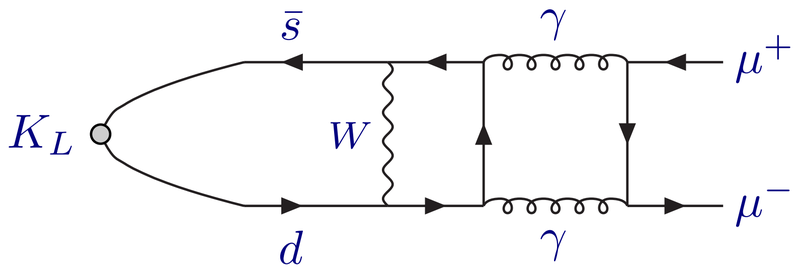
3) **Theoretical questions: lattice**
    a) Prospect of lattice results for kaons in the next 5 - 10 years
    b) What can we still learn from kaons within SM?
    c) Can we predict epsilonK at the level of the experimental uncertainty (~ 0.5%), what is needed for that, and is it worth the effort?
    d) How well can the radiative decays be predicted and for which channels? Any show-stoppers? What precision is conceivable on a 5-10y time scale?
    e) Which channels will likely be addressed for QCD+QED+strongIB in the coming decade?
    f) How well/which rare decays, not golden channels, can be predicted ?
    g) What are the prospects for predictions for hyperon and eta decays?
    h) What would a wish list for experiment look like now and for the coming decade, in order to make best use of experimental and theory (lattice) efforts?
    i) Interplay lattice theory for K+ → pi+ l+ l-/KS->pi0 l+ l-: how to optimize the knowledge from both approaches
    j) Interplay lattice theory for KL → mu+mu- |_LD : how to optimize the knowledge from both approaches

$K_L \to \mu^+\mu^-$

- I have not been able to discuss the long-term RBC-UKQCD project, computing the two-photon contribution to the decay $K_L \to \mu^+\mu^-$ from diagrams such as



- Experimental result:
  $B(K_L \to \mu^+\mu^-) = (6.84 \pm 0.11) \times 10^{-9}$

- A number of preparatory/exploratory studies have been performed:

  - A calculation of the amplitude for the related, but simpler process $\pi^0 \to e^+e^-$ has been performed with physical pion masses, on a series of lattices so that the continuum limit can be taken:

    $$\mathrm{Re}\,A = 18.60\,(1.19)_{\mathrm{stat}}\,(1.04)_{\mathrm{syst}}\,\mathrm{eV}, \quad \mathrm{Im}\,A = 32.59\,(1.50)_{\mathrm{stat}}\,(1.65)_{\mathrm{syst}}\,\mathrm{eV}\,, \quad \frac{\mathrm{Re}\,A}{\mathrm{Im}\,A} = 0.571\,(10)_{\mathrm{stat}}\,(4)_{\mathrm{syst}}$$

    to be compared to the experimental numbers:

    $$\mathrm{Re}\,A = 24.10\,(2.0)\,\mathrm{eV}, \quad \mathrm{Im}\,A = 35.07\,(37)\,\mathrm{eV}$$

    N.H.Christ, X.Feng, L.Jin, C.Tu and Y.Zhao, 2208.03834

  - A strategy and exploratory calculation of the amplitude for the CP-concerning contribution to the amplitude for the $K_L \to \gamma\gamma$ decay was also presented. N.H.Christ and Y.Zhao, PoS (Lattice 2021) 2022 451

  - At Lattice 2023 an update on the project was presented by En-Hung Chao with a focus on the $\pi\pi\gamma$ intermediate state.

Decays with a γ^* in the intermediate state

- In exploratory computations with unphysical quark masses two groups have calculate the amplitudes/rates for the decays $K^+ \to \ell^+ \nu_\ell \gamma^* \to \ell^+ \nu_\ell \ell'^+ \ell'^-$, with both $\ell \neq \ell'$ and $\ell = \ell'$.

X.-Y.Tuo et al. , arXiv:2103.11331
G.Gagliardi et al., arXiv:2202.03883

  - In these computations the masses are such that $m_K < 2m_\pi$.
  - For physical masses a complication occurs when $q^2 > 4m_\pi^2$, where q is the momentum of the γ^*.
  - Interesting methods, based on the evaluation of the spectral density, are being developed, to deal with this (particularly for the decays of heavy mesons.
  - Comments from lattice theorists???
  - Which quantities with a virtual photon should we be considering?

Thank you so much for organising this most stimulating and interesting workshop

So much exciting kaon physics to look forward to !!!