

# $\tau$ -data based evaluation of $a_{\mu}^{\text{HVP,LO}}$

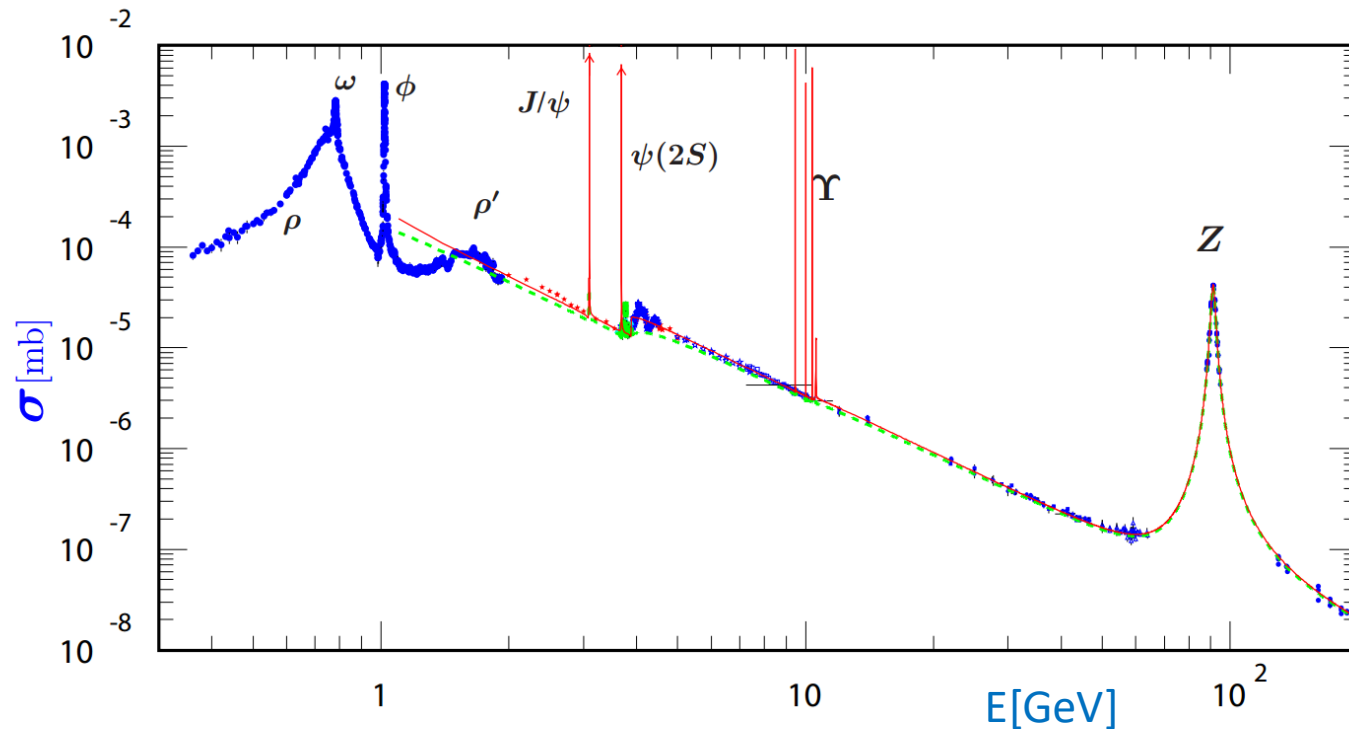
Pablo Roig  
Cinvestav (Mexico City)

XVII International Workshop on Tau Lepton Physics,  
Louisville, Kentucky, USA, 4-8 Dec. 2023

# $a_\mu^{\text{HVP}}$ in the SM: $e^+e^-$ data-driven

White Paper'20, Snowmass document and refs. therein

$$a_\mu^{\text{HVP,LO}} = \frac{1}{4\pi^3} \int_{s_{thr}}^{\infty} ds K(s) \sigma_{e^+e^- \rightarrow \text{hadrons}}^0(s) \quad \text{Both } K \text{ \& } \sigma \text{ go as } 1/s \text{ enhancing low-E contributions}$$

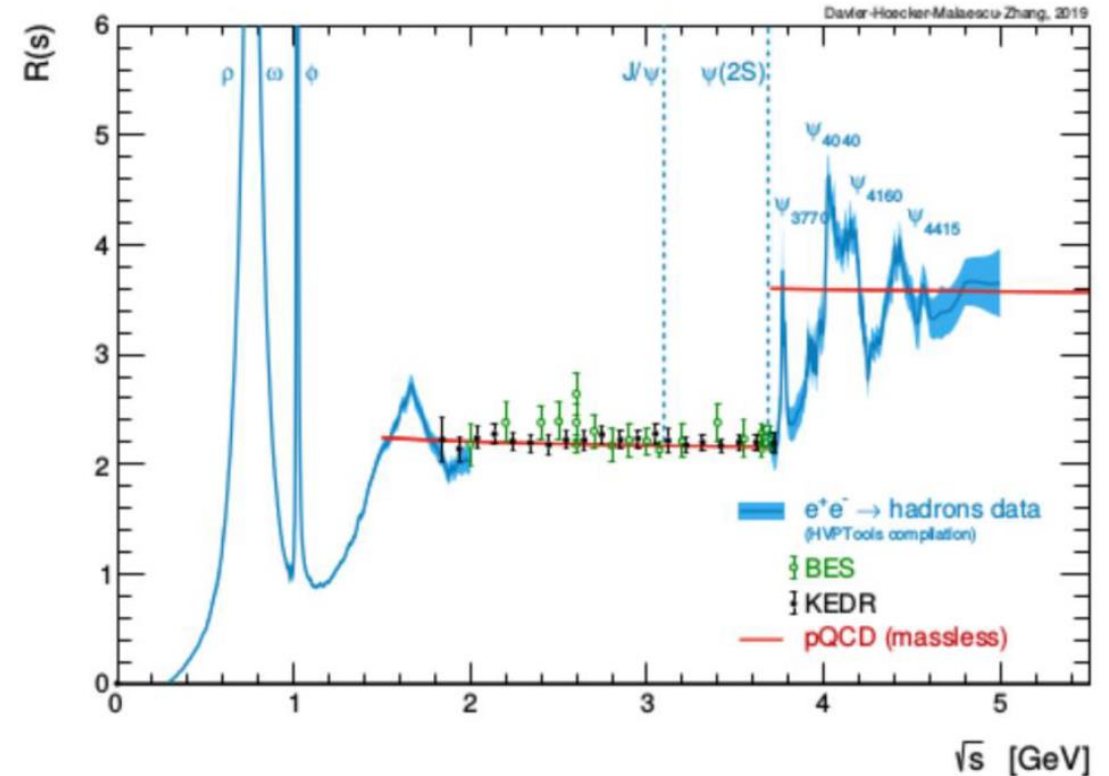
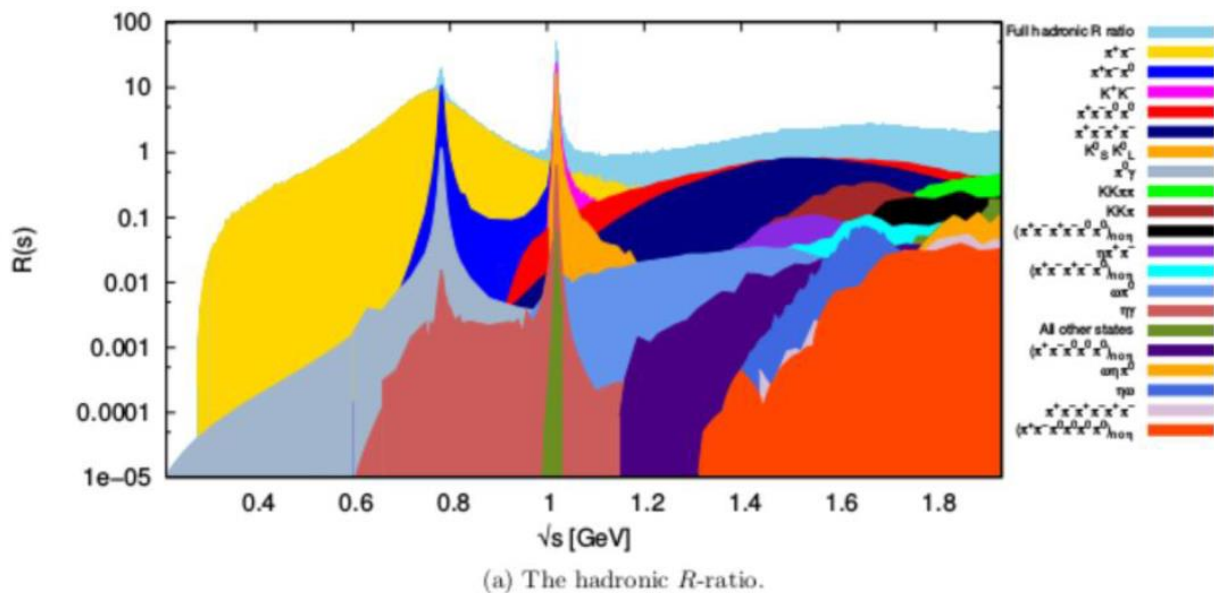


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# $a_\mu^{\text{HVP}}$ in the SM: $e^+e^-$ data-driven

$$R = \frac{\sigma^{(0)}(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$$

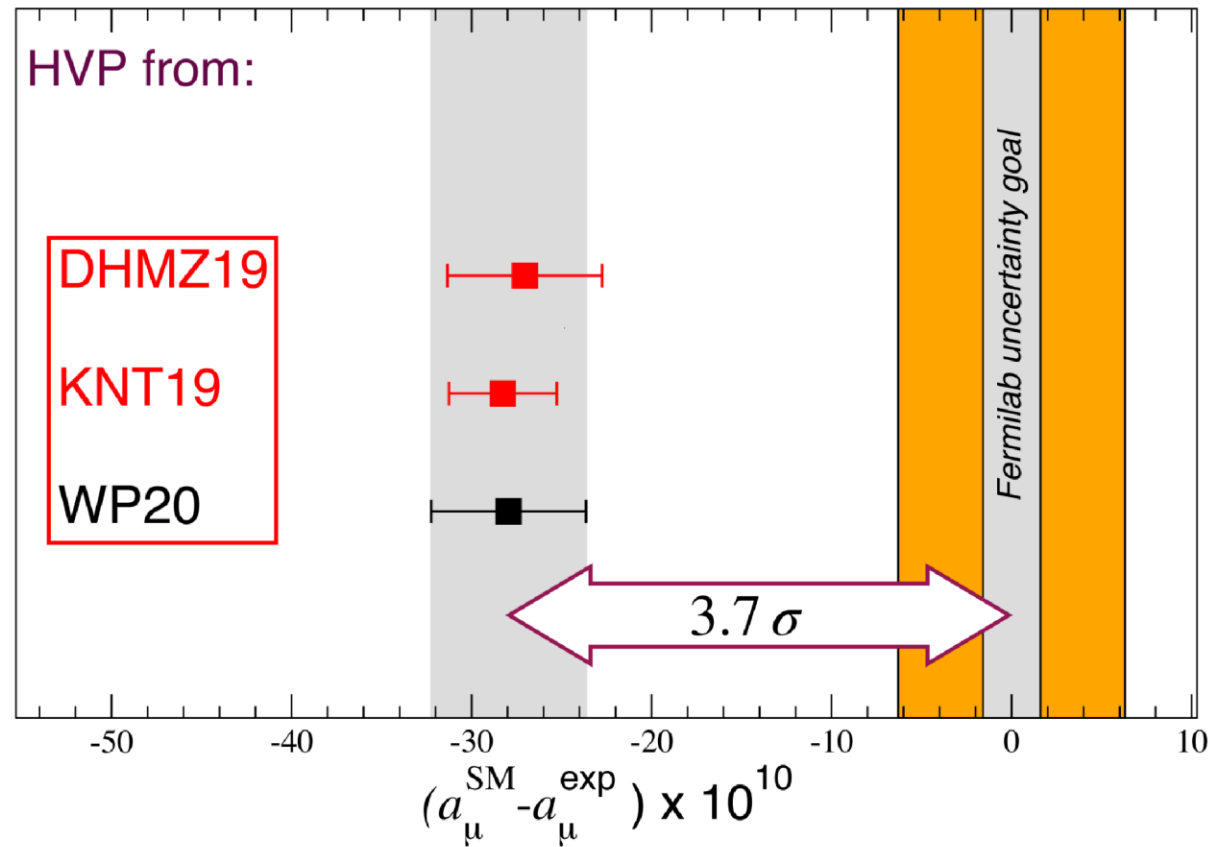
- Oversimplifying: precise measurements for  $e^+e^- \rightarrow \text{hadrons}$  or the  $R$ -ratio



Figs in KNT'18/DHMZ'19 (left/right)

# $a_\mu$ in the SM using $e^+e^-$ data-driven for HVP

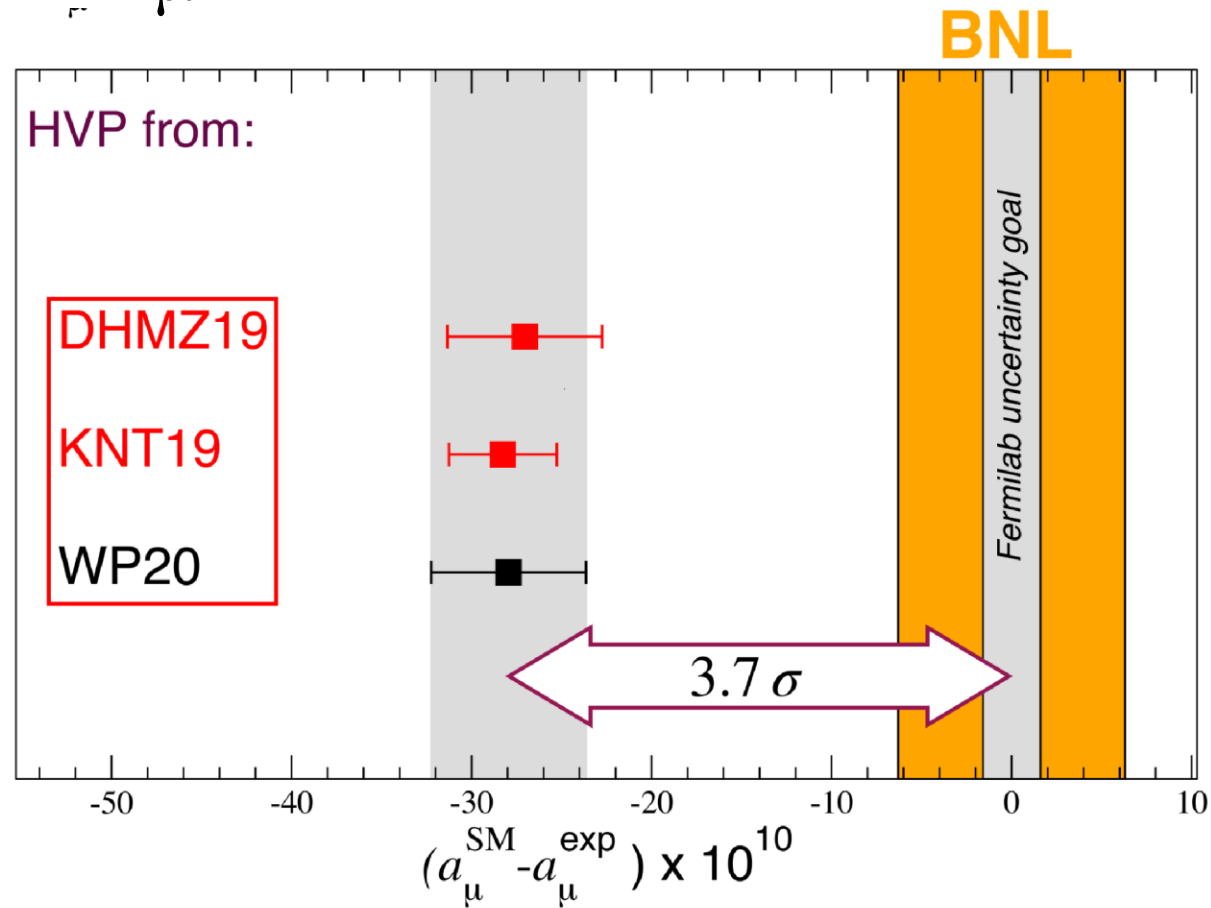
BNL



Davier-Höcker-Malaescu-Zhang'19 & Keshavarzi-Nomura-Teubner'19 drive the White Paper'20 combination

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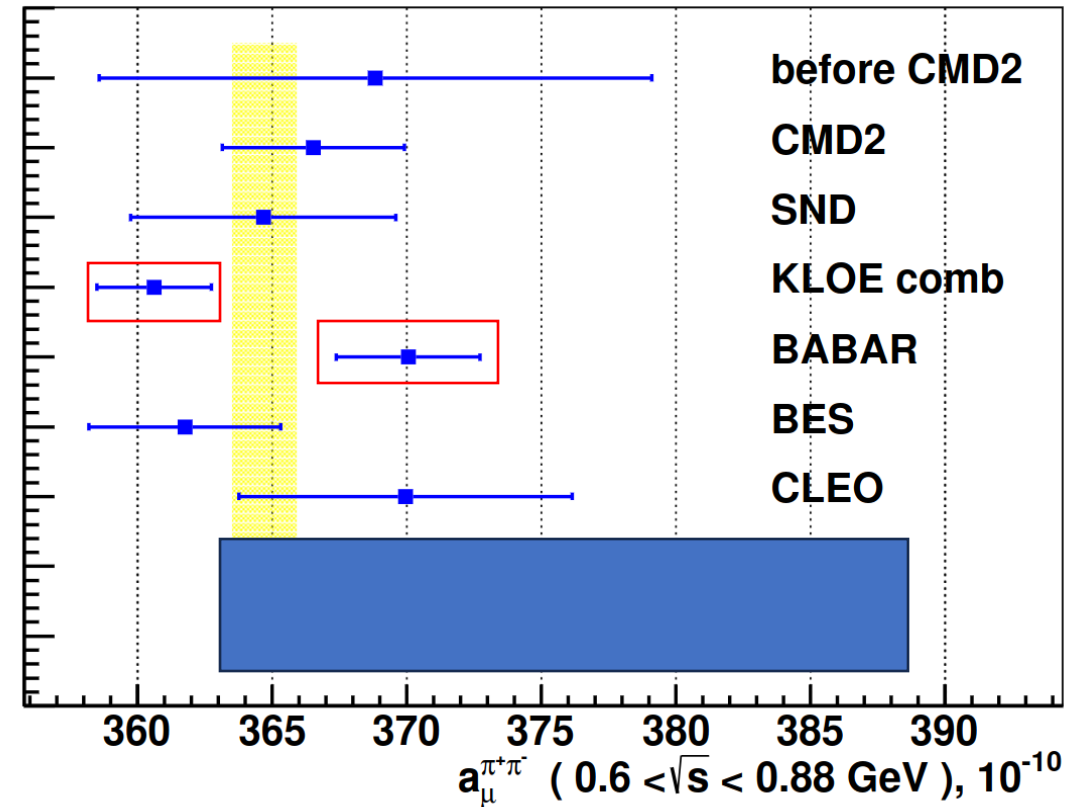
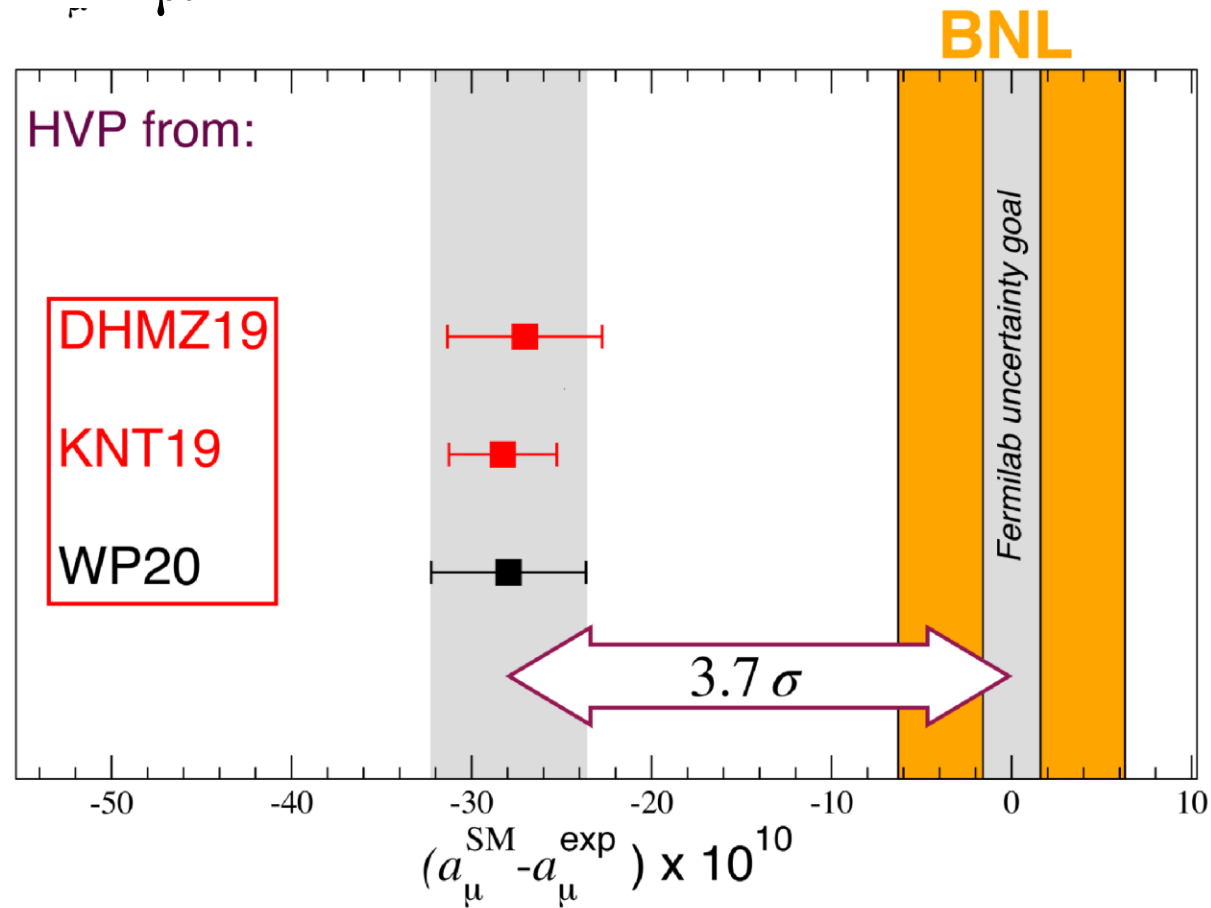


Figure 36: The  $\pi^+\pi^-(\gamma)$  contribution to the  $a_\mu^{\text{had},LO}$  from the energy range  $0.6 < \sqrt{s} < 0.88 \text{ GeV}$  obtained from the CMD-3 data and the results of the other experiments.

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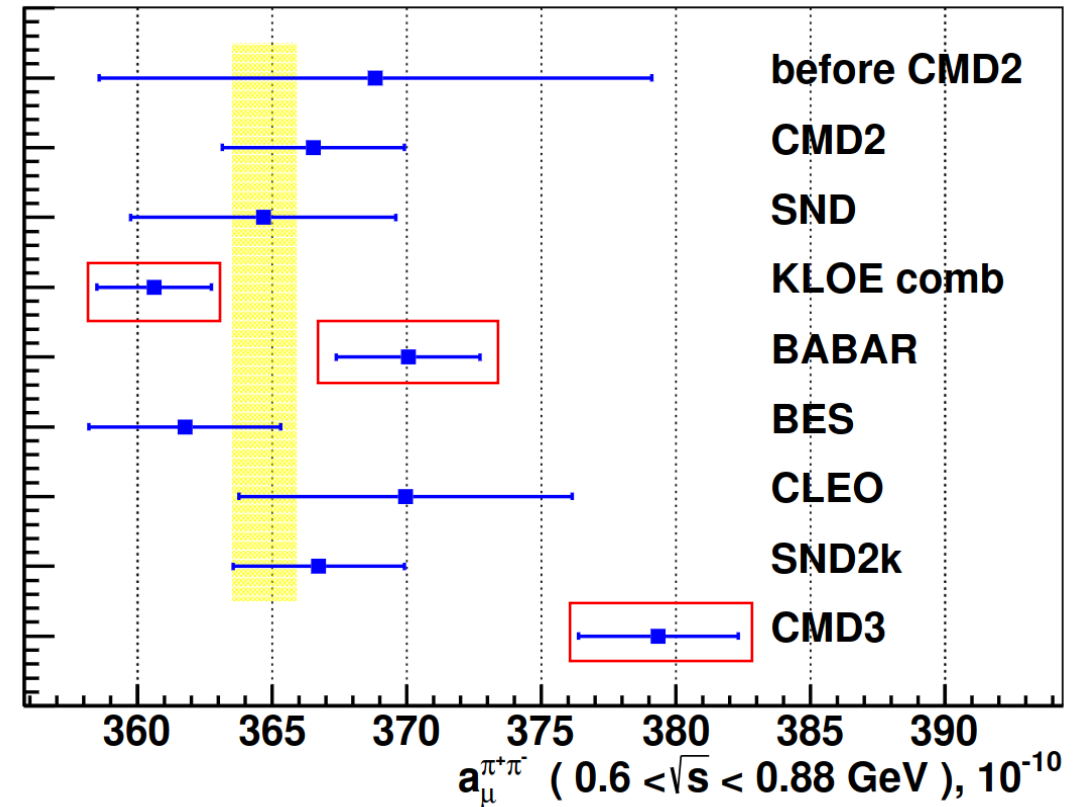


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# $a_{\mu}^{\text{HVP}}$ in the SM: Our $\tau$ -data based prediction

Miranda-Roig'20

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(Alemany, Davier, Höcker '97)

Alternative evaluation possible using semileptonic tau decay data, specifically  $2\pi$  ( $4\pi$ ) channel. Requires isospin breaking (IB).

$$\sigma_{\pi\pi}^0 = \left[ \frac{K_\sigma(s)}{K_\Gamma(s)} \frac{d\Gamma_{\pi\pi[\gamma]}}{ds} \right] \frac{R_{IB}(s)}{S_{EW}},$$

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Kinematics  
& global cts.

Measurement

Short-distance EW RadCor

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Kinematics & global cts.      Measurement      Short-distance EW RadCor      Final-state Rad      Kinematics

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Measurement
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➔

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Final-state Rad
Kinematics

(Cirigliano-Ecker-Neufeld '01)

The ratio of neutral to charged current di-pion form factors ( $F_V/f_+$ ) and the long-distance em RadCor ( $G_{EM}$ ) are challenging.

# $a_\mu^{\text{HVP}}$ in the SM: Our $\tau$ -data based prediction

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Kinematics & global cts.
Measurement
Short-distance EW RadCor
Final-state Rad
Kinematics

The ratio of neutral to charged current di-pion form factors ( $F_V/f_+$ ) and the long-distance em RadCor ( $G_{EM}$ ) are challenging.

- The  $S_{EW}$  contribution  $S_{EW} = 1.0233$  gives  $\Delta a_\mu^{\text{HVP},LO} = -119.6 \times 10^{-11}$ , consistent with earlier determinations (using slightly different values of  $S_{EW}$ ) and with a negligible error.
- The phase space (PS) correction induces  $\Delta a_\mu^{\text{HVP},LO} = -74.5 \times 10^{-11}$  (trivially in agreement with previous computations), again with tiny uncertainties.
- The final state radiation (FSR, which is formally  $NLO$ ) yields  $\Delta a_\mu^{\text{HVP},LO} = +45.5(4.6) \times 10^{-11}$ , in accord with ref. [67] (its value was not quoted in ref. [62]).

[62] Cirigliano-Ecker-Neufeld'02  
 [67] Davier-...-López Castro-...-Toledo et al.'09

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# $a_\mu^{\text{HVP}}$ in the SM: Our $\tau$ -data based prediction

Miranda-Roig'20

$$\sigma_{\pi\pi}^0 = \left[ \frac{K_\sigma(s)}{K_\Gamma(s)} \frac{d\Gamma_{\pi\pi[\gamma]}}{ds} \right] \frac{R_{IB}(s)}{S_{EW}},$$

Kinematics & global cts.
Measurement
Short-distance EW RadCor

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Final-state Rad
Kinematics

The ratio of neutral to charged current di-pion form factors ( $F_V/f_+$ ) and the long-distance em RadCor ( $G_{EM}$ ) are challenging.

This correction was  $+(61 \pm 26 \pm 3) \cdot 10^{-11}$  in [62] and  $+(86 \pm 32 \pm 7) \cdot 10^{-11}$  in [67], in agreement (despite the big errors) with our FF2 and FF1 determinations, respectively.

$$\Delta a_\mu^{\text{HVP},LO} = +40.9(48.9) \times 10^{-11} \quad \Delta a_\mu^{\text{HVP},LO} = +77.6(24.0) \times 10^{-11}$$

[62] Cirigliano-Ecker-Neufeld'02  
 [67] Davier-...-López Castro-...-Toledo et al.'09

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Kinematics & global cts.
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The ratio of neutral to charged current di-pion form factors ( $F_V/f_+$ ) and the long-distance em RadCor ( $G_{EM}$ ) are challenging.

- Finally, we get  $(-15.9^{+5.7}_{-16.0}) \cdot 10^{-11}$  ( $(-76 \pm 46) \cdot 10^{-11}$ ) for the  $G_{EM}(s)$  correction at  $\mathcal{O}(p^4)$  ( $\mathcal{O}(p^6)$ ), versus  $-10 \cdot 10^{-11}$  in [62] and  $-37 \cdot 10^{-11}$  in [65] (from the last two results,  $(-19.2 \pm 9.0) \cdot 10^{-11}$  was used in [67]). ( $\omega \rightarrow \pi^0 \gamma$  contribution was subtracted from [65]'s)

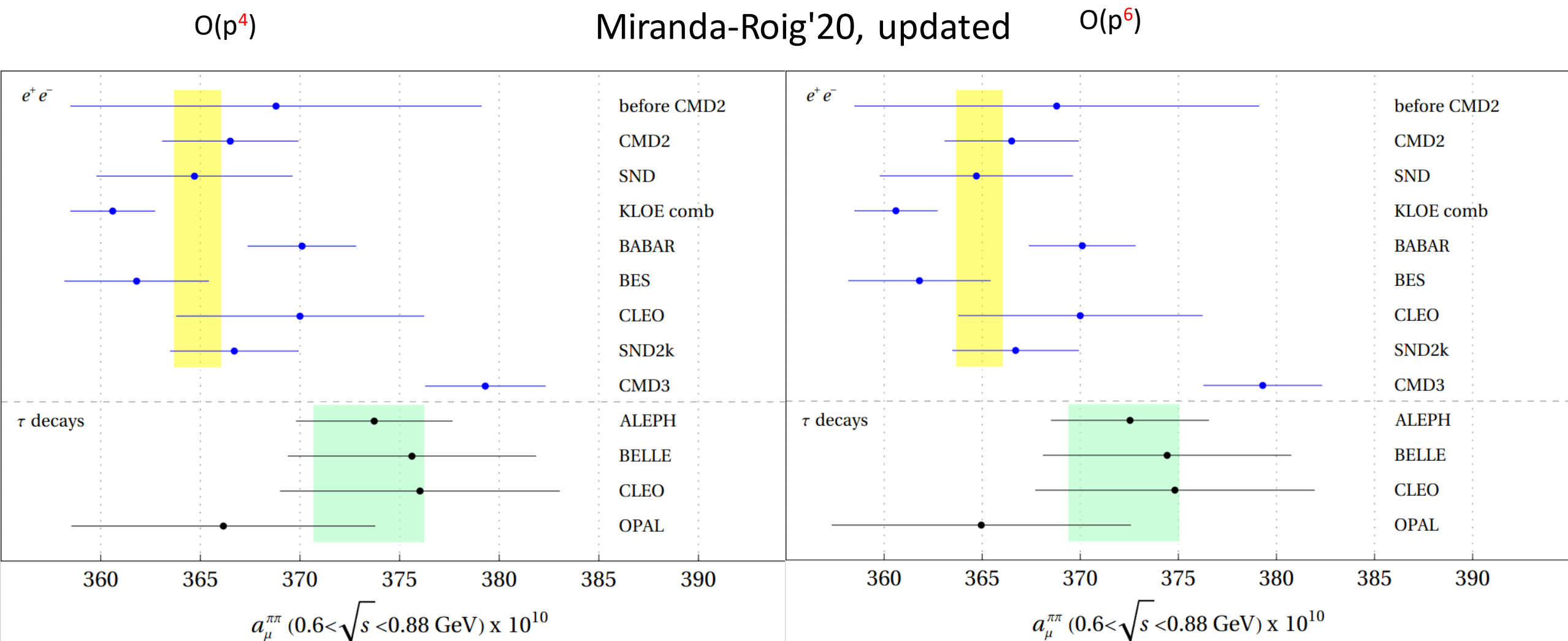
[62] Cirigliano-Ecker-Neufeld'02  
 [67] Davier-...-López Castro-...-Toledo et al.'09

[65] Florez Baez- Flores Tlalpa-López Castro-Toledo '06

Consistent results found in Esparza-Arellano—Rojas—Toledo'23

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# $a_\mu^{\text{HVP}}$ in the SM: Our $\tau$ -data based prediction



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# $a_\mu^{\text{HVP}}$ in the SM: Window quantities

Blum, Boyle, Gülpers, Izubuchi, Jin, Jung, Jüttner, Lehner, Portelli, Tsang (RBC, UKQCD), '18

$$\Theta_{\text{SD}}(t) = 1 - \Theta(t, t_0, \Delta),$$

$$\Theta_{\text{win}}(t) = \Theta(t, t_0, \Delta) - \Theta(t, t_1, \Delta),$$

$$\Theta_{\text{LD}}(t) = \Theta(t, t_1, \Delta), \quad t_0 = 0.4 \text{ fm}, \quad t_1 = 1.0 \text{ fm}, \quad \Delta = 0.15 \text{ fm}.$$

$$\Theta(t, t', \Delta) = \frac{1}{2} \left( 1 + \tanh \frac{t - t'}{\Delta} \right),$$

# $a_{\mu}^{\text{HVP}}$ in the SM: Window quantities

Colangelo, El-Khadra, Hoferichter, Keshavarzi, Lehner, Stoffer, Teubner'22

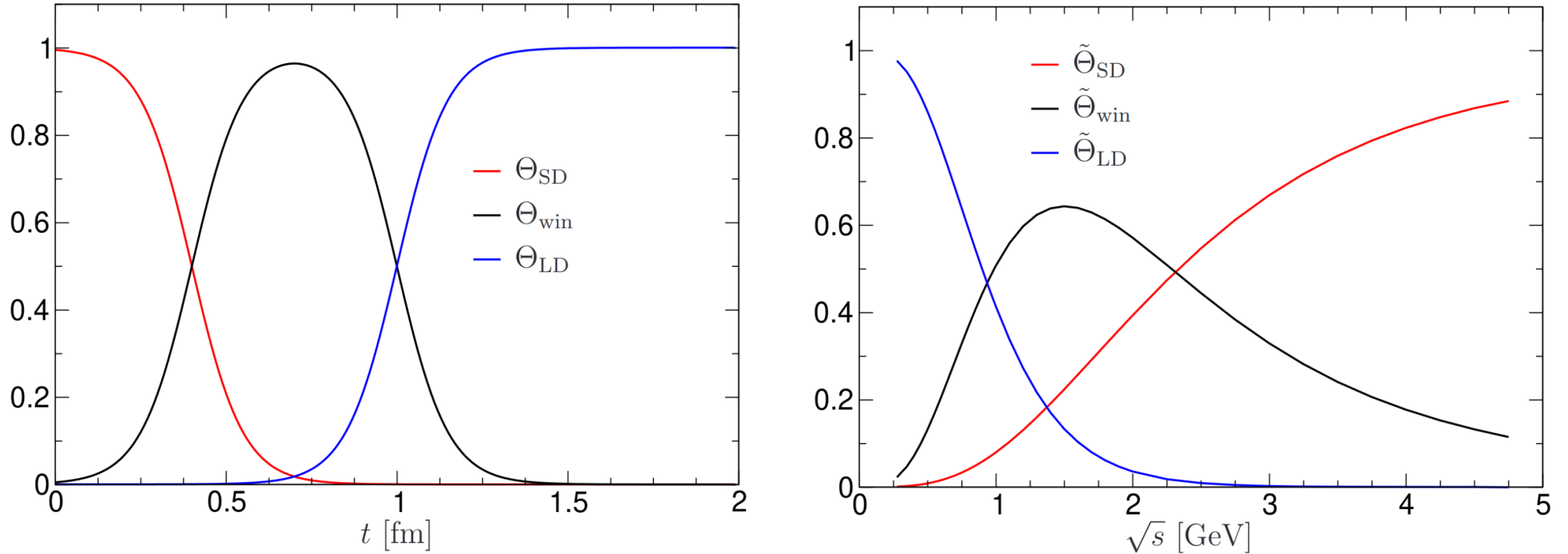
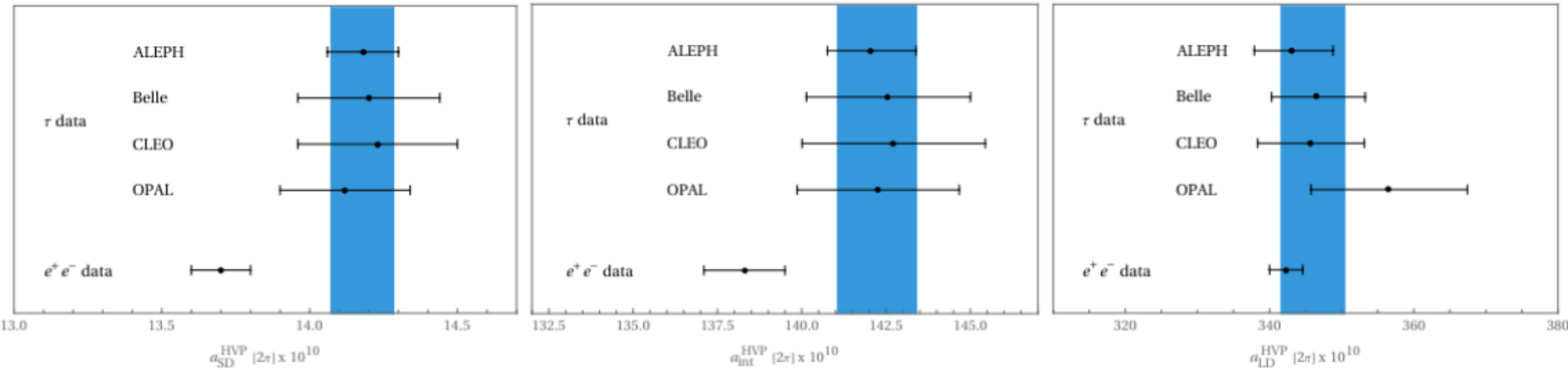


Figure 1: Short-distance, intermediate, and long-distance weight functions in Euclidean time (left), and their correspondence in center-of-mass energy (right).

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# $a_{\mu}^{\text{HVP}}$ in the SM: Our $\tau$ -data based prediction

Masjuan-Miranda-Roig'23

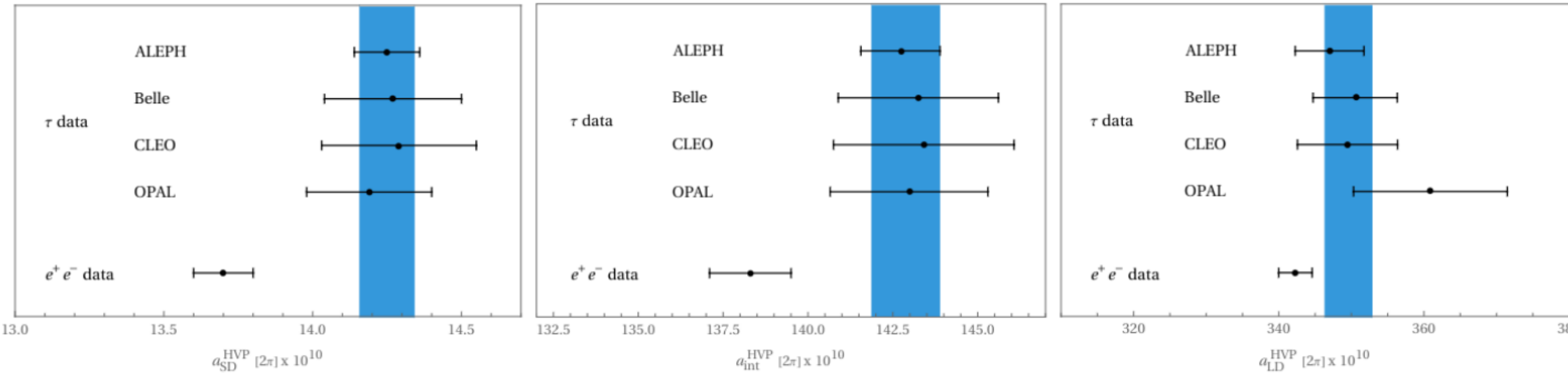


Window quantities @ $O(p^6)$  below 1 GeV. Blue band is  $\tau$ -data average.  $e^+e^-$  number taken from Colangelo et al.'22

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Masjuan-Miranda-Roig'23



Window quantities @ $\mathcal{O}(p^4)$  below 1 GeV. Blue band is  $\tau$ -data average.  $e^+e^-$  number taken from Colangelo et al.'22

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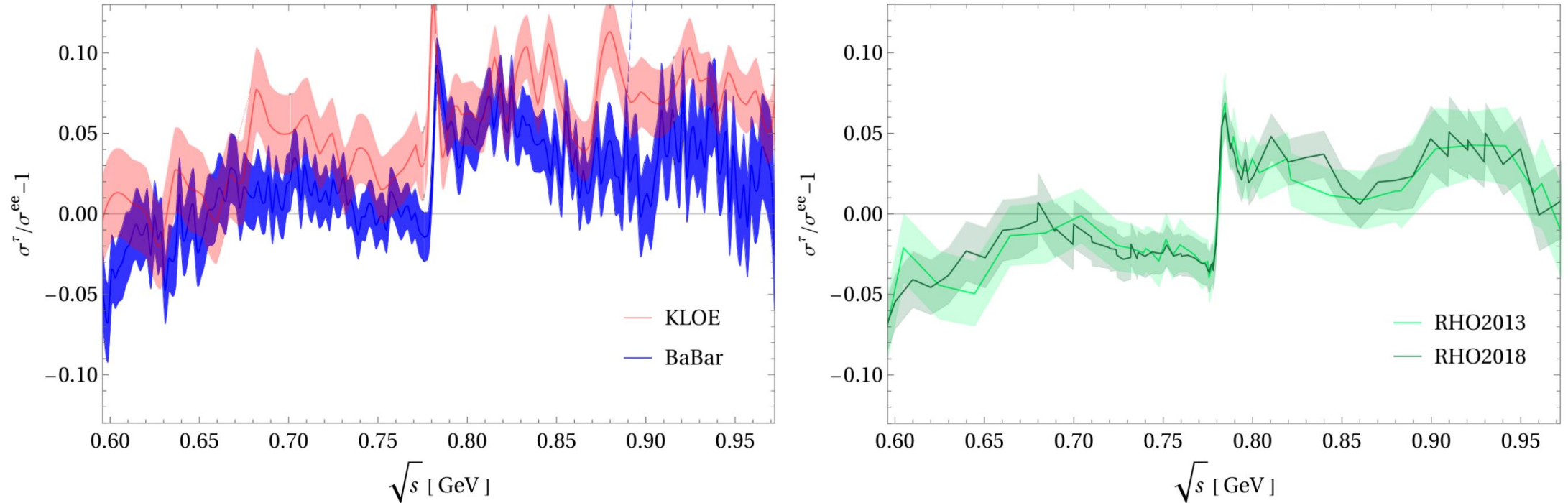


Figure 3: Comparison between the  $\tau$  (after IB corrections) and  $e^+e^- \rightarrow \pi^+\pi^-$  spectral function using the ISR measurements from BABAR [78] and KLOE [76] (left-hand) and the energy-scan measurements from CMD-3 [67] (right-hand).

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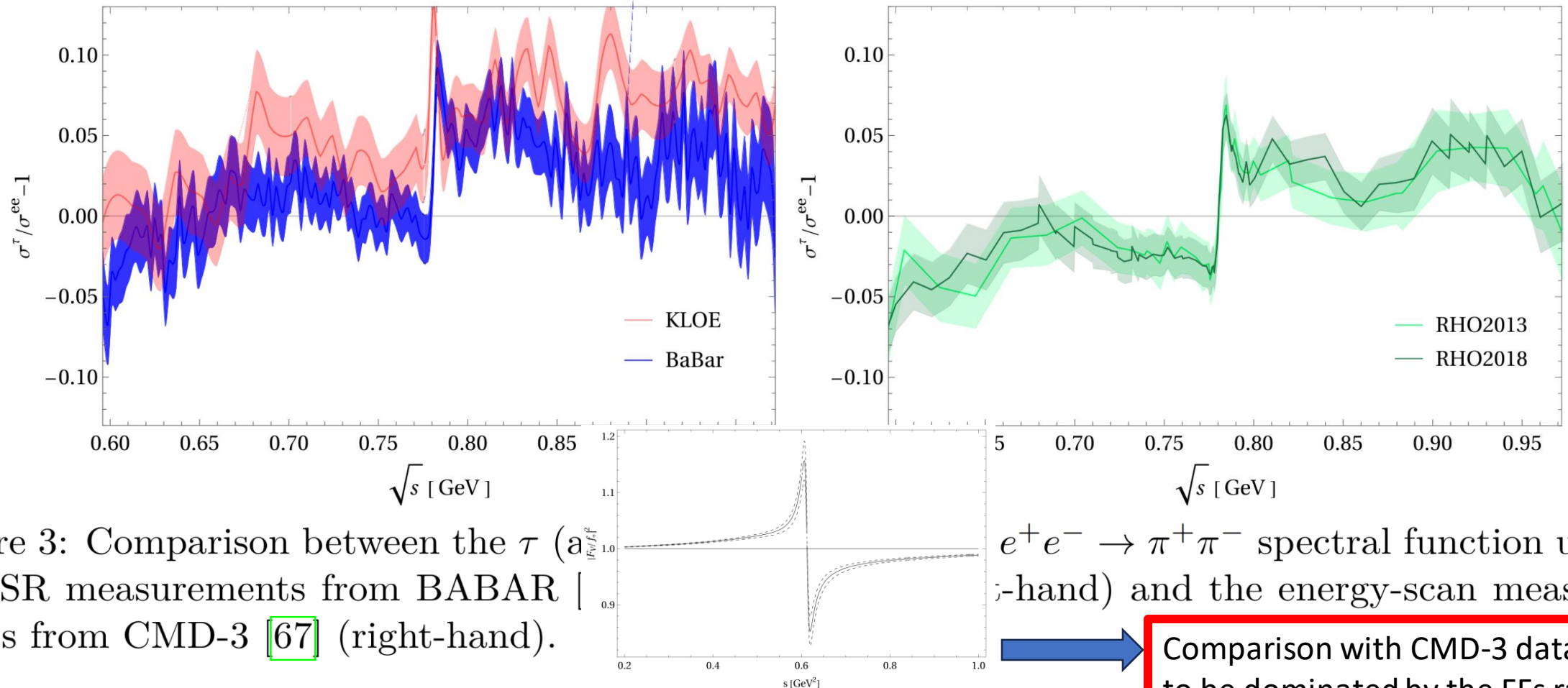


Figure 3: Comparison between the  $\tau$  ( $a_{\mu}^{\text{HVP}}$ ) using the ISR measurements from BABAR [67] (right-hand).

$e^+e^- \rightarrow \pi^+\pi^-$  spectral function using  $\tau$ -hand)

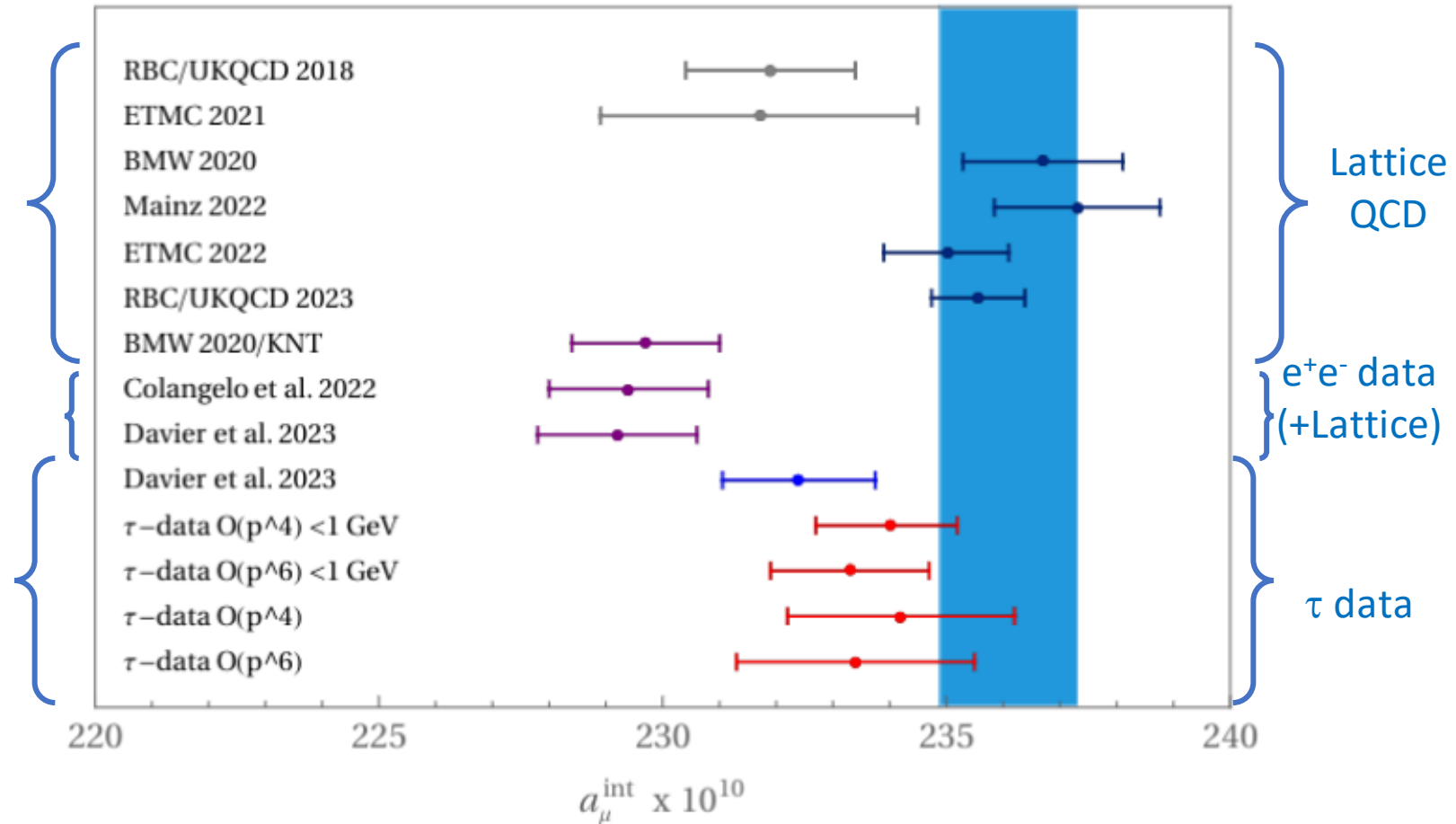
Comparison with CMD-3 data seems to be dominated by the FFs ratio!!

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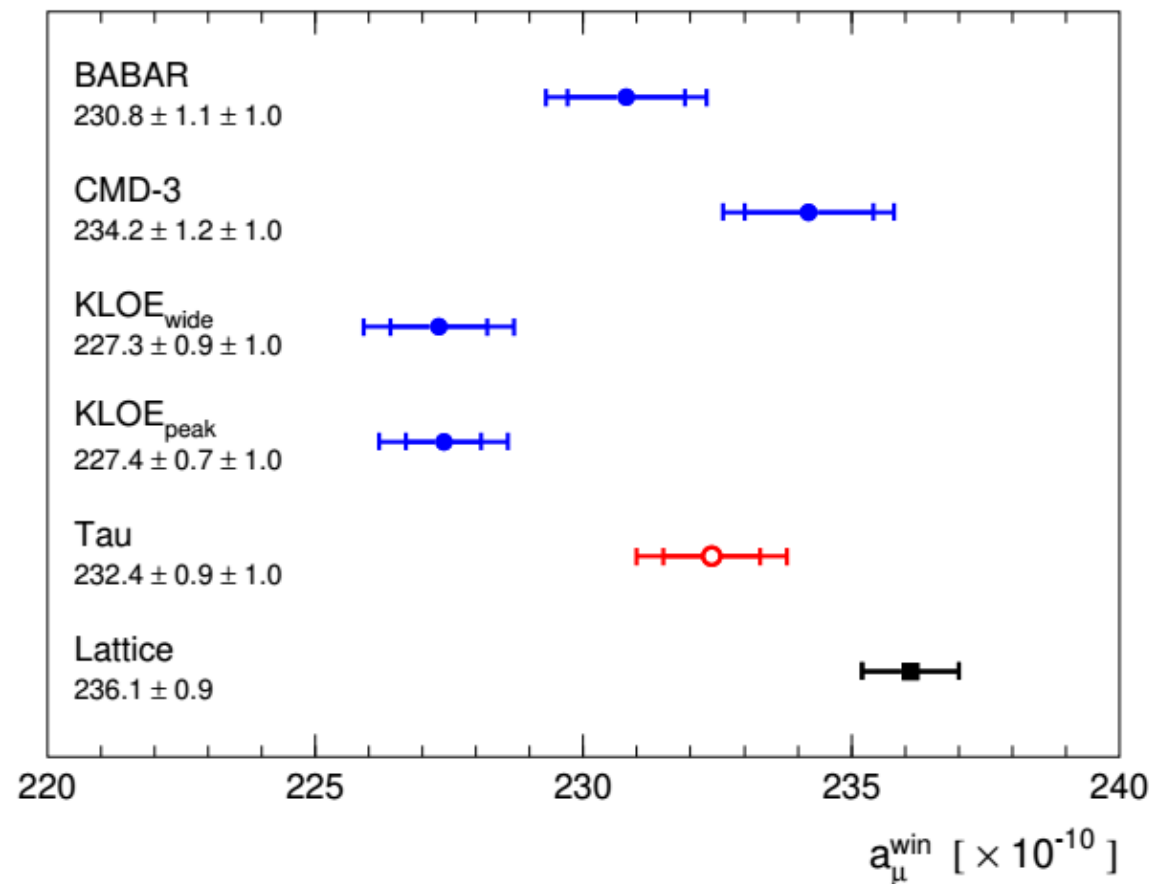
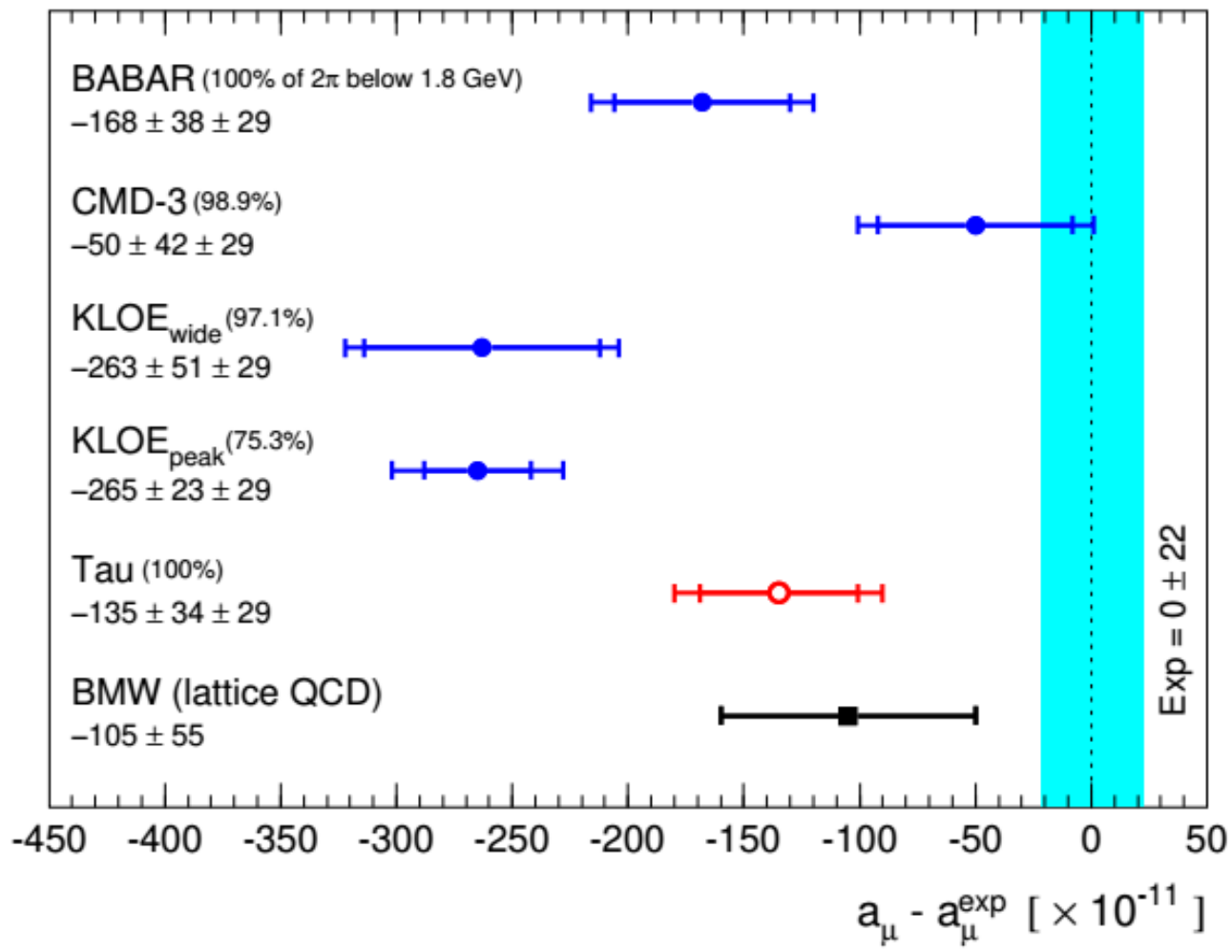
Masjuan-Miranda-Roig'23

Comparison of the total intermediate window contribution to  $a_\mu^{\text{HVP,LO}}$ . Blue band corresponds to lattice average excluding first two results (currently superseded).



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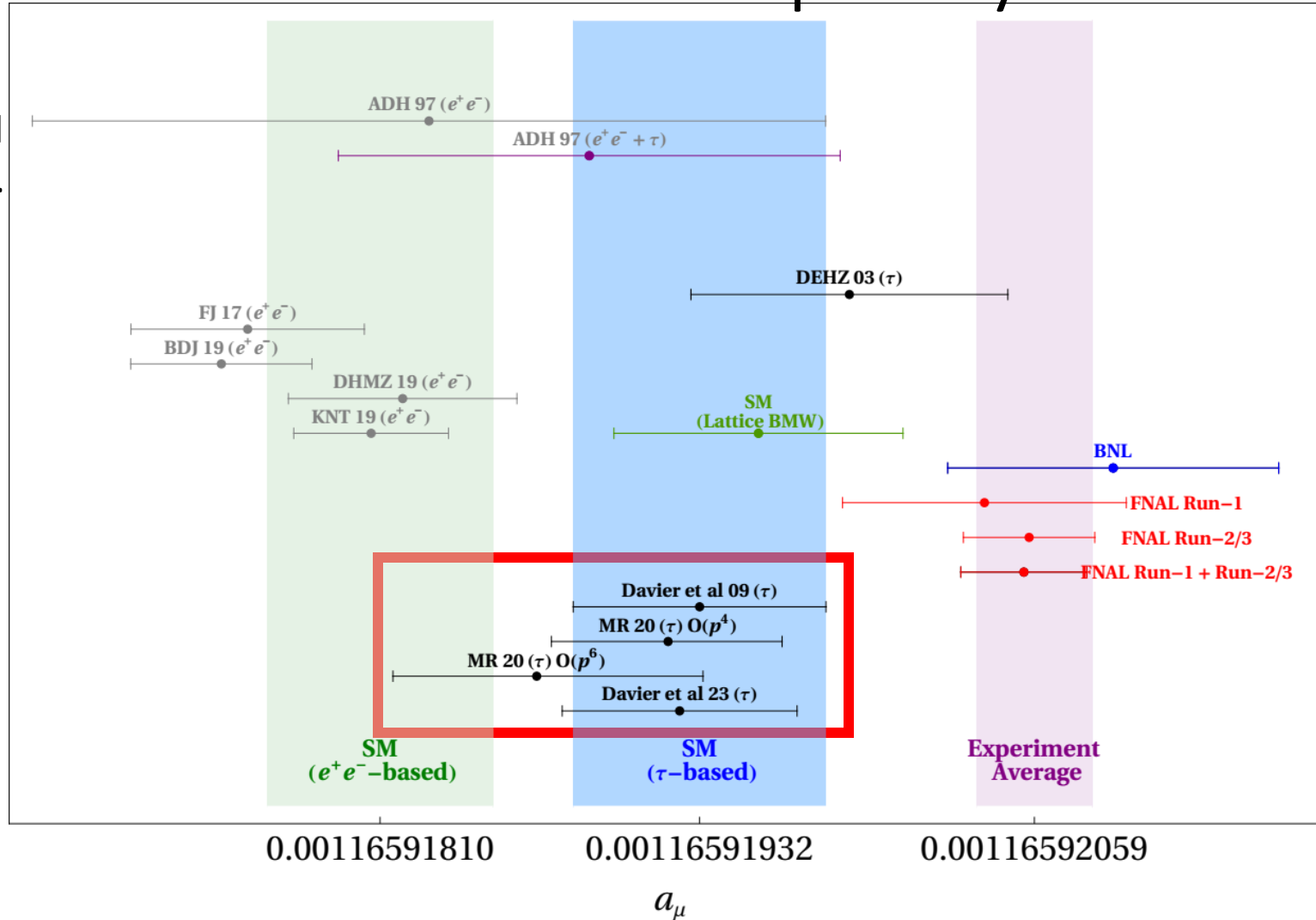
# Today's paper: 2312.02053 Davier, Höcker, Lutz, Malaescu & Zhang





# Their discrepancy (Updated using Miranda-Roig'20)

- Experimental result seems extremely reliable. Uncertainty will decrease by  $\sim \frac{1}{2}$  in 2025.

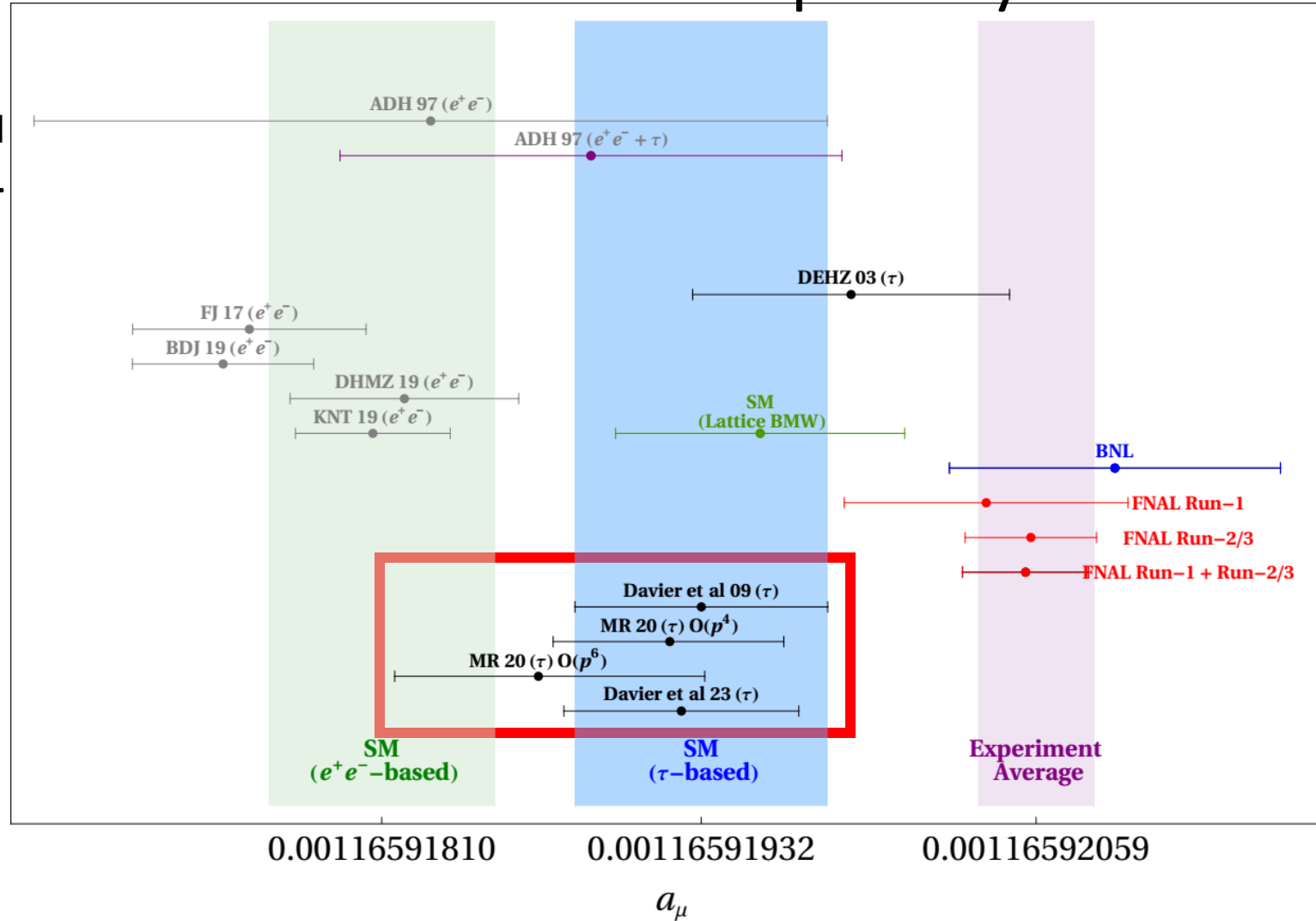


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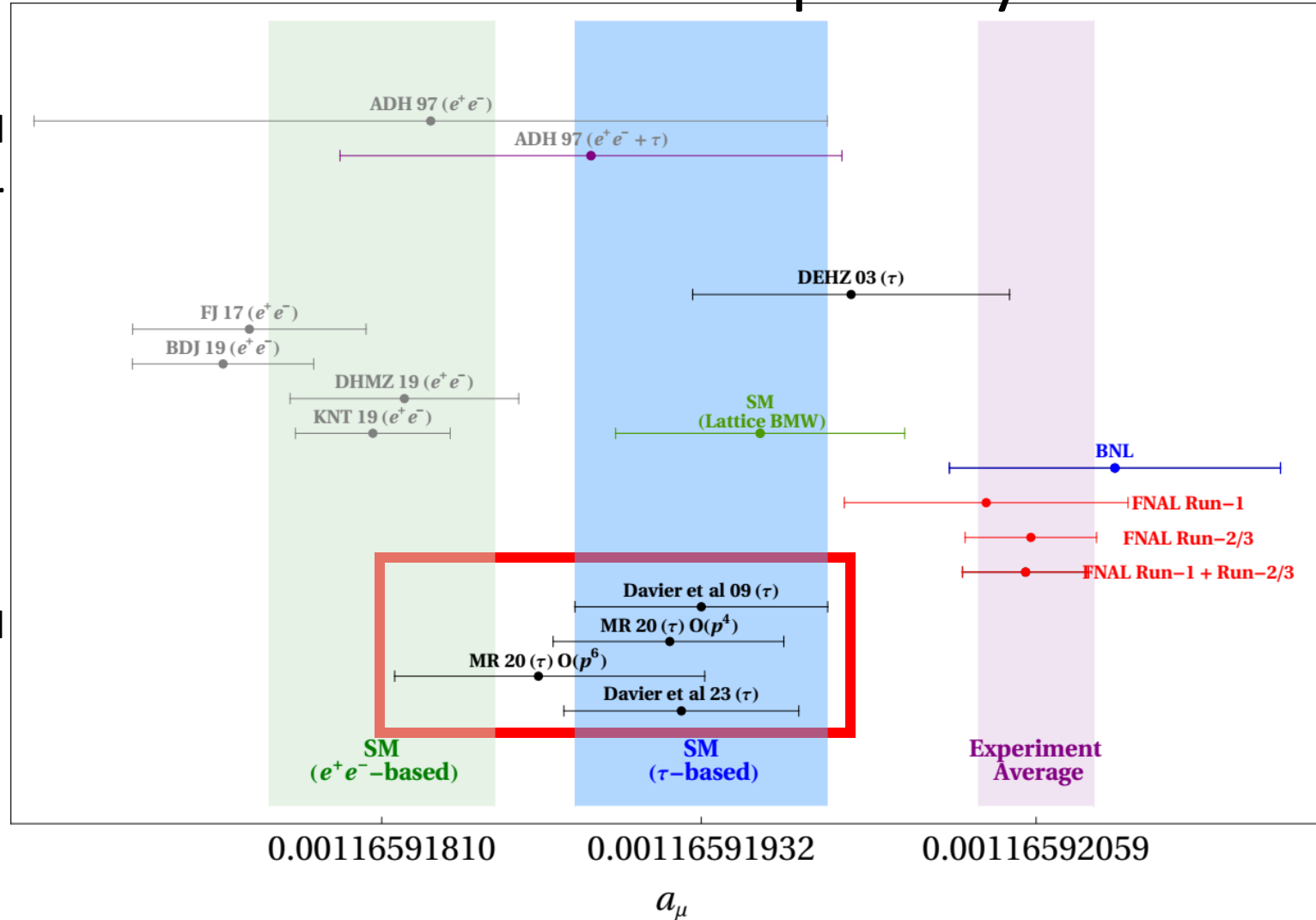
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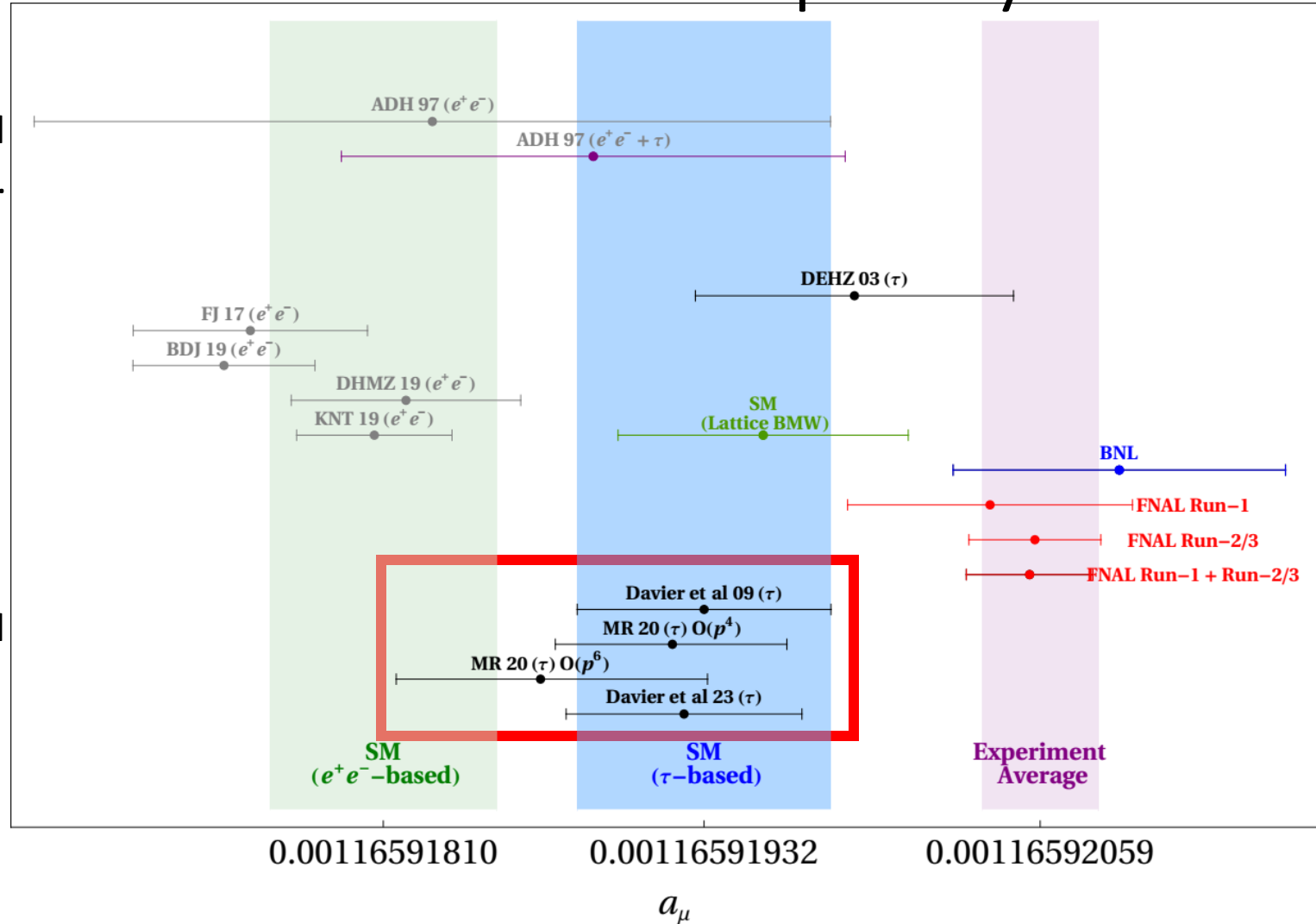
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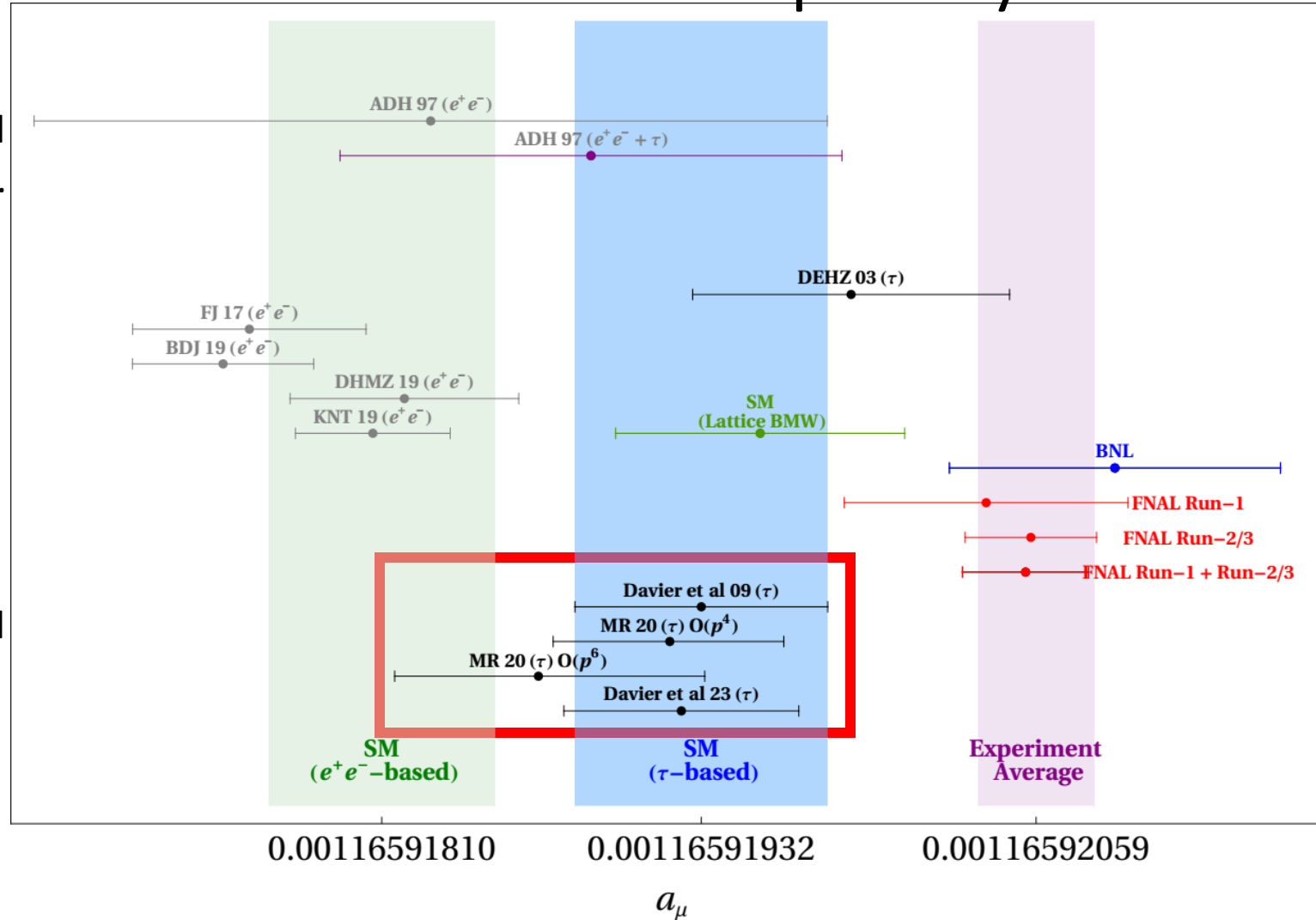
- Tau data based results have been more consistent over time: between experiments and among groups using them (with slightly different IB corr.). These also include: Cirigliano-Ecker-Neufeld'01,02; Maltman'05, Maltman-Yorke'06,'11; reanalyses by Davier's group (including today's); Esparza-Arellano—Rojas—Toledo'23, ...

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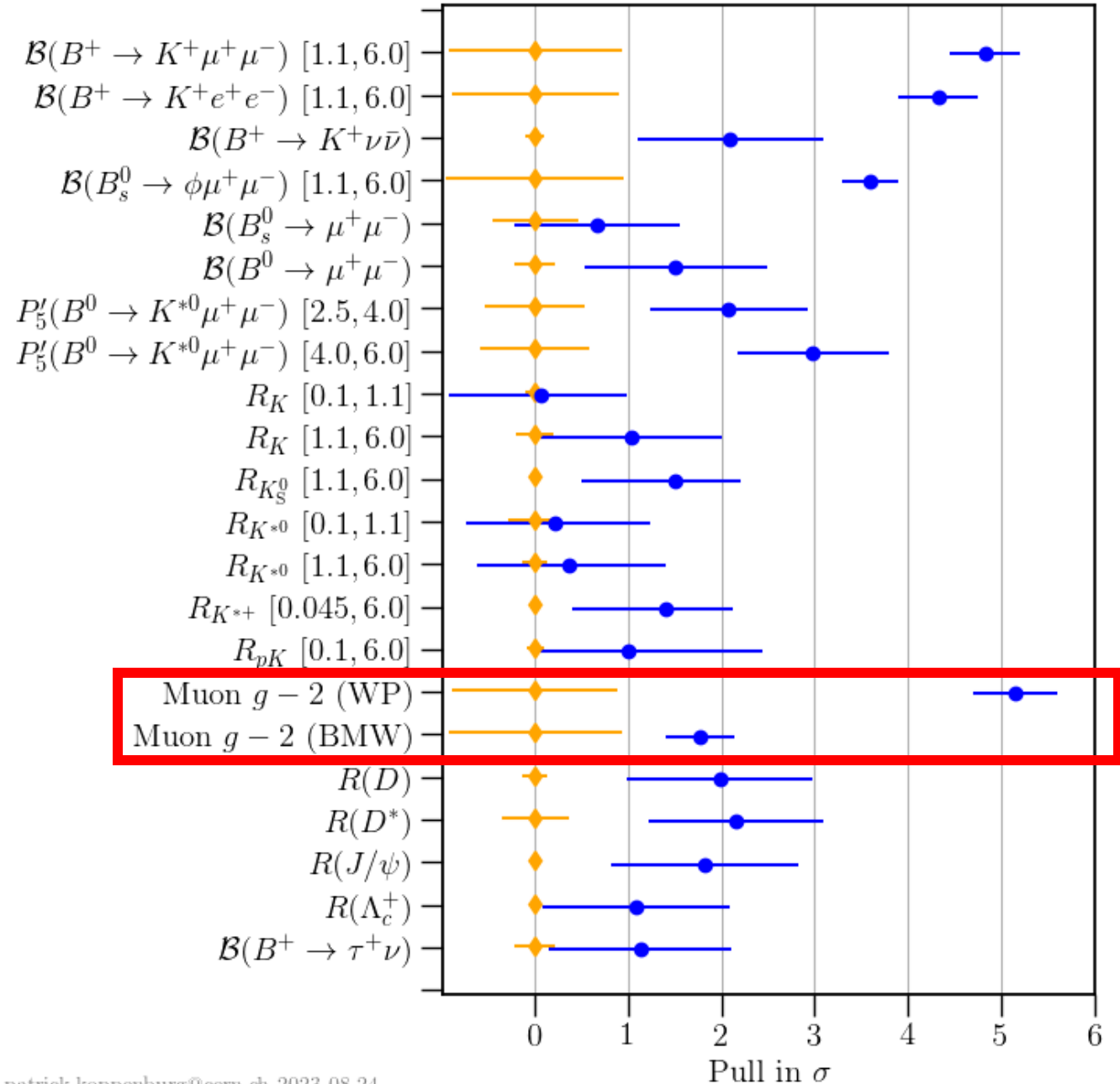
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Pablo Roig (Cinvestav, Mexico City)

# ANOMALIES & THEIR PULL



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# Outlook (Largely from Theory Initiative web)

- See **VI plenary workshop of the Muon g-2 Theory Initiative** in Bern, <http://muong-2.itp.unibe.ch/> (4-8 Sept. '23).
- A new analysis of the cross section based on the **full statistics** collected by the **BABAR** experiment is underway.
- The **SND** 2020 analysis was based on 10% of the available statistics. An **update** using the full data set is in preparation.
- **Improved BES III** data between 2&5 GeV will be published.
- The **largest KLOE** data set ('04-'05) will be **analyzed**. Its statistics is 7 times larger than their published results together.
- **Radiative corrections and Monte Carlo generators**, in particular for the crucial di-pion channel, **are being scrutinized**. This includes the calculation and implementation of higher-order and structure-dependent corrections.
- **New lattice-QCD results** for the total HVP contribution and the long-distance window observable with a precision comparable to BMW and the data-driven approach will be available by 2025.
- **Other window quantities** and related observables will be analyzed.
- **Belle-II** will soon release their 3pi analysis & by 2025 the  $\pi\pi$  one.
- The **MUonE** experiment at CERN will provide an independent and competitive method to compute the HVP contribution to the muon g-2, based on the high-precision measurement of the shape of the differential cross section of muon-electron elastic scattering as a function of the space-like squared momentum transfer. It would take data from 2026 on.
- **Lattice QCD** will make substantial progress in the evaluation of the **IB corrections** needed to use tau data for the HVP contribution.
- **Belle-II** shall improve the measurement of di-pion **tau** decays.
- ...

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Nos complace anunciarle que el Jurado Calificador de los Premios Arturo Rosenblueth 2023, ha designado a las y los ganadores a las mejores tesis de doctorado del Cinvestav.

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Autor:  
Jesús Alejandro Miranda Hernández  
Director de tesis:  
Dr. Pablo Roig Garcés  
Título de tesis:  
Análisis de Teoría de campo efectiva de sondas de precisión a bajas energías en la búsqueda de nueva física  
Programa: Doctorado en Ciencias en la especialidad de Física  
Departamento: Física

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Dr. Antony Boucard Jr.  
Título:  
Identificación de un eje molecular de patogenidad dado por defectos de señalización en la vía G13 como resultado de la alteración del receptor Lphn3 por variantes asociadas al Trastorno por Déficit de Atención con Hiperactividad  
Programa: Doctorado en Ciencias en la especialidad de Biología Celular  
Departamento: Biología Celular

**ÁREA DE TECNOLOGÍA Y CIENCIAS DE LA INGENIERÍA (TCI)**

Autor:  
Juan Pablo Flores Flores  
Director de tesis:  
Dr. Rafael Martínez Guerra  
Título de tesis:  
Controladores dinámicos para sistemas distribuidos enteros y fraccionales  
Programa: Doctorado en Ciencias en la especialidad de Control Automático  
Departamento: Control Automático

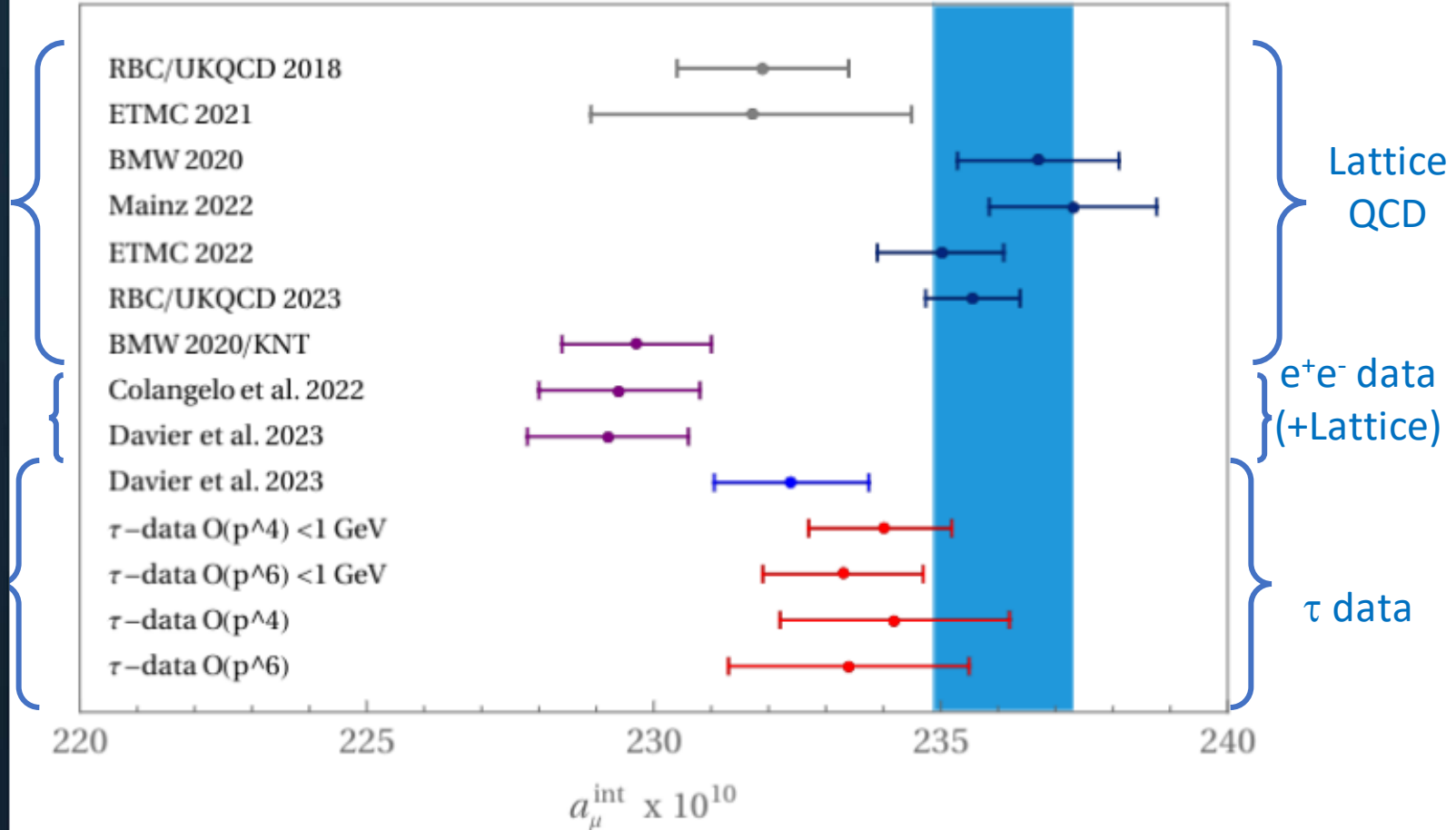
**ÁREA DE CIENCIAS SOCIALES Y HUMANIDADES (CSH)**

El premio fue declarado desierto.

La ceremonia de entrega de los premios Arturo Rosenblueth se llevará a cabo el día 5 de diciembre a las 17:00 horas.

<https://sinac.cinvestav.mx/Rosenblueth/>

Miranda-Roig'20  
Masjuan-Miranda-Roig'23



Pablo Roig (Cinvestav, Mexico City)