Binned Log-Likelihood Template Parameter Fitting In Action Applied to a $\nu_e CC\pi^+$ ND280 Analysis

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- Example 'in action' applied to a ν_eCC pion production analysis at T2K.
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- This technique has been used for several recent T2K measurements; two of which have primary authors present at NuXTract:

→ ν_μCC$1\pi^+$ using transverse kinematic imbalance, Phys. Rev. D **103**, 112009 (2021).

→ ND280-INGRID joint ν_μCC0π, arXiv:2303.14228.

→ $\nu_\mu(\bar{\nu}_\mu)$ CC-COH pion production, arXiv:2308.16606.

→ Complete list of publications in M. Buizza Avanzini's talk.

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Many more at late-stage, including the analysis discussed in this talk!

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- We want to measure the cross section, σ

$$\left(\frac{d\sigma}{dx}\right)_i = \frac{\hat{N}_i^{\text{sig}}}{\epsilon_i \Phi N_T} \frac{1}{\Delta x_i}, \tag{1}$$

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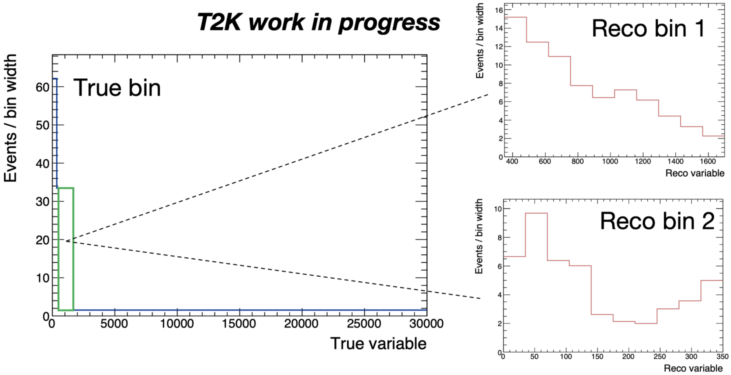
- Number of events in true bin i and reconstructed bin j:

$$N_i^{\text{true}} = \sum_j U_{ij} N_j^{\text{reco}}, \qquad N_j^{\text{reco}} = \sum_i U_{ij}^{-1} N_i^{\text{true}}. \tag{2}$$

- **Unfolding** is where we find U_{ij} by deconvolving resolution effects.

Introduction

- Template fitting:
  - Assign normalisation weights, c_i, to each truth space bin
  - c_i are iteratively changed during the fitting process
  - Changes in c_i will increase or decrease N^{sig} in a true bin
  - Observe effects on reconstructed distributions



Template Fitting in Action

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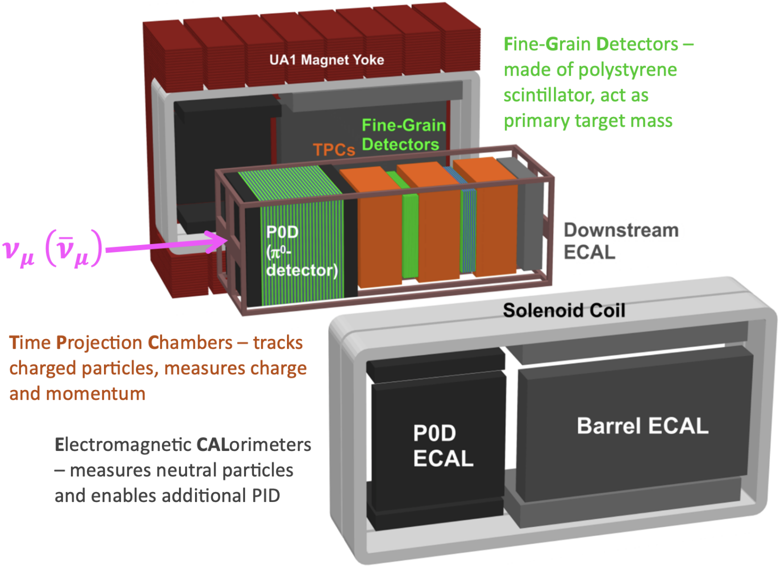
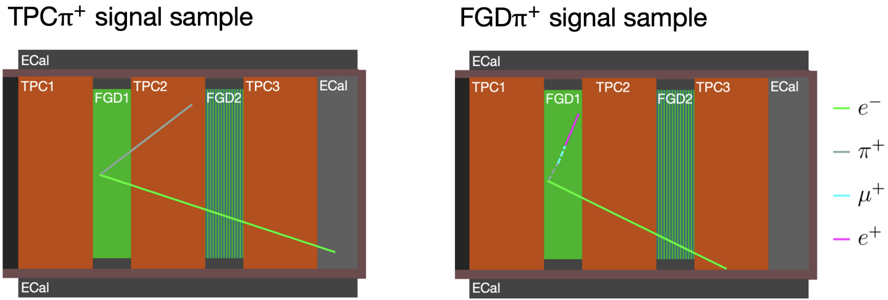


Fine-**G**rain **D**etectors – made of polystyrene scintillator, act as primary target mass

Time **P**rojection **C**hambers – tracks charged particles, measures charge and momentum

Electromagnetic **CAL**orimeters – measures neutral particles and enables additional PID

- Example applied to ν_eCC pion production analysis at ND280:



Signal definition: $\boxed{\nu_e + C \to e^- + \pi^+ + X}$

Only 355 events expected for runs 2-8, $\sim$ 100 selected

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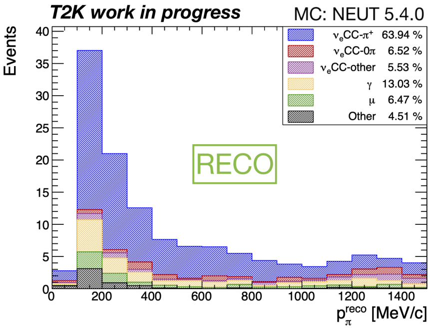

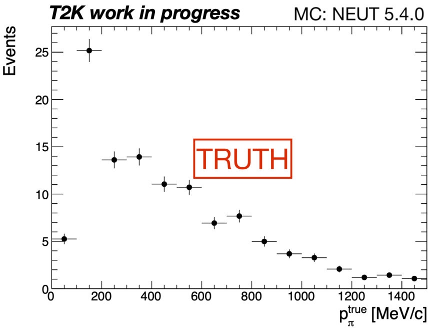

Input variables: $p_e, \cos\theta_e, p_\pi, \cos\theta_\pi$

4D binning:[1] $\Delta x_{ijkl} = \Delta p_{e,i} \Delta\theta_{e,j} \Delta p_{\pi,k} \Delta\theta_{\pi,l}$

[1] See talk on Thursday by **S. Jenkins** for exploration of higher dimensional efficiency corrections

- Template and nuisance parameters in expected event rate model:

$$N_j^{\text{exp}} = \sum_i^{\text{true}} \left[c_i \left(N_i^{\text{MC, sig}} \prod_a^{\text{model}} w\left(a, \vec{x}\right) \right) + \sum_{ik}^{\text{bkg}} N_{ik}^{\text{MC, bkg}} \prod_a^{\text{model}} w\left(a, \vec{x}\right) \right] t_{ij} d_j \sum_n^{E_\nu} v_{in} f_n. \tag{3}$$

$$\begin{aligned} c_i &: \text{template parameters} \\ w(a, \vec{x}) &: \text{model systematics} \\ d_j &: \text{detector systematics} \\ v_{in} f_n &: \text{flux systematics} \end{aligned} \tag{4}$$

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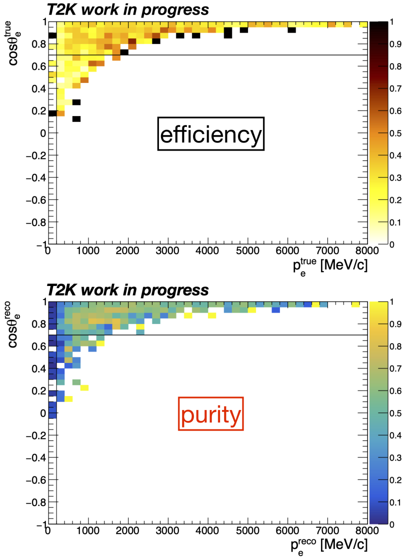
- Other terms: t_{ij} accounts for detector smearing effects, $N_{i,ik}^{\text{MC, sig(bkg)}}$ is the number of signal or background MC events, E_ν is neutrino energy.

- Parameters are iteratively changed to yield best data-MC agreement for reconstructed variables.

- c_i: template parameters
  - Each parameter corresponds to a true bin
  - Weights signal events only
- $w(a, \vec{x})$: model systematics
  - Typically 10-40 depending on backgrounds
  - Weights applied event-by-event using cubic splines
- d_j: detector systematics
  - Each parameter corresponds to a reco bin.
  - Normalisation weights
- $v_{in} f_n$: flux systematics
  - Between 7 and 50 depending on primary neutrino flavour(s)
  - Normalisation weights binned by E_ν



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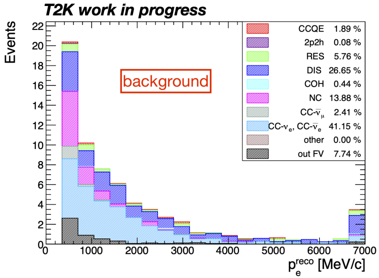

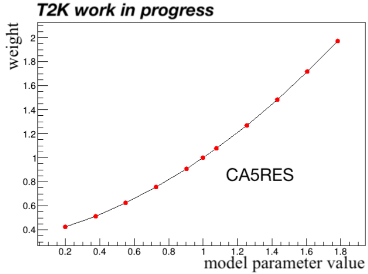

Template Fitting in Action

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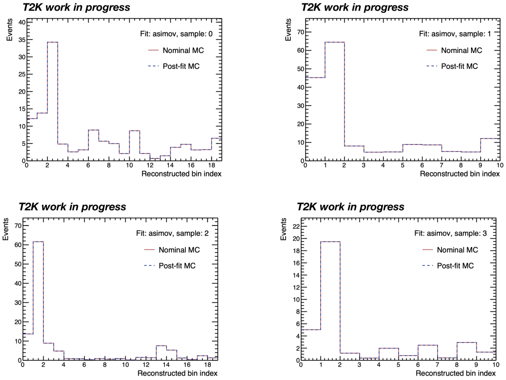
- Four samples corresponding to two signal-enriched samples and their sidebands:



- Strongly analysis-dependent, ranges from one bin to hundreds.

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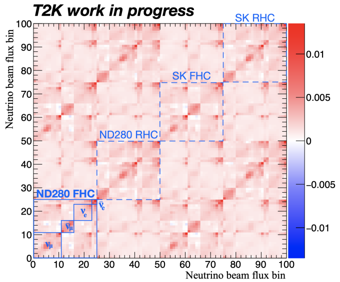
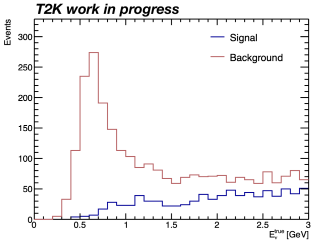
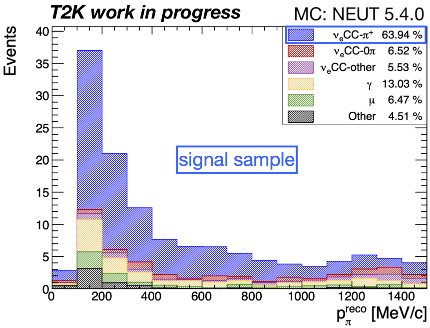
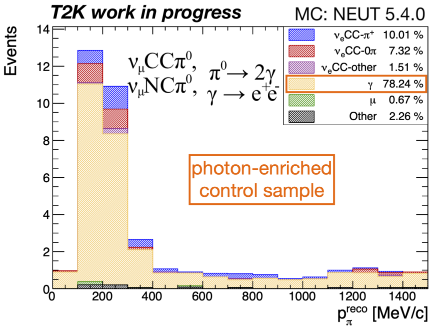




- Template parameters have no prior uncertainty $\rightarrow$ free to alter.
- Nuisance parameters constrained with control samples.





- Note the physics for the control sample may not perfectly match the backgrounds in the signal sample.

- The fitting process minimises the **total chi-square** to yield the best fit parameters:

$$\boxed{\chi^2 = \chi^2_{\mathrm{stat}} + \chi^2_{\mathrm{syst}} = -2\ln\mathcal{L}_{\mathrm{stat}} - 2\ln\mathcal{L}_{\mathrm{syst}}} \tag{5}$$

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$$-2\ln\mathcal{L}_{\text{stat}} = \chi^2_{\text{stat}} = \sum_j^{\text{bins}} 2\left(\beta_j N_j^{\text{exp}} - N_j^{\text{obs}} + N_j^{\text{obs}} \ln\frac{N_j^{\text{obs}}}{\beta_j N_j^{\text{exp}}} + \frac{(\beta_j - 1)^2}{2\sigma_j^2}\right), \quad (6)$$

where β_j is the Beeston-Barlow scaling factor.

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- Systematic penalty terms are incurred for moving systematic parameters away from their nominal values:

$$-2\ln\mathcal{L}_{\text{syst}} = \chi^2_{\text{syst}} = \sum_p \left(\vec{p} - \vec{p}_{\text{prior}}\right)\left(V^{\text{syst}}_{\text{cor}}\right)^{-1}\left(\vec{p} - \vec{p}_{\text{prior}}\right), \tag{7}$$

- Minimisation of total chi-square performed using `Minuit2`[2].

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- We systematically check the fitter outputs for a wide range of pseudo data studies:

Name	Notes
Asimov	Data is the nominal MC
Stat. & syst. fluctuations	Fit and XSec χ^2 coverage checked
Enhanced signal	All signal events re-weighted by $+20\%$
Enhanced DIS	All DIS events re-weighted by $+20\%$
Enhanced flux	All events re-weighted by $+20\%$
Alternate event generator	`GENIE` MC sample used as data
Low Q^2 SPP suppression	CC1π-RES with $Q^2 < 0.7$ GeV^2 re-weighted
SPP adversarial	CC1π with $p_\pi < 0.3$ GeV re-weighted
Martini 1π	Alternative pion production model
1π hadron kinematics	Alternative pion production model
Low E_ν excess	Events with $E_\nu < 1250$ MeV re-weighted
RS $\rightarrow$ DCC re-weight	Alternative pion production model

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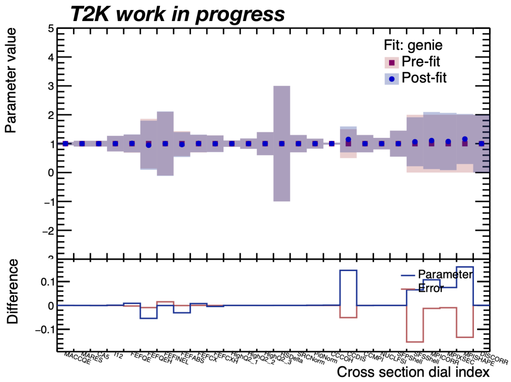
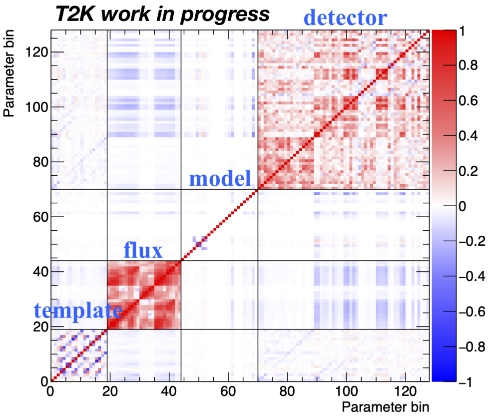
Subsequent slides will use `GENIE` MC as an example pseudo data study.

- Fitting software produces a set of parameters, parameter errors and their correlations:



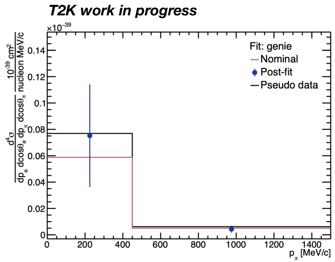


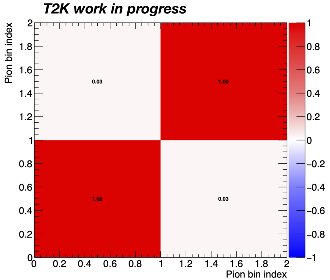
- Fitted to fake data generated using an alternate event model (`GENIE`).

Cross Section Extraction

Cross Section Extraction

- Post-fit parameters and their errors correspond to an uncertainty on $\hat{N}^{\text{sig}}$.
- Template fitting also predicts ϵ and Φ.
- Towards a cross section result:
  - Use post-fit covariance matrix to generate random parameter values and values for additional parameters
  - Repeat process for a large number of toy experiments ($\gtrsim 1000$)
  - Calculate a cross section for each toy; uncertainty constructed from toy distribution
- Example (right) for projected pion cross section.





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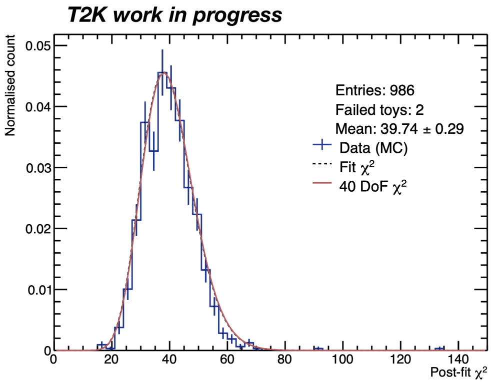
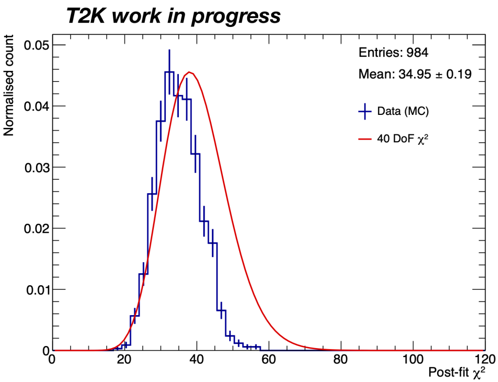
- **MINUIT:**
  - Error logs can indicate failed fits, forced positive-definite Hessians, output χ^2 values by sample at each iteration, current estimated distance to the minimum, etc.

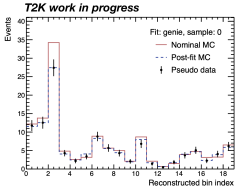
- Example of coverage study:



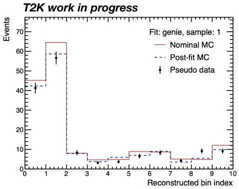


- Left: good coverage, right: over-coverage, errors are too large for our true values.
- Pointed towards problems caused by the flux binning being too fine.

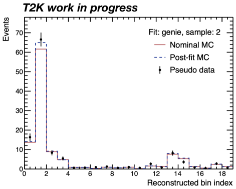
- Can assess the fit performance for reproducing pseudo data distributions:
- Example of reconstructed event distributions:



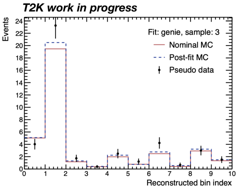
Signal sample I



Signal sample II



Control sample I



Control sample II

- Good overall agreement between post-fit values and fake data.
- Can quantify the fit performance by calculating χ^2 values.

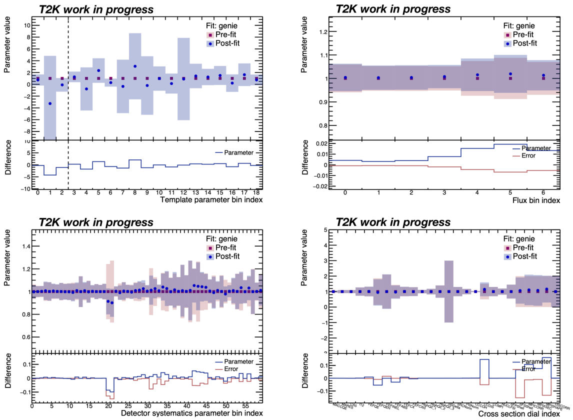
Summary

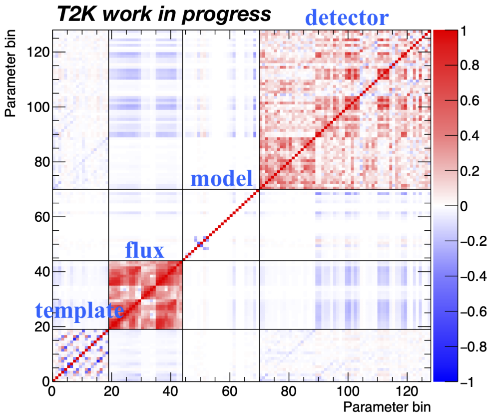
- Template fitting is a useful unfolding strategy for extracting neutrino cross sections.
- Many ongoing analyses use this technique at T2K in various fitters.
- Plentiful diagnostics useful for identifying issues at all stages of fitting and cross section extraction.

For more information, lots of experts on template fitting are present at NuXTract!

Backup

- Fitting software produces a set of parameters, parameter errors and their correlations:





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