Quantum Sensing for Particle Physics

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Summary

- Introduction to the features and advantages of quantum sensing
- Examples of different types of quantum sensors
- Some examples of applications of quantum sensing, both fundamental and practical
- A bit about my work on quantum sensing, developing dark matter and gravitational wave detectors based on atom interferometry
- Sharing our recent experience with applying quantum optimal control to atom interferometer experiments
	- Open-loop and closed-loop control
	- Combining automated algorithms with human intuition
- Fundamental sensitivity limits of quantum sensors
	- The standard quantum limit
	- How to circumvent the standard quantum limit using entanglement
	- The Heisenberg limit

HEP Science in Various Quantum Sensor Energy Ranges

BRN Study for HEP Detector Research and Development, 2019

Features of Quantum Sensing

- Quantum sensors takes advantage of one or more aspects of quantum mechanics
	- Quantum superposition
	- Detection of jumps between quantized energy levels
	- **Entanglement**
- In many cases, leveraging these quantum features can offer improved sensitivity in measuring quantities of interest
- Quantum sensors are often highly reproducible
	- In many cases, rely upon relatively simple systems such as ensembles of atoms or molecules
	- Identical particles: useful for making "copies" of quantum sensors based on simple systems
	- Valuable for understanding and mitigating systematic errors

Quantum Superposition

- Quantum superposition: objects can have quantum mechanical wavefunctions that span multiple quantum states, said to be in a superposition of these states. Some examples:
	- Superposition of different positions in space
	- Superposition of different internal energy levels
	- Superpositions of different polarizations (photons)

• *How might we leverage quantum superposition for sensing purposes?*

Sensing with Quantum Superpositions

Make an interference measurement between the different states in a quantum superposition

Fringes from interfering different spatial components of an atom's wavefunction

- Will reveal *phase* that has evolved between the different states
- This phase is often highly sensitive to a variety of quantities we might want to measure
- A simple and broadly applicable example: Ramsey interferometry with two-level quantum systems

The Quantum Two-Level System

• Key building block for understanding quantum sensors

- Two states with transition frequency ω_0
- Coupling matrix element Ω
- Example: two states coupled by an electric dipole interaction, with interaction Hamiltonian term $H_{\text{int}} = -\hat{\mu} \cdot \mathbf{E}$ for electric dipole operator $\hat{\mu}$ and applied time-varying electric field:

$$
\mathbf{E} = \mathbf{E_0} \cos (\phi - \omega_0 t)
$$

 ϕ corresponds to the phase of driving electric field at the location of the two-level system

$$
\Omega \equiv -\frac{1}{\hbar} \bra{2} \hat{\boldsymbol{\mu}} \cdot \mathbf{E} \ket{1}
$$

The Quantum Two-Level System $\begin{aligned}\n\begin{pmatrix}\n\omega_0 \\
\omega_0 \\
\end{pmatrix} \mathbf{E} &= \mathbf{E_0} \cos(\phi - \omega_0 t) \qquad \Omega \equiv -\frac{1}{\hbar} \langle 2 | \hat{\boldsymbol{\mu}} \cdot \mathbf{E} | 1 \rangle\n\end{aligned}$

• General form of wavefunction, where now we pull phase factors from the free evolution of the system into the definition of the states (interaction picture)

 $|\psi(t)\rangle = c_1(t) |1\rangle + c_2(t) |2\rangle$

• Substitute into Schroedinger equation to get dynamical equations of motion for coefficients

$$
\frac{dc_1(t)}{dt} = -ic_2(t)\Omega\cos(\phi - \omega_0 t)e^{-i\omega_0 t} = -ic_2(t)\frac{\Omega}{2}(e^{-i\phi} + e^{-i(2\omega_0 t - \phi)})
$$

$$
\frac{dc_2(t)}{dt} = -ic_1(t)\Omega\cos(\phi - \omega_0 t)e^{i\omega_0 t} = -ic_1(t)\frac{\Omega}{2}(e^{i\phi} + e^{i(2\omega_0 t - \phi)})
$$

• *What approximation might we be able to make to simplify these equations, assuming that the transition frequency* ω_0 *is much larger than the drive strength* Ω

The Quantum Two-Level System

$$
\begin{aligned}\n\begin{aligned}\n\left| \begin{array}{c} 2 \rangle \\
\omega_0 \end{array} \right| \mathbf{E} \end{aligned} &= \mathbf{E}_0 \cos \left(\phi - \omega_0 t \right) \quad \Omega \equiv -\frac{1}{\hbar} \langle 2 | \hat{\boldsymbol{\mu}} \cdot \mathbf{E} | 1 \rangle \quad |\psi(t) \rangle = c_1(t) \left| 1 \right\rangle + c_2(t) \left| 2 \right\rangle \\
\frac{dc_1(t)}{dt} &= -ic_2(t) \Omega \cos \left(\phi - \omega_0 t \right) e^{-i\omega_0 t} = -ic_2(t) \frac{\Omega}{2} \left(e^{-i\phi} + \frac{e^{-i(2\omega_0 t - \phi)}}{e^{-i(2\omega_0 t - \phi)}} \right) \\
\frac{dc_2(t)}{dt} &= -ic_1(t) \Omega \cos \left(\phi - \omega_0 t \right) e^{i\omega_0 t} = -ic_1(t) \frac{\Omega}{2} \left(e^{i\phi} + \frac{e^{i(2\omega_0 t - \phi)}}{e^{-i(2\omega_0 t - \phi)}} \right)\n\end{aligned}
$$

• Rotating wave approximation: ignore rapidly oscillating exponential terms, since the contributions from these will approximately average to 0:

$$
\frac{dc_1(t)}{dt} = -i\frac{\Omega}{2}e^{-i\phi}c_2(t)
$$

$$
\frac{dc_2(t)}{dt} = -i\frac{\Omega}{2}e^{i\phi}c_1(t),
$$

Solutions to simplified equations:

$$
c_1(t) = c_1(0)\cos\left(\frac{\Omega t}{2}\right) - ie^{-i\phi}c_2(0)\sin\left(\frac{\Omega t}{2}\right)
$$

$$
c_2(t) = -ie^{i\phi}c_1(0)\sin\left(\frac{\Omega t}{2}\right) + c_2(0)\cos\left(\frac{\Omega t}{2}\right)
$$

Quantum Beam Splitters and Mirrors

$$
c_1(t) = c_1(0)\cos\left(\frac{\Omega t}{2}\right) - ie^{-i\phi}c_2(0)\sin\left(\frac{\Omega t}{2}\right)
$$

$$
c_2(t) = -ie^{i\phi}c_1(0)\sin\left(\frac{\Omega t}{2}\right) + c_2(0)\cos\left(\frac{\Omega t}{2}\right)
$$

• Probability sinusoidally oscillates between the two states. If all probability initially in state 1, then:

Beam splitter or $\pi/2$ -pulse "splits" quantum state into 50/50 superposition of the two states: 1

$$
|1\rangle \longrightarrow \frac{1}{\sqrt{2}} (|1\rangle - ie^{i\phi} |2\rangle)
$$

$$
|2\rangle \longrightarrow \frac{1}{\sqrt{2}} (-ie^{-i\phi} |1\rangle + |2\rangle)
$$

Quantum Beam Splitters and Mirrors

$$
c_1(t) = c_1(0)\cos\left(\frac{\Omega t}{2}\right) - ie^{-i\phi}c_2(0)\sin\left(\frac{\Omega t}{2}\right)
$$

$$
c_2(t) = -ie^{i\phi}c_1(0)\sin\left(\frac{\Omega t}{2}\right) + c_2(0)\cos\left(\frac{\Omega t}{2}\right)
$$

• Probability sinusoidally oscillates between the two states. If all probability initially in state 1, then:

Mirror or π -pulse "reflects" quantum amplitude from one state to the other:

$$
|1\rangle \longrightarrow -ie^{i\phi} |2\rangle
$$

$$
|2\rangle \longrightarrow -ie^{-i\phi} |1\rangle
$$

Ramsey Interferometry

- Uses quantum superposition and interference to determine frequency difference between two quantum states
- Important tool for atomic clocks
- Depending on the system under consideration, this frequency difference can depend on variety of quantities we might want to measure, such as:
	- Magnetic and electric fields
	- Fundamental constants such as the fine structure constant or electron mass, which could undergo small oscillations induced by certain dark matter candidates
	- **Temperature**
	- Pressure
	- Interactions with surrounding particles

Ramsey Sequence

Steps

- 1. Initial $π/2$ -pulse to create superposition of two states
- 2. free evolution for a time T
- $-$ 3. Final $\pi/2$ -pulse to make quantum amplitudes in the two states interfere
- 4. Measure relative populations of two states, which reveals evolved phase
- Consider a driving field oscillating at frequency ω close to (but not necessarily identical to) the transition frequency ω_0 , approximate pulses as instantaneous for simplicity
- For a pulse applied at time t, imprinted phase from $\pi/2$ -pulse will be

$$
\phi(t) = -(\omega - \omega_0)t
$$

$$
|1\rangle \longrightarrow \frac{1}{\sqrt{2}} (|1\rangle - ie^{i\phi(t)}|2\rangle)
$$

$$
|2\rangle \longrightarrow \frac{1}{\sqrt{2}} (-ie^{-i\phi(t)}|1\rangle + |2\rangle)
$$

• Recall that time-dependence from free evolution factored into state definitions, and that the full phase of the drive field is

$$
\phi(t)-\omega_0 t=-\omega t
$$

Ramsey Sequence

• Initial pulse at time t=0 creates the following superposition

$$
|1\rangle \longrightarrow \frac{1}{\sqrt{2}} (|1\rangle - i |2\rangle)
$$

• Final $\pi/2$ -pulse at time t= T applies the following transformation

$$
|1\rangle \longrightarrow \frac{1}{\sqrt{2}} (|1\rangle - ie^{-i(\omega - \omega_0)T} |2\rangle)
$$

\n
$$
|2\rangle \longrightarrow \frac{1}{\sqrt{2}} (-ie^{i(\omega - \omega_0)T} |1\rangle + |2\rangle)
$$

\nFinally,
$$
\frac{1}{2} \left[\left(1 - e^{i(\omega - \omega_0)T}\right) |1\rangle - i \left(1 + e^{-i(\omega - \omega_0)T}\right) |2\rangle \right]
$$

Ramsey Sequence

• Measure populations in the two states. Probability of ending up in either state is:

$$
p_1 = \frac{1}{2} - \frac{1}{2}\cos(\Delta\phi)
$$

\n
$$
p_2 = \frac{1}{2} + \frac{1}{2}\cos(\Delta\phi)
$$

\n
$$
\Delta\phi = (\omega - \omega_0)T
$$

- For a well-known drive frequency, measurement of phase shift allows transition frequency to be determined
- Alternatively, if transition frequency is well-known (e.g., it is a stable atomic transition), can measure and apply feedback to stabilize drive frequency
	- The stable drive frequency can then serve as a stable reference for a clock
	- This is the idea behind how an atomic clock works

NIST-F1 Cs Atomic Clock

Atomic Clock Technology

- State-of-the-art atomic clocks leverage quantum coherence of states coupled by ultranarrow optical atomic transitions
- Large transition energy, narrow linewidth, and long coherence time (tens of seconds for atoms trapped in optical lattices): extremely precise comparison of fractional frequency difference between different clocks
- Better than 1 part in 10^{20} fractional frequency comparison, allows measurement of gravitational redshift across a single atom cloud
- Clock comparison measurements are a powerful tool to search for wavelike dark matter

Arvanitaki et al., PRD 91, 015015 (2015) Kennedy et al., PRL 125, 201302 (2020) Bothwell et al., Nature 602, 420 (2022) Zheng et al., Nature 602, 425 (2022)

Atomic Clock Applications

- Stable time reference to which clocks around the world can be synchronized
- GPS
- Tests of special and general relativity
	- As an example of sensitivity, special relativistic and also gravitational time dilation is important to consider for GPS
- Stabilizing lasers for use in precision spectroscopy, valuable for a variety of fundamental physics tests
- Searches for wavelike, ultralight dark matter

gps.gov

Dark Matter

We know it's there, but what is it?

Gravitational lensing that is not explained by luminous matter

Other evidence includes dynamics of galaxy interactions, structure of cosmic microwave background, etc…

Ultralight dark matter

Ultralight DM acts as a coherent, wavelike background field

Example for scalar DM field:

$$
\mathcal{L} = +\frac{1}{2}\partial_{\mu}\phi\partial^{\mu}\phi - \frac{1}{2}m_{\phi}^{2}\phi^{2} - \sqrt{4\pi G_{N}}\phi \left[d_{m_{e}}m_{e}\bar{e}e - \frac{d_{e}}{4}F_{\mu\nu}F^{\mu\nu}\right] + \dots
$$

\n
$$
\phi(t, \mathbf{x}) = \phi_{0}\cos\left[m_{\phi}(t - \mathbf{v} \cdot \mathbf{x}) + \beta\right] + \mathcal{O}\left(|\mathbf{v}|^{2}\right) \qquad \phi_{0} \propto \sqrt{\rho_{\text{DM}}} \qquad \text{DM mass}
$$

\n
$$
\phi(t, \mathbf{x}) = \phi_{0}\cos\left[m_{\phi}(t - \mathbf{v} \cdot \mathbf{x}) + \beta\right] + \mathcal{O}\left(|\mathbf{v}|^{2}\right) \qquad \phi_{0} \propto \sqrt{\rho_{\text{DM}}} \qquad \text{DM mass}
$$

DM coupling causes time-varying atomic energy levels:

Atomic Clock Searches for Ultralight Dark Matter

- Proposal by Arvanitaki et al., PRD 91, 015015 (2015)
- Measure frequency ratios of atomic clocks using different types of atoms
- Search sensitivity beyond previous limits recently demonstrated (Kennedy et al., PRL 125, 201302 (2020))

Ramsey Interferometry to Search for and Electron Electric Dipole Moment (EDM)

- Electron EDM searches
	- Provide a precise probe of time-reversal (T) symmetry
	- Sources of T-violation (corresponding to CP violation due to CPT symmetry) beyond those in the standard model predicted by many theories to explain the observed matter-antimatter asymmetry
	- Measurements have already ruled out many proposed standard model extensions
- Ramsey interferometry carried out with polar molecules, for which internal electric field and therefore sensitivity to electron EDM is greatly enhanced
- Next generation experiment under construction in Gabrielse labs at Northwestern

ACME Collaboration, Nature 562, 355 (2018)

 $|d_e|$ < 1.1 × 10⁻²⁹e cm

Searching for New Physics with Electron EDM Measurements

- Existing measurements place bounds on many theories
- Next generation measurements underway
- New techniques to further enhance sensitivity, including:
	- Quantum solids: polar molecules trapped in solid matrix for greatly increased particle number (Vutha et al., Atoms 6, 3 (2018))
	- Ultracold, laser-cooled polar molecules; potentially trapped in optical lattice (Fitch et al., Quantum Science and Technology 6, 014006 (2021))

Regan et al., PRL 88, 071805 (2002) Hudson et al., Nature 473, 7348 (2011) Cairncross et al., PRL 119, 153001 (2017) New measurement: Roussy et al., Science 381, 46 (2023)

Resonant Detection of Axion-Mediated Forces

- QCD axion: strong physics motivation
	- Strong CP problem
	- Excellent dark matter candidate
- ARIADNE: NMR with spin-polarized He-3 to search for QCD-axionmediated spin dependent forces (axion mass range $0.1 - 10$ meV)
- Rotating source mass with sprockets resonantly drives spin-precession from QCD-axion-induced interactions
	- Sprocket teeth pass by at nuclear Larmor precession frequency

Arvanitaki and Geraci, PRL 113, 161801 (2014) ARIADNE collaboration, arXiv:1710.05413v1

Nitrogen Vacancy (NV Centers)

- Nitrogen vacancy defects in diamond form a relatively simple quantum system embedded in a solid
- Can perform Ramsey interferometry with two energy levels of this system
- Applications include
	- Nanoscale magnetometry
	- Nanoscale pressure sensing
	- Nanoscale strain sensing
	- Nanoscale temperature sensing
	- Directional dark matter detection

nist.gov

Directional Dark Matter Detection with NV Centers

- Proposal by Rajendran et al., PRD 96, 035009 (2017)
- Weakly interacting massive particles (WIMPs) are one of the leading dark matter candidates
- A limiting factor in WIMP searches is backgrounds due to neutrinos from the Sun
- Directional detection could provide a way reject this background, since solar neutrinos come from a particular direction while incident WIMPs expected to have a different angular distribution
- NV centers can perform nanoscale strain measurements to determine directionality of damage tracks in detector

FIG. 1: Left: Event identified by conventional methods. Right: Section of interest separately studied by superresolution methods.

Quantum Sensor Networks of Optical Atomic Magnetometers

- GNOME: global network of optical atomic magnetometers to search for transient exotic spin couplings by
- Example: domain walls of axion-like fields
- Look at correlations between spatially separated sensors

Budker and Romalis, Nat. Phys. 3, 227 (2007) Afach et al., Physics of the Dark Universe 22, 162 (2018)

Matter Wave Interferometry

- Massive objects such as neutrons, atoms, and molecules can exist in quantum superpositions of two spatially distinct trajectories
- Recombine trajectories to make interferometer
- Sensitive to a variety of quantities of interest, for example:
	- Accelerations and rotations, useful for inertial navigation
	- Gravity: gravity mapping and fundamental tests of gravity
	- Measurements of atomic and molecular polarizabilities
	- Measurements of the fundamental constants (e.g., fine structure constant, Newton's gravitational constant)
	- Searches for ultralight dark matter
	- Gravitational wave detection in new frequency bands
	- Fundamental tests of quantum mechanics: can we create quantum superpositions arbitrarily large size, or are there fundamental limitations to this?

Atom interferometry

http://scienceblogs.com/principles/2013/10/22/quantum-erasure/ http://www.cobolt.se/interferometry.html

Atom optics using light

(1) Light absorption (transition from ground to excited state):

(2) Stimulated emission (transition from excited to ground state):

$$
\lim_{\hbar k} \sum \left(\frac{v}{\hbar k} \right)^{v} = \hbar k/m
$$

Atom optics using light

(1) Light absorption:

Rabi oscillations

Our Work

- Large scale atom interferometry
- Foundational Quantum Science
- Dark Matter Detectors
- Gravitational Wave Detectors
- Enhancing Atom Interferometers with Robust Quantum Control

Foundational Quantum Science

Can quantum superposition extend to distance and time scales of everyday life, many meters and tens of seconds?

Various models that limit the size/duration/mass of superposition have been proposed

Can test by implementing such superpositions in an atom interferometer

> Arndt and Hornberger, Nature Physics 2014 Bassi et al., RMP 2013 Nimmrichter and Hornberger, PRL 2013 Altimirano et al., Classical and Quantum Gravity, 2018 Bassi et al., Classical and Quantum Gravity, 2017

Dark Matter

We know it's there, but what is it?

Galactic rotation curves not consistent with luminous matter only

Gravitational lensing that is not explained by luminous matter

Other evidence includes dynamics of galaxy interactions, structure of cosmic microwave background, etc…

One well-motivated candidate: wavelike dark matter, coherently oscillating field Can lead to small oscillations in atomic energy levels

Gravitational Wave Detection

$$
ds^{2} = dt^{2} - (1 + h\sin(\omega(t - z)))dx^{2} - (1 - h\sin(\omega(t - z)))dy^{2} - dz^{2}
$$

strain

frequency

Gravitational waves science:

New carrier for astronomy: Generated by moving mass instead of electric charge *Probing the early universe*: Can see to the earliest times in the universe, study corresponding high energy scales *Tests of gravity*: Extreme systems (e.g., black hole binaries) test general relativity

Laser Interferometer Detectors

Gound-based detectors: e.g.: LIGO, VIRGO, GEO, proposed ET (> ~3 Hz)

Space-based detector: planned LISA mission $(1 \text{ mHz} - 100 \text{ mHz})$, also proposals to extend LISA concept to higher frequencies (e.g., DECIGO)

Gravitational wave frequency bands

There is a gap between the LIGO and LISA detectors (\sim 0.3 Hz – 3 Hz).

Mid-band Science

Mid-band discovery potential

-Historically every new band/modality has led to discovery -Observe LIGO sources when they are younger

Cosmological signals that give insight into high energy physics

-operating in mid-band instead of lower frequencies advantageous for avoiding white dwarf confusion noise

Optimal for sky localization (Graham and Jung, PRD 2018)

Predict *when* and *where* events will occur (before they reach LIGO band) Observe run-up to coalescence using electromagnetic telescopes

Astrophysical Sources

White dwarf binaries (Type IA supernovae), black hole binaries, intermediate mass black holes, and neutron star binaries

MAGIS-100 detector at Fermilab

Abe et al., Quantum Science and Technology 6, 044003 (2021)

MAGIS-100 Experiment

- 17 modules, each with magnetic shielding, vacuum pipe, current-carrying wires for generating bias magnetic field
- Laser lab at top of shaft (currently undergoing construction)
- Three Sr atom sources over 100 m baseline, local optical lattices can launch atoms from each source
- High-power laser system with agile frequency control, spatially filtered beam mode, and precisely controlled pointing via tip-tilt mirrors
- Construction and testing of subsystems underway

Global Efforts in Long-Baseline Atom Interferometry

- Growing global community in this area

- Example: CERN Very-Long-Baseline Terrestrial Atom Interferometry Workshop, March 13-14 (2023): ~200 registered participants.<https://indico.cern.ch/event/1208783/>

MAGIS-100 in US (Abe et al., QST 6, 044003 (2021))

AION in UK (Badurina et al., J. of Cosmology and Astrophysics 05 (2020) 011)

MIGA in France (Canuel et al., Scientific Reports 8, 14064 (2018))

ZAIGA in China (Zhan et al., International Journal of Modern Physics D 29, 1940005 (2020))

VLBAI in Germany (Hartwig et al., New Journal of Physics 17, 035011 (2015))

- Community is also studying prospects for future space-based detectors (Abou El-Neaj et al., EPJ Quantum Technology 7, 6 (2020)); synergies with related proposals using optical lattice clocks in space (Kolkowitz et al., PRD 94, 124043 (2016))

Measurement Concept

Essential Features

- 1. Light propagates across the baseline at a constant speed
- 2. Atoms are good clocks and good inertial proof masses (freely falling in vacuum, not mechanically connected to Earth).
- 3. Clocks read transit time signal over baseline
- 4. GW changes number of clock ticks associated with transit by modifying light travel time across baseline, DM changes number of clock ticks by modulating clock frequency (i.e., atomic transition frequency)
- 5. Many pulses sent across baseline (large momentum transfer) to coherently enhance signal

Yu and Tinto, GRG 2011; Graham et al., PRL 2013; Arvanitaki et al., PRD 2018

Clock Atom Interferometry

Perform atom interferometry using narrow optical clock transition of Sr with a long-lived excited state (natural lifetime >100 s)

Can have long lived superpositions of ground + excited state with a large energy difference, useful for very precise timing measurements

Difference from atomic clock? -atoms not trapped, center-of-mass motion from photon momentum kicks

Hu et al., PRL 2017; Rudolph et al., PRL 2020

Robust Quantum Control

Want to apply many laser pulses to enhance atom interferometer sensitivity

Pulse fidelity is never perfect due to experimental nonidealities and tradeoffs

Robust quantum control: design sequences of pulses that have increased robustness against experimental errors

Sources of Errors

- Detuning errors
	- Doppler shifts from velocity spread of atom cloud
	- Laser frequency instability

- Amplitude/Rabi frequency errors
	- Laser intensity noise
	- Inhomogeneities in laser beam
	- Atom cloud expansion

$$
\beta = -0.2
$$

 $\beta = 0$

Closed-loop vs. Open-loop Optimization

Open-loop control: simulation-based optimization -Advantage: does not require experimental run time

-Disadvantage: may not capture all aspects of experiment if model is not perfect

Closed-loop control: optimize based on real-time experimental data

-Challenge: experimental noise can make optimization more difficult

Resonant Atom Interferometry

Multiple interferometer loops (N loops) can amplify signal by a factor of 2N at the resonance frequency

- Analogy to lock-in detection and dynamical decoupling
- Comes at the cost of decreased bandwidth

Implement using MAGIS laser system with Sr-88 atom interferometer at Northwestern, using singlephoton transitions on 689 nm line

Experimental Challenges

- Experimental tradeoffs and imperfections limit mirror pulse efficiency
- Desire for higher laser intensity to mitigate various errors
- Reducing beam size increases intensity but also increases intensity inhomogeneity across atom cloud
- How small one can make atom cloud faces limitations
	- Cloud expansion over long measurement times
	- Lensing sequences trade larger cloud size for smaller velocity spread
- Experiments on resonant atom interferometry with intensity inhomogeneity similar to what expected for long-baseline atom interferometer
- 90% pulse transfer efficiency
- Interferometer signal rapidly decays after only 10 loops

Overcoming the Challenge: Applying Robust Quantum Control to Multipath Interference

– Due to mirror pulse inefficiencies, atoms branch off into many "stray paths"

– Multipath interference often purposely avoided in atom interferometry since it can substantially complicate system dynamics

Leverage multipath interference to generate desired signal in resonant atom interferometer despite essentially all of atoms being lost to stray paths (i.e., probability of atom being kicked by every mirror pulse to stay on "ideal paths" is very small)

Overcoming the Challenge: Applying Robust Quantum Control to Multipath Interference

– Quantum control: use phases of various mirror pulses as control variables (a, b, c, d, ….)

a | b | c | d | e | f | g | h …

- Open-loop approach: Choose pulse phases to minimize a specified cost function based on simulations of experiment
- Closed-loop approach: use input from experimental data

Strategy for Optimizing Pulse Phases

- Repeat groups of pulses that maximize probability that stray paths constructively recombine into "central family" of trajectories that overlap with ideal paths
- Destructive interference suppresses probability of atoms spreading out away from ideal paths
	- Assign cost function associated with this spreading

Simplest sequence: all phases 0

Interference Fringes

Contrast and phase determined from fitting interference fringe of a phase scan over multiple shots

Thousandfold Resonant Phase Amplification

As a signal, add alternating laser phase to pulse sequence δφ

N=504 loops, interferometer phase ϴ scales as 2Nδφ

Despite essentially all atoms being in stray paths, contrast remains within order 1 factor of fundamental spontaneous emission contrast limit up to ~500 loops

3

 $\overline{2}$

0

 -2

 -3

 -3

 -2

 θ [rad]

Phase Resolution

Sensitivity demonstration: measured digitization floor of our DDS (direct digital synthesis) frequency synthesizer—DDS phase is imprinted on optical phase via acousto-optic modulator

Phase is digitized into 96 µrad steps (corresponding to 16 bits)

Closed-loop Control Workflow

Fully automated process

Closed Loop Control: Basic Gradient Ascent

Ť

$$
g_{n,i} = \frac{c'_{n,i} - c_{n,i}}{\epsilon_n}
$$

$$
\Theta_{n,i+1} = \Theta_{n,i} + \gamma g_n,
$$

- $n -$ denotes the nth parameter (for us this is $n = X, Y$)
- i iteration of the optimization process
- $g_{n,i}$ component of the gradient in the nth parameter at the ith interation
- $c_{n,i}$ cost value of the nth parameter at the ith interation
- $c'_{n,i}$ cost value of the dithered nth parameter at the ith interation
- ϵ_n dither range of nth parameter
- $\Theta_{n,i}$ value of nth parameter at the ith iteration
- γ learning rate of the optimizer

Gradient ascent + momentum

Direction of Motion Friction Friction

Averages noise without redundant measurements

$$
g_{n,i} = \frac{c_{n,i} - c_{n,i}}{\epsilon_n}
$$

\n
$$
\Theta_{n,i+1} = \Theta_{n,i} + p_{n,i} + \gamma g_{n,i}
$$

\n
$$
p_{n,i+1} = \gamma_p (p_{n,i} + \gamma g_{n,i})
$$

 $p_{n,i}$ – momentum in the nth parameter at the ith interation γ_p - decay of the momentum ranging from 0 to 1 (For us its 0.4)

Gradient ascent + momentum + step limit

Prevents the system from getting thrown off when measuring large gradients near a maximum

$$
g_{n,i} = \frac{c'_{n,i} - c_{n,i}}{\epsilon_n}
$$

\n
$$
g'_{n,i} = \frac{g_{n,i}}{\sqrt{\left(\frac{\gamma * g_{n,i}}{\epsilon_n}\right)^2 + 1}}
$$

\n
$$
\Theta_{n,i+1} = \Theta_{n,i} + p_{n,i} + \gamma g'_{n,i}
$$

\n
$$
p_{n,i+1} = \gamma_p (p_{n,i} + \gamma g'_{n,i})
$$

 $g'_{n,i}$ – attenuated component of the gradient in the nth parameter at the ith interation

Closed-loop Control Example

Run at 256 loops

Impose constraints based on patterns that had been observed to work well: Parameter space XY represents pulses of symmetry that follows $XY(Y+\pi)(X+\pi)+$ mirror

Use simulations to guide initial guesses

Fundamental Sensitivity Limits for Quantum Sensors

- Related to fundamental uncertainty relations that arise in quantum mechanics
- Standard quantum limit applies to phase estimation in the absence of entanglement
- Can surpass the standard quantum limit using entanglement, however still reach Heisenberg limit
- Same broad considerations relevant for two-level systems, electromagnetic fields, quantum noise in amplifiers, etc… (further applications, e.g. in axion searches)

The Standard Quantum Limit

- For an interferometer, we would like to estimate the phase evolved between the two branches of the wave function based on the probability of detecting a particle in one state vs. the other after the final beam splitter
- Say that we perform this measurement for N uncorrelated particles, each with the same evolved phase ϕ
- Probability of finding a given particle in output state 1

$$
p_1 = \frac{1}{2} - \frac{1}{2}\cos{(\phi)}
$$

• Mean number of particles found in state 1

$$
\bar{N}_1 = Np_1
$$

- *Which statistical distribution should we use to describe the probabilities of finding a given number of particles in state 1?*
- *How would we expect the uncertainty in the evolved phase to scale with N?*

The Standard Quantum Limit

- The *binomial distribution* is appropriate here
- Based on the properties of this distribution, the standard deviation in the number of particles found in state 1 is

$$
\delta N_1 = \sqrt{N p_1 (1 - p_1)} = \frac{\sqrt{N}}{2} |\sin (\phi)|
$$

• Applying uncertainty propagation to the equation $\bar{N}_1 = Np_1$, we obtain the additional relation

$$
\delta N_1 = \frac{N}{2} \left| \sin \left(\phi \right) \right| \delta \phi
$$

• The uncertainty in the estimated phase is thus

$$
\delta\phi=1/\sqrt{N}
$$

Number-Phase Uncertainty Relation

- It turns out that there is a general relationship between the uncertainty in the number of excitations of a quantum system and the uncertainty in the phase of the quantum state
- Consider that each excitation corresponds to energy $\hbar\omega$
	- Could corresponds to photons each of energy $\hbar\omega$
	- Could correspond to how many atoms in a two-level system are in the excited state, with transition energy $\hbar\omega$
- Back-of-the-envelope derivation from energy-time uncertainty principle

$$
\delta E \delta t \gtrsim \hbar \qquad \qquad \delta E = \delta N \hbar \omega \qquad \delta t = \delta \phi / \omega
$$

$$
\delta N \delta \phi \gtrsim 1
$$

• Can also be derived more formally

Number-Phase Uncertainty Relation

- For typical uncorrelated systems, such as nominally classical light sources or collections of uncorrelated two-level systems, the state of the system is well described as a *coherent state*
- Coherent states reach the lower bound of the number-phase uncertainty relation, with the following balance between the uncertainty in the number of excitations and the uncertainty in the phase

$$
\delta N \sim \sqrt{N} \qquad \qquad \delta \phi \sim 1/\sqrt{N}
$$

- Can trade off greater uncertainty in number for reduced uncertainty in phase by employing entanglement, often referred to as *squeezed states*
- For N particles, maximum possible uncertainty in number of excitations is N
- Implies the *Heisenberg limit* for phase resolution:

$$
\delta\phi\sim 1/N
$$

Squeezing to Reduce Quantum Noise

Quantum projection noise limit for interferometric measurements: for N uncorrelated particles (atoms, photons,…), phase resolution limited to

$$
\delta\phi=1/\sqrt{N}
$$

Can entangle particles to induce correlations that allow better phase resolution, fundamentally limited by the Heisenberg limit (potential for dramatic improvement)

 $\delta \phi \sim 1/N$

Example of spin squeezing: reduce uncertainty in (i.e., squeeze) one spin component of N-particle ensemble, corresponding increase in uncertainty in another spin component to maintain uncertainty relations

-squeeze the component that is measured to infer interferometer phase

Hosten et al., Nature 529, 505 (2016) Cox et al., PRL 116, 093602 (2016)

Squeezing Example

- Entanglement between atoms used for a Ramsey sequence generated by using the interaction between atoms and light in an optical cavity
- Phase uncertainty, and resulting population uncertainty in interferometer output ports, reduced by a factor of ~10 beyond standard quantum limit
- Another example: squeezing used to improved sensitivity of LIGO (Aasi et al., Nature Photonics 7, 613 (2013))