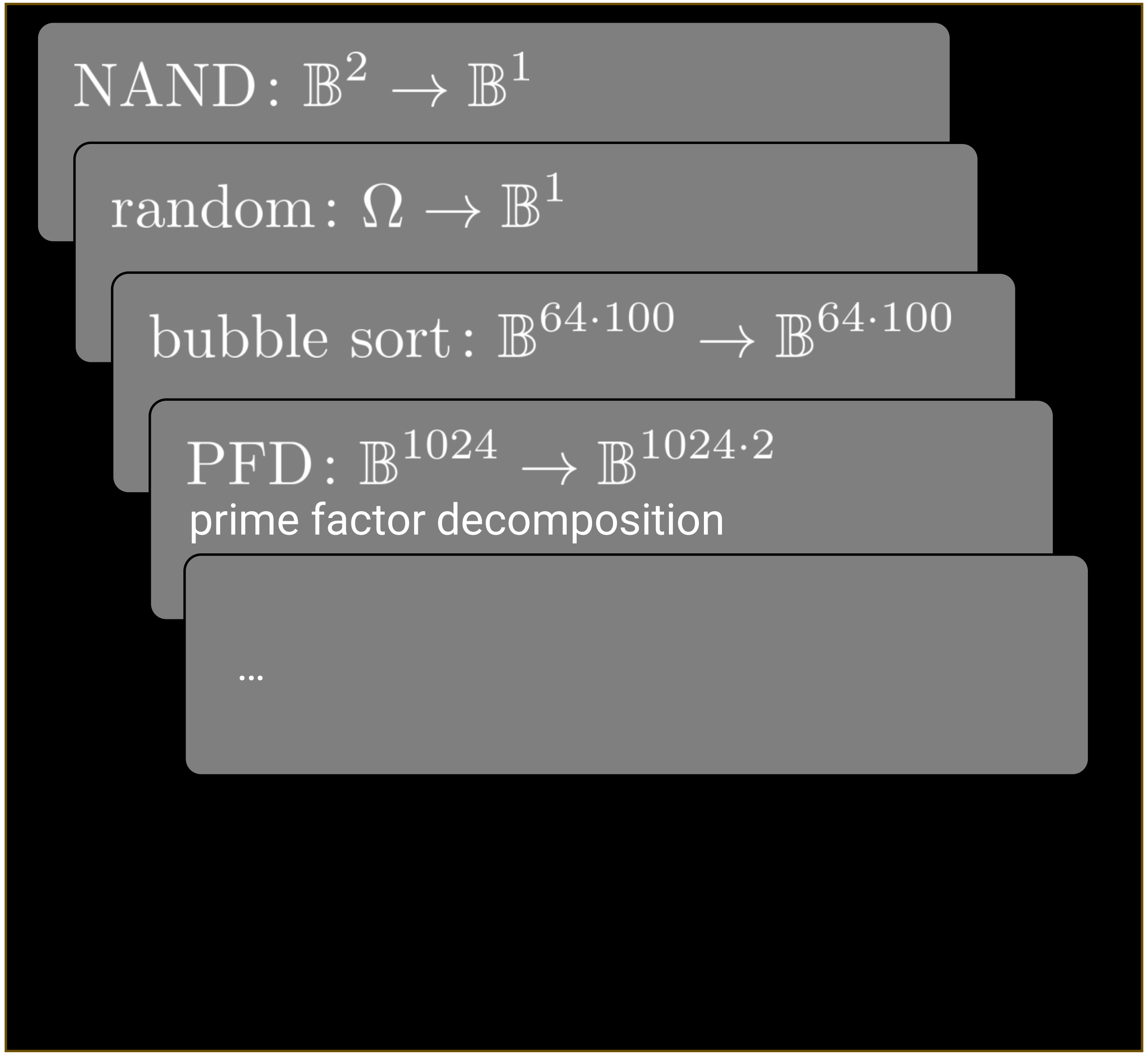




programmable computer

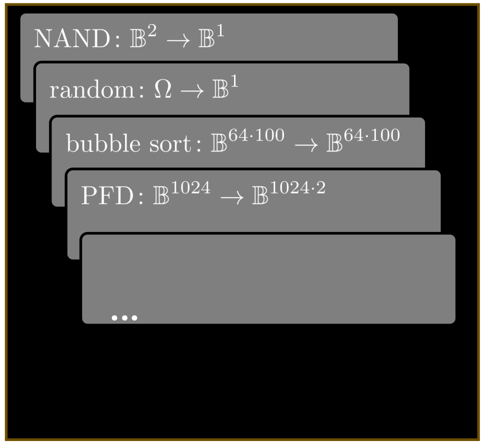
a black box* that can implement** any*** given *f*



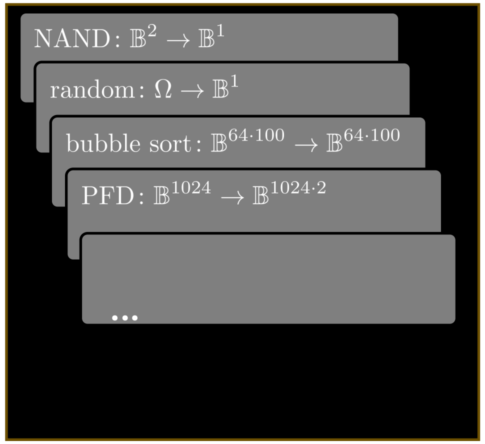
* a “device” obeying the laws of physics
** with input & output sizes $n, m \leq N_0$
*** output is provided in finite time

assessing performance

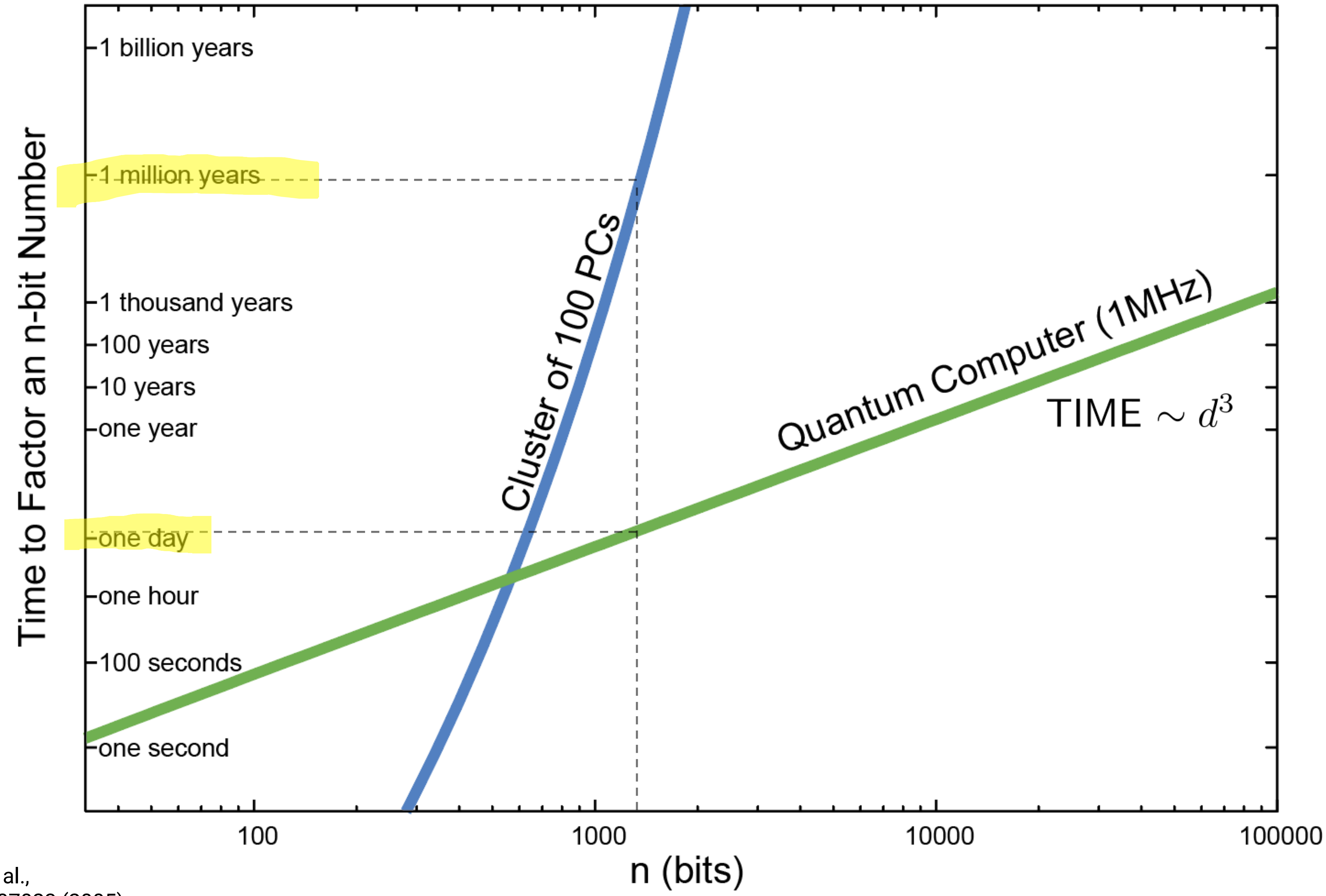
COMPUTER 1



COMPUTER 2



Quantum computation: Shor's algorithm



Peter Shor

based on:
van Meter et al.,
quant-ph/0507023 (2005)