Discussion (SM predictions – continuum)

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Kaons@CERN 2023

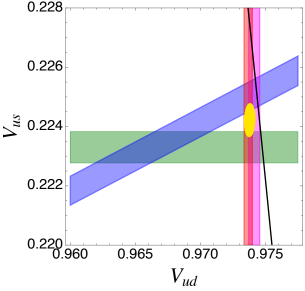
Sep 13, 2023

Kaons within the SM

Q1

What can we still learn from kaons within the SM?

- **CKM parameters**: Cabibbo anomaly, Re λ_t, Im λ_t
- But is this really the main point? Rather, kaon decays as sensitive laboratories for BSM searches (already true for test of CKM unitarity)



Cirigliano, Crivellin, MH, Moulson 2023

$K_L \to \mu^+\mu^-$: amplitude decomposition

Q2

How well can the long-distance–short-distance ambiguity in $K_L \to \mu^+\mu^-$ be resolved?

work in progress with Bai-Long Hoid and Jacobo Ruiz de Elvira

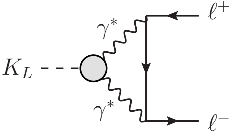
- **Reduced amplitude $\mathcal{A}_\ell$**

$$R_L^\ell = \frac{\mathrm{Br}[K_L \to \ell^+\ell^-]}{\mathrm{Br}[K_L \to \gamma\gamma]} = 2\sigma_\ell(M_K^2)\left(\frac{\alpha}{\pi}r_\ell\right)^2 \big|\mathcal{A}_\ell(M_K^2)\big|^2 \qquad r_\ell = \frac{m_\ell}{M_K} \qquad \sigma_\ell(M_K^2) = \sqrt{1-\frac{4m_\ell^2}{M_K^2}}$$

$$\mathrm{Im}_{\gamma\gamma}\mathcal{A}_\ell(M_K^2) = \frac{\pi}{2\sigma_\ell(M_K^2)}\log\big[y_\ell(M_K^2)\big] \qquad y_\ell(M_K^2) = \frac{1-\sigma_\ell(M_K^2)}{1+\sigma_\ell(M_K^2)}$$

$$\mathrm{Re}\,\mathcal{A}_\ell(M_K^2) = \frac{1}{\sigma_\ell(M_K^2)}\bigg[\mathrm{Li}_2\big[-y_\ell(M_K^2)\big] + \frac{1}{4}\log^2\big[y_\ell(M_K^2)\big] + \frac{\pi^2}{12}\bigg] + 3\log\frac{m_\ell}{\mu} - \frac{5}{2} + \chi(\mu)$$

- Contributions to $\chi(\mu)$
  - Long-distance SM
  - Short-distance SM
  - BSM



- Experimental status:

$$\mathrm{Re}\,\mathcal{A}_\mu^{\mathrm{exp}}(M_K^2) = \pm 1.16(24)$$

$K_L \to \mu^+\mu^-$: short-distance in SM

- Short-distance contribution in SM Isidori, Unterdorfer 2004, Gorbahn, Haisch 2006

$$\chi_{\mathrm{SD}}^{\mathrm{SM}} = -\tilde{\kappa}\Big(\mathrm{Re}\,\lambda_t Y(x_t) + \lambda^4 \mathrm{Re}\,\lambda_c P_c\Big) = -1.83(7)$$

$$|\tilde{\kappa}| = \sqrt{\frac{M_K}{16\pi\Gamma[K_L \to \gamma\gamma]}}\,\frac{\sqrt{2}\,G_F M_K F_K \alpha(M_Z)}{\sin^2\theta_W \alpha(0)} = 4988.4(23.6)$$

using $P_c = 0.111(10)$ Gorbahn, Haisch 2006, $Y(x_t) = 0.931(5)$ Brod, Stamou 2023

- Relative sign to LD contribution Isidori, Unterdorfer 2004:
  - Need to know sign of $K_L \to \gamma\gamma$ amplitude
  - Dominated by π^0 contribution

$$c(0,0)\big|_{\pi^0} = \frac{2G_8 F_\pi \alpha}{\pi}\,\frac{M_K^2}{M_K^2 - M_\pi^2}$$

$$\big|c(0,0)\big|_{\pi^0} \simeq 4.2 \times 10^{-9}\,\mathrm{GeV}^{-1} \qquad |c(0,0)|_{\mathrm{exp}} = 3.389(14) \times 10^{-9}\,\mathrm{GeV}^{-1}$$

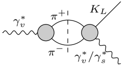
$\hookrightarrow$ assume $\mathrm{sgn}(\mathcal{A}[K_L \to \gamma\gamma]) = \mathrm{sgn}(\mathcal{A}[K_L \to \pi^0 \to \gamma\gamma])$

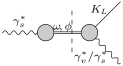
  - Assume factorization $\mathrm{sgn}(\langle\pi^0|\mathcal{H}_W|K_L\rangle) = \mathrm{sgn}(\sum_i C_i(\mu)\langle\pi^0|O_i(\mu)|K_L\rangle_{N_c\to\infty})$

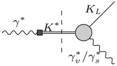
$\hookrightarrow$ need to put $G_8 < 0$

$K_L \to \mu^+\mu^-$: long-distance contribution

- Need $K_L \to \gamma^*\gamma^*$ form factor







$$\mathcal{A}^{\mu\nu}[K_L \to \gamma^*(q_1,\mu)\gamma^*(q_2,\nu)] = i\epsilon^{\mu\nu\alpha\beta} q_{1\alpha} q_{2\beta}\, c(q_1^2,q_2^2)$$

- Lots of experience from $\pi^0,\eta,\eta' \to \gamma^*\gamma^*$ in the context of HLbL for $(g-2)_\mu$
- Strategy:
  - Normalization from on-shell process
  - Constraints from leptonic decays, $K_L \to \ell^+\ell^-\gamma$ etc.
  - Constraints from hadronic decays, $K_L \to \pi^+\pi^-\gamma$, via dispersion relation

    ↪ access to data for spectrum?
  - Matching to asymptotic constraints, formulated as a dispersion relation

$$c^{\text{asym}}(q_1^2,q_2^2) = \frac{16\alpha G_F V_{us}^* V_{ud} F_K}{9\pi\sqrt{2}} \big[C_2(\mu) + 3C_1(\mu)\big] \Big[I(q_1^2,q_2^2) + T(q_1^2) + T(q_2^2)\Big]$$

$$I(q_1^2,q_2^2) = \frac{m_c^2}{\pi} \int_{s_c}^{\infty} dx \frac{2\pi \log \frac{1+\sigma_c(x)}{1-\sigma_c(x)}}{(x-q_1^2)(x-q_2^2)}$$

↪ major uncertainty from running of $C_i(\mu)$

$K_L \to \mu^+\mu^-$: long-distance–short-distance ambiguity

- Need to predict LD contribution to $\chi(\mu)$ to extract BSM constraint
- Dispersive representation includes $\pi\pi\gamma$ and $3\pi\gamma$ intermediate states

$$\text{Im}\,\mathcal{A}_\mu(M_K^2) = -5.20(0) \qquad \text{Im}_{\gamma\gamma}\mathcal{A}_\mu(M_K^2) = -5.21$$

$\hookrightarrow$ imaginary part completely dominated by $\gamma\gamma$ cut

- Real part currently being finalized (numbers preliminary!)

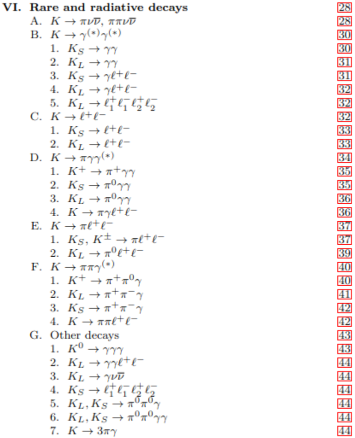
$$\text{Re}\,\mathcal{A}_\mu(M_K^2) = 0.xx(8)_{\text{disp}}(47)_{\text{asym}}(17)_{\text{exp}}$$

- Asymptotic matching:
  - Use LL RG for $C_i(\mu)$ with $\mu^2 = \frac{q_1^2+q_2^2}{2}$
  - Keep $C_i(\mu)$ constant below scale $\mu_{\text{cut}} \in [2,4]$ GeV

Radiative channels

Q3

How well can the radiative decays be predicted in the continuum and for which channels (now and future)?



Cirigliano, Ecker, Neufeld, Pich, Portolés 2012

Limitations of conventional methods

Q4

Are there any limitations going forward where conventional methods hit a wall? What is the strategy going forward?

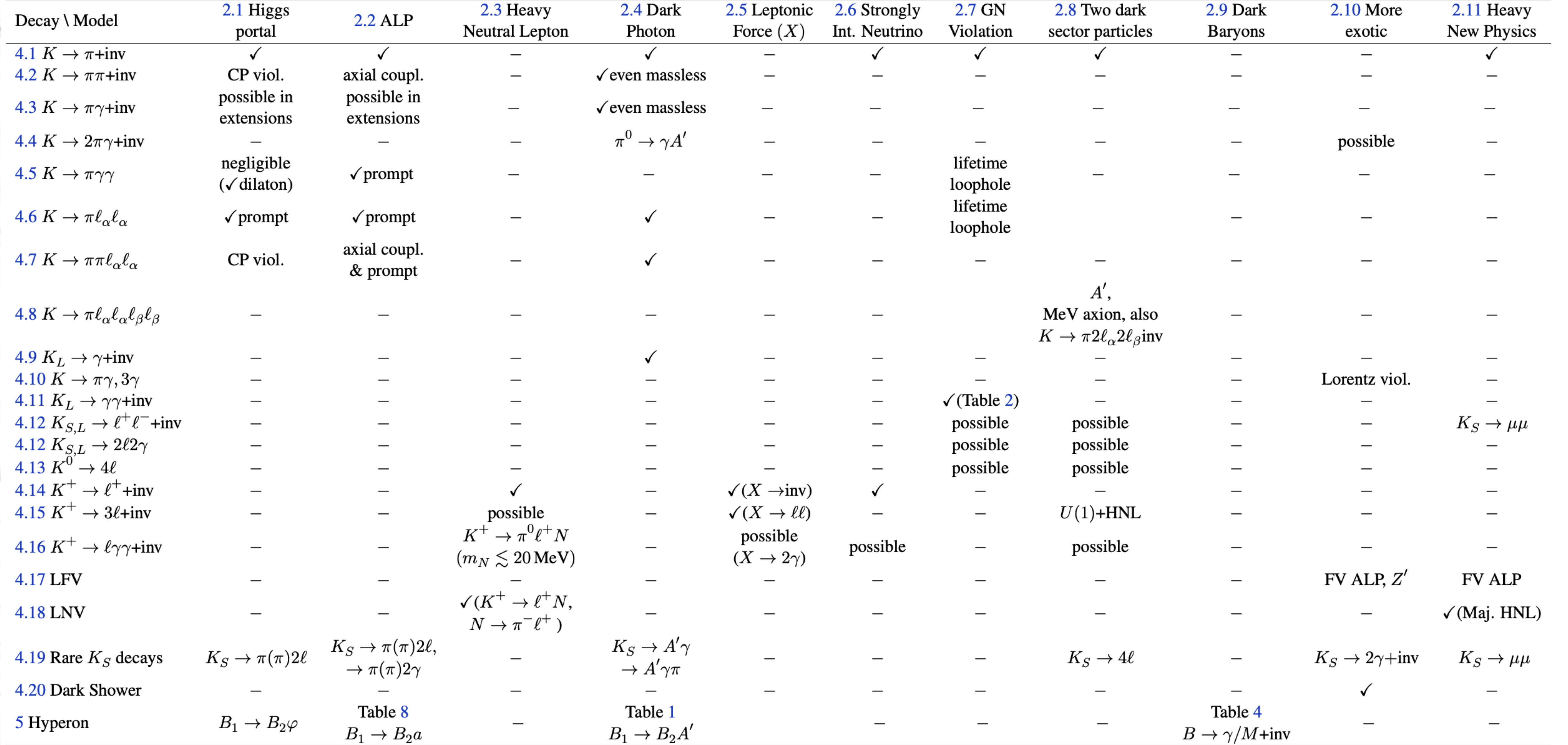
- What are the "conventional methods"? If ChPT, possible strategies include:
  - Dispersion relations to extend range in energy (unitarization) talk by P. Stoffer
  - Dispersive techniques to constrain low-energy constants, e.g., radiative corrections to $K_{\ell 3}$ decays Seng, Galviz, Gorchtein, Meißner 2021, 2022
  - Matching to lattice QCD to determine low-energy constants
  - Combination of the above

Exotics/forbidden

Q5

Are there any searches for exotics/forbidden that are particularly interesting?

- Are there any searches for exotics/ forbidden that are particularly interesting?

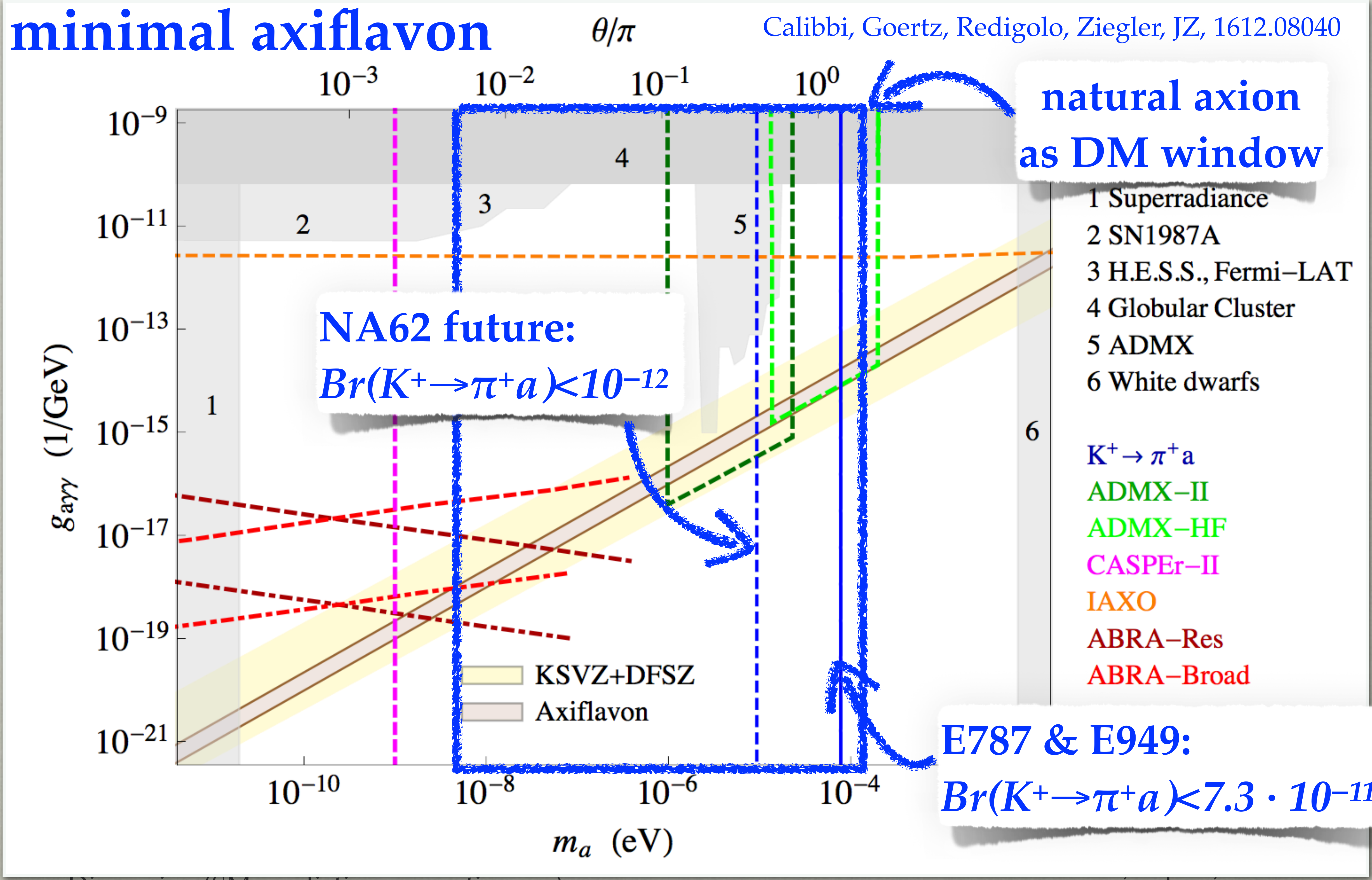


Decay \ Model	2.1 Higgs portal	2.2 ALP	2.3 Heavy Neutral Lepton	2.4 Dark Photon	2.5 Leptonic Force (X)	2.6 Strongly Int. Neutrino	2.7 GN Violation	2.8 Two dark sector particles	2.9 Dark Baryons	2.10 More exotic	2.11 Heavy New Physics
4.1 $K \to \pi$+inv	✓	✓	–	✓	–	✓	✓	✓	–	–	✓
4.2 $K \to \pi\pi$+inv	CP viol.	axial coupl.	–	✓even massless	–	–	–	–	–	–	–
4.3 $K \to \pi\gamma$+inv	possible in extensions	possible in extensions	–	✓even massless	–	–	–	–	–	–	–
4.4 $K \to 2\pi\gamma$+inv	–	–	–	$\pi^0 \to \gamma A'$	–	–	–	–	–	possible	–
4.5 $K \to \pi\gamma\gamma$	negligible (✓dilaton)	✓prompt	–	–	–	–	lifetime loophole	–	–	–	–
4.6 $K \to \pi\ell_\alpha\ell_\alpha$	✓prompt	✓prompt	–	✓	–	–	lifetime loophole		–	–	–
4.7 $K \to \pi\pi\ell_\alpha\ell_\alpha$	CP viol.	axial coupl. & prompt	–	✓	–	–	–	–	–	–	–
4.8 $K \to \pi\ell_\alpha\ell_\alpha\ell_\beta\ell_\beta$	–	–	–	–	–	–		A', MeV axion, also $K \to \pi 2\ell_\alpha 2\ell_\beta$inv	–	–	–
4.9 $K_L \to \gamma$+inv	–	–	–	✓	–	–	–	–	–	–	–
4.10 $K \to \pi\gamma, 3\gamma$	–	–	–	–	–	–	–	–	–	Lorentz viol.	–
4.11 $K_L \to \gamma\gamma$+inv	–	–	–	–	–	–	✓(Table 2)	–	–	–	–
4.12 $K_{S,L} \to \ell^+\ell^-$+inv	–	–	–	–	–	–	possible	possible	–	–	$K_S \to \mu\mu$
4.12 $K_{S,L} \to 2\ell 2\gamma$	–	–	–	–	–	–	possible	possible	–	–	–
4.13 $K^0 \to 4\ell$	–	–	–	–	–	–	possible	possible	–	–	–
4.14 $K^+ \to \ell^+$+inv	–	–	✓	–	✓($X \to$inv)	✓	–	–	–	–	–
4.15 $K^+ \to 3\ell$+inv	–	–	possible	–	✓($X \to \ell\ell$)	–	–	$U(1)$+HNL	–	–	–
4.16 $K^+ \to \ell\gamma\gamma$+inv	–	–	$K^+ \to \pi^0\ell^+N$ ($m_N \lesssim 20\,\text{MeV}$)	–	possible ($X \to 2\gamma$)	possible	–	possible	–	–	–
4.17 LFV	–	–	–	–	–	–	–	–	–	FV ALP, Z'	FV ALP
4.18 LNV	–	–	✓($K^+ \to \ell^+N$, $N \to \pi^-\ell^+$)	–	–	–	–	–	–	–	✓(Maj. HNL)
4.19 Rare K_S decays	$K_S \to \pi(\pi)2\ell$	$K_S \to \pi(\pi)2\ell$, $\to \pi(\pi)2\gamma$	–	$K_S \to A'\gamma$ $\to A'\gamma\pi$	–	–	–	$K_S \to 4\ell$	–	$K_S \to 2\gamma$+inv	$K_S \to \mu\mu$
4.20 Dark Shower	–	–	–	–	–	–	–	–	–	✓	–
5 Hyperon	$B_1 \to B_2\varphi$	Table 8 $B_1 \to B_2 a$	–	Table 1 $B_1 \to B_2 A'$		–	–	–	Table 4 $B \to \gamma/M$+inv	–	–

Are there any searches for exotics / forbidden that are particularly interesting?

- how to rank models
  - simplicity?
  - does it solve any other problem?
  - interesting signatures (signature-building)?
  - no stone left unturned...
- each approach has problems
- for presentation purposes makes sense
  - highlight models that are not kaon centric, plot in parameter space (comparison with ATLAS, CMS, LHCb,....)
  - use PBC benchmarks (comparison with SHiP)

SEARCHING FOR AXIONS/ AXIFLAVONS



minimal axiflavon

Calibbi, Goertz, Redigolo, Ziegler, JZ, 1612.08040

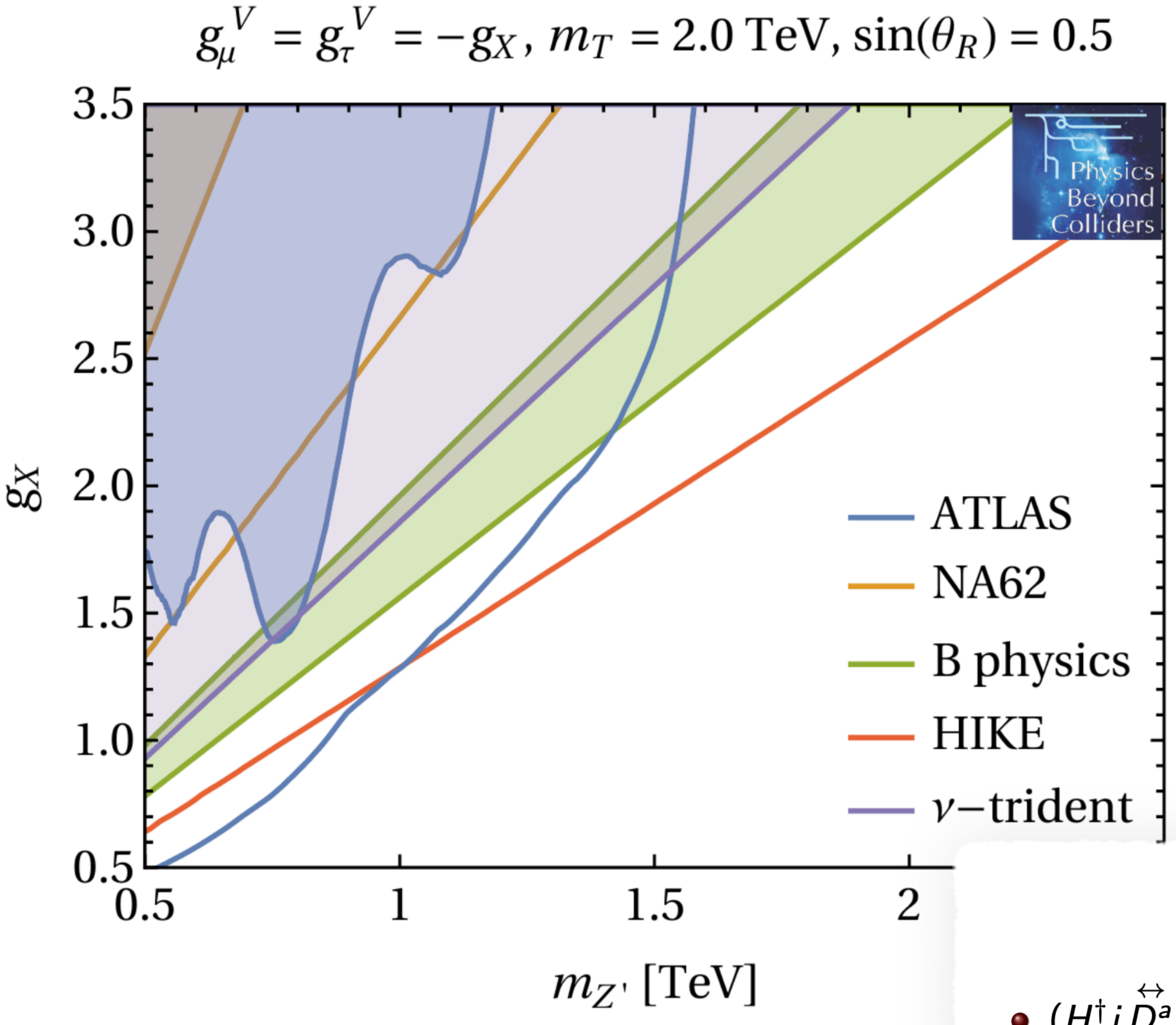
natural axion as DM window

NA62 future:
$Br(K^+ \to \pi^+ a) < 10^{-12}$

E787 & E949:
$Br(K^+ \to \pi^+ a) < 7.3 \cdot 10^{-11}$

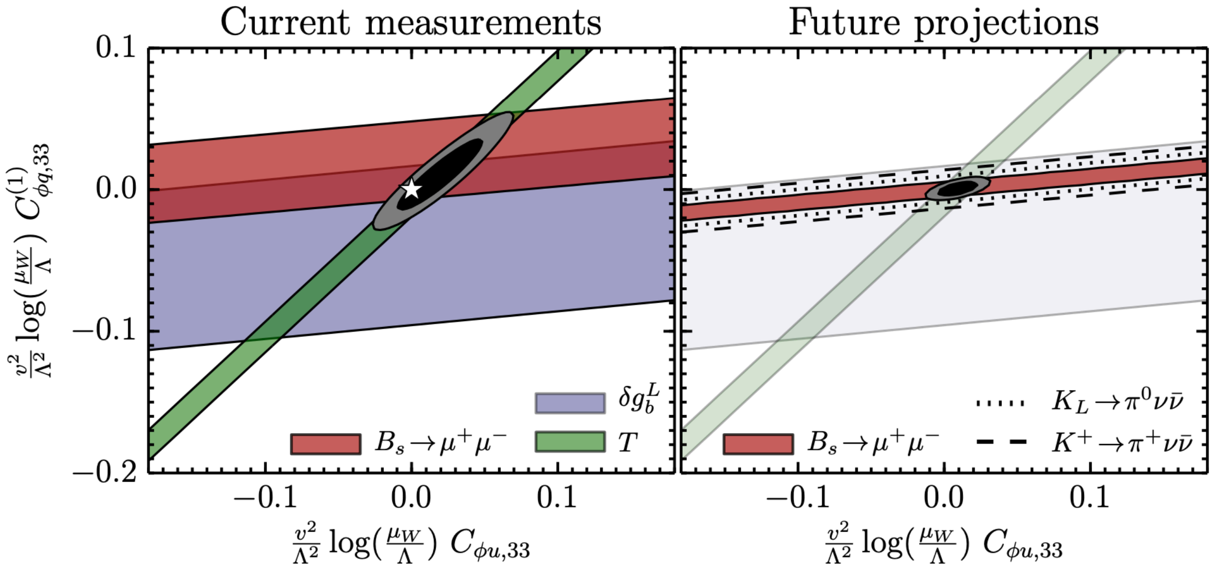
Discussion (SM predictions -- continuum)

Z′



ttZ couplings

- $(H^\dagger i \overset{\leftrightarrow}{D}{}^a_\mu H)(\bar{Q}_{L,3}\gamma^\mu \sigma^a Q_{L,3})$, $(H^\dagger i \overset{\leftrightarrow}{D}_\mu H)(\bar{Q}_{L,3}\gamma^\mu Q_{L,3})$, $(H^\dagger i \overset{\leftrightarrow}{D}_\mu H)(\bar{t}_R\gamma^\mu t_R)$

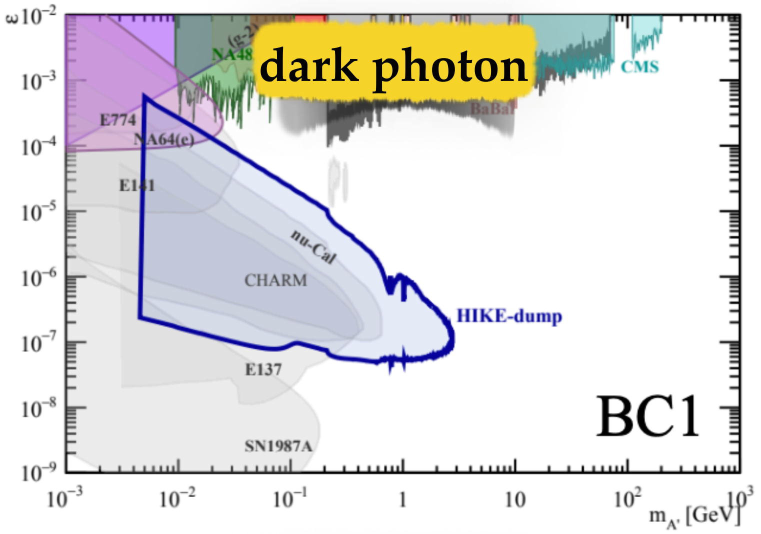


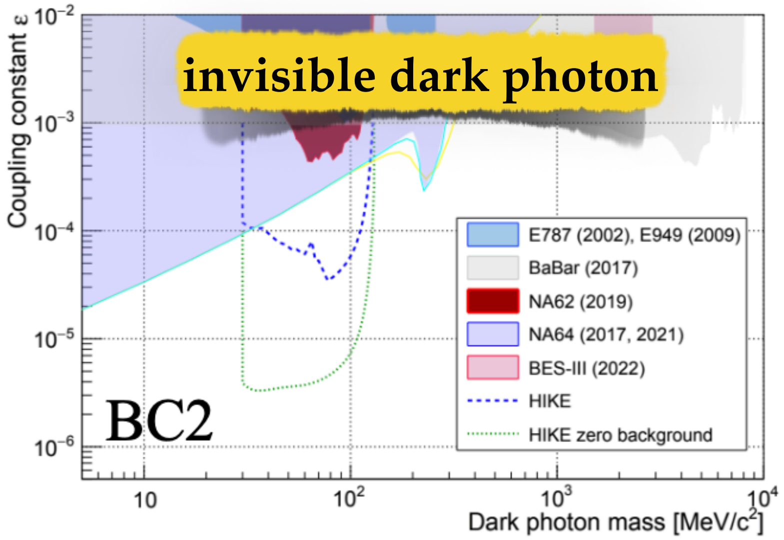
[Brod, Greljo, Stamou, Uttayarat 1408.0792]

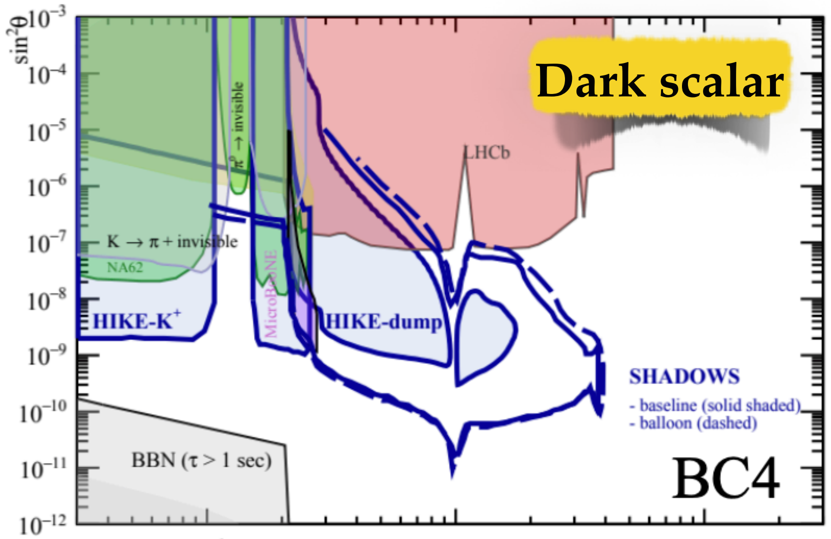
M. Hoferichter, J. Zupan,
Discussion (SM predictions -- continuum)

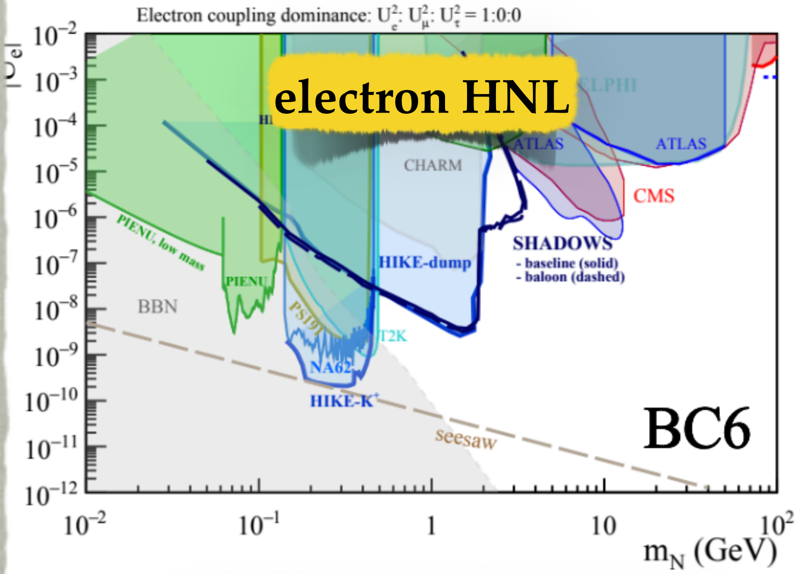
Feebly-interacting particles @ HIKE

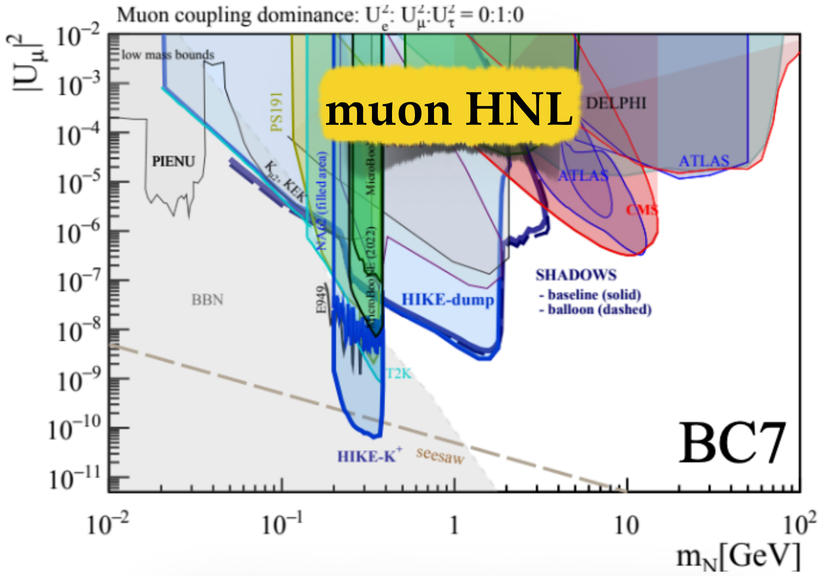
HIKE sensitive to all BC benchmarks except BC3 & BC5

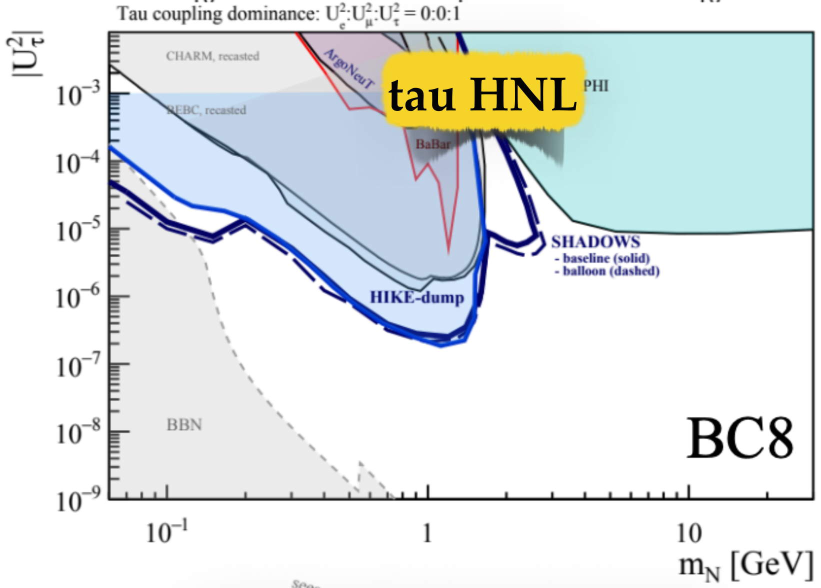


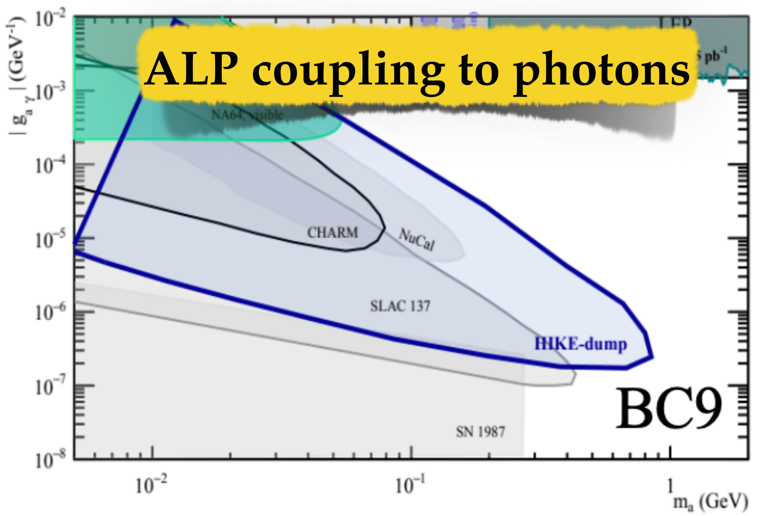


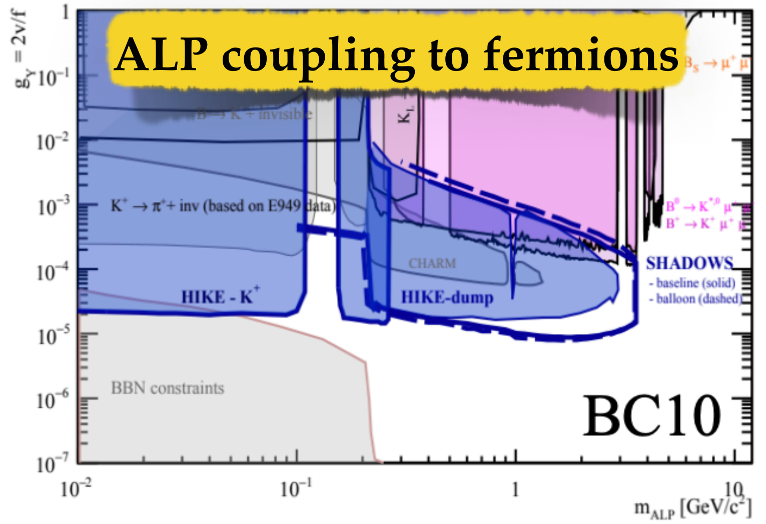


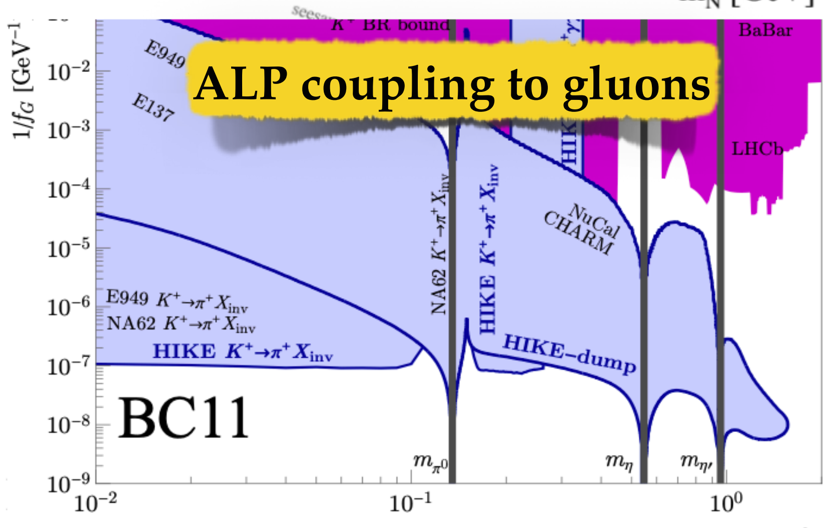












Discussion (SM predictions -- continuum)

Theoretical uncertainties

Q6

What are the leading theoretical uncertainties to be addressed in the kaon sector?

- Depends on channel:
    - Parametric uncertainties
    - Long-range contributions
    - Perturbative corrections

Hyperon and η decays

Q7

What are the prospects for predictions for hyperon and η decays?

- **η decays** Gan, Kubis, Passemar, Tulin 2022
  - Pretty good, lots of work done recently for $g-2$ and JEF/REDTOP
- **Hyperons**
  - Tough to compete with kaon decays for $|V_{us}|$, otherwise complementarity already discussed yesterday
  - In general, precise predictions for baryonic systems more challenging than for mesons (both for continuum and lattice)

Wish list

Q8

What would a wish list for experiment look like now and for the coming decade, in order to make best use of experimental and theory (continuum) efforts?