

GRAPH NEURAL NETWORKS

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RECAP

- Last time, we covered:
	- History of CNNs
	- CNN Ingredients
	- CNN use cases, and examples of these cases in physics
	- High level thoughts about how to apply a CNN to your dataset.
- This time:
	- Briefly, how to train a CNN on simulation that doesn't quite match data.
	- Then, all about GNNs.

USING AI TO SORT CLASSIFY SIGNAL/BACKGROUND

- Input size $40X40x110$ (voxel size $10x10x5$ mm³)
- Energy of every event normalized to 1 (so the network does not have information about total event energy)
- \sim 500000 fiducial events, 35% signal
- Network is **sparse 3D** convolutions

[https://link.springer.com/article/10.1007/JHEP01\(2021\)189](https://link.springer.com/article/10.1007/JHEP01(2021)189)

[Source](https://github.com/coreyjadams/NEXT_SparseEventID) Code

DATA DRIVEN EVALUATION

DATA VS MC

Prediction on data is biased towards lower values

Significant disagreement and performance hit on data

[https://link.springer.com/article/10.1007/JHEP01\(2021\)189](https://link.springer.com/article/10.1007/JHEP01(2021)189)

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From classical analysis we know there are some MC/data differences, e.g. track length, blob energy...

We apply on-the-fly augmentation to the training data to prevent the network from using these features **DATA AUGMENTATION TO PREVENT OVERFITTING**
We apply on-the-fly augmentation
are some MC/data differences, e.g.
track length, blob energy...
during training.

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HOW TO QUANTIFY THE OVERFITTING?

- Can we know if it will work ¹⁰⁰⁰ before we look at the interesting data? Can we know if augmentation is improving things?
- $Yes we can use energy$ 200 distance to compare the flattened layers in the sidebands around the interesting data!
- More about energy [distance](http://pages.stat.wisc.edu/~wahba/stat860public/pdf4/Energy2/ARSIA-final-sub.pdf)

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$$
E_{n,m}(X,Y):=2A-B-C\\
$$

$$
A:=\tfrac{1}{nm}\sum\limits_{i=1}^{n}\sum\limits_{j=1}^{m}\|x_{i}-y_{j}\|, B:=\tfrac{1}{n^{2}}\sum\limits_{i=1}^{n}\sum\limits_{j=1}^{n}\|x_{i}-x_{j}\|, C:=\tfrac{1}{m^{2}}\sum\limits_{i=1}^{m}\sum\limits_{j=1}^{m}\|y_{i}-y_{j}\|
$$

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TRACK ENERGY DISTANCE DURING TRAINING

The energy distance is significantly lower with augmentation than without – this implies that the features learned by the network are much more closely related between data/MC

[Source](https://github.com/coreyjadams/NEXT_SparseEventID) Code

 Ω 0.0

 0.2

 0.4

NN prediction

 0.6

 0.8

 10

TAKEAWAYS

- When your training data does not perfectly match the target data:
	- Understand the discrepancies dead pixels? Different sensor responses?
	- Understand the symmetries: is there a "direction" to events? Does dropping pixels from the training data change the labels?
	- If you can remake the training data to match the target data great!
	- If you can't, use augmentation on the fly.
		- Already a well known technique in Comp Sci.
		- Focus on label preserving augmentation techniques.
	- Use sidebands and statistical measurements (energy distance not unique) to quantify discrepancies and know if you are improving things!

GRAPH NEURAL NETWORKS

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WHAT MAKES GRAPHS DIFFERENT?

- Graphs are a collection of:
	- Nodes Individual locations that represent some piece of local information
	- Edges Connections between two nodes
	- Globals information that applies to the entire graph.

(Left) 3d representation of the Citronellal molecule (Center) Adjacency matrix of the bonds in the molecule (Right) Graph representation of the molecule.

<https://distill.pub/2021/gnn-intro/>

IMAGES ARE GRAPHS!

<https://distill.pub/2021/gnn-intro/>

GRAPHS ARE MORE GENERAL THAN IMAGES

Generalize images by relaxing requirements:

- non-uniform pixel spacing
- non-uniform connections
- can be directed
- computationally, must be permutation invariant

GRAPHS ARE MORE VARIED

Connections in a social network are graph data.

The "type" of connections (friend / watch / like) is an example of an edge feature.

PHYSICS GRAPHS ARE MORE IMAGE-Y

We often use graphs when images don't make sense:

- How to embed circular/cylindrical data into an image?
- What if pixels are a differnt size?
- What about GEANT output, directed particle interactions at specified locations?

GRAPH MATH

- Graphs can be large, and often sparse.
	- Nodes can be stored in arrays of [n_nodes, n node features]
	- Edges can be stored in arrays of [n_edges, n_edge_features]
	- The adjacency matrix is a sparse connectivity between nodes

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GRAPH OPERATIONS

If Graphs are generalizations of images, what's the generalization of convolutions?

GRAPH OPERATIONS

Formally, if h_k ⁽ⁱ⁻¹⁾ represents the node k at layer i-1:

 a_k ⁱ = AGGREGATE(h_u ⁽ⁱ⁻¹⁾, u connected to k) h_k ⁱ = COMBINE(h_k ⁽ⁱ⁻¹⁾, a_k ⁱ) ⁱ)

Most graph convolutions can be written like this (assuming all edges have no features, like images)

How Powerful are Graph Neural [Networks?](https://arxiv.org/pdf/1810.00826.pdf)

GRAPH OPERATIONS

After multiple layers of graph AGGREGATE and COMBINE, you can READOUT your graph information for the prediction of the network.

Possible graph-level readout: POOL over the nodes

(aka, sum(nodes, axis=0) where nodes has shape $[n]$ nodes, n features])

AGGREGATE, COMBINE, READOUT all often have learnable parameters, typically as MLPs. READOUT is almost certainly required to be permutation-invariant.

How Powerful are Graph Neural [Networks?](https://arxiv.org/pdf/1810.00826.pdf)

WHAT ABOUT THE EDGES?

For nodes h and edges e:

$$
m_v^{t+1} = \sum_{w \in N(v)} M_t(h_v^t, h_w^t, e_{vw})
$$

$$
h_v^{t+1} = U_t(h_v^t, m_v^{t+1})
$$

Where M, and U are learnable "message" and "update" functions. (Upon close inspection, the previous operations are variants of Message Passing!)

Neural Message Passing for Quantum

[Chemistry](https://arxiv.org/pdf/1704.01212v2.pdf)

BUILDING YOUR GNN

- Most parameters of a GNN come from the learnable parameters of aggregation, message creation, etc.
- Normalization of your data is still valuable!
- Pooling of information **between nodes**, with or without messages, is critical for the network.
- Pooling of the **graph**, however, is not a well defined operation in many cases!

GRAPH POOLING

Graph Pooling is an ill-defined, ambiguous task.

	No-pool	DiffPool	MinCut	NMF	LaPool	TopK	SAGPool	NDP	Graclus
Colors-3	$40.8{\pm}2.1$	$55.2 + 1.5$	$60.1 + 4.0$	$29.7 + 1.7$	44.9 ± 1.0	$26.9 + 40$	$34.4 + 5.2$	$25.4 + 1.8$	$29.5 + 20$
Triangles	93.5 ± 0.7	91.3 ± 0.2	$95.3{\scriptstyle~ \pm 0.5}$	58.1 \pm 5.2	88.8 ± 0.8	$75.2 + 17.3$	$80.3 + 8.6$	$75.3 + 10$	$71.4 + 1.7$
Proteins	$68.8 + 28$	70.0 ± 0.6	$73.8 + 0.8$	$68.9 + 3.4$	72.9 ± 2.0	$71.3 + 0.8$	$73.7 + 0.8$	$68.4 + 3.4$	$72.6 + 1.1$
Enzymes	83.6 ± 2.0	$72.4 + 3.9$	$83.6 + 0.6$	$32.4 + 8.1$	$85.0 + 1.2$	$81.0 + 0.4$	68.8 ± 18.8	$84.8 + 32$	$85.4 + 4.1$
DD	$81.1 + 0.4$	$75.6 + 1.8$	$82.5 + 0.9$	OOR	OOR	$80.4 +0.9$	$79.0 + 27$	$79.6 + 12$	$78.3 + 29$
Mutagen.	$78.0 + 1.6$	$76.2 + 1.4$	$73.9 + 1.6$	70.3 ± 1.6	75.3 ± 0.1	$75.8 + 14$	$76.9 + 1.4$	$76.9 + 10$	$74.2 + 0.5$
ModelNet	$81.0 + 0.5$	$70.4 + 24$	$75.9 + 1.2$	OOR	OOR	74.1 ± 3.0	71.9 ± 2.6	$77.1 + 26$	$83.9 + 1.9$
Rank		4.43	2.57	7.14	4.29	4.71	3.86	4.29	4.29

Table 6: Accuracy on the graph classification benchmarks.^[4]

Recent results call into question if it is worth pooling at all...

[https://arxiv.org/pdf/2110.05292.pdf](https://arxiv.org/pdf/2110.05292.pdf
)

GRAPH U NETS

- Downsampling learns a trainable projection layer, per downsample, to select nodes.
- Also used in a sigmoid to gate information flow (and make it differentiable!)

<https://arxiv.org/abs/1905.05178>

GRAPH U NETS

Table 4. Results of inductive learning experiments in terms of graph classification accuracies on D&D, PROTEINS, and COLLAB datasets. g-U-Nets denotes our proposed graph U-Nets model.

Results improve upon contemporary models but not in all cases....

<https://arxiv.org/abs/1905.05178>

CONNECTING YOUR GNN

- Usually, defining nodes is easy in graphs, especially physics graphs.
	- Typically have some "position" (x/y/z or similar) and some "feature".
	-
	- You can often just concatenated these together and have node features.• Much like MLPs and dense neural networks, feature normalization can be useful.
- Connecting nodes and defining edges:
	- Sometimes easy (particle flow data)
	- Sometimes ambiguous.
	- When in doubt, try k-NN algorithm
	- Usually, don't connect all-to-all!
- Defining Edge features?
	- If nothing else, displacement vector of features

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Icecube, at the south pole, has regular but not rectangular sensor layouts - CNNs are not a great fit. GNNs instead are an improvement.

<https://arxiv.org/pdf/1809.06166.pdf>

 $GConv(\mathbf{X}^{(t)}) = [\mathbf{A} \mathbf{X}^{(t)}, \ \mathbf{X}^{(t)}] (\mathbf{a}^{(t)})^{\top} + b^{(t)} \mathbf{1},$

Both GNNs and CNNs (on transformed
ata) are tested, using a Graph Convolution
perator. GNNs significantly outperform
CNNs.
 $\frac{\text{# events per year}}{\text{Signal} \quad \text{Background}}$ Signal.Noise $\frac{2}{5}$ 0.2 data) are tested, using a Graph Convolution operator. GNNs significantly outperform CNNs.

<https://arxiv.org/pdf/1809.06166.pdf>

$$
\tilde{x}_j = \sum_{i=1}^{N_{\text{neighbors}}} \text{MLP}(x_j, x_j - x_i),
$$

2022 Update from Icecube extends GNN usecase to lower energy events.

<https://arxiv.org/abs/2209.03042>

Improvement in classification metrics as well as in regression variables (energy, angles) compared to traditional reconstruction.

EXAMPLE: SEGMENTATION WITH GNNS

Challenge: 2D scintillator data projected to 3D images can produce fake 3D hits from <u>crosstalk</u> or coincidence hits (ghost) - can a GNN label everything correctly looking at all 3D hits?

Graph neural network for 3D classification of ambiguities and optical crosstalk in [scintillator-based](Graph neural network for 3D classification of ambiguities and optical crosstalk in scintillator-based neutrino detectors.) neutrino detectors.

EXAMPLE: SEGMENTATION WITH GNNS

 3 layers of GraphSAGE + aggregation + fully connected + per-node segmentation

TABLE IV: Mean efficiencies and purities of voxel classification for the GNN and a simple charge cut.

Graph neural network for 3D classification of ambiguities and optical

crosstalk in [scintillator-based](Graph neural network for 3D classification of ambiguities and optical crosstalk in scintillator-based neutrino detectors.) neutrino detectors.

coordinates features k -NN k -NN indices edge feature Linear **BatchNorm** ÷ ReLU Linear **BatchNorm** ReLU Linear BatchNorm ReLU Aggregation ReLU

Challenge: Jet images are $\frac{k = 16, C = (64, 64, 64)}{1}$ sparse and CNNs are highly $\sqrt{\frac{EdgeConv Block}{k = 16, C = (128, 128, 128)}}$ inefficienct - can GNNs improve over $CNNs$?

Input features include Fully Connected Fully Connected 256, ReLU, Dropout = 0.1 significant derived physics information.

Jet [Tagging](https://arxiv.org/pdf/1902.08570.pdf) via Particle Clouds

Jet [Tagging](https://arxiv.org/pdf/1902.08570.pdf) via Particle Clouds

But, it is hard to compete against vendor optimized convolutional kernels for performance

Jet [Tagging](https://arxiv.org/pdf/1902.08570.pdf) via Particle Clouds

EXAMPLE: THEORY WITH GNNS

- Variational Monte Carlo is a numerical technique for solutions to the Schrodinger Equation:
	- Requires an "ansatz" aka a trial wavefunction that can be optimized.
	- The wavefunction must obey physial principles (twice differentiable, continous, antisymmetric under exchange of Fermions)
	- The wavefunction must be sufficiently general to capture all the physics of the system.

$$
\frac{\langle \Psi_V | H | \Psi_V \rangle}{\langle \Psi_V | \Psi_V \rangle} = E_V \geq E_0
$$

Graph Neural Networks are an ideal candidate for building an ansatz.

<https://github.com/google-deepmind/ferminet>

EXAMPLE: FERMINET

Ferminet solves molecular physics by encoding electron locations in a dynamic graph.

<https://github.com/google-deepmind/ferminet>

EXAMPLE: FERMINET

- Nuclei locations are "constants" while electron positions are inputs to the network.
- Two-body correlations directly learned by the $\qquad \qquad \circ \circ \circ \circ$ network.
- Ferminet beats traditional methods of encoding $\frac{a}{\frac{a}{\omega}}$ wavefunctions by applying permutation
invariant methods to the narticle positions invariant methods to the particle positions.
- Antisymmetry is enforced through the use of \qquad _{0.005} Slater Determinants - the permutation invariance of GNNs is critical to constructing a physical wavefunction.

EXAMPLE: VARIATIONAL MONTE CARLO

$$
H_{LO} = -\sum_{i} \frac{\vec{\nabla}_{i}^{2}}{2m_{N}} + \sum_{i < j} (C_{1} + C_{2} \vec{\sigma}_{i} \cdot \vec{\sigma}_{j}) e^{-r_{ij}^{2} \Lambda^{2}/4} + D_{0} \sum_{i < j < k} \sum_{\text{cyc}} e^{-\left(r_{ik}^{2} + r_{ij}^{2}\right) \Lambda^{2}/4}, \qquad (
$$

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Simpler than FermiNet: dump a bunch of protons and neutrons together in an all-to-all GNN and solve for the minimum energy.

$$
\mathcal{F}(\mathbf{x}_1,\ldots,\mathbf{x}_A)=\rho_{\mathcal{F}}\left(\sum_{\mathbf{x}_i}\phi_{\mathcal{F}}(\mathbf{x}_i)\right)\quad \mathcal{F}=\mathcal{U},\mathcal{V}.
$$

[https://arxiv.org/abs/2007.14282](https://arxiv.org/abs/2007.14282
)

EXAMPLE: VARIATIONAL MONTE CARLO

"Ultra cold Fermi Gas" - apply a message passing GNN to transform electron positions and features (spin) before applying an antisymmetric function ("pfaffian").

<https://arxiv.org/pdf/2305.08831.pdf>

WRITING A GNN FOR YOURSELF

- Use the libraries out there!
	- Pytorch [Geometric](https://pytorch-geometric.readthedocs.io/en/latest/)
		- Dynamic dispatch (just like torch), fast and easy to use
		- Probably leaves performance on the table
	- [Jraph](https://github.com/google-deepmind/jraph) (JAX)
		- Disclaimer: I haven't used it!
		- Probably very fast, due to JIT compilation
		- Pads data to enable static runtime shapes (required in JAX).
	- TF [GNN](https://github.com/tensorflow/gnn)
		- It also exists?

WRAPPING UP GNNS

- If you want to apply machine learning and your dataset *doesn't* have rectangular structure, Graph Neural Networks can be a powerful tool.
- Compared to CNNs, there is much less consensus about what makes a good GNN.
	- Part of this is because the variety in graphs is much much larger than in image data!
- What I've covered is really the tip of the iceberg:
	- Graph [Transformers](https://arxiv.org/abs/2012.09699v2)
	- Graph Residual [Networks](https://arxiv.org/pdf/1909.05729.pdf)
	- Going [Deeper](https://arxiv.org/pdf/2007.09296.pdf) with GNNs
- All of the challenges with CNNs may still apply! (data/mc in particular)

THANK YOU FOR LISTENING, AND I HOPE IT WAS USEFUL!

QUESTIONS?

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