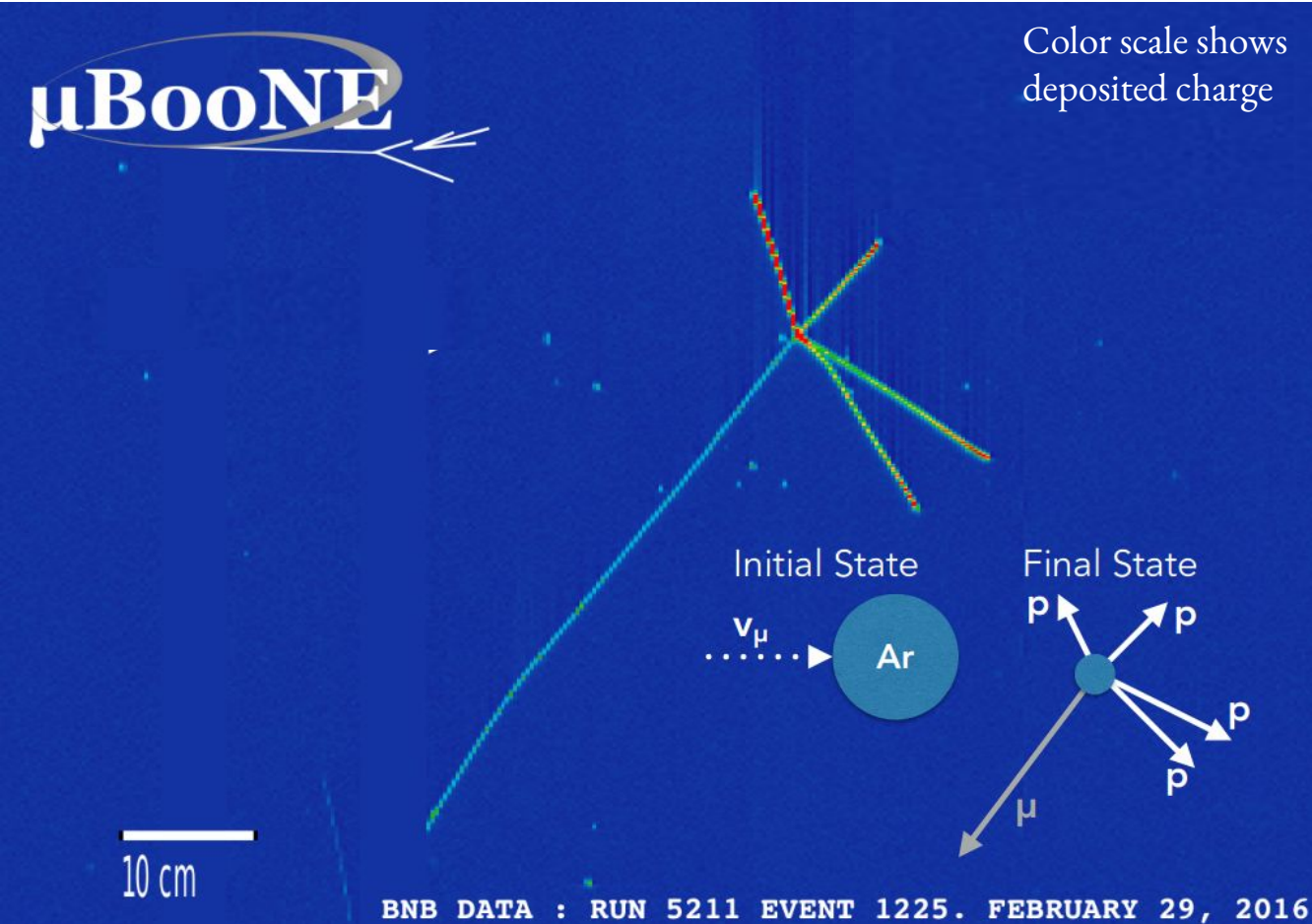


MicroBooNE Cross Section Extraction Techniques

Afroditi Papadopoulou apapadopoulou@anl.gov
on behalf of the MicroBooNE Collaboration
10/3/2023, NuXTract, CERN



MicroBooNE Data Events



- Largest available neutrino-argon data set with ~500k recorded neutrino interactions
- Over 10 released and more than 30 active MicroBooNE cross section analyses
- Multiple cross section extraction techniques investigated

Already Public Results

CC inclusive

- 1D & 2D ν_μ CC inclusive @ BNB
[Phys. Rev. Lett. 123, 131801 \(2019\)](#)
- 1D ν_μ CC E_ν @ BNB
[Phys. Rev. Lett. 128, 151801 \(2022\)](#)
- 3D CC E_ν @ BNB
[arXiv:2307.06413](#), submitted to PRL
- 1D ν_e CC inclusive @ NuMI
[Phys. Rev. D105, L051102 \(2022\)](#)
[Phys. Rev. D104, 052002 \(2021\)](#)

Pion production

- ν_μ NC π^0 @ BNB
[Phys. Rev. D 107, 012004 \(2023\)](#)

CC0 π

- 1D ν_e CCNp0 π @ BNB
[Phys. Rev. D 106, L051102 \(2022\)](#)
- 1D & 2D ν_μ CC1p0 π Kinematic Imbalance @ BNB
[Phys. Rev. Lett. 131, 101802 \(2023\)](#)
[Phys. Rev. D 108, 053002 \(2023\)](#)
- 1D ν_μ CC1p0 π @ BNB
[Phys. Rev. Lett. 125, 201803 \(2020\)](#)
- 1D ν_μ CC2p @ BNB
[arXiv:2211.03734](#), submitted to PRL
- 1D ν_μ CCNp0 π @ BNB
[Phys. Rev. D102, 112013 \(2020\)](#)

Rare channels

- η production @ BNB
[arXiv:2305.16249](#), submitted to PRL
- Λ production @ NuMI
[Phys. Rev. Lett. 130, 231802 \(2023\)](#)



Already Public Results

CC inclusive

- 1D & 2D ν_μ CC inclusive @ BNB
[Phys. Rev. Lett. 123, 131801 \(2019\)](#)

- 1D ν_μ CC E_ν @ BNB
[Phys. Rev. Lett. 128, 151801 \(2022\)](#)

- 3D CC E_ν @ BNB
[arXiv:2307.06413](#), submitted to PRL

- 1D ν_e CC inclusive @ NuMI
[Phys. Rev. D105, L051102 \(2022\)](#)

- [Phys. Rev. D104, 052002 \(2021\)](#)

Pion production

- ν_μ NC π^0 @ BNB
[Phys. Rev. D 107, 012004 \(2023\)](#)

Techniques discussed by
Nitish/Xin & Afro

CC0 π

- 1D ν_e CCNp0 π @ BNB
[Phys. Rev. D 106, L051102 \(2022\)](#)

- 1D & 2D ν_μ CC1p0 π Kinematic Imbalance @ BNB
[Phys. Rev. Lett. 131, 101802 \(2023\)](#)
[Phys. Rev. D 108, 053002 \(2023\)](#)

- 1D ν_μ CC1p0 π @ BNB
[Phys. Rev. Lett. 125, 201803 \(2020\)](#)

- 1D ν_μ CC2p @ BNB
[arXiv:2211.03734](#), submitted to PRL

- 1D ν_μ CCNp0 π @ BNB
[Phys. Rev. D102, 112013 \(2020\)](#)

Rare channels

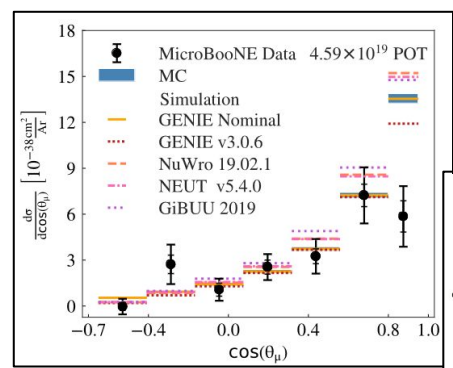
- η production @ BNB
[arXiv:2305.16249](#), submitted to PRL

- Λ production @ NuMI
[Phys. Rev. Lett. 130, 231802 \(2023\)](#)



Techniques

- Single Bin Extraction
- Bayesian Approach
- Forward folding
- Effective efficiency
- D'Agostini

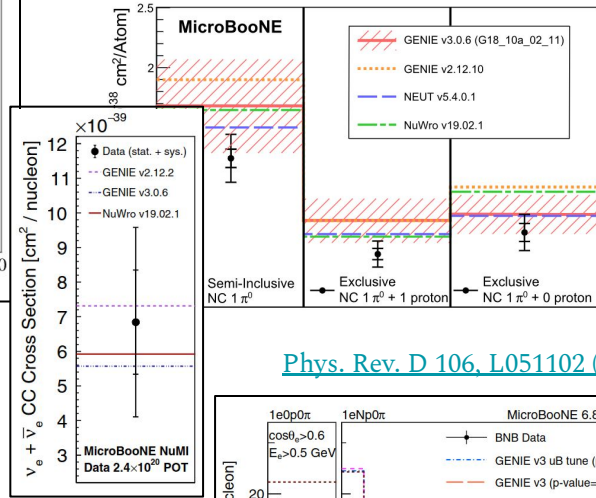


[Phys. Rev. Lett. 125, 201803 \(2020\)](#)

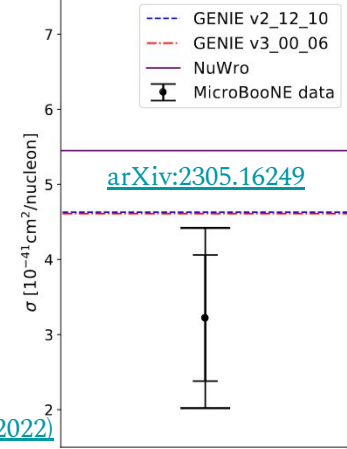
[Phys. Rev. D104, 052002 \(2021\)](#)

[Phys. Rev. D102, 112013 \(2020\)](#)

[Phys. Rev. D 107, 012004 \(2023\)](#)



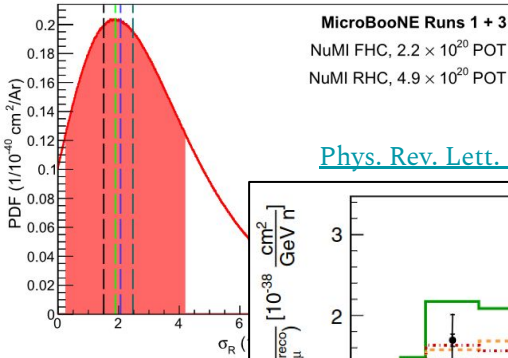
[Phys. Rev. D 106, L051102 \(2022\)](#)



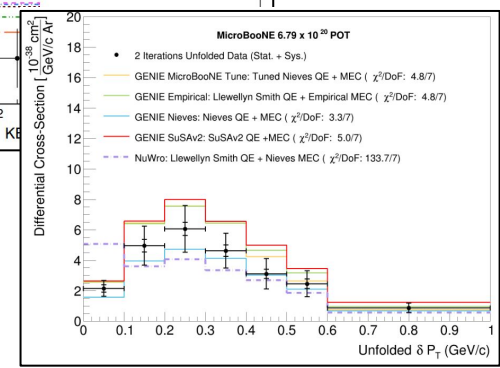
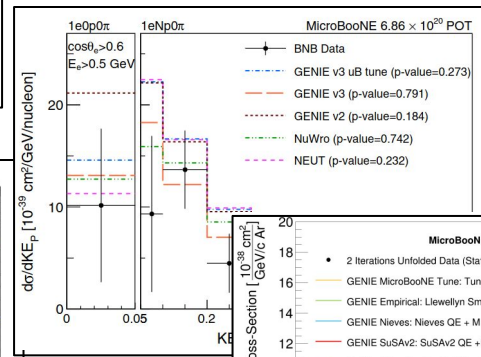
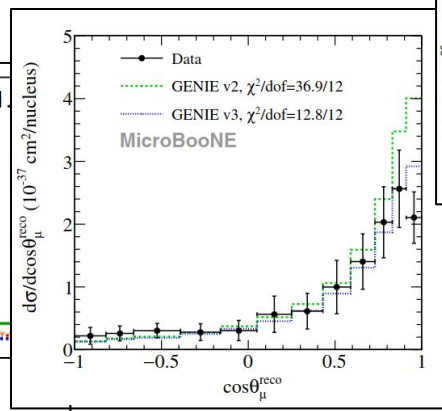
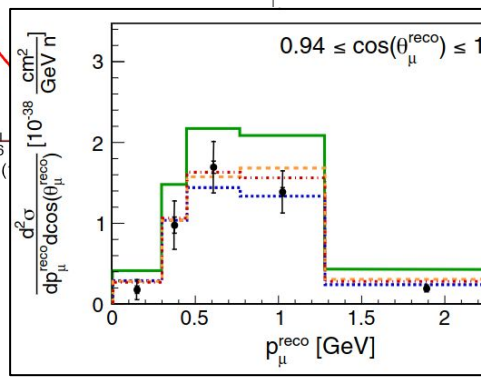
[arXiv:2211.03734](#)

[Phys. Rev. Lett. 130, 231802 \(2023\)](#)

- Posterior Dist.
- 68% CR
- MicroBooNE GENIE Tune
- NuWro Default
- NuWro $M_A = 0.5$ GeV
- NuWro $M_A = 1.5$ GeV



[Phys. Rev. Lett. 123, 131801 \(2019\)](#)



Single Bin Extraction

- Total flux-integrated cross section formula
- Results reported in true space to allow for theory comparisons
- Used in statistically limited analyses
- Plan for differential results with full data set (2x current stats)
- Systematics by allowing ϵ , Φ , and B to fluctuate

$$\sigma = \frac{N - B}{\epsilon \times N_{\text{target}} \times \Phi_{\nu}}$$

Observed data \rightarrow N

Number of expected cosmic and ν -induced backgrounds \rightarrow B

Signal selection efficiency \rightarrow ϵ

Number of argon targets in fiducial volume \rightarrow N_{target}

Integrated flux \rightarrow Φ_{ν}

Bayesian Method

- Use Bayesian method for propagating data / MC statistical uncertainties
- Restricted (R) phase space total cross section due to momenta thresholds
- $\Gamma = 0.64 = \Lambda \rightarrow p + \pi^-$
- Obtain posterior distribution on the background acceptance and efficiency using [TEfficiency](#) class from ROOT

$$\sigma_R = \frac{N_{\text{obs}} - B}{T\Phi\Gamma\epsilon}$$

$$\varphi_\epsilon(\epsilon) = P(\epsilon | \epsilon_{\text{MC}})$$

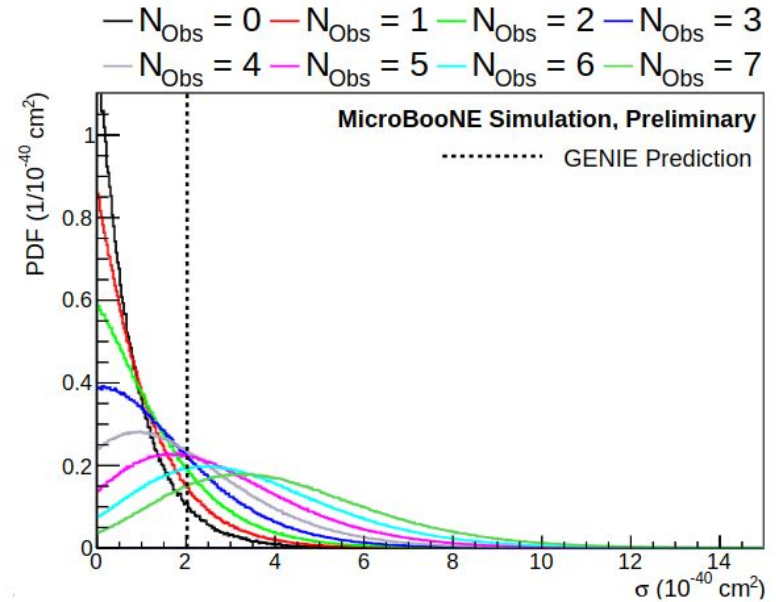
$$\varphi_B(B) = P(B | B_{\text{MC}})$$

Bayesian Method

- Posterior distribution on data event rate

$$P(N|N_{\text{obs}}) = \frac{P(N_{\text{obs}}|N)P(N)}{\int P(N_{\text{obs}}|N)P(N)dN}$$

- Use uniform priors
- Throw many values of ε , B and N from their respective posterior distributions
- Systematic uncertainties are included by throwing fluctuations of these, using the covariance matrix of B , ε and Φ
- Build the posterior distribution on σ_R



Bayesian posterior distributions
on extracted cross section for a
given number of data events

Forward Folding

- Lives in reconstructed space
- Avoids biases introduced by unfolding
- Requires **overall efficiency** (selection eff x acceptance) in reconstructed bin i (MC)
- Smearing matrix **S** included in data release to allow testing against other theory predictions
- Does not propagate MC stat uncertainties on smearing matrix ([arXiv:2112.09194](https://arxiv.org/abs/2112.09194))
- Systematics by allowing $\tilde{\epsilon}$, Φ , and B to fluctuate

$$\left\langle \frac{d\sigma}{dp_\mu^{\text{reco}}} \right\rangle_i = \frac{N_i - B_i}{\tilde{\epsilon}_i N_{\text{target}} \cdot \Phi_{\nu_\mu} \cdot (\Delta p_\mu)_i}$$

$$N_i^{\text{reco}} = \sum_j S_{ij} N_j^{\text{true}} \quad \tilde{\epsilon}_i = \frac{\sum_j S_{ij} N_j^{\text{sel}}}{\sum_j S_{ij} N_j^{\text{gen}}}$$

$S_{ij} = P(\text{observed in bin } i \mid \text{true value in bin } j)$

Smearing Matrix (p_μ)

Reco Bin	1	2	3	4	5	6	OF
OF	0.00	0.00	0.00	0.00	0.00	0.02	0.14
6	0.02	0.01	0.01	0.01	0.09	0.52	0.54
5	0.04	0.01	0.01	0.12	0.66	0.36	0.21
4	0.07	0.04	0.17	0.72	0.22	0.08	0.09
3	0.19	0.25	0.67	0.13	0.02	0.02	0.02
2	0.47	0.64	0.13	0.01	0.01	0.00	0.00
1	0.21	0.05	0.00	0.00	0.00	0.00	0.00
True Bin	1	2	3	4	5	6	OF

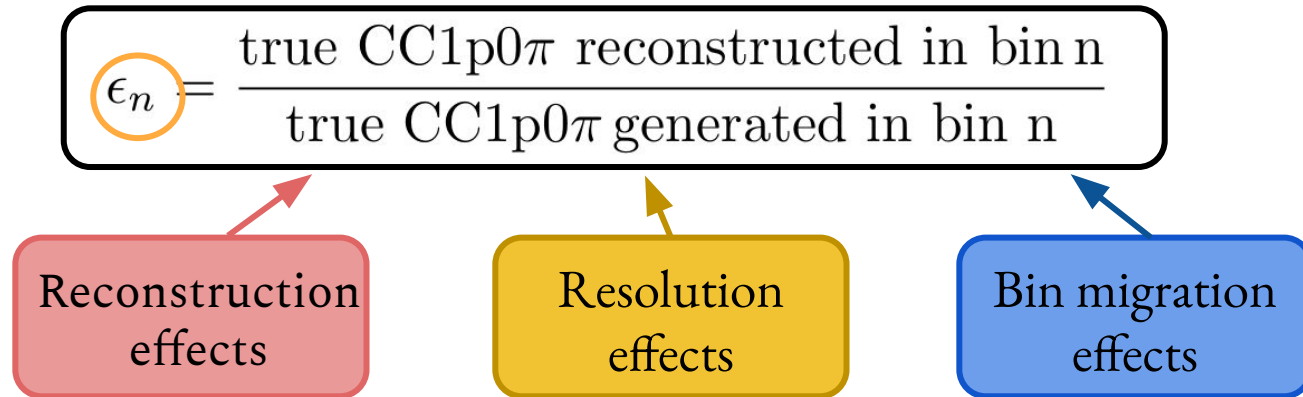
[Phys. Rev. Lett. 123, 131801 \(2019\)](https://arxiv.org/abs/2112.09194)
[Phys. Rev. D102, 112013 \(2020\)](https://arxiv.org/abs/2112.09194)

Effective Efficiency

- Unfolded in true space
- Model dependent
- No impact on the analysis conclusions due to large uncertainties
- New variations of the analysis with smaller uncertainties used Wiener SVD
- Systematics by allowing ϵ , Φ , and B to fluctuate

$$\frac{d\sigma}{dX_n} = \frac{N_n^{\text{on}} - N_n^{\text{off}} - B_n}{\epsilon_n \cdot \Phi_\nu \cdot N_{\text{target}} \cdot \Delta_n}$$

X = kinematic variable of interest



D'Agostini

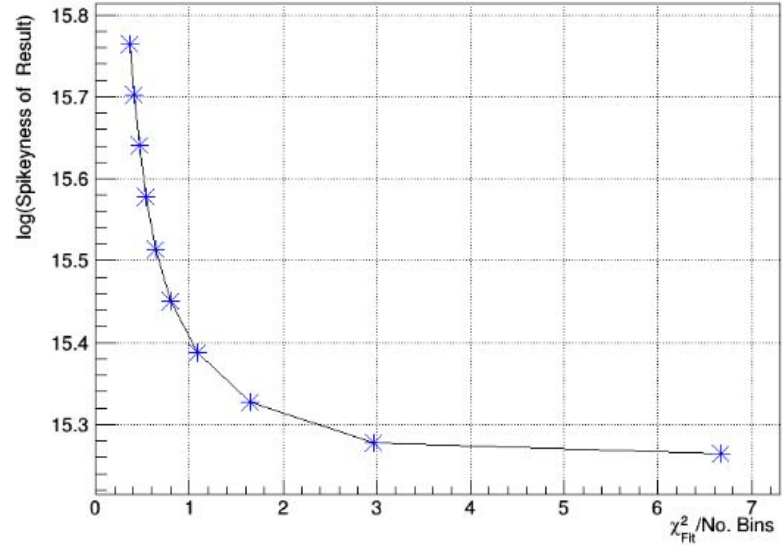
- Lives in true space
- Number of iterations determined based on L-curve and “spikiness vs χ^2 ”
- **S** = smearing matrix
- **P** = prior / best guess for truth
- Systematics by allowing ϵ , Φ , and **B** to fluctuate

Unfolding matrix

$$\left\langle \frac{d\sigma}{dx} \right\rangle_i = \frac{\sum_j U_{ij} (n_j - b_j)}{N_{\text{target}} \times \phi \times (\Delta x)_i}$$

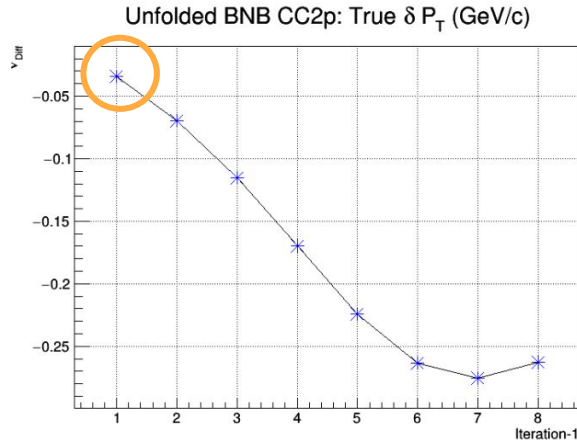
$$U_{T,R} = \frac{S_{T,R} \times P_T}{\sum_R S_{T,R} \times P_T}$$

Unfolded BNB CC2p: True δP_T (GeV/c)



D'Agostini

- Lives in true space
- Number of iterations determined based on L-curve and “spikiness vs χ^2 ”
- **S** = smearing matrix
- **P** = prior / best guess for truth
- Systematics by allowing ϵ , Φ , and **B** to fluctuate



$$\nabla_{Up}(i) = \frac{\text{spikiness}(i+1) - \text{spikiness}(i)}{\chi_{\text{Fit}}^2(i+1) - \chi_{\text{Fit}}^2(i)}$$

$$\nabla_{Down}(i) = \frac{\text{Spikiness}(i) - \text{Spikiness}(i-1)}{\chi_{\text{Fit}}^2(i) - \chi_{\text{Fit}}^2(i-1)}$$

$$\nabla_{Diff}(i) = \nabla_{Up}(i) - \nabla_{Down}(i)$$

Look for iteration with **difference** close to 0

Wealth Of Cross Section Results To Follow!



CC inclusive

- ν_{μ} CC inclusive @ NuMI
- ν_e/ν_{μ} ratios @ BNB, NuMI
- 3D E_{ν} , E_{μ} , hadronic energy @ NuMI & BNB
- anti- ν_e @ NuMI

Pion production

- ν_{μ} CC1 π^+ @ BNB, NuMI
- ν_{μ} CCN π @ NuMI
- 1D ν_{μ} CC π^0 @ BNB
- 2D ν_{μ} CC/NC π^0 @ BNB
- 2D $\nu_{e,\mu}$ NC π^0 @ BNB

CC0 π

- 2D ν_{μ} CC1p0 π Generalized Kinematic Imbalance @ BNB
- ν_{μ} CC0 π inclusive @ BNB
- 2D ν_{μ} CCNp0 π @ BNB
- 1D ν_e CC0 π Np @ NuMI
- 1D ν_{μ} NC1p0 π @ BNB

Rare & novel channels

- ν_{μ} CC Kaon @ BNB, NuMI
- MeV-scale Physics in MicroBooNE
- Neutrons @ BNB



Thank you!

Backup Slides

TABLE IV. Tuned parameter values and uncertainties after fitting to T2K $\text{CC}0\pi$ data for the nominal simulation and three tunes that build to the final four parameter tune. Note that postfit χ^2 values are quoted here only for the 58 bins included in the fit (excluding the highest muon momentum bin in each $\cos\theta$ bin), and using diagonal elements of the covariance matrix only. In the text and figures, pre- and postfit χ^2 comparisons are also quoted for the full T2K dataset of 67 bins. “Norm.” is an abbreviation for normalization.

	MaCCQE fitted value	CC2p2h Norm. fitted value	CCQE RPA Strength fitted value	CC2p2h Shape fitted value	T2K $\chi^2_{\text{diag}}/N_{\text{bins}}$
Nominal (untuned)	0.961242 GeV	1	100%	0	106.7/58
Fit MaCCQE + CC2p2h Norm.	1.14 ± 0.07 GeV	1.61 ± 0.19	100% (fixed)	0 (fixed)	71.8/58
Fit MaCCQE + CC2p2h Norm + CCQE RPA Strength	1.18 ± 0.08 GeV	1.12 ± 0.38	$(64 \pm 23)\%$	0 (fixed)	69.7/58
Fit MaCCQE + CC2p2h Norm + CCQE RPA Strength + CC2p2h Shape	1.10 ± 0.07 GeV	1.66 ± 0.19	$(85 \pm 20)\%$	$1^{+0}_{-0.74}$	52.5/58

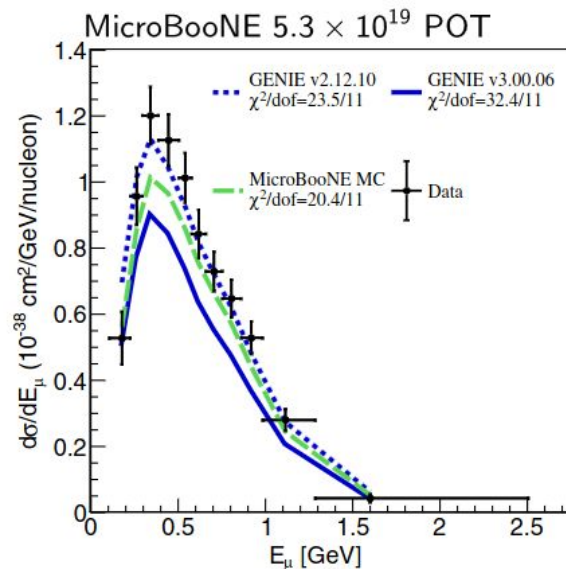
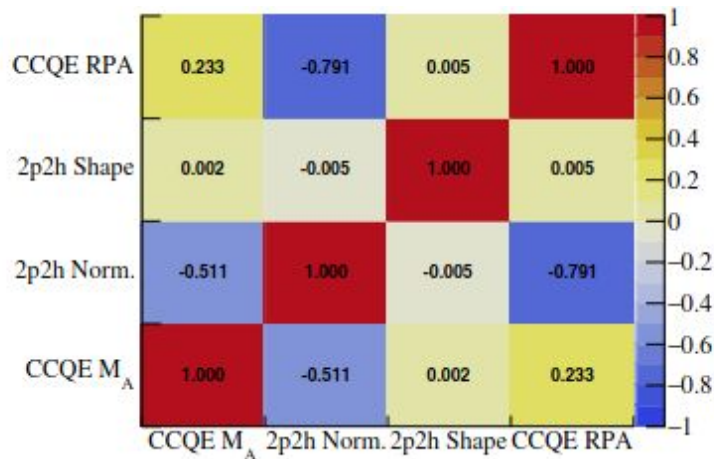


FIG. 7. Correlations between parameters after fitting to T2K $\text{CC}0\pi$ data.

Nuclear Effects in Event Generators

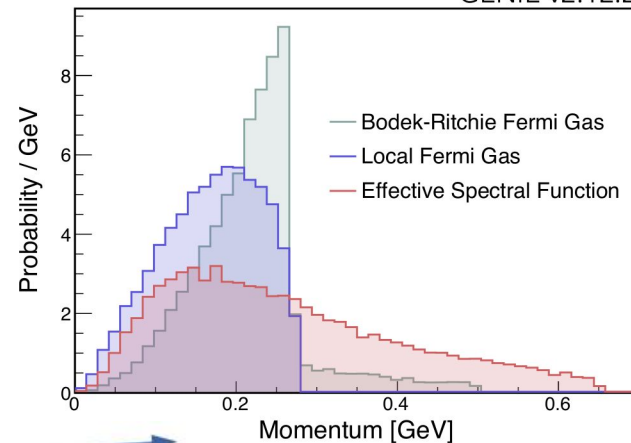
- Fermi motion
- Final state interactions
- Meson exchange currents
- ...

} Known unknowns that need to be accurately simulated

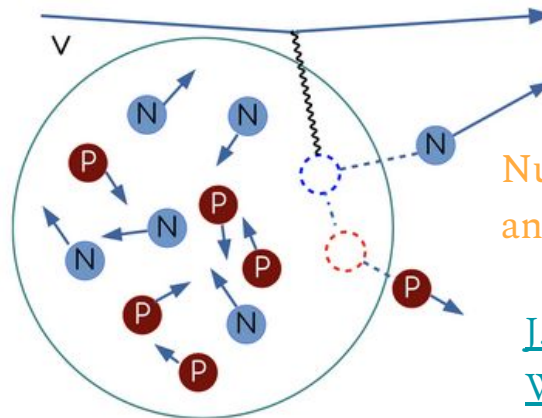
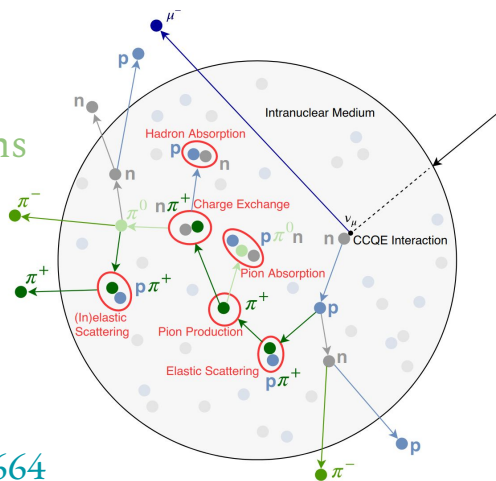
[Rev. Mod. Phys. 89, 045002 \(2017\)](#)

Struck nucleon motion in argon

GENIE v2.12.2



Hadron reinteractions

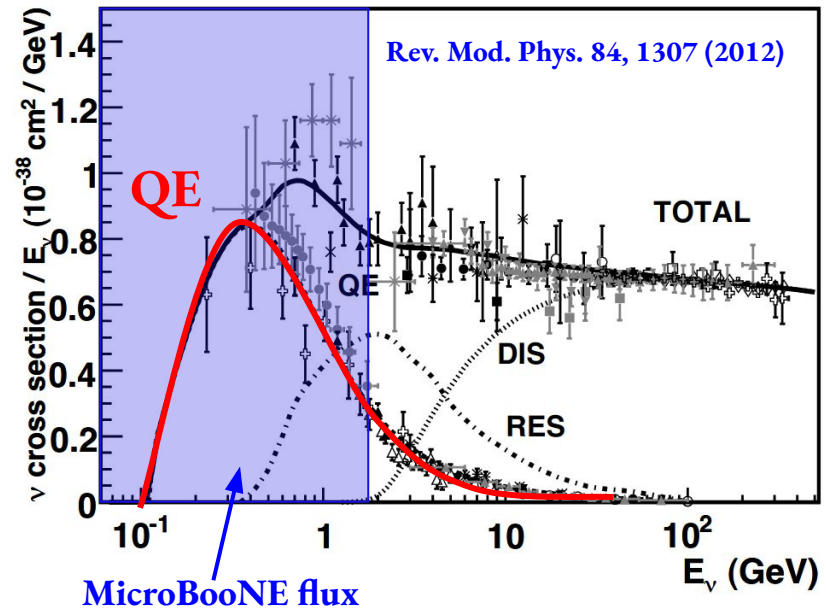
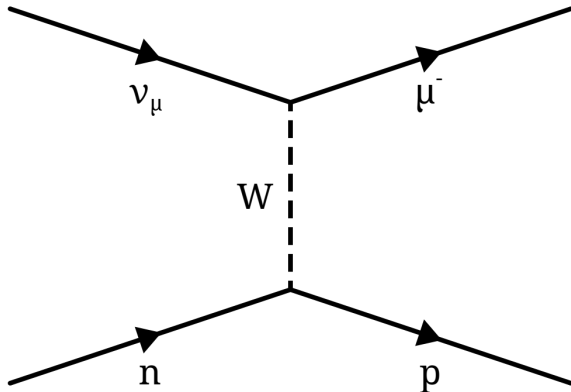


Nucleon-nucleon relative angle and momenta

[J. Wolcott](#)

[Wine & Cheese Seminar](#)

Single-Proton Knockout



- Dominated by Charged Current Quasi-elastic (CCQE) interactions
- Simple single muon-proton events
- Dominant at MicroBooNE energies

TKI Neutrino Measurements

Experiment	Target	References
T2K	CH	Phys.Rev.D 103 11, 112009 (2021) Phys. Rev. D 98, 032003 (2018)
MINERvA	CH	Phys. Rev. Lett. 121, 022504 (2018) Phys. Rev. D 101, 092001 (2020) Phys. Rev. D 102, 072007 (2020)

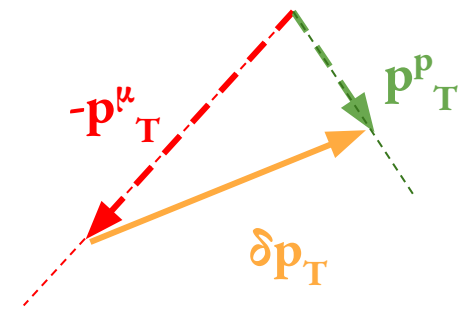
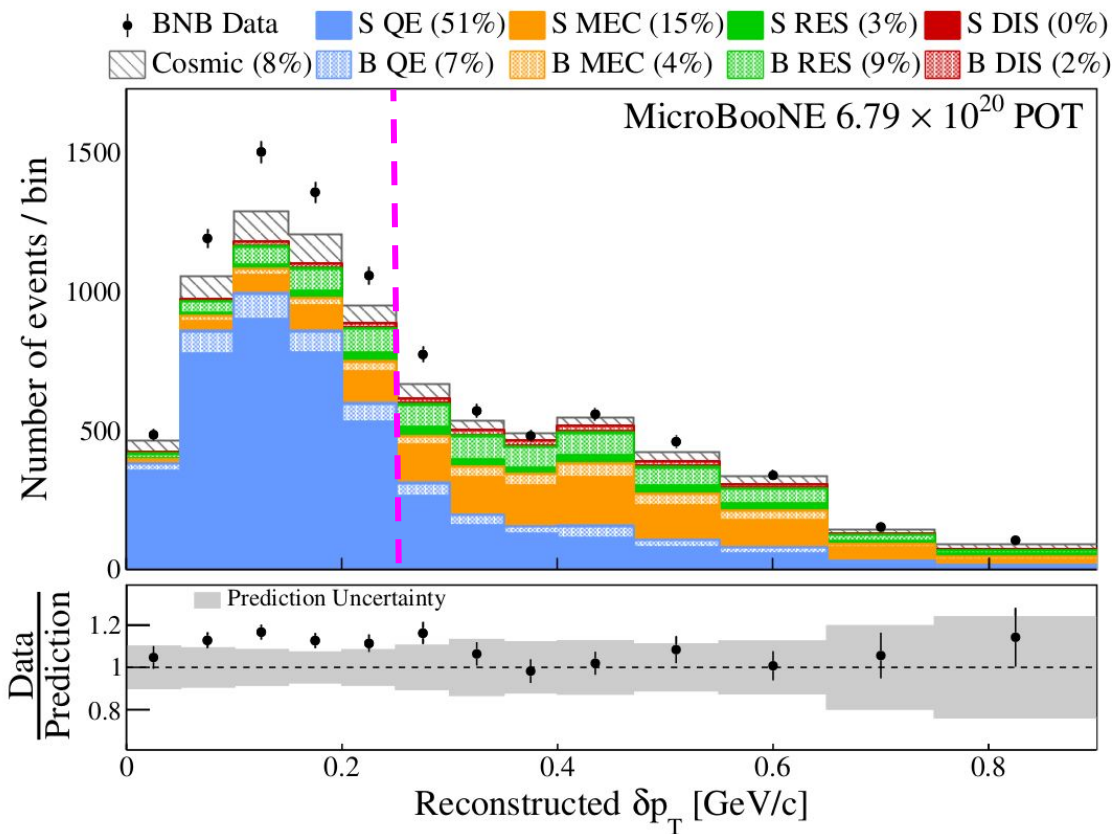
But none on argon up to now!

TKI Neutrino Measurements

Experiment	Target	References
T2K	CH	Phys.Rev.D 103 11, 112009 (2021) Phys. Rev. D 98, 032003 (2018)
MINERvA	CH	Phys. Rev. Lett. 121, 022504 (2018) Phys. Rev. D 101, 092001 (2020) Phys. Rev. D 102, 072007 (2020)
MicroBooNE	Ar	arXiv:2301.03706 (accepted to PRL) arXiv:2301.03700 (accepted to PRD)

First single- and double-differential single-proton cross section measurements on argon in transverse kinematic imbalance

Transverse Missing Momentum δp_T



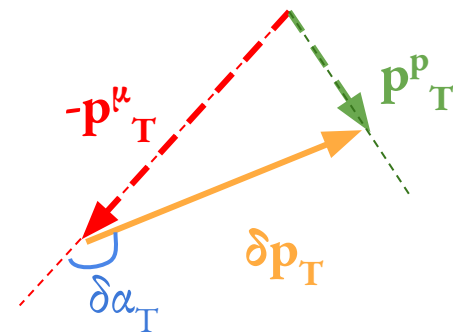
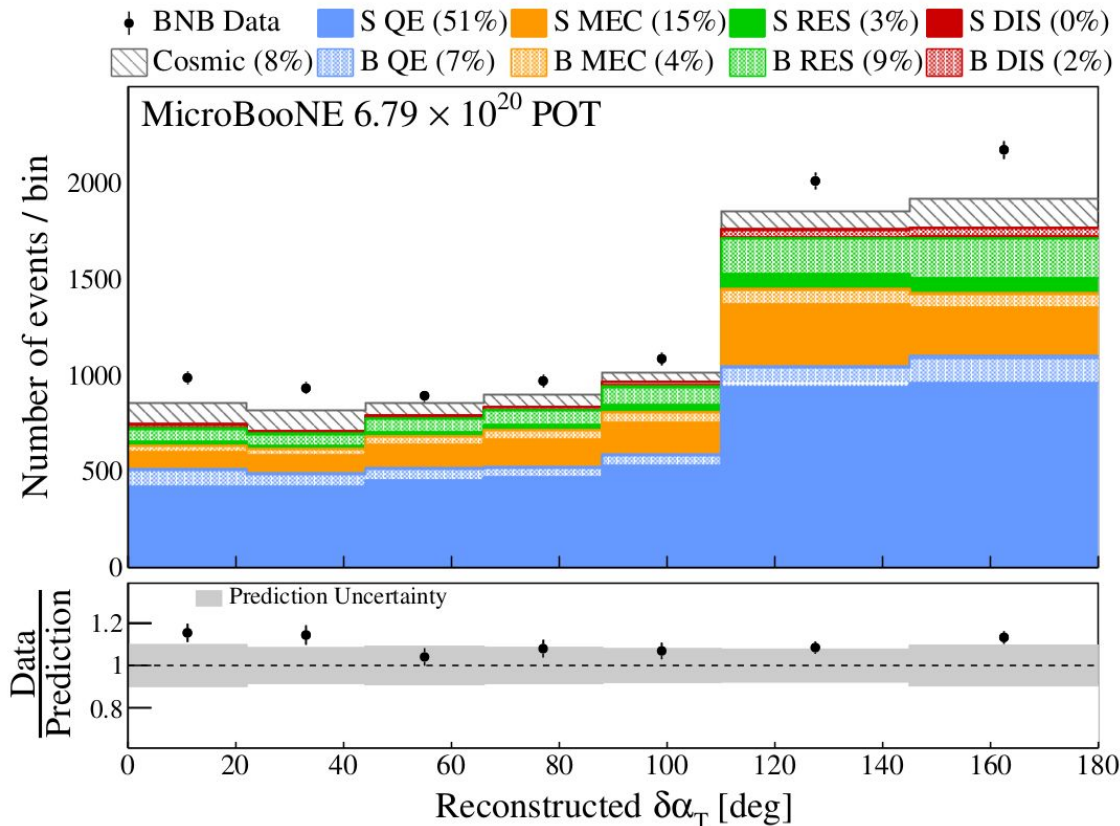
- S = Signal, B = Background
- **QE** dominance in peak below Fermi momentum (~ 250 MeV/c)
- **MEC/RES** mainly in high momentum tail

[arXiv:2301.03700](https://arxiv.org/abs/2301.03700) (accepted to PRD)

* [Phys. Rev. D 105, 072001 \(2022\)](https://arxiv.org/abs/2007.07200)

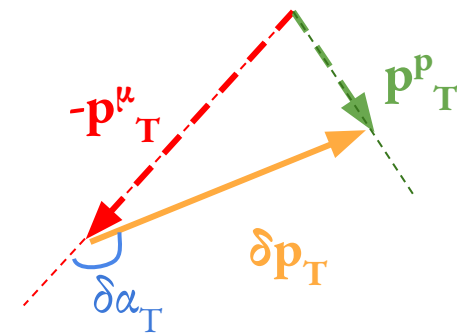
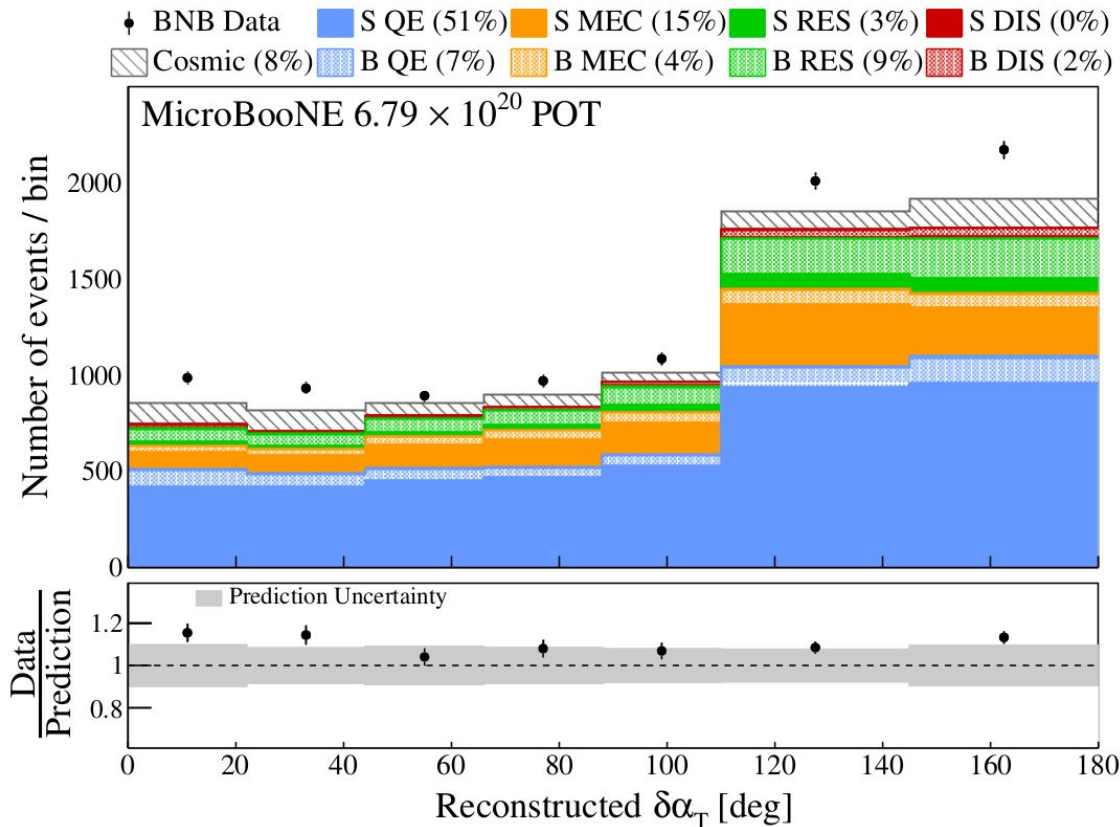
GENIE v3.0.6 G18_10a_02_11b + tune*
Nieves QE & MEC, Berger Sehgal RES

Transverse Orientation $\delta\alpha_T$



- $\delta\alpha_T$ asymmetry due to proton FSI
- **MEC/RES** fractional contribution enhanced in $\sim 180^\circ$ region

Transverse Orientation $\delta\alpha_T$



Need to move from event distributions to cross sections \rightarrow Wiener-SVD unfolding

[JINST 12 P10002 \(2017\)](#)

More details in backup slides

[arXiv:2301.03706](#) (accepted to PRD)

* [Phys. Rev. D 105, 072001 \(2022\)](#)

Cross Section Extraction with Wiener SVD Unfolding

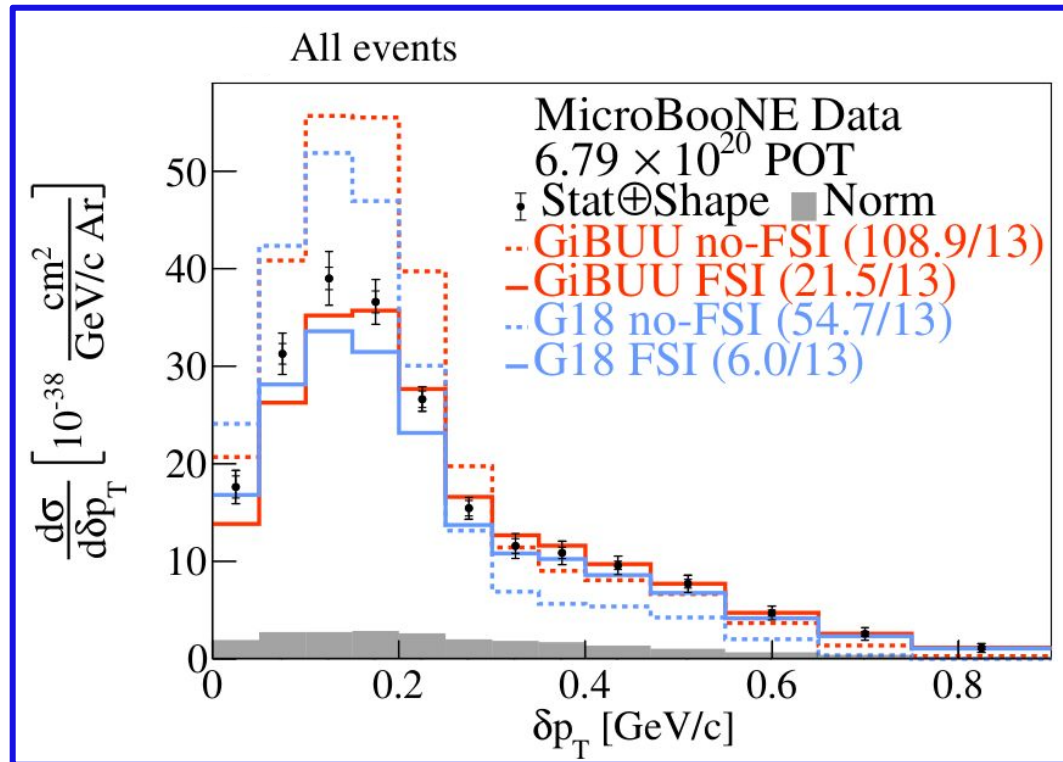
[JINST 12 P10002 \(2017\)](#)

Output quantities in regularized space

- Unfolded data spectrum

- Smearing Matrix A_C

*Applied on theory predictions and included in data release



Cross Section Extraction with Wiener SVD Unfolding

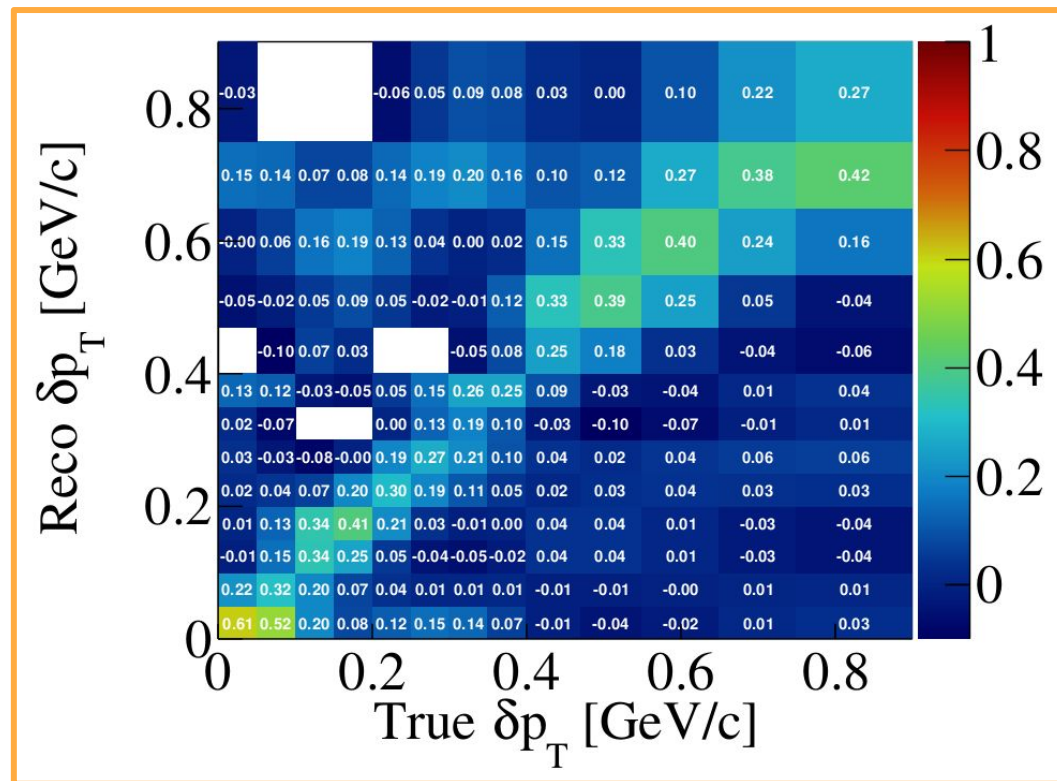
[JINST 12 P10002 \(2017\)](#)

Output quantities in regularized space

- Unfolded data spectrum

- Smearing Matrix A_C

*Applied on theory predictions and included in data release

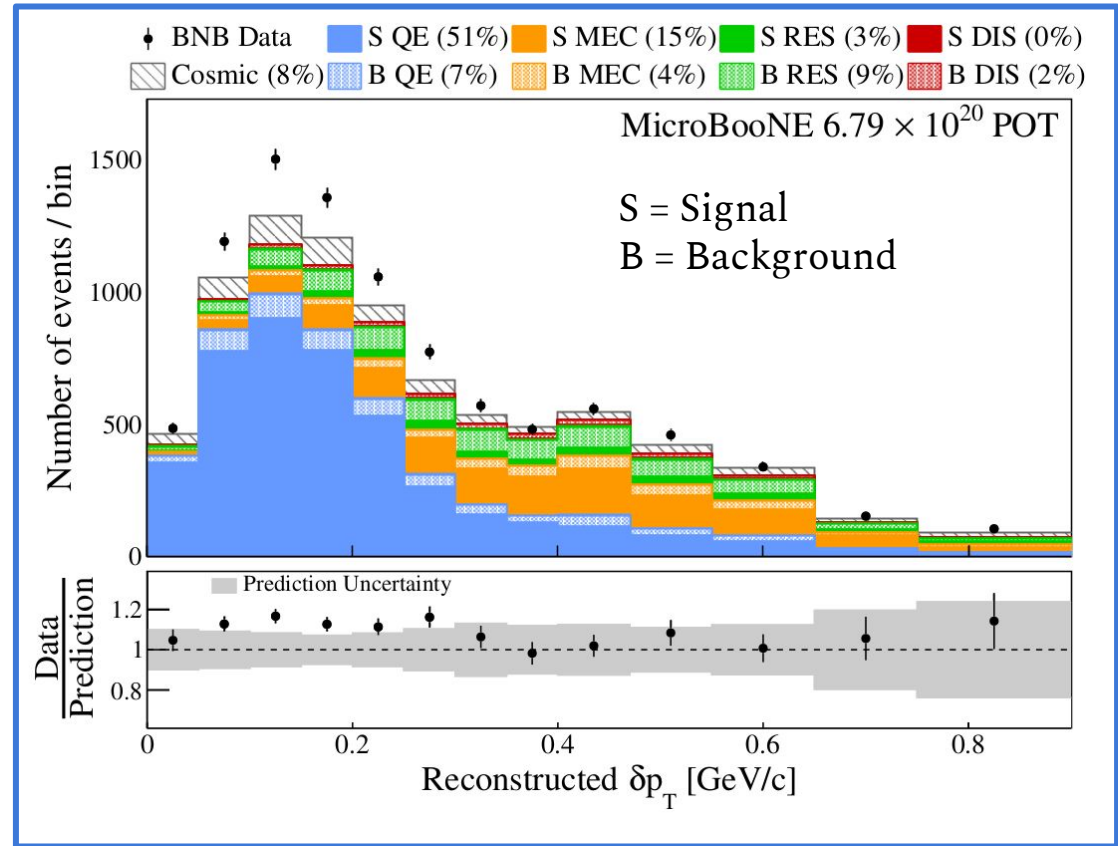


Cross Section Extraction with Wiener SVD Unfolding

[JINST 12 P10002 \(2017\)](#)

Input Quantities

- Measurement (Data)
- Background (Cosmics + MC)
- Response Matrix (MC)
- Total Covariance Matrix (MC)



Cross Section Extraction with Wiener SVD Unfolding

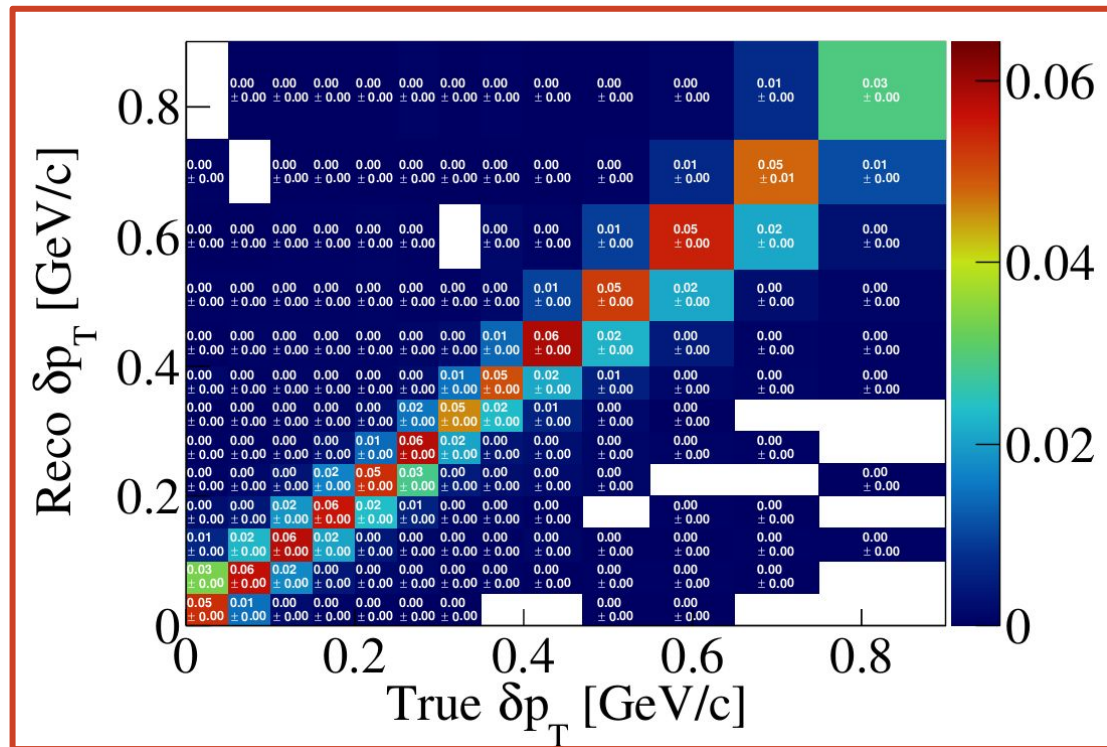
[JINST 12 P10002 \(2017\)](#)

Input Quantities

- Measurement (Data)
- Background (MC)
- Response Matrix (MC)
- Total Covariance Matrix (MC)

Probability that a generated event is reconstructed and selected

Diagonal matrix with flat ~6% efficiency



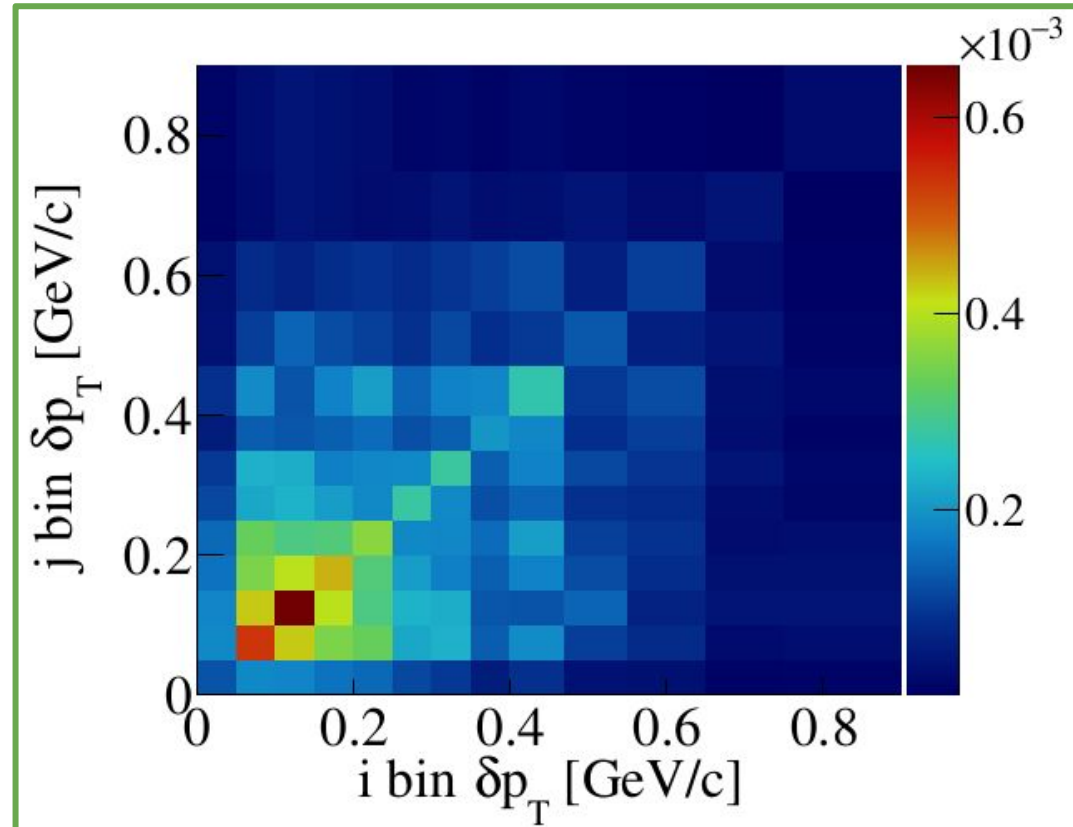
Cross Section Extraction with Wiener SVD Unfolding

[JINST 12 P10002 \(2017\)](#)

Input Quantities

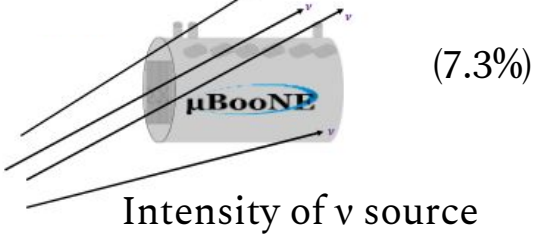
- Measurement (Data)
- Background (MC)
- Response Matrix (MC)
- Total Covariance Matrix (MC)

Includes information on statistical and systematic uncertainties

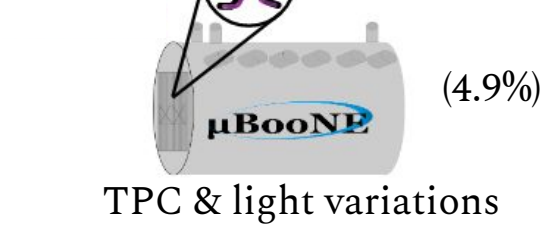


Uncertainties

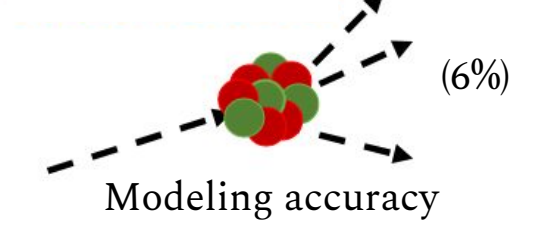
Flux



Detector



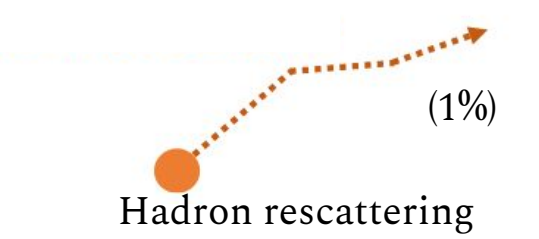
Cross section



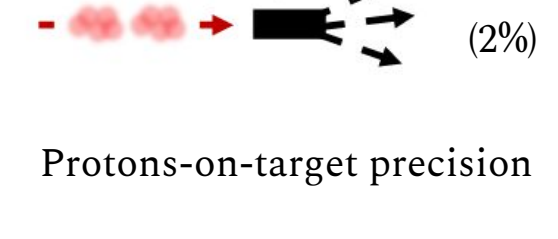
Dirt



Reinteractions



POT counting



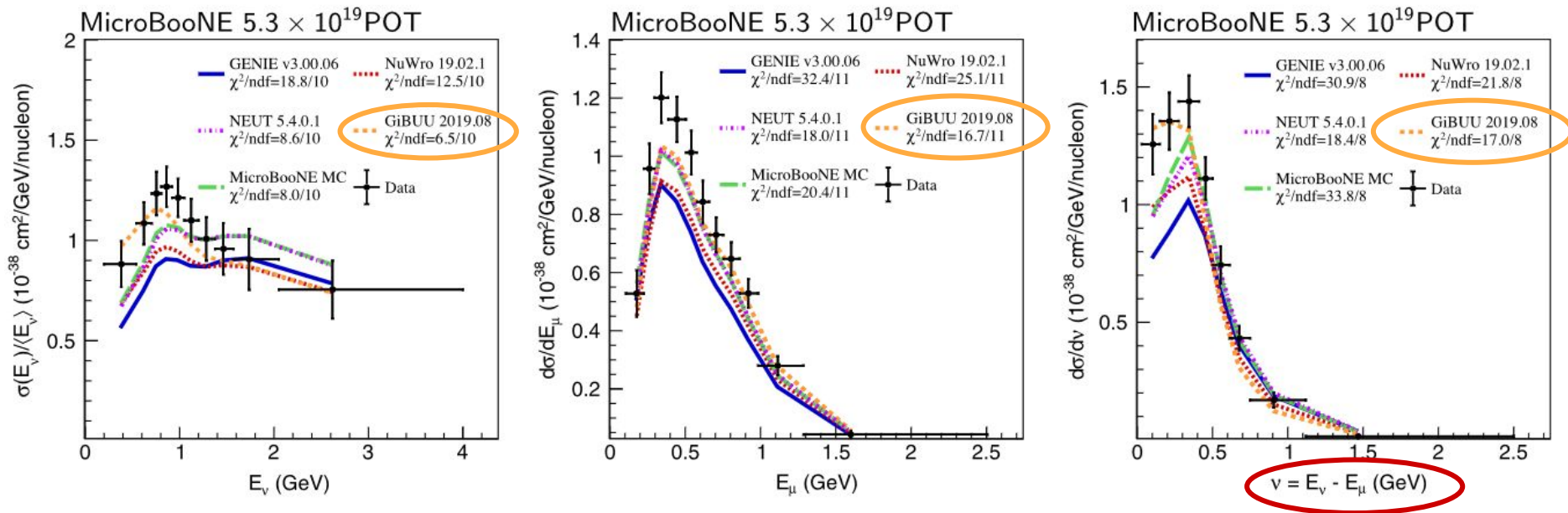
- + Statistical (1.5%)
- + Number of argon targets (1%)

Total (11%)

Systematics-dominated analysis

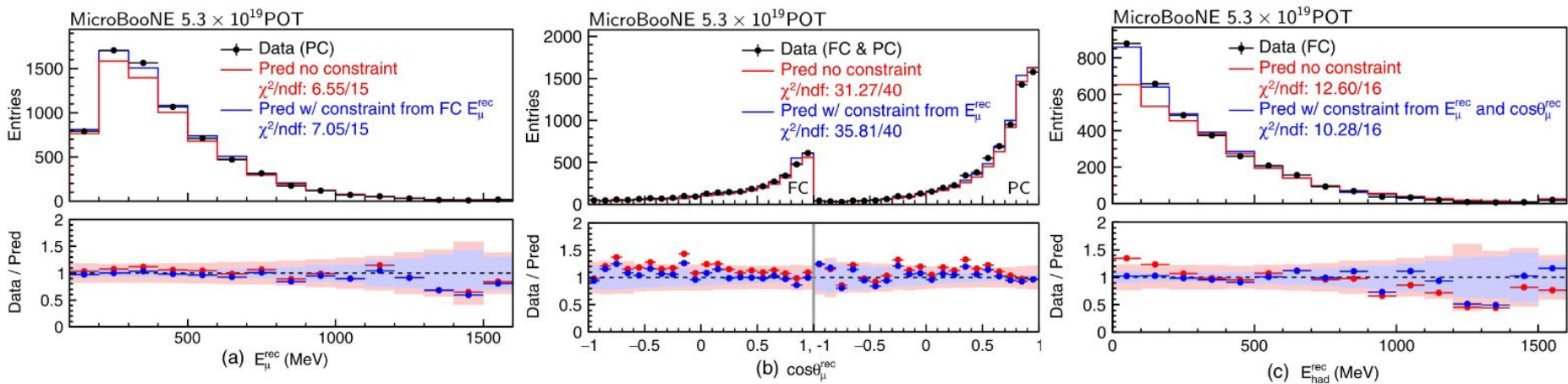
ν_μ CC Inclusive 1D

- Unfolding using Wiener-SVD [JINST 12 P10002 \(2017\)](#)
- First ever measurement of cross section as a function of **energy transfer**
- **GiBUU** results in best performance

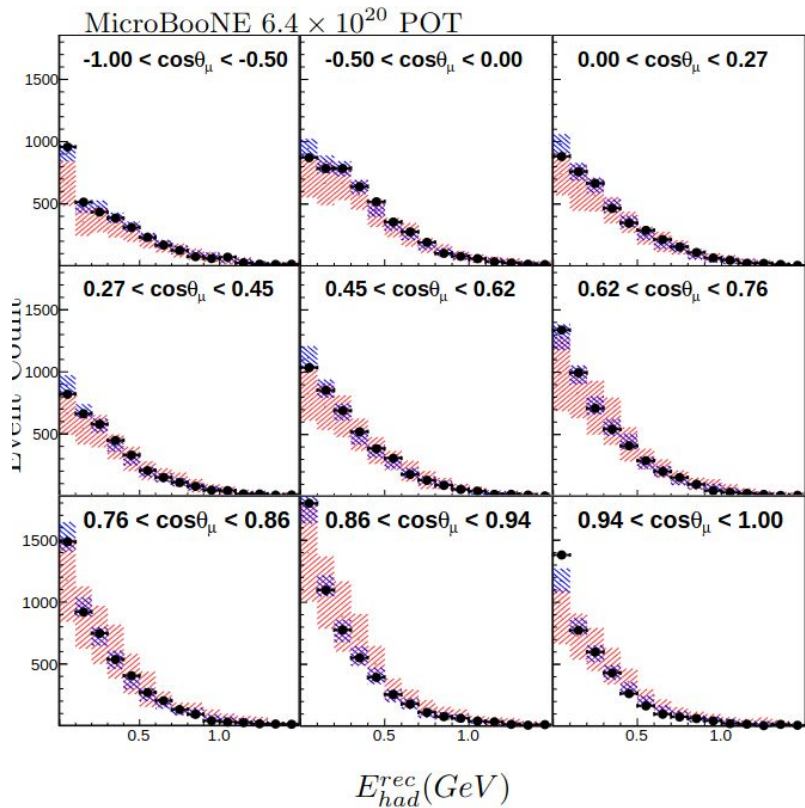


ν_μ CC Inclusive 1D

- Observe the visible hadronic energy, muon energy and direction
- Use them to test model validity due to missing hadronic energy



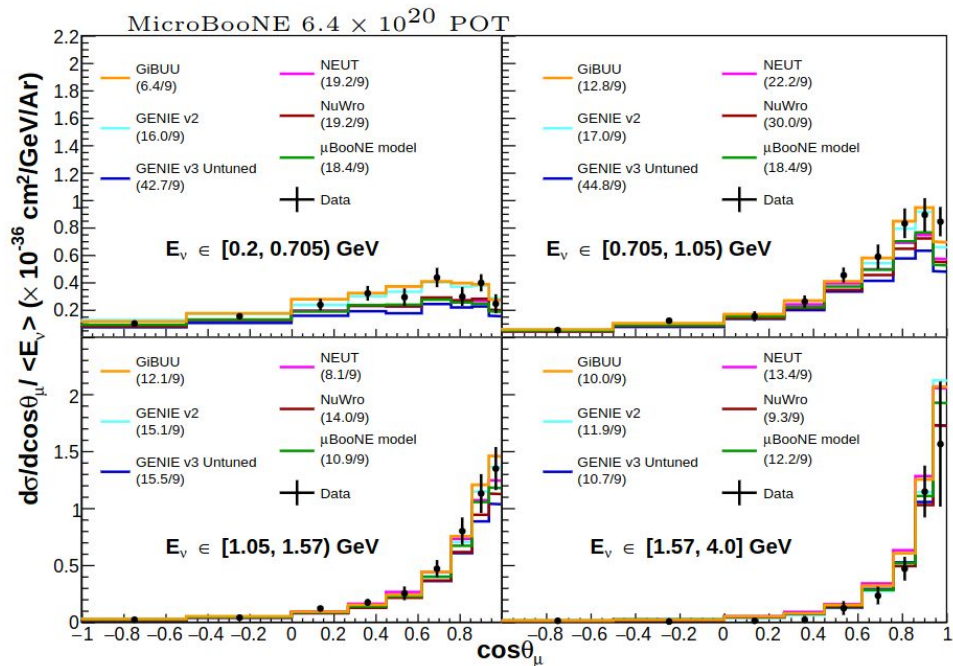
ν_μ CC Inclusive 3D



● Data

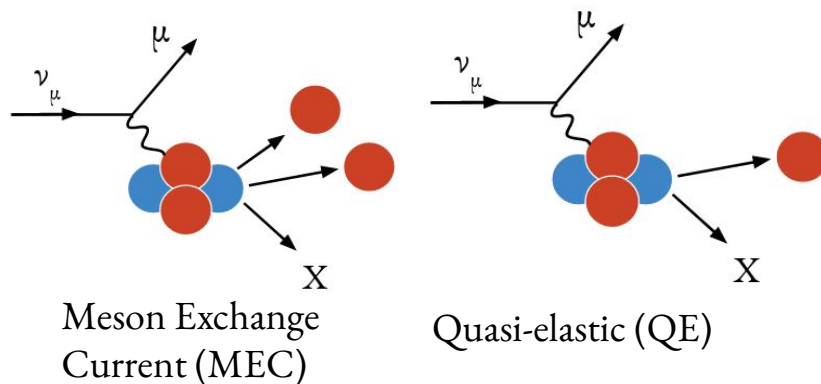
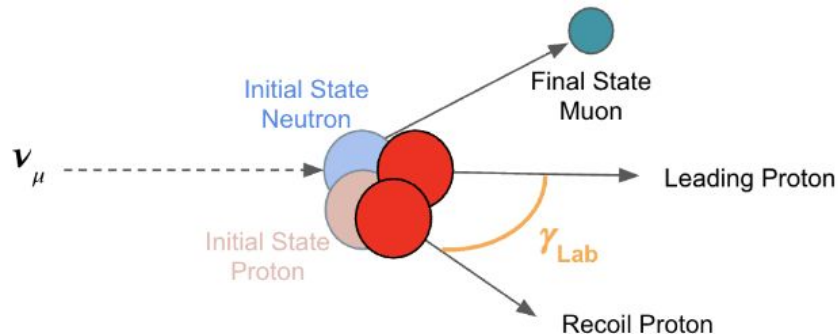
▨ Pred no constraint
 $\chi^2/ndf = 119.54/144$

▨ Pred w/ constraint
 $\chi^2/ndf = 123.07/144$



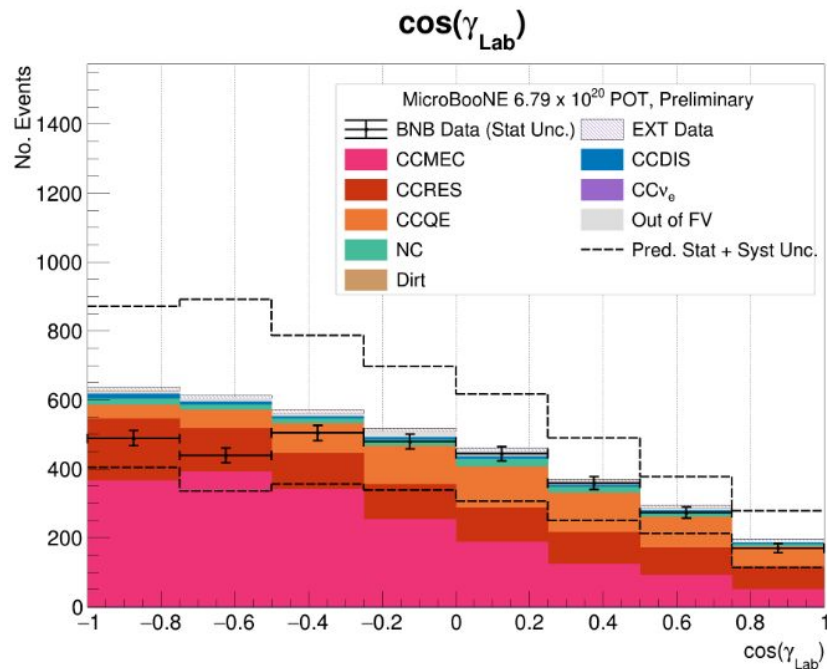
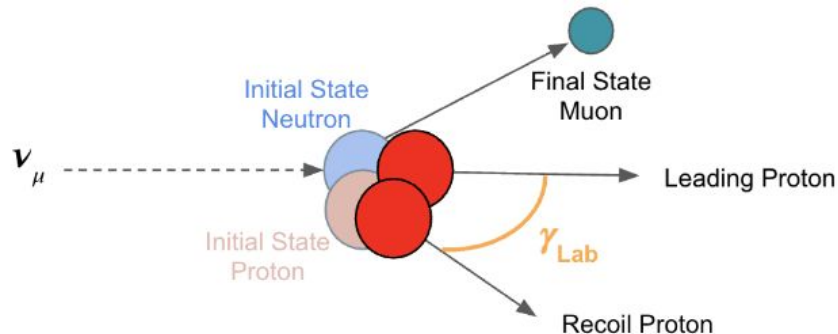
ν_{μ} CC2p0 π

- First neutrino-argon cross sections for an exclusive 2p final state
 - Various observables studied
- γ_{Lab} : angle between the two protons
 - Sensitive to modeling choices for MEC and QE



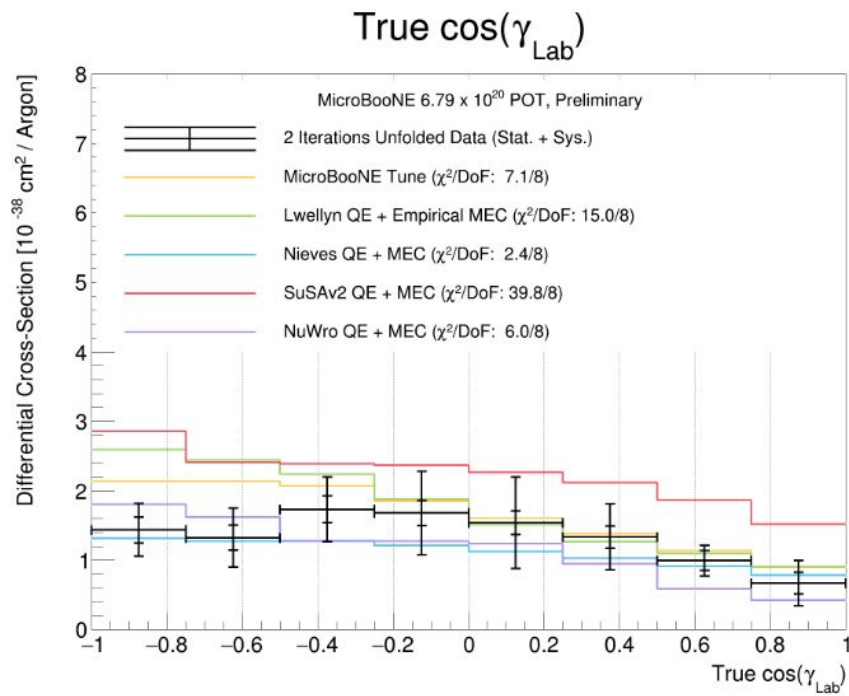
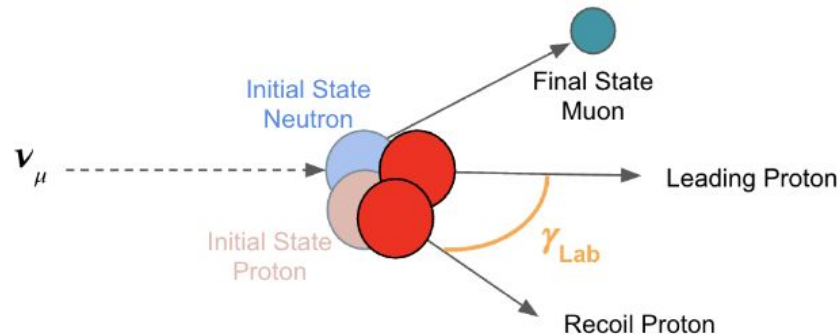
ν_{μ} CC2p0 π

- First neutrino-argon cross sections for an exclusive 2p final state
 - Various observables studied
- γ_{Lab} : angle between the two protons
 - Sensitive to modeling choices for MEC and QE

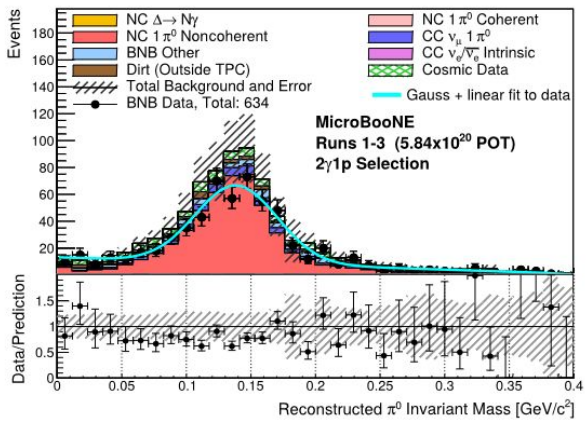


ν_{μ} CC2p0 π

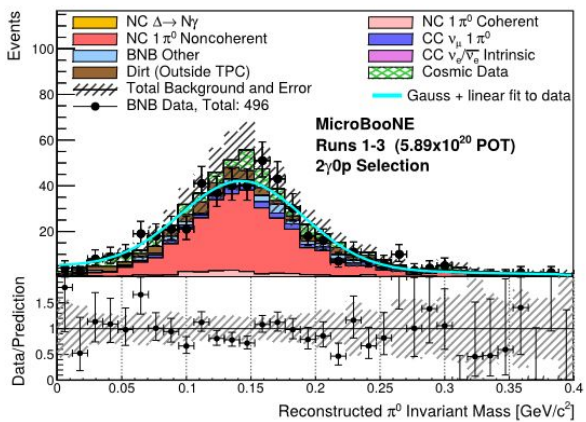
- First neutrino-argon cross sections for an exclusive 2p final state
 - Various observables studied
- γ_{Lab} : angle between the two protons
 - Sensitive to modeling choices for MEC and QE
- Data-MC shape & normalization differences identified



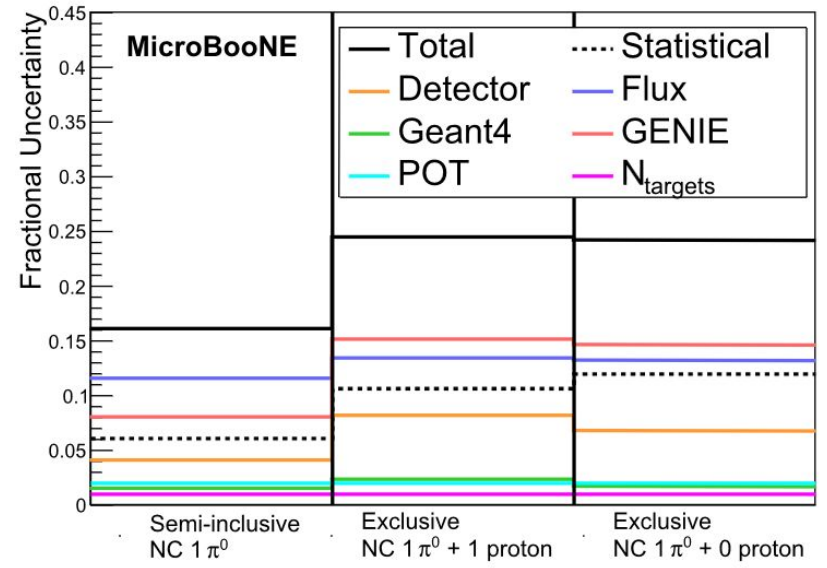
NC π^0



(a) $2\gamma 1p$



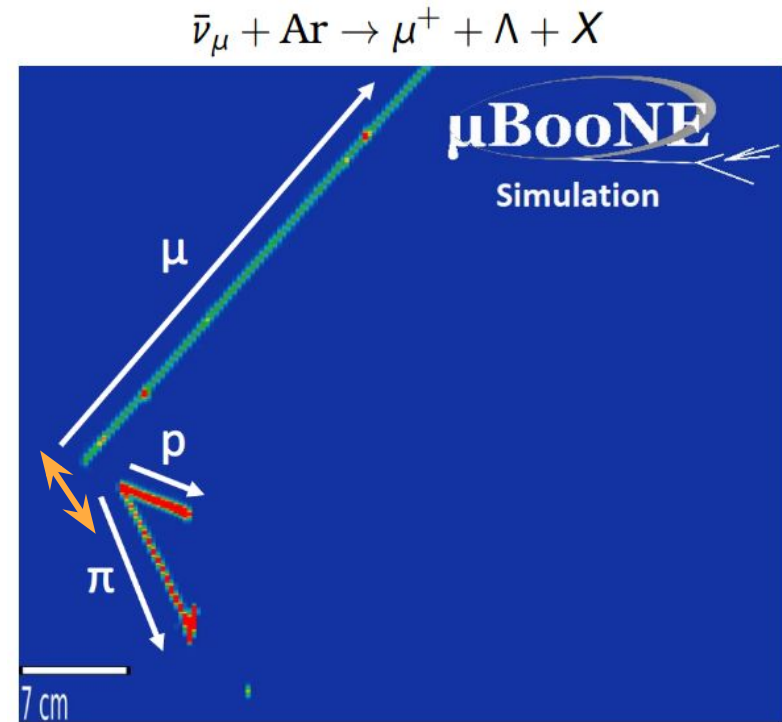
(b) $2\gamma 0p$



Λ Production

Event Selection

- Selection identifies a muon candidate and a proton-pion candidate pair
- Proton-pion “island” activity **separated** from muon candidate
- Proton-pion kinematics consistency with Λ baryon decay



Λ Production

Λ baryon decay consistency

- Keeping events with
 $1.09 < \text{invariant mass } W < 1.14 \text{ GeV}/c^2$
and angular deviation $\alpha < 14^\circ$
- After selection

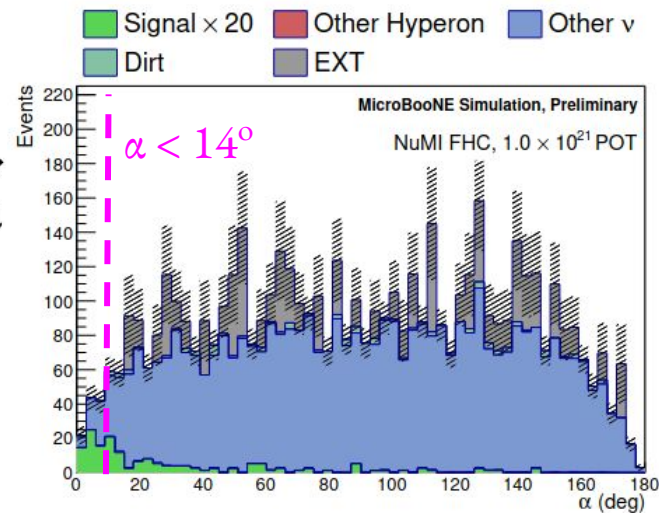
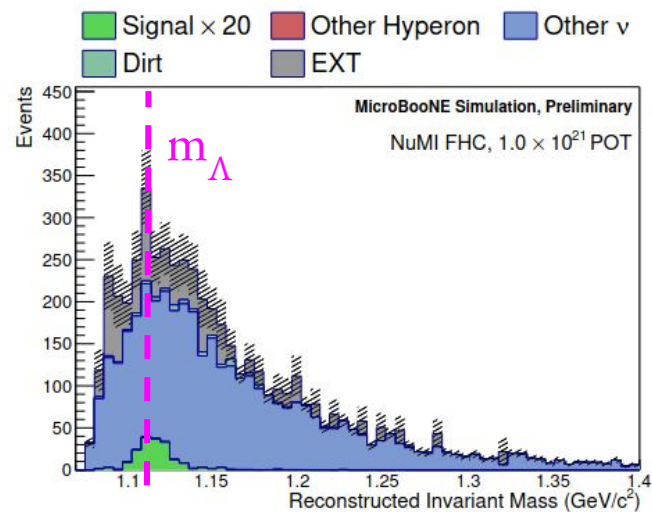
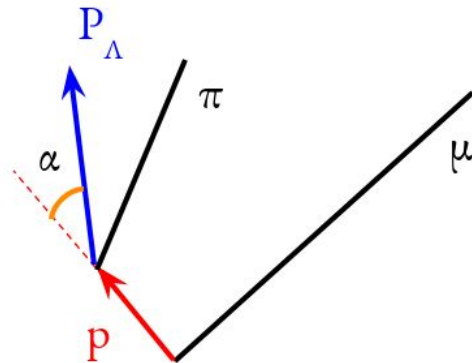
Signal = 2.5 events

Bkg = 2.8 events

when initially

Signal = ~ 40 events

Bkg = $\sim 2\text{M}$ events



Also see poster by [C.Thorpe](#)

MICROBOONE-NOTE-1097-PUB

Bayesian Method / Λ production

Partial Phase Space Definition

As the selection can only identify Λ baryons that decay into a proton and π^- with momenta above detection thresholds, the cross section must be corrected to only include this portion of the phase space. Our restricted phase space cross section, σ_* , is related to the total cross section for quasi-elastic Λ production by:

$$\sigma_* = F\sigma, \quad (1)$$

$$F = \frac{1}{\sigma} \int_0^\infty f(p_\Lambda) \frac{d\sigma}{dp_\Lambda} dp_\Lambda. \quad (2)$$

$f(p_\Lambda)$ is the fraction of Λ baryons decaying via $\Lambda \rightarrow p + \pi^-$ that will be above the detection thresholds. This function is shown in Fig. 3, and may be calculated with:

$$f(p_\Lambda) = \begin{cases} 0 & \text{if } A > B \\ \frac{B-A}{2} & \text{Otherwise} \end{cases}, \quad (3)$$

$$A = \max \left(\frac{\sqrt{M_p^2 + |p_p^{\text{thresh}}|^2} - \gamma E_p}{\beta \gamma p}, -1 \right), \quad (4)$$

$$B = \min \left(\frac{-\sqrt{M_\pi^2 + |p_\pi^{\text{thresh}}|^2} + \gamma E_\pi}{\beta \gamma p}, 1 \right), \quad (5)$$

$$E_p = \sqrt{M_p^2 + p^2}, \quad (6)$$

$$E_\pi = \sqrt{M_\pi^2 + p^2}. \quad (7)$$

M_p and M_π are the rest masses of the proton and π^- respectively, $p_p^{\text{thresh}} = 0.3$ GeV, $p_\pi^{\text{thresh}} = 0.1$ GeV are the detection thresholds, $p = 0.101$ GeV is the momentum the decay products are emitted with in the Λ baryon's rest frame, β is the boost factor of the Λ baryon in the detector's frame, and $\gamma = 1/\sqrt{1 - \beta^2}$. Natural units are used.

Bayesian Method / Λ production

Studying the selection efficiency as a function of the Λ baryon's momentum, it is shown there is some shape to this distribution.

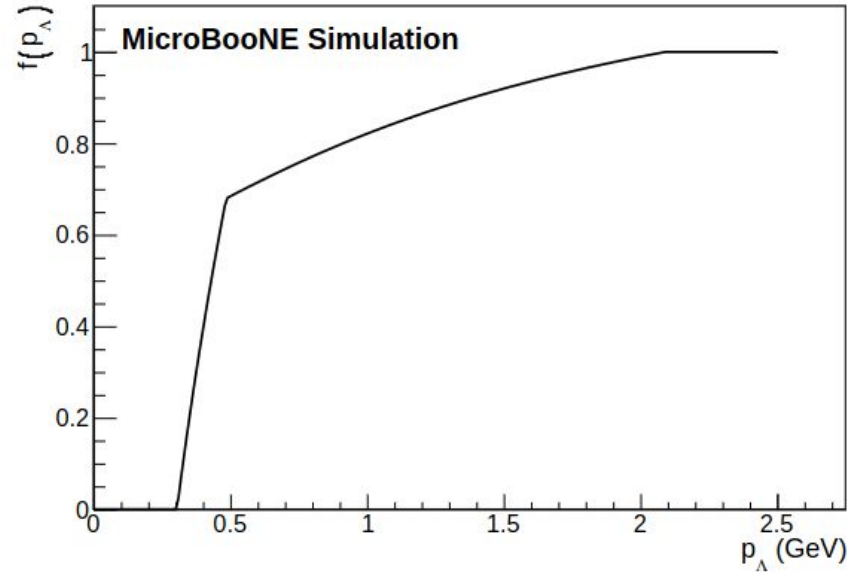


Figure 3: The function $f(p_\Lambda)$ featured in equations 2 and 3. The two discontinuities in gradient occur when one, and then both, particles produced in the decay are always above the detection threshold.

Bayesian Method / Λ production

We adopt a Bayesian approach to propagate the statistical and systematic uncertainties through this formula: Posterior distributions describing the MC statistical uncertainties on the efficiency and selected background are generated using the TEfficiency class from Root [13]:

$$\phi_{\epsilon}(\epsilon) = P(\epsilon|\epsilon_{MC}) \quad (25)$$

$$\phi_B(B) = P(B|B_{MC}) \quad (26)$$

ϵ and B are hypothetical true efficiency and background rates and ϵ_{MC} and B_{MC} and the efficiency and background rates observed in the MC. The posterior distribution on the number of events observed in data is:

$$P(N|N_{Obs}) = \frac{P(N_{Obs}|N)P(N)}{\int_a^b P(N_{Obs}|N)P(N)dN} \quad (27)$$

Where N_{Obs} is the number of events observed in data and N is a hypothetical true event rate. $P(N)$ is the Bayesian prior, a uniform distribution on the interval $[a, b]$, for which we use $[0, 20]$ here. $P(N_{Obs}|N)$ is the Poisson probability of observing N_{Obs} counts with a mean count of N .

Bayesian Method / Λ production

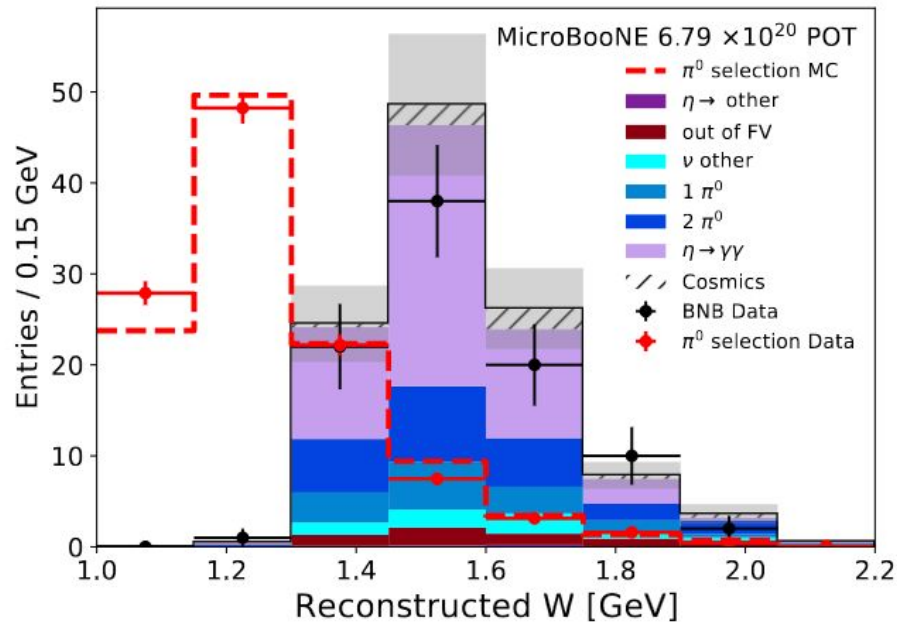
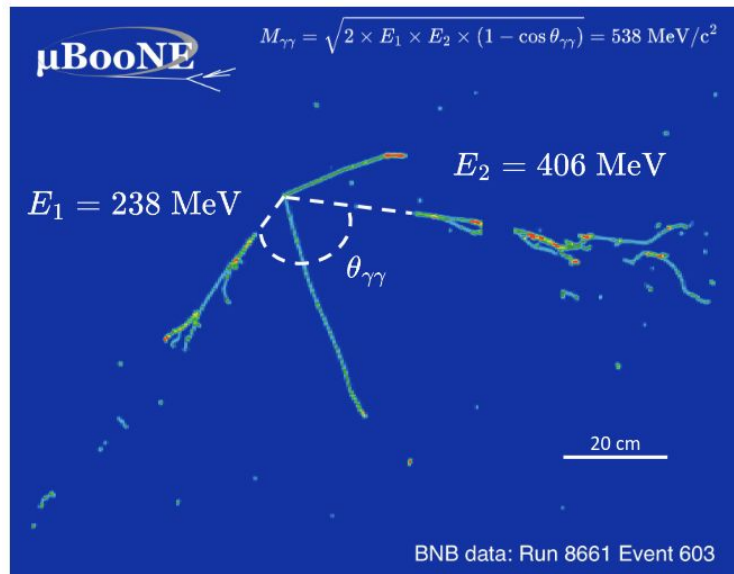
We build the posterior distribution of σ_* using a Monte Carlo method: For a fixed value of N_{Obs} , we draw many values of ϵ , B and N from their respective distributions. Systematic uncertainties are included by drawing three smearing parameters, α_ϵ , α_Φ and α_B from a three dimensional Gaussian parameterised by the covariance matrix between ϵ , Φ and B , shown in tables 6a, 6b and 6c for the different procedures for extracting uncertainties on the background. For each set of six parameters, the cross section is calculated:

$$\sigma_* = \frac{N - (B + \alpha_B)}{T(\Phi + \alpha_\Phi)\Gamma(\epsilon + \alpha_\epsilon)} \quad (28)$$

By repeatedly calculating σ_* and binning the results, we obtain the posterior distribution, containing all statistical and systematic uncertainties, shown for both data taking periods with

η Meson Production

- Unique probe of higher resonances such as $N(1535)$
- Identified via decay to 2γ with invariant mass of 548 MeV
- Include protons to estimate reconstructed invariant mass of hadronic system



D'Agostini

- Lives in true space
- Number of iterations determined based on L-curve and “spikiness vs χ^2 ”
- **S** = smearing matrix
- **P** = prior / best guess for truth

Unfolding matrix

$$\left\langle \frac{d\sigma}{dx} \right\rangle_i = \frac{\sum_j U_{ij}(n_j - b_j)}{N_{\text{target}} \times \phi \times (\Delta x)_i}$$

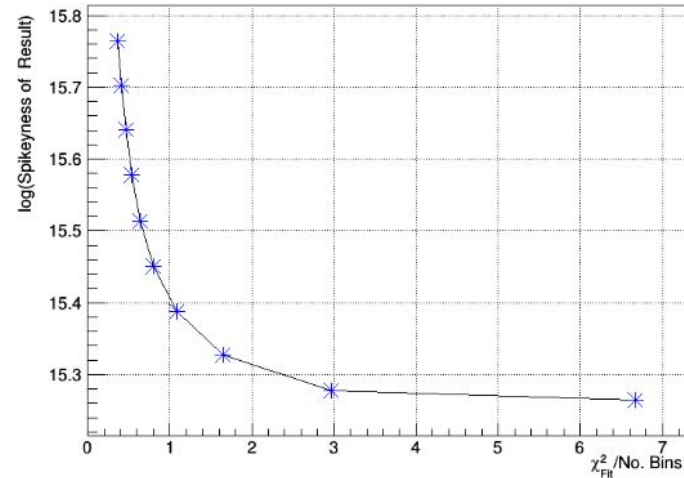
$$U_{T,R} = \frac{S_{T,R} \times P_T}{\sum_R S_{T,R} \times P_T}$$

$$\text{Spikiness} = \sum_{j=1}^{(\# \text{ of Bins})+1} (U(i)_j - U(i)_{j-1})^2$$

$$\chi_{\text{Fit}}^2 = \sum_{j=1}^{(\# \text{ of Bins})+1} \frac{(U(i)_j - U(i-1)_j)^2}{U(i)_j}$$

events in true bin j after i iterations

Unfolded BNB CC2p: True δP_T (GeV/c)



[Phys. Rev. D 106, L051102 \(2022\)](#)

[arXiv:2211.03734](#)

[PhD Thesis](#)