

GRAPH NEURAL NETWORKS

COREY ADAMS

Computational Scientist Physicist



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12/10/23 San Juan, PR

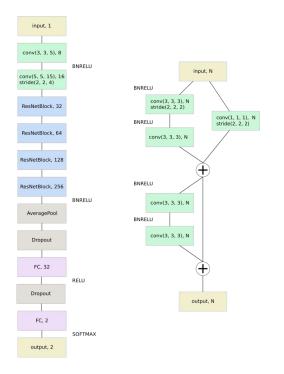
RECAP

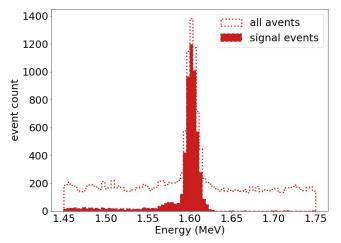
- Last time, we covered:
 - History of CNNs
 - CNN Ingredients
 - CNN use cases, and examples of these cases in physics
 - High level thoughts about how to apply a CNN to your dataset.
- This time:
 - Briefly, how to train a CNN on simulation that doesn't quite match data.
 - Then, all about GNNs.





USING AI TO SORT CLASSIFY SIGNAL/BACKGROUND





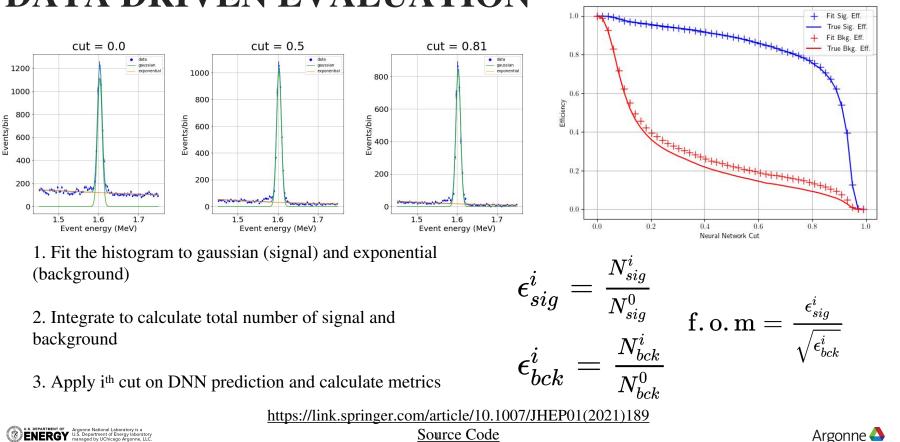
- Input size 40X40x110 (voxel size 10x10x5 mm³)
- Energy of every event normalized to 1 (so the network does not have information about total event energy)
- ~500000 fiducial events, 35% signal
- Network is sparse 3D convolutions

https://link.springer.com/article/10.1007/JHEP01(2021)189



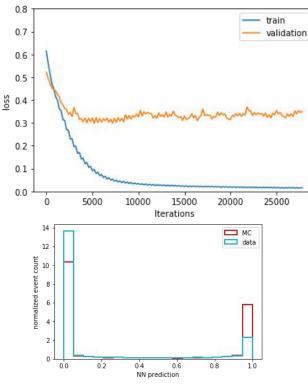
Source Code

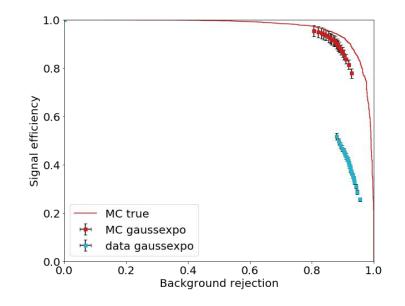




DATA DRIVEN EVALUATION

DATA VS MC





Prediction on data is biased towards lower values

Significant disagreement and performance hit on data

https://link.springer.com/article/10.1007/JHEP01(2021)189

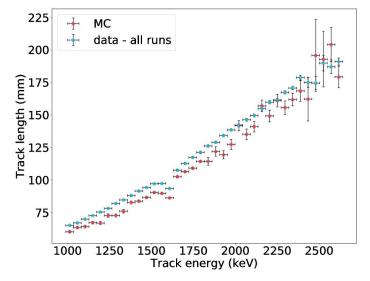


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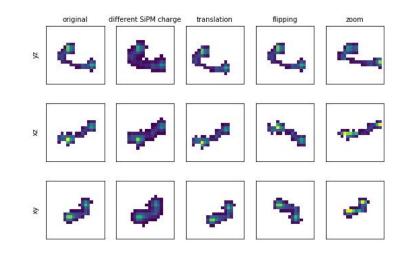


DATA AUGMENTATION TO PREVENT OVERFITTING

From classical analysis we know there are some MC/data differences, e.g. track length, blob energy...



We apply on-the-fly augmentation to the training data to prevent the network from using these features during training.



https://link.springer.com/article/10.1007/JHEP01(2021)189



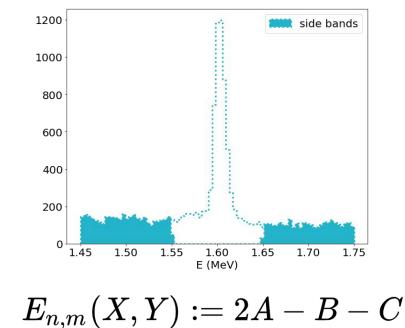
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HOW TO QUANTIFY THE OVERFITTING?

- Can we know if it will work **before** we look at the interesting data? Can we know if augmentation is improving things?
- Yes we can use <u>energy</u> <u>distance</u> to compare the flattened layers in the sidebands around the interesting data!
- More about energy distance

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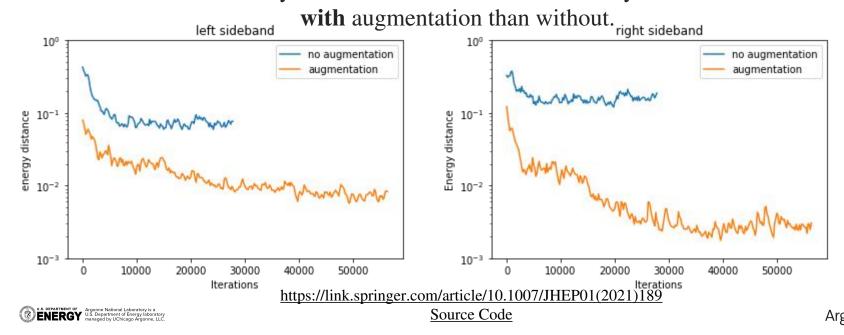
 $A := rac{1}{nm}\sum_{i=1}^n \sum_{i=1}^m \|x_i - y_j\|, B := rac{1}{n^2}\sum_{i=1}^n \sum_{i=1}^n \|x_i - x_j\|, C := rac{1}{m^2}\sum_{i=1}^m \sum_{i=1}^m \|y_i - y_j\|$

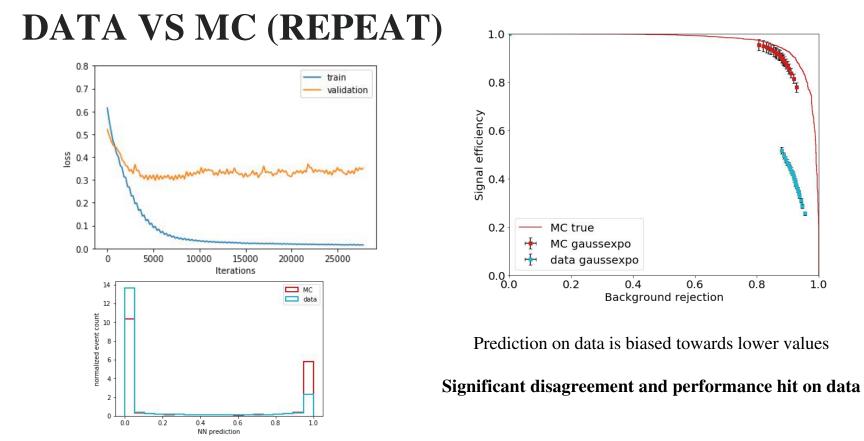
https://link.springer.com/article/10.1007/JHEP01(2021)189 Source Code



TRACK ENERGY DISTANCE DURING TRAINING

The energy distance is significantly lower with augmentation than without – this implies that the features learned by the network are much more closely related between data/MC





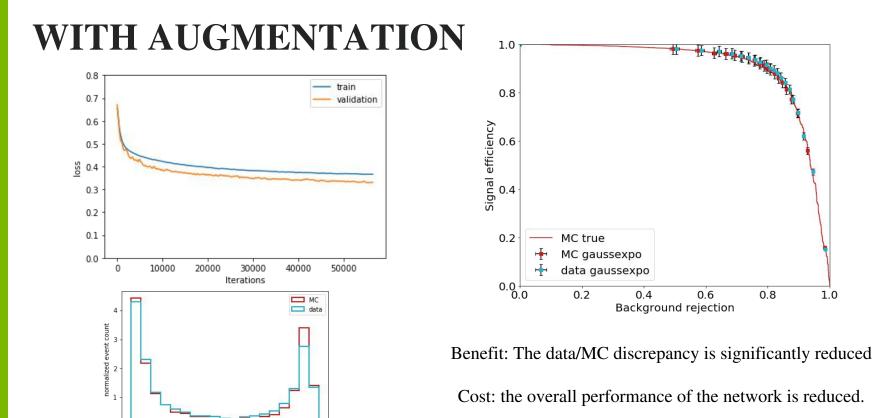
https://link.springer.com/article/10.1007/JHEP01(2021)189



Source Code



1.0



Argonne

0 - 0.0

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0.2

0.4

NN prediction

0.6

0.8

10

TAKEAWAYS

- When your training data does not perfectly match the target data:
 - Understand the discrepancies dead pixels? Different sensor responses?
 - Understand the symmetries: is there a "direction" to events? Does dropping pixels from the training data change the labels?
 - If you can remake the training data to match the target data great!
 - If you can't, use augmentation on the fly.
 - Already a well known technique in Comp Sci.
 - Focus on label preserving augmentation techniques.
 - Use sidebands and statistical measurements (energy distance not unique) to quantify discrepancies and know if you are improving things!





GRAPH NEURAL NETWORKS

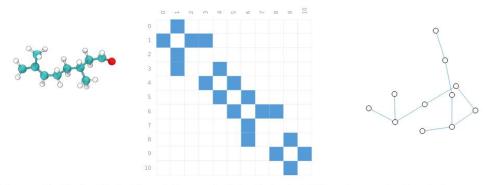


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WHAT MAKES GRAPHS DIFFERENT?

- Graphs are a collection of:
 - Nodes Individual locations that represent some piece of local information
 - Edges Connections between two nodes
 - Globals information that applies to the entire graph.



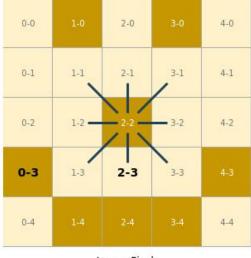
(Left) 3d representation of the Citronellal molecule (Center) Adjacency matrix of the bonds in the molecule (Right) Graph representation of the molecule.

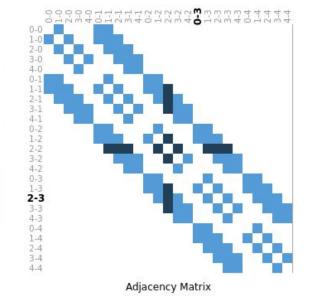


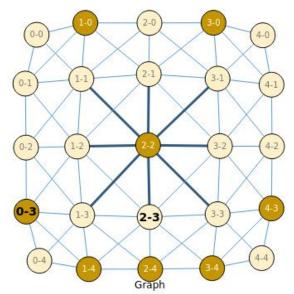
https://distill.pub/2021/gnn-intro/



IMAGES ARE GRAPHS!









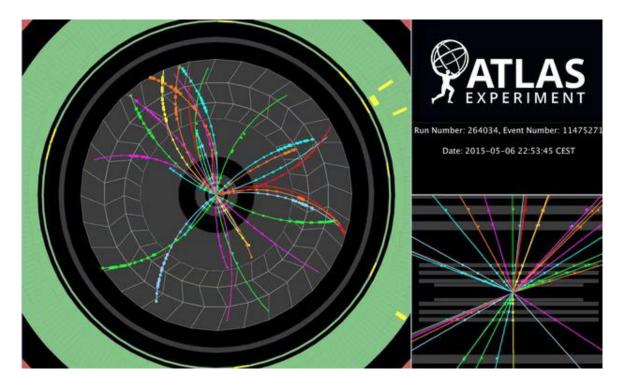




GRAPHS ARE MORE GENERAL THAN IMAGES

Generalize images by relaxing requirements:

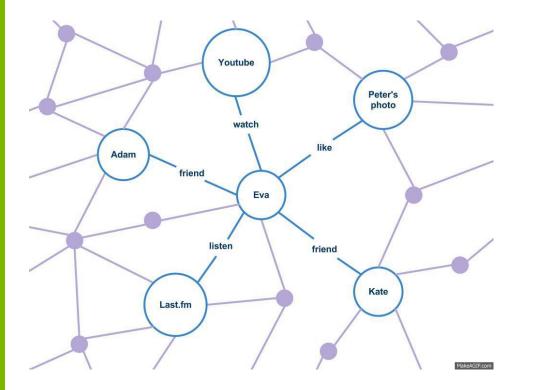
- non-uniform pixel spacing
- non-uniform connections
- can be **directed**
- computationally, must be permutation invariant







GRAPHS ARE MORE VARIED



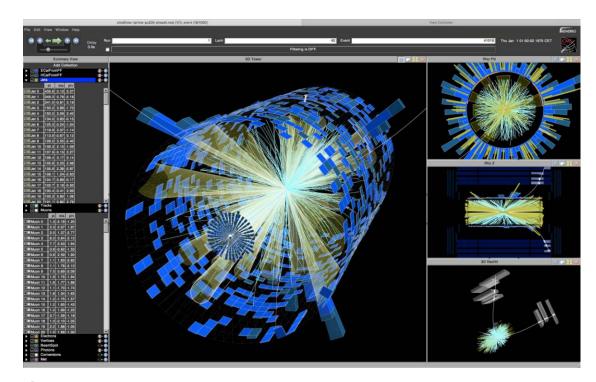
Connections in a social network are graph data.

The "type" of connections (friend / watch / like) is an example of an edge feature.





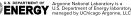
PHYSICS GRAPHS ARE MORE IMAGE-Y



We often use graphs when images don't make sense:

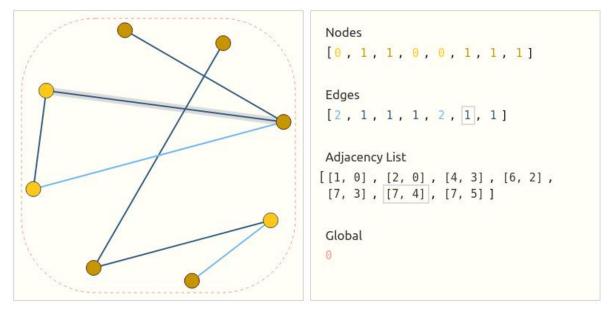
- How to embed circular/cylindrical data into an image?
- What if pixels are a differnt size?
- What about GEANT output, directed particle interactions at specified locations?

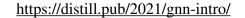




GRAPH MATH

- Graphs can be large, and often **sparse**.
 - Nodes can be stored in arrays of [n_nodes, n_node_features]
 - Edges can be stored in arrays of [n_edges, n_edge_features]
 - The adjacency matrix is a sparse connectivity between nodes

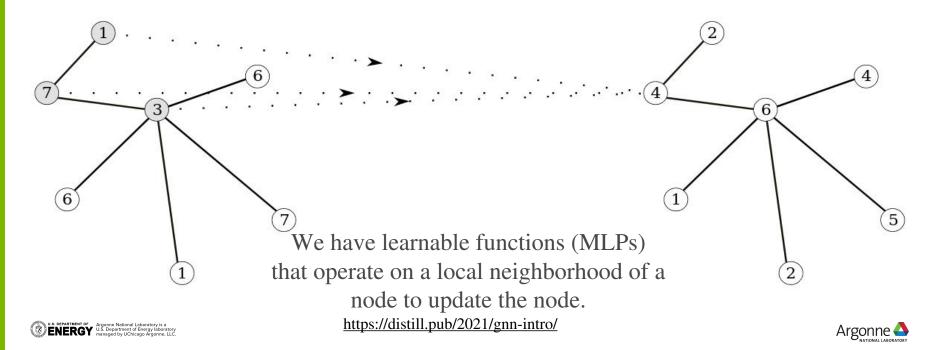






GRAPH OPERATIONS

If Graphs are generalizations of images, what's the generalization of convolutions?



GRAPH OPERATIONS

Formally, if $h_k^{(i-1)}$ represents the node k at layer i-1:

 $a_k^{i} = AGGREGATE(h_u^{(i-1)}, u \text{ connected to } k)$ $h_k^{i} = COMBINE(h_k^{(i-1)}, a_k^{i})$

Most graph convolutions can be written like this (assuming all edges have no features, like images)





GRAPH OPERATIONS

After multiple layers of graph AGGREGATE and COMBINE, you can READOUT your graph information for the prediction of the network.

Possible graph-level readout: POOL over the nodes

(aka, sum(nodes, axis=0) where nodes has shape [n_nodes, n_features])

AGGREGATE, COMBINE, READOUT all often have learnable parameters, typically as MLPs. READOUT is almost certainly required to be **permutation-invariant.**

How Powerful are Graph Neural Networks?





WHAT ABOUT THE EDGES?

For nodes h and edges e:

$$m_v^{t+1} = \sum_{w \in N(v)} M_t(h_v^t, h_w^t, e_{vw})$$
$$h_v^{t+1} = U_t(h_v^t, m_v^{t+1})$$

Where M, and U are learnable "message" and "update" functions. (Upon close inspection, the previous operations are variants of Message Passing!)

Neural Message Passing for Quantum



<u>Chemistry</u>



BUILDING YOUR GNN

- Most parameters of a GNN come from the learnable parameters of aggregation, message creation, etc.
- Normalization of your data is still valuable!
- Pooling of information **between nodes**, with or without messages, is critical for the network.
- Pooling of the **graph**, however, is not a well defined operation in many cases!





GRAPH POOLING

Graph Pooling is an ill-defined, ambiguous task.

	No-pool	DiffPool	MinCut	NMF	LaPool	ТорК	SAGPool	NDP	Graclus
Colors-3	$40.8_{\pm 2.1}$	55.2 ±1.5	60.1 ±4.0	29.7 ±1.7	44.9 ± 1.0	26.9 ±4.0	34.4 ±5.2	25.4 ±1.8	29.5 ±2.0
Triangles	93.5 ± 0.7	91.3 ± 0.2	95.3 ±0.5	58.1 ± 5.2	88.8 ± 0.8	75.2 ± 17.3	$80.3{\scriptstyle~\pm 8.6}$	75.3 ± 1.0	71.4 ±1.7
Proteins	68.8 ± 2.8	70.0 ± 0.6	73.8 ±0.8	68.9 ± 3.4	72.9 ±2.0	71.3 ± 0.8	73.7 ±0.8	68.4 ± 3.4	72.6 ±1.1
Enzymes	83.6±2.0	72.4 ±3.9	83.6 ±0.6	32.4 ± 8.1	85.0 ±1.2	81.0 ± 0.4	$68.8{\scriptstyle~\pm18.8}$	84.8 ±3.2	85.4 ±4.1
DD	81.1 ± 0.4	75.6 ±1.8	82.5 ±0.9	OOR	OOR	80.4 ± 0.9	79.0 ± 2.7	79.6 ±1.2	78.3 ±29
Mutagen.	78.0±1.6	76.2 ± 1.4	73.9 ±1.6	70.3 ± 1.6	75.3 ± 0.1	75.8 ± 1.4	76.9 ± 1.4	76.9 ± 1.0	74.2 ±0.5
ModelNet	$81.0{\scriptstyle \pm 0.5}$	$70.4{\scriptstyle~\pm2.4}$	$75.9{\scriptstyle~\pm1.2}$	OOR	OOR	$74.1{\scriptstyle~\pm3.0}$	$71.9{\scriptstyle~\pm 2.6}$	$77.1{\scriptstyle~\pm 2.6}$	83.9 ± 1.9
Rank		4.43	2.57	7.14	4.29	4.71	3.86	4.29	4.29

Table 6: Accuracy on the graph classification benchmarks.

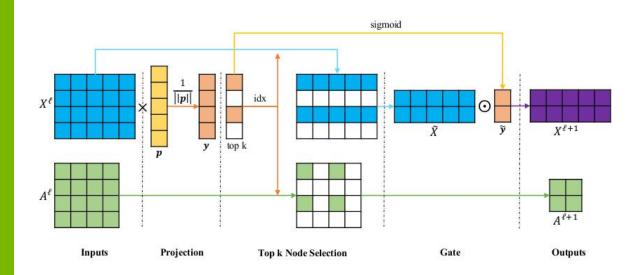
Recent results call into question if it is worth pooling at all...

https://arxiv.org/pdf/2110.05292.pdf





GRAPH U NETS



- Downsampling learns a trainable projection layer, per downsample, to select nodes.
- Also used in a sigmoid to gate information flow (and make it differentiable!)

https://arxiv.org/abs/1905.05178





GRAPH U NETS

Table 4. Results of inductive learning experiments in terms of graph classification accuracies on D&D, PROTEINS, and COLLAB datasets. g-U-Nets denotes our proposed graph U-Nets model.

Models	D&D	PROTEINS	COLLAB
PSCN (Niepert et al., 2016)	76.27%	75.00%	72.60%
DGCNN (Zhang et al., 2018)	79.37%	76.26%	73.76%
DiffPool-DET (Ying et al., 2018)	75.47%	75.62%	82.13%
DiffPool-NOLP (Ying et al., 2018)	79.98%	76.22%	75.58%
DiffPool (Ying et al., 2018)	80.64%	76.25%	75.48%
g-U-Nets (Ours)	82.43%	77.68%	77.56%

Results improve upon contemporary models but not in all cases....

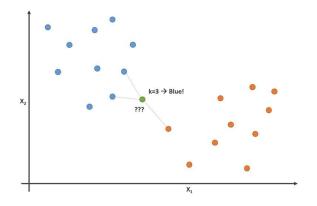
https://arxiv.org/abs/1905.05178





CONNECTING YOUR GNN

- Usually, defining nodes is easy in graphs, especially physics graphs.
 - Typically have some "position" (x/y/z or similar) and some "feature".
 - You can often just concatenated these together and have node features.
 - Much like MLPs and dense neural networks, feature normalization can be useful.
- Connecting nodes and defining edges:
 - Sometimes easy (particle flow data)
 - Sometimes ambiguous.
 - When in doubt, try k-NN algorithm
 - Usually, don't connect all-to-all!
- Defining Edge features?
 - If nothing else, displacement vector of features

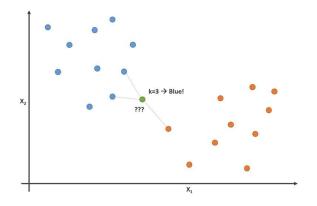






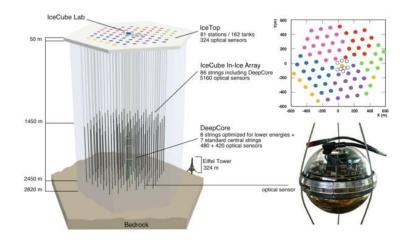
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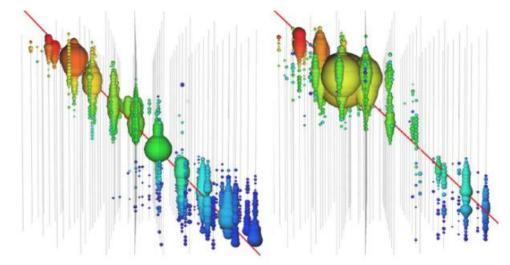








Icecube, at the south pole, has regular but not rectangular sensor layouts - CNNs are not a great fit. GNNs instead are an improvement.



https://arxiv.org/pdf/1809.06166.pdf

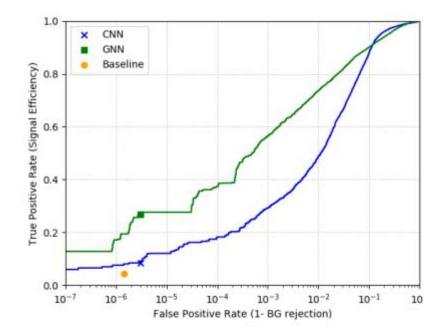




$$\operatorname{GConv}(\mathbf{X}^{(t)}) = [\mathbf{A}\mathbf{X}^{(t)}, \ \mathbf{X}^{(t)}](\mathbf{a}^{(t)})^\top + b^{(t)}\mathbf{1},$$

Both GNNs and CNNs (on transformed data) are tested, using a Graph Convolution operator. GNNs significantly outperform CNNs.

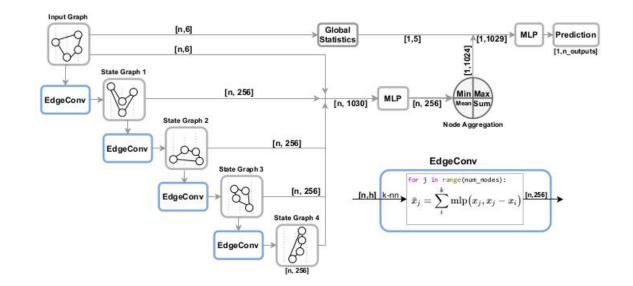
	# eve		
Method	Signal	Background	Signal:Noise
Physics Baseline	0.922	0.934	0.987
3D CNN	1.815	1.937	0.937
GNN	5.772	1.937	2.980



https://arxiv.org/pdf/1809.06166.pdf







$$\tilde{x}_j = \sum_{i=1}^{N_{\text{neighbors}}} \text{MLP}(x_j, x_j - x_i),$$

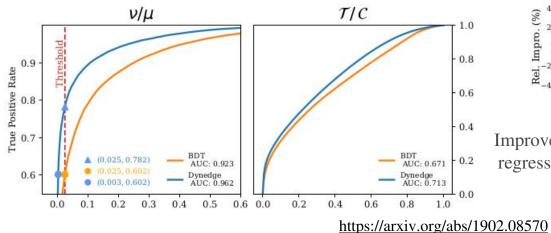
2022 Update from Icecube extends GNN usecase to lower energy events.

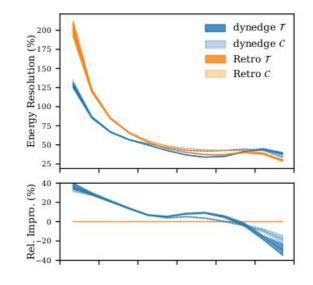
https://arxiv.org/abs/2209.03042





Targets	Description	Residual Definition
ν/μ	Classification of neutrino vs. muon events	
E	Deposited energy of neutrino interaction	$R_E = \log_{10}(E_{\text{reco}}) - \log_{10}(E_{\text{true}})$
θ, ϕ	Zenith and azimuth angles of neutrino	$R_{\text{angle}} = \text{angle}_{\text{reco}} - \text{angle}_{\text{true}}$
r	Direction vector of neutrino	$R_{\vec{r}} = \arccos \frac{\vec{r}_{reco} \cdot \vec{r}_{true}}{ \vec{r}_{reco} \vec{r}_{true} }$
V _{xyz}	Vertex position of neutrino interaction	$R_{V_{\rm xyz}} = \vec{P}_{\rm reco} - \vec{P}_{\rm true} $
\mathcal{T}/C	Classification into tracks and cascades	<u>ज</u> ार्थ



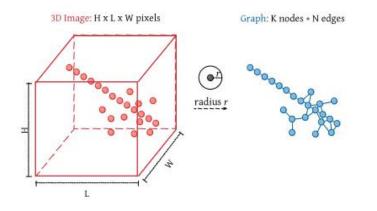


Improvement in classification metrics as well as in regression variables (energy, angles) compared to traditional reconstruction.

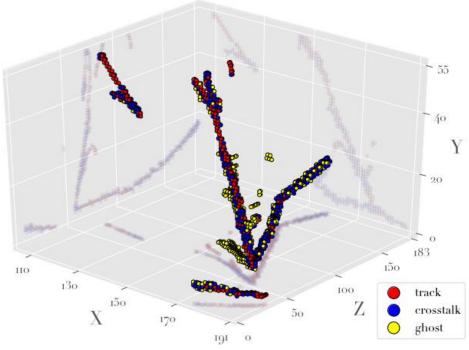
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EXAMPLE: SEGMENTATION WITH GNNS



Challenge: 2D scintillator data projected to 3D images can produce fake 3D hits from <u>crosstalk</u> or coincidence hits (<u>ghost</u>) - can a GNN label everything correctly looking at all 3D hits?

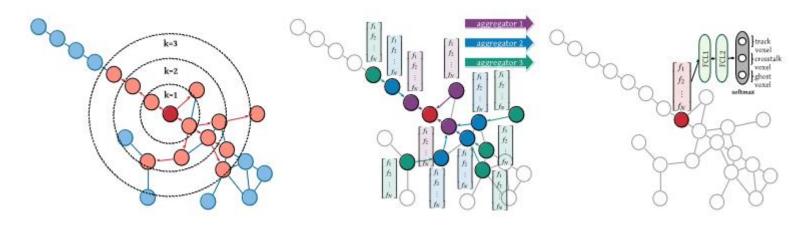




<u>Graph neural network for 3D classification of ambiguities and optical</u> crosstalk in scintillator-based neutrino detectors.



EXAMPLE: SEGMENTATION WITH GNNS



3 layers of GraphSAGE + aggregation + fully connected + per-node segmentation

G	NN		Chai	ge Cut		
	Track	Other		Track	Other	
Efficiency	94%	96%	Efficiency	93%	80%	
Purity	96%	95%	Purity	80%	91%	

TABLE IV: Mean efficiencies and purities of voxel classification for the GNN and a simple charge cut.

Graph neural network for 3D classification of ambiguities and optical



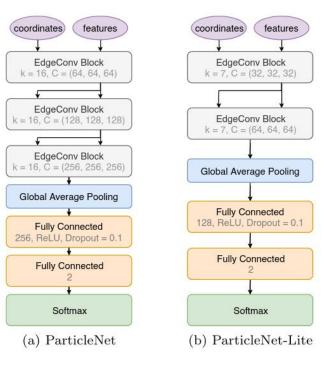
crosstalk in scintillator-based neutrino detectors.



coordinates features k-NN k-NN indices Linear BatchNorm ReLU Linear BatchNorm ReLU Linear BatchNorm ReLU Aggregation æ ReLU

Challenge: Jet images are sparse and CNNs are highly inefficienct - can GNNs improve over CNNs?

Input features include significant derived physics information.





Jet Tagging via Particle Clouds



In tests, ParticleNet
outperforms other
methods in accuracy
metrics.

	Parameters	Time (CPU) [ms]	Time (GPU) [ms]
ResNeXt-50	1.46M	7.4	0.22
P-CNN	348k	1.6	0.020
PFN	82k	0.8	0.018
ParticleNet-Lite	26k	2.4	0.084
ParticleNet	366k	23	0.92

6. 6.	Accuracy	AUC	$1/\varepsilon_b$ at $\varepsilon_s = 50\%$	$1/\varepsilon_b$ at $\varepsilon_s = 30\%$
ResNeXt-50	0.936	0.9837	302 ± 5	1147 ± 58
P-CNN	0.930	0.9803	201 ± 4	759 ± 24
PFN		0.9819	247 ± 3	888 ± 17
ParticleNet-Lite	0.937	0.9844	325 ± 5	1262 ± 49
ParticleNet	0.940	0.9858	397 ± 7	$\bf 1615 \pm 93$



Jet Tagging via Particle Clouds



But, it is hard to compete against vendor optimized convolutional kernels for performance

	Parameters	Time (CPU) [ms]	Time (GPU) [ms]
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Jet Tagging via Particle Clouds



EXAMPLE: THEORY WITH GNNS

- Variational Monte Carlo is a numerical technique for solutions to the Schrodinger Equation:
 - Requires an "ansatz" aka a trial wavefunction that can be optimized.
 - The wavefunction must obey physial principles (twice differentiable, continous, antisymmetric under exchange of Fermions)
 - The wavefunction must be sufficiently general to capture all the physics of the system.

$$\frac{\langle \Psi_V | H | \Psi_V \rangle}{\langle \Psi_V | \Psi_V \rangle} = E_V \ge E_0$$

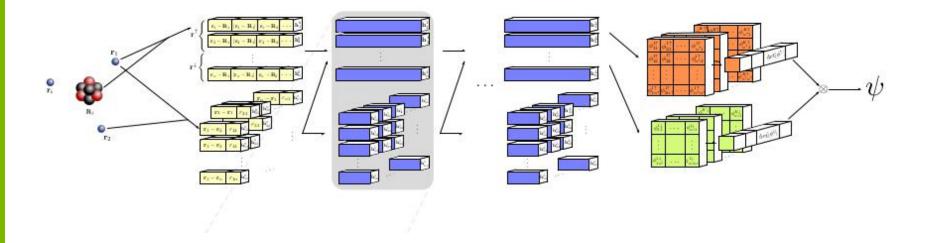
Graph Neural Networks are an ideal candidate for building an ansatz.



https://github.com/google-deepmind/ferminet



EXAMPLE: FERMINET



Ferminet solves molecular physics by encoding electron locations in a dynamic graph.

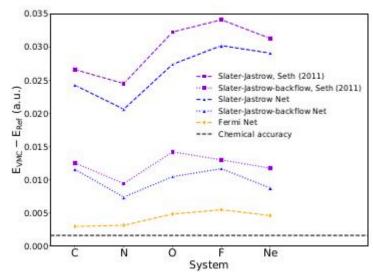


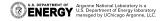
https://github.com/google-deepmind/ferminet



EXAMPLE: FERMINET

- Nuclei locations are "constants" while electron positions are inputs to the network.
- Two-body correlations directly learned by the network.
- Ferminet beats traditional methods of encoding wavefunctions by applying permutation invariant methods to the particle positions.
- Antisymmetry is enforced through the use of Slater Determinants - the permutation invariance of GNNs is critical to constructing a physical wavefunction.



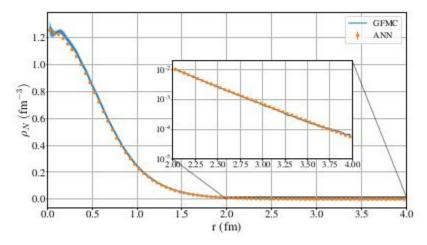


https://github.com/google-deepmind/ferminet



EXAMPLE: VARIATIONAL MONTE CARLO

$$H_{LO} = -\sum_{i} \frac{\vec{\nabla}_{i}^{2}}{2m_{N}} + \sum_{i < j} \left(C_{1} + C_{2} \, \vec{\sigma_{i}} \cdot \vec{\sigma_{j}} \right) e^{-r_{ij}^{2} \Lambda^{2}/4} + D_{0} \sum_{i < j < k} \sum_{\text{cyc}} e^{-\left(r_{ik}^{2} + r_{ij}^{2}\right) \Lambda^{2}/4}, \qquad ($$



Simpler than FermiNet: dump a bunch of protons and neutrons together in an all-to-all GNN and solve for the minimum energy.

$$\mathcal{F}(\mathbf{x}_1,\ldots,\mathbf{x}_A) = \rho_{\mathcal{F}}\left(\sum_{\mathbf{x}_i} \phi_{\mathcal{F}}(\mathbf{x}_i)\right) \quad \mathcal{F} = \mathcal{U}, \mathcal{V}.$$

	Λ	VMC-ANN	VMC-JS	GFMC	GFMC_{c}
^{2}H	4 fm^{-1}		-2.223(1)	-2.224(1)	10
п	$6~{\rm fm}^{-1}$	-2.224(4)	-2.220(1)	-2.225(1)	-
^{3}H	4 fm^{-1}	-8.26(1)	-7.80(1)	-8.38(2)	-7.82(1)
п	$6 \ {\rm fm}^{-1}$	-8.27(1)	-7.74(1)	-8.38(2)	-7.81(1)
4 He	4 fm^{-1}	-23.30(2)	-22.54(1)	-23.62(3)	-22.77(2)
пе	6 fm^{-1}	-24.47(3)	-23.44(2)	-25.06(3)	-24.10(2)

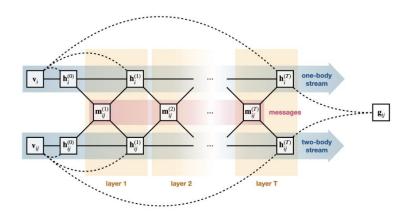


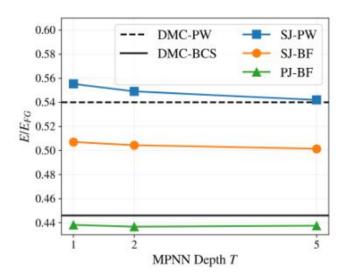
https://arxiv.org/abs/2007.14282



EXAMPLE: VARIATIONAL MONTE CARLO

"Ultra cold Fermi Gas" - apply a message passing GNN to transform electron positions and features (spin) before applying an antisymmetric function ("pfaffian").







https://arxiv.org/pdf/2305.08831.pdf



WRITING A GNN FOR YOURSELF

- Use the libraries out there!
 - <u>Pytorch Geometric</u>
 - Dynamic dispatch (just like torch), fast and easy to use
 - Probably leaves performance on the table
 - <u>Jraph</u> (JAX)
 - Disclaimer: I haven't used it!
 - Probably very fast, due to JIT compilation
 - Pads data to enable static runtime shapes (required in JAX).
 - <u>TF GNN</u>
 - It also exists?





WRAPPING UP GNNS

- If you want to apply machine learning and your dataset *doesn't* have rectangular structure, Graph Neural Networks can be a powerful tool.
- Compared to CNNs, there is much less consensus about what makes a good GNN.
 - Part of this is because the variety in graphs is much much larger than in image data!
- What I've covered is really the tip of the iceberg:
 - Graph Transformers
 - Graph Residual Networks
 - Going Deeper with GNNs
- All of the challenges with CNNs may still apply! (data/mc in particular)





THANK YOU FOR LISTENING, AND I HOPE IT WAS USEFUL!

QUESTIONS?



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