The latest calculations for rare kaon processes

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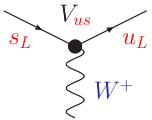
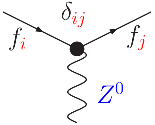
Neutral & Charged Current Interactions

Mass ≠ flavour eigenstates





SM: Only charged currents change the flavour ($\propto V_{us}$)

SM: Neutral currents do not change the flavour (i=j) at tree-level

$$V_{\text{CKM}} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho+i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

- SM: Yukawas only source of flavour & CP violation
- CKM parametrises CP & flavour violation
- First row from tree-level semi-leptonic decays

Charged Current decays

- $K_{\ell 2}$ and $K_{\ell 3}$ extraction of $\lambda = |V_{us}|$

$$\Gamma(K^0 \to \pi^- \ell^+ \nu_\ell(\gamma)) = \frac{G_F^2 m_K^5}{128\pi^3} |V_{us}|^2 \; S_{EW} \;\; |f_+^{K^0\pi^-}(0)|^2 \; I_{K^0\ell}^{(0)} \left(1 + \delta_{EM}^{K^0\ell} + \delta_{SU(2)}^{K^0\pi^-}\right).$$

- QED: ξPT [Seng et.al.'2019,Cirigliano et.al.'23] and Lattice [Carrasco et.al.'15,DiCarlo et.al.'19]
- EW corrections in W-Mass scheme [Marciano, Sirlin]
- EFT Approach [Gorbahn et.al.'22,Cirigliano et.al.'23]
- $|V_{ud}|$, extracted from nuclear β decays [Hardy,Towner'20],
- $\Delta_{CKM} \equiv 1 - |V_{ud}|^2 - |V_{us}|^2 - \mathcal{O}(|V_{ub}|^2) = 0.$

(Effective) Interaction

- $\mathcal{H}(x) = 4\,\frac{G_F}{\sqrt{2}} C_O\ V_{ud}^* O(x)$
- $O(x) = \left(\bar{d}(x)\gamma^\mu P_L u(x)\right)\left(\bar{\nu}_l(x)\gamma_\mu P_L l(x)\right)$
- SD in W-Mass scheme:
  $\frac{1}{k^2} \to \frac{1}{k^2 - M_W^2} - \frac{M_W^2}{k^2 - M_W^2}\frac{1}{k^2} = \gamma_> - \gamma_<$
  - UV poles → Absorbed into G_F from muon decay
  - Combining with SU(3) current algebra → QCD corrections to S_{EW}
  - No scale separation and α^2 log and $1/s_w^2$ effects
- EFT: scale separation, $\mathcal{O}(\alpha^2)$, match to Lattice

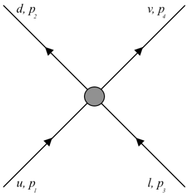
Effective Theory Calculation

- Decoupling Theorem (Renormalization [Collins]):

$$\langle \ell_3|T|K\rangle = 4G_F/\sqrt{2}C(\mu_W)\langle \ell_3|O|K\rangle(\mu_W) + \mathcal{O}(p_\ell^2/M_W^2)$$

- Determine $C(\mu_W)$ in perturbation theory
- Use RGE to run $\langle \ell_3|T|K\rangle =$ $4G_F/\sqrt{2}C(\mu_W)U(\mu_W,\mu_{Lat})\langle \ell_3|O|K\rangle(\mu_{Lat}) + \mathcal{O}(p_\ell^2/M_W^2)$
- Determine $\langle\ \ell_3\ |\ \mathrm{O}\ |\ \mathrm{K}\ \rangle$ using symmetries and data or Lattice calculation
- Lattice: have to convert Lattice to continuum scheme
- Residual μ_W and μ_{Lat} dependence reduces at N^n LO

Lattice Renormalisation



- off-shell renormalisation conditions
  - $\mathrm{RI}^{(\prime)} - \mathrm{MOM}$: $p_1 = p_2 = p_3 = p_4 = p, \quad p^2 = -\mu^2$
  - $\mathrm{RI} - \mathrm{SMOM}$:
    $p_1 = p_3, \quad p_2 = p_4, \quad p_1^2 = p_2^2 = -\mu^2, \quad p_1 \cdot p_2 = -\frac{1}{2}\mu^2$

Choice of Projector

- $\sigma^A \equiv \frac{1}{4\,p^2}\mathrm{Tr}(S_A^{-1}(p)\not{p}) \overset{\text{A=RI}}{=} 1, \quad \lambda^A \equiv \Lambda^A_{\alpha\beta\gamma\delta}\mathcal{P}^{\alpha\beta\gamma\delta}$

- $\Lambda^b = \Lambda^{b,\mu}(p) \otimes \gamma_\mu P_L + \mathcal{O}(\alpha)$, only 2 form factors in RIMOM $\Lambda^{b,\mu}(p) = F_1(p)\gamma^\mu P_L + F_2(p)p^\mu \not{p}/p^2 P_L$

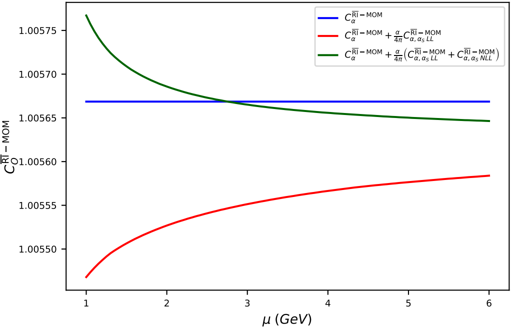
- Choose $\mathcal{P} = -\frac{1}{12\,p^2}(\not{p}P_R \otimes \not{p}P_R + p^2/2\gamma^\nu P_R \otimes \gamma_\nu P_R)$

  - Projects out $F_1(p)$ → no pure QCD corrections

- $C_O^{\overline{\mathrm{MS}}\to RI} = \lambda^{\overline{\mathrm{MS}}}\left(\sigma_u^{\overline{\mathrm{MS}}}\sigma_d^{\overline{\mathrm{MS}}}\sigma_\ell^{\overline{\mathrm{MS}}}\right)^{1/2}$

$\overline{RI}$ and $\overline{MS}$ Wilson coefficients

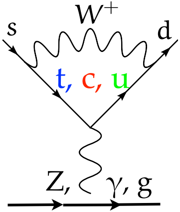
Including 2-loop EW matching and 3-loop RGE [MG, SJ, Moretti, EM]



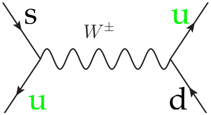
For V_{ud} we have $C_O(m_c) = 1.00754$ [Cirigliano et.al.'23] $\longrightarrow$ $C_O(m_c) = 1.00794$

$K \to \pi \bar{\nu} \nu$

Rare Kaon Decays: CKM Structure



Using the GIM mechanism, we can eliminate either $V_{cs}^* V_{cd}$ or $V_{us}^* V_{ud} \rightarrow - V_{cs}^* V_{cd} - V_{ts}^* V_{td}$



Z-Penguin and Boxes (high virtuality): power expansion in: $A_c - A_u \propto 0 + O(m_c^2/M_W^2)$

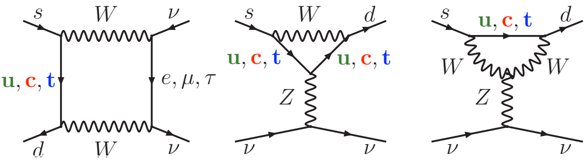
γ/g-Penguin (expand in mom.): $A_c - A_u \propto O(\mathrm{Log}(m_c^2/m_u^2))$

$$\mathrm{Im} V_{ts}^* V_{td} = -\mathrm{Im} V_{cs}^* V_{cd} = \mathcal{O}(\lambda^5) \qquad \mathrm{Im} V_{us}^* V_{ud} = 0$$

$$\mathrm{Re} V_{us}^* V_{ud} = -\mathrm{Re} V_{cs}^* V_{cd} = \mathcal{O}(\lambda^1) \qquad \mathrm{Re} V_{ts}^* V_{td} = \mathcal{O}(\lambda^5)$$

- $K \rightarrow \pi \bar{\nu} \nu$ (from Z & Boxes): Clean and suppressed

$K \to \pi\bar{\nu}\nu$ at M_W



$$x_i = \frac{m_i^2}{M_W^2}$$

$$\sum_i V_{is}^* V_{id} F(x_i) = V_{ts}^* V_{td} (F(x_t) - F(x_u)) + V_{cs}^* V_{cd} (F(x_c) - F(x_u))$$

Quadratic GIM: $\lambda^5 \frac{m_t^2}{M_W^2}$ $\quad \lambda \frac{m_c^2}{M_W^2} \ln \frac{M_W}{m_c}$ $\quad \lambda \frac{\Lambda_{QCD}^2}{M_W^2}$

Matching (NLO +EW):

Operator Mixing (RGE)

ChiPT & Lattice

$$Q_\nu = (\bar{s}_L \gamma_\mu d_L)(\bar{\nu}_L \gamma^\mu \nu_L)$$

- Below the charm: Only Q_ν, ME from K_{l3}
- semi-leptonic $(\bar{s}\gamma_\mu u_L)(\bar{\nu}\gamma^\mu \ell_L)$ operator: χ PT gives small contribution (10% of charm contribution)

Leading Effective Hamiltonian for $\mu < m_c$

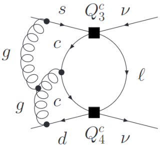
SM: $\nu\bar{\nu}$ are only invisibles $\Rightarrow$ no γ-Penguin $\Rightarrow$

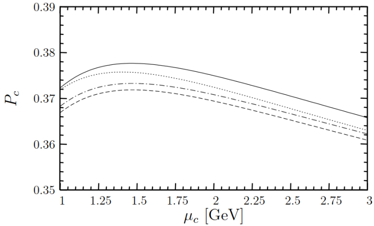
$$\mathcal{H}_{\text{eff}} = \frac{\sqrt{2}\alpha G_F}{\pi \sin^2\theta_w} \sum_{\ell=e,\mu,\tau} (\lambda_c X^\ell + \lambda_t X_t)(\bar{s}_L \gamma_\mu d_L)(\bar{\nu}_{\ell L}\gamma^\mu \nu_{\ell L}) + \text{h.c.}$$

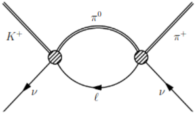
generated by highly virtual particles + tiny light quark contribution $\Rightarrow$ clean & CKM suppressed ($\lambda_i = V_{is}^* V_{id}$).

- X_t known at NLO QCD [Buchalla,Buras;Misiak,Urban'99] and two-loop EW [Brod et.al.'10]:
  $X_t = 1.462 \pm 0.017_{\text{QCD}} \pm 0.002_{\text{EW}}$
- $P_c = \lambda^{-4}(\frac{2}{3}X^e + \frac{1}{3}X^\tau)$ at NNLO QCD [Buras et.al.'05] + NLO EW [Brod et.al.'08]

Higher order corrections for P_c

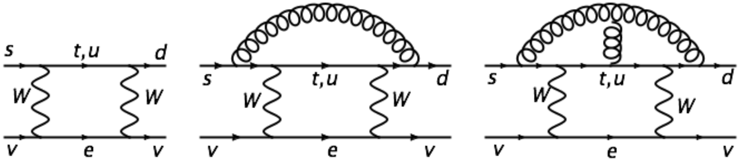






- GIM in the EFT: (Charm - Up) only $m_c^2 G_F^2 Q_\nu$ above m_c
  - $P_c = (0.2255/\lambda)^4 \times (0.3604 \pm 0.0087)$ @ NNLO+EW [Brod et.al.'21]
- Small corrections from higher dimensional operator + light quarks below m_c
  - ξ PT matching logs: δ $P_{c,u}$ [Isidori et.al.'05]
  - Future Lattice: [Bai et.al.'18]

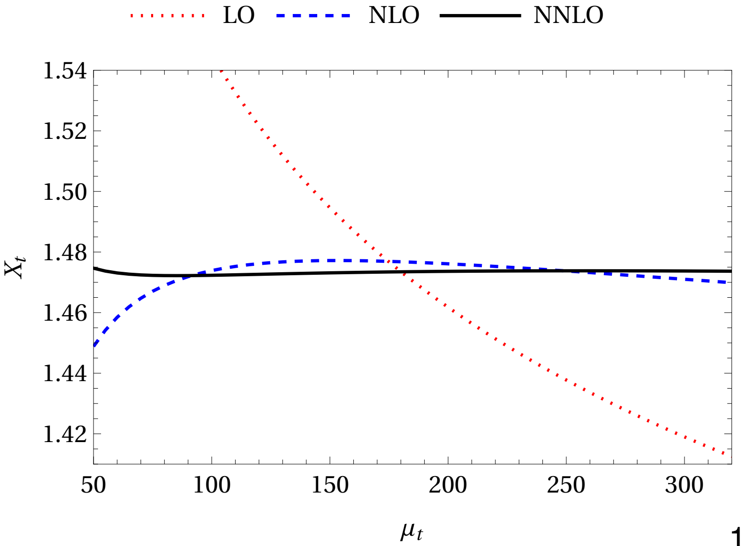
Higher order corrections for X_t



- $X_t = X_{\mathrm{t}}^{\mathrm{NLO}} + X_{\mathrm{t}}^{\mathrm{EW}} = 1.469(30)$ up to now
- NNLO Penguin [Hermann, Misiak, Steinhauser]
- NNLO-Boxes [Cerda-Sevilla, Gorbahn, Leak; Yu, Stamou] (related to electron-boxes): Use known master integrals and numerical evaluation
- Matching result should be independent of μ_t (order by order)

Scale Dependence @ NNLO

- Residual μ_t dependence estimates uncertainty
- Reduces from ±1% @NLO → ± 0.1% @NNLO



$K \to \pi\nu\bar{\nu}$ Branching Ratios

- Matrix elements from $K_{\ell 3}$ including strong and em iso-spin breaking [0705.2025] $\kappa_+, \kappa_L, \Delta_{EM}$

$$\kappa_+ = \frac{s_w^{-2}\lambda^8\alpha(M_Z)^2}{7.5248\cdot 10^{-9}} \times 0.5173(25)\times 10^{-10},\ \Delta_{EM} = -0.003$$

- indirect CP violation contribution given by r_{ϵ_K}

$$\mathrm{Br}_{K^+} = \kappa_+(1+\Delta_{\mathrm{EM}})\left[\left(\frac{\mathrm{Im}\lambda_t}{\lambda^5}X_t\right)^2 + \left(\frac{\mathrm{Re}\lambda_c}{\lambda}\left(P_c+\delta P_{c,u}\right) + \frac{\mathrm{Re}\lambda_t}{\lambda^5}X_t\right)^2\right].$$

$$\mathrm{Br}_{K_L} = \kappa_L r_{\epsilon_K}\left(\frac{\mathrm{Im}\lambda_t}{\lambda^5}X_t\right)^2, \quad \kappa_L = \frac{s_w^{-2}\lambda^8\alpha(M_Z)^2}{7.5248\cdot 10^{-9}}\times 2.231(13)\times 10^{-10}$$

CKM parameters for $\mathcal{H}_{\text{eff}}$

- We need Imλ_t, Reλ_t and Reλ_c:
- Use exact Wolfenstein parameterization (PDG)
  - $s_{12} = \lambda$, $s_{23} = A\lambda^2$, $s_{13}e^{i\delta} = A\lambda^3(\rho + i\eta)$:

$$\text{Im}\lambda_t = A^2\bar{\eta}\lambda^5 + \dots, \text{Re}\lambda_c = -\lambda + \dots$$

$$\text{Re}\lambda_t = A^2\lambda^5(\bar{\rho} - 1) + \dots$$

PDG	$\bar{\rho}$	$\bar{\eta}$	λ	A
2020	0.141(17)	0.357(11)	0.22650(48)	0.790(17)
2022	0.159(10)	0.348(10)	0.22500(67)	$0.826^{+0.018}_{-0.015}$

- PDG fit assumes SM ⇒ unbiased SM prediction

$K \to \pi\nu\bar{\nu}$ in the Standard Model

- 2105.02868 Standard Model Prediction

$$\begin{aligned} \mathrm{BR}(K^+ \to \pi^+\nu\bar{\nu}) &= 7.73(16)_{SD}(25)_{LD}(54)_{para.} \times 10^{-11}\,, \\ \mathrm{BR}(K_L \to \pi^0\nu\bar{\nu}) &= 2.59(6)_{SD}(2)_{LD}(28)_{para.} \times 10^{-11}\,. \end{aligned}$$

- will update [preliminary numerics:]

$$\begin{aligned} \mathrm{BR}(K^+ \to \pi^+\nu\bar{\nu}) &= 8.25(11)_{SD}(25)_{LD}(57)_{para.} \times 10^{-11}\,, \\ \mathrm{BR}(K_L \to \pi^0\nu\bar{\nu}) &= 2.83(1)_{SD}(2)_{LD}(30)_{para.} \times 10^{-11}\,. \end{aligned}$$

- NA62 collaboration
  $\mathrm{BR}(K^+ \to \pi^+\nu\bar{\nu}) = (10.6^{+3.4}_{-3.4}|_{\mathrm{stat}} \pm 0.9_{\mathrm{syst}}) \times 10^{-11}$
- JPARC-KOTO has $\mathrm{BR}(K_L \to \pi^0\nu\bar{\nu}) \leq 2.0 \times 10^{-9}$

$$K \to \mu^+\mu^-$$

$K \to \mu^+\mu^-$

SD: $Y_t + Y_{NNLO}$ from SM $\mathcal{L}_{eff}^{d=6}$ gives $\ell = 0$. $[\mathcal{O}(\lambda_t, \frac{m_c^2}{M_W^2}\lambda_c)]$

LD: from γ-γ loop is CP conserving: $\mathcal{O}(\frac{\alpha_{QED}}{4\pi})$

Setting ϵ_K to zero, K_L and K_S are CP eigenstates:

K_i^{CP}	BR_{exp}	$(\mu^+\mu^-)^-_{\ell=0}$	$(\mu^+\mu^-)^+_{\ell=1}$
$K_L^{(-)}$	$6.84(11) \times 10^{-9}$	$\lambda_t, \frac{m_c^2}{M_W^2}\lambda_c, \gamma$-$\gamma$	(~~CP~~) $\simeq 0$
$K_S^{(+)}$	$< 2.1 \times 10^{-10}$	~~CP~~: Im λ_t	γ-γ

ℓ is not experimentally accessible, but interference term in time dependent decay sensitive to *SD* [D'Ambrosio, Kitahara '17]

$K(t) \to \mu^+\mu^-$

$$\frac{d\Gamma}{dt} \propto C_L e^{-\Gamma_L} + C_S e^{-\Gamma_S} + 2[C_{sin}\sin(\Delta M t) + C_{cos}\cos(\Delta M t)]e^{-\Gamma t}$$

$$C_{sin/cos} = Im/Re\left\{(A_0^S)^* A_0^L + (A_1^S)^* A_1^L\right\}$$

$$\frac{BR(K_S \to \mu^+\mu^-)^{(pert)}_{\ell=0}}{BR(K_L \to \mu^+\mu^-)} = \frac{\tau_S}{\tau_L}\frac{|A_0^S A_0^L|^2}{|A_0^L|^4} = \frac{\tau_S}{\tau_L}\frac{C_{int}^2}{C_L^2}$$

where we assumed $A_1^L = 0$, $\epsilon_K = 0$

- Measurement of $BR(K_S \to \mu^+\mu^-)_{\ell=0}$ [Dery et.al.'21]

NNLO, EW and $\epsilon_K = 0$

Higher order corrections from $b \to s\ell^+\ell^-$ for 5 flavour

{NNLO QCD [Hermann et.al.'13]; 2-loop EW [Bobeth et.al.'13] RGE [Bobeth et.al.'03]}

$$BR(K_S \to \mu^+\mu^-)^{(pert)}_{\ell=0} = 1.70(02)_{QCD/EW}(01)_{f_K}(19)_{param.}\times 10^{-13}$$

indirect CP violation mixes γ-γ: $A_0^S \to A_0^S + \epsilon_K A_0^L$

$$\frac{BR(K_S \to \mu^+\mu^-)_{\ell=0}}{BR(K_S \to \mu^+\mu^-)^{(pert)}_{\ell=0}} = 1 + \sqrt{2}|\epsilon_K \frac{|A_0^L|}{|A_0^S|}(\cos\phi_0 - \sin\phi_0)$$

[Brod, Stamou'22] and can shift by $\mathcal{O}(3\%)$, while ϕ_0 can be obtained from $K_L \to \mu^+\mu^-$ and $K_L \to \gamma\gamma$ [Dery et al.'22]

$$\epsilon_K$$

CP violation in $K \to \pi\pi$

- Experimental definition using $\eta_{ij} = \dfrac{\langle \pi^i \pi^j | K_L \rangle}{\langle \pi^i \pi^j | K_S \rangle}$

$$\epsilon_K = (2\eta_{+-} + \eta_{00})/3\ , \qquad \epsilon' = (\eta_{+-} - \eta_{00})/3$$

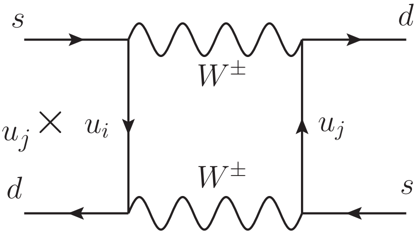
- ϵ_K theory expression $\epsilon_K \simeq \dfrac{\langle (\pi\pi)_{I=0} | K_L \rangle}{\langle (\pi\pi)_{I=0} | K_S \rangle} =$

$$e^{i\phi_\epsilon} \sin\phi_\epsilon \frac{1}{2} \arg\left(\frac{-M_{12}}{\Gamma_{12}}\right) = e^{i\phi_\epsilon} \sin\phi_\epsilon \left(\frac{\mathrm{Im}(M_{12})^{Dis}}{\Delta M_K} + \xi\right)$$

$$\langle K^0 | H^{|\Delta S|=2} | \bar{K}^0 \rangle \to \mathrm{Im}(M_{12})^{Dis}\ , \ \frac{\mathrm{Im}\langle (\pi\pi)_{I=0} | K^0 \rangle}{\mathrm{Re}\langle (\pi\pi)_{I=0} | K^0 \rangle} \to \xi\ , \ \phi_\epsilon \equiv \arctan \frac{\Delta M_K}{\Delta \Gamma_K / 2}$$

Kaon Mixing: CKM Structure

$$\sum_{u_i,u_j}^{u,c,t} \lambda_{u_i}\lambda_{u_j} \times$$



	Im	Re	$\mathcal{O}$
λ_t^2	$\sim \lambda^{10}$	$\sim \lambda^{10}$	m_t^2/M_W^2
$\lambda_c\ \lambda_t$	$\sim \lambda^6$	$\sim \lambda^6$	$m_c^2/M_W^2 \ln(m_t/m_c)$
λ_c^2	$\sim \lambda^6$	$\sim \lambda^2$	m_c^2/M_W^2
$\lambda_u\ \lambda_t$	$\sim \lambda^6$	$\sim \lambda^6$	$m_c^2/M_W^2 \ln(m_t/m_c)$
λ_u^2	0	$\sim \lambda^2$	m_c^2/M_W^2

Where $\lambda_i = V_{id}V_{is}^*$, $\lambda \equiv |V_{us}| \sim 0.2$ and we eliminated either: $\lambda_u = -\lambda_c - \lambda_t$ or $\lambda_c = -\lambda_u - \lambda_t$.

$\Delta S = 2$ Hamiltonian - Phase (In)Dependence

- Recall $\epsilon_K \propto \arg(-M_{12}/\Gamma_{12})$
- Trick: pull out λ_u^* and $(\lambda_u^*)^2$ from $H^{\Delta S=1}$ and $H^{\Delta S=2}$:
- Rephaseing invariant: $\lambda_i \lambda_j^* = V_{id} V_{is}^* V_{jd}^* V_{js}$
- One Operator: $Q_{S2} = (\bar{s}_L \gamma_\mu d_L) \otimes (\bar{s}_L \gamma^\mu d_L)$

$$\mathcal{H}_{f=3}^{\Delta S=2} = \frac{G_F^2 M_W^2}{4\pi^2 (\lambda_u^*)^2} Q_{S2} \left\{ f_1 \mathcal{C}_1(\mu) + iJ\left[f_2\, \mathcal{C}_2(\mu) + f_3\, \mathcal{C}_3(\mu)\right] \right\} + \text{h.c.}$$

- $f_1 = |\lambda_u|^4$, $f_2 = 2\mathrm{Re}(\lambda_t \lambda_u^*)$ and $f_3 = |\lambda_u|^2$

$\mathrm{Im}M_{12}$ without ΔM_K pollution

- Using CKM unitarity and the PDG convention we can also write (as used in Lattice [Christ et.al.]):

$$\mathcal{H}^{\Delta=2}_{f=3} = \frac{G_F^2 M_W^2}{4\pi^2}\left[\lambda_u^2 C^{uu}_{S2}(\mu)+\lambda_t^2 C^{tt}_{S2}(\mu)+\lambda_u\lambda_t C^{ut}_{S2}(\mu)\right]Q_{S2}+\mathrm{h.c.}$$

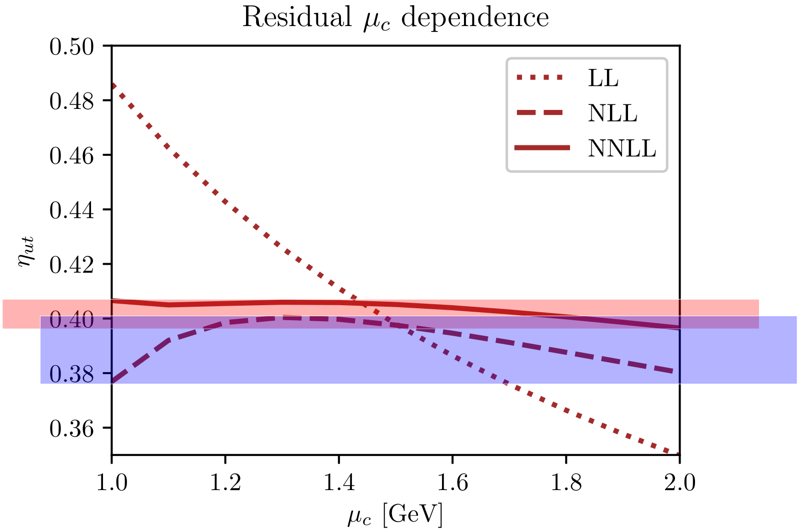
- Now real $\mathrm{Re}M_{12}$ and $\mathrm{Im}M_{12}$ are disentangled

$$C^{uu}_{S2} \equiv \mathcal{C}_1, \quad C^{tt}_{S2} \equiv \mathcal{C}_2, \quad C^{ut}_{S2} \equiv \mathcal{C}_3$$

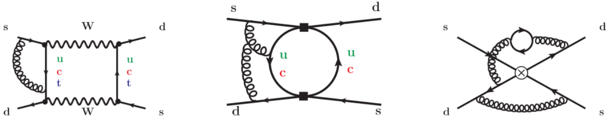
$$\begin{aligned} \mathcal{C}_3 &\leftarrow (A_{tu} - A_{tc} + A_{cc} - A_{cu}) \leftarrow \\ &\leftarrow (A_{uu} - 2A_{cu} + A_{cc}) - (A_{tc} - A_{tu} + A_{uu} - A_{cu}) \end{aligned}$$

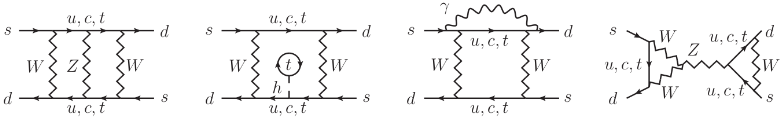
- NNLO QCD corrections [Brod et.al.'10,Brod et.al.'11] to C^{ct}_{S2} absorbed into η_{ut} [Brod et.al.'19]

Residual scale dependence



Further Improvements

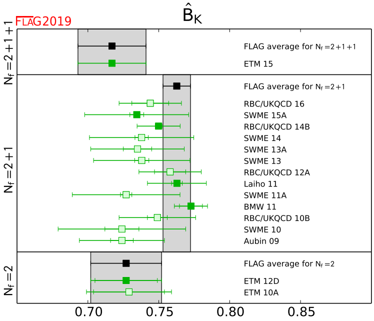




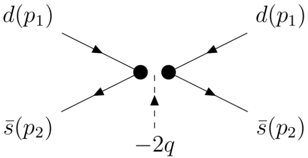
$$|\epsilon_K| = 2.170\,(65)_{\text{pert.}}(76)_{\text{non-pert.}}(153)_{\text{param.}} \times 10^{-3}$$

- $\hat{B}_K = \dfrac{3}{2f_K^2 M_K^2}\langle \bar{K}^0 | Q^{|\Delta S=2|} | K^0 \rangle u^{-1}(\mu_{\text{had}})$ from Lattice

$\hat{B}_K$

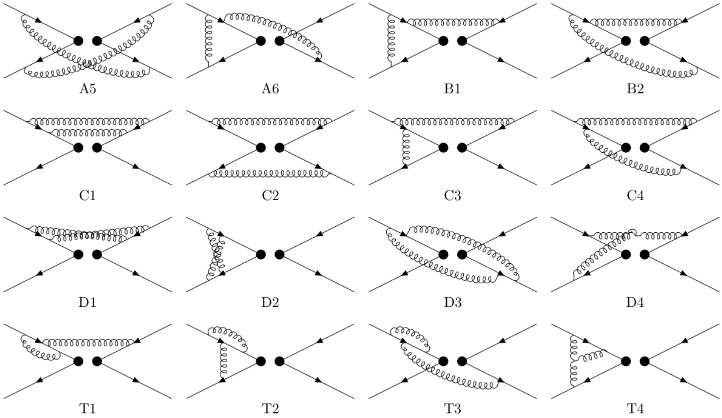


E.g. RBC UKQCD uses SMOM kinematics



- Define momentum-space subtraction schemes
- Projected renormalised Green's function $P_{(\gamma_\mu)}(\Lambda_R) \rightarrow$
- $Z_{Q_{S2}}^{(\gamma_\mu,\gamma_\mu)} = \left(Z_q^{(\gamma_\mu)}\right)^2 \dfrac{1}{P_{(\gamma_\mu)}(\Lambda_B)}$
- $Z_{Q_{S2}}^{(\gamma_\mu,\gamma_\mu)} / Z_{Q_{S2}}^{\overline{\mathrm{MS}}}$ converts between Lattice and continuum

SMOM $\hat{B}_K$ @ NNLO



- Use projectors to find $\Lambda^{ijkl}_{\alpha\beta\gamma\delta}$ at 2-loops
- Integrals reduce to scalar off-shell 4-point functions

Combine with Lattice values

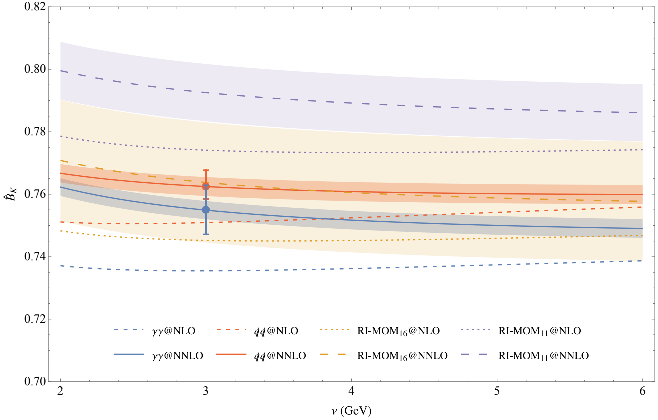
- The result should be independent of the matching scale

$$B_K^{(X,Y)}(|p|)C_{B_K}^{(X,Y)}(|p|,\mu)u^{-1}(\mu)u(\mu_0)$$

- study scale variation setting $\mu_0 = |p|$

- Compute $\hat{B}_K$ for f=3,4 for SMOM, RIMOM, RI'MOM

$\hat{B}_K$ at NNLO



Numerics of NNLO result by [MG, Kvedaraitė, Jäger]

Back to $K \to \pi\nu\bar{\nu}$

- CKM factors of ϵ_K and $K \to \pi\nu\bar{\nu}$ are quite similar
  - In fact $BR(K \to \pi\nu\bar{\nu})/|\epsilon_K|^{0.82}$ is effectively V_{cb} independent [Buras, Venturini '21] for current theory calculations
- This provides an excellent null test of the Standard Model
- Multiplying with $|\epsilon_K|^{0.82}$ results in a model prediction for $BR(K \to \pi\nu\bar{\nu})$ that does not aim to explain other observables with 5% uncertainty
- But parametric and theory uncertainties are something different

Conclusions

- Measurement of $K \to \pi \bar{\nu} \nu$ can be compared with precise theory prediction.
- New formula for ϵ_K allows for better theory control.
- NNLO calculations for electroweak and lattice-continuum matching will increase precision.
- Precision test of the standard model (EFT)