Monte Carlos for tau lepton – Standard Model and New Physics signatures.

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- One of the main purpose of High Energy accelerator experiments is to provide confrontation of theory and measurements in ever new realms.

- Any new agreement extend theory applicability domain, any discrepancy hint to unexplained. That means needs of better calculations or new deeper theory.

- Because easy things for present day accelerators are already completed. One has to search for small variation over large Standard Model distributions.

- My talk concentrate on the following points :

(i) KKMC for $e^+e^- \to \tau^+\tau^-(n\gamma)$; τ decays included
(ii) Emission of additional pairs of SM and New Physics
(iii) Anomalous magnetic and electric dipole moments
(iv) Arrangements for τ decays.
(v) Arrangements for software.

(i) KKMC for $e^+e^- \to \tau^+\tau^-(n\gamma)$

- **Main design targets:**
  (i) precision simulation of lepton pair production
  (ii) flexible set up initialization for user defined hadronic currents.
- The C++ version is available, better arrangements for beam-spread, continuity of numerical tests was essential ingredient of work: Comput.Phys.Commun. 283 (2023) 108556,
- work on enhanced precision of QED part started (2303.14260 Phys Lett B in print), but road toward third order matrix elements is long and perilous.
- New authors are entering, preservation of skills, continuity and new challenges.
- The C++ version is not yet adopted to lower energies (just $\Pi_{\gamma\gamma}(s)$ and dispersion relations).
- User service issues: help in installations and set ups.
- The F77 version as installed in Belle2 I take care of myself.

(ii) Emission of additional pairs of SM and New Physics

- Pair emission is the necessary step toward third order matrix element installation. From algorithmic point of view, treated as radiative corrections it is incompatible with exponentiation, because corresponding crude distribution does not feature conformal symmetry. This symmetry is essential in building relation between crude level Monte Carlo with matrix elements of eikonal form, and of line-shape generation for initial state bremsstrahlung.
- Final state pair emission can be achieved by simultaneous running of KKMC of appropriate part of loop vertex correction switched on, and of simultaneous run of four fermion Monte Carlo KORALW.
- Alternatively, for final state pair emissions, Monte Carlo of the after-burn type, `PHOTOS` is available since Comput.Phys.Commun. 199 (2016) 86. Program enables full phase-space coverage. Tests with Matrix element simulations indicate that precision is sufficient for today applications.
- some studies of systematic are included in: Eur.Phys.J.C 83 (2023) 1
- Further work for FCC precision is of course needed.

(ii) Emission of additional pairs of SM and New Physics

- Nonetheless some extensions beyond Standard Model are available already now.

  • For τ decays: Comput.Phys.Commun. 283 (2023) 108592,

  • for production process Contribution to TAU2021: 2111.05914. It is about emission of dark photons (dark scalars) decaying to light lepton pairs.

- Extensions of KKMC Monte Carlo for four fermion final states (processes mediated by ZH pair) is a possible future continuation of this development path.

- But it is long way to achieve FCC precision.

(iii) Anomalous magnetic and electric dipole moments

- The g-2 measurements implied interests in anomalous dipole moments.
- The question arise how to simulate their impact on differential distributions. We addressed this point with the help of algorithms enabling calculation of event weights: ratios of matrix elements for production and decay of tau lepton pairs with other the one without dipole moment effects.
- First it was done for lower energies $e^+e^- \to \tau^+\tau^-$ processes, as of Belle 2, Phys.Rev.D 106 (2022) 11, 113010
- Later for higher energies, as of Z peak and above. 2307.03526
- In the second paper, elements of such calculations are prepared for $pp \to \tau^+\tau^- X$ as well.

(iv) Arrangements for τ decays.

- The process of migration to C++ is advancing well. The C++ Versions of PHOTOS Comput.Phys.Commun. 199 (2016) 86 and KKMC Comput.Phys.Commun. 283 (2023) 108556 are published.
- TAUOLA for τ decays is prepared for such transformation too. It internal structure was prepared: Comput.Phys.Commun. 232 (2018) 220 already some time ago. Now improvements/fits in Belle 2 hands ...
- Program was prepared to run with user provided hadronic currents, which could be coded in C++ and activated with redefinition of pointers.
- That temporary solution was partly abandoned. Step back was due to requirements of Belle software organization.
- My experience with PHOTOS is that the best is to translate program with somebody's else participation. It can not go too fast, because existing arrangements for future and for tests can be easily lost.
- That double authoring is useful, even if partner choose other path of life.
- It is profitable for future takeover of the projects, independently who it will be.

(v) Arrangements for software.

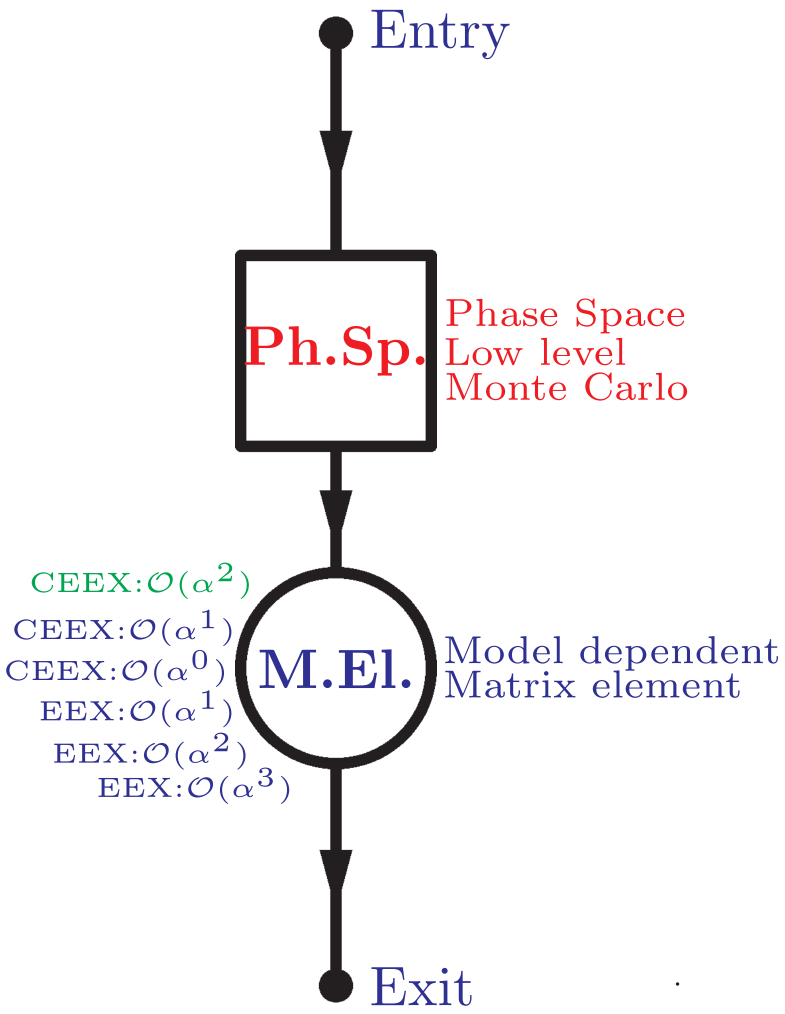
- The software for Belle 2 is working well and I am involved personally. Main master is Swagato Banerjee.
- Installation and initialization for other platforms pose sometimes problems especially for C++ version of KKMC. Proper loading of root library was bringing difficulties and it was reported to us.
- That is one of the most visible aspect of the man power issue for our community.
- but passing expertise to the next generation is potentially larger issue, even if its challenges are not imminent.

Now some details ...

I will skip or cover most of these points briefly, and will focus a bit more on anomalous dipole moments only.

KKMC for $e^+e^- \to \tau^+\tau^-(n\gamma)$

KKMC basis: "matrix element $\times$ full and exact phase space" it is NOT shower-like algorithm !



- Phase-space simulator: basis for "raw events" (sampling internal wights for intermediate resonances/singularities).
- Tower of simplified/improvable structures: 1-dim spectrum $\to$ eikonal matrix elements (full phase space) $\to$ QED bremsstrahlung part $\to$ remaining electroweak QCD parts $\to$ pair emissions $\to$ Beware of presamplers: same CW-complexes for crude and final space.
- Library of Matrix Elements; input for "model weight"; independent module
- Used for LEP precision Monte Carlos, e.g. KKMC. Now it is used for Belle (FCC ...) for τ lepton pair production with decays and multi-photon radiation.
- Details: Phys. Rev. D41 (1990) 1425.

Phase space is treated exactly , no approximations are used. But Effects of pair emissions is not large, matrix elements are simplified (in improvable way though)!

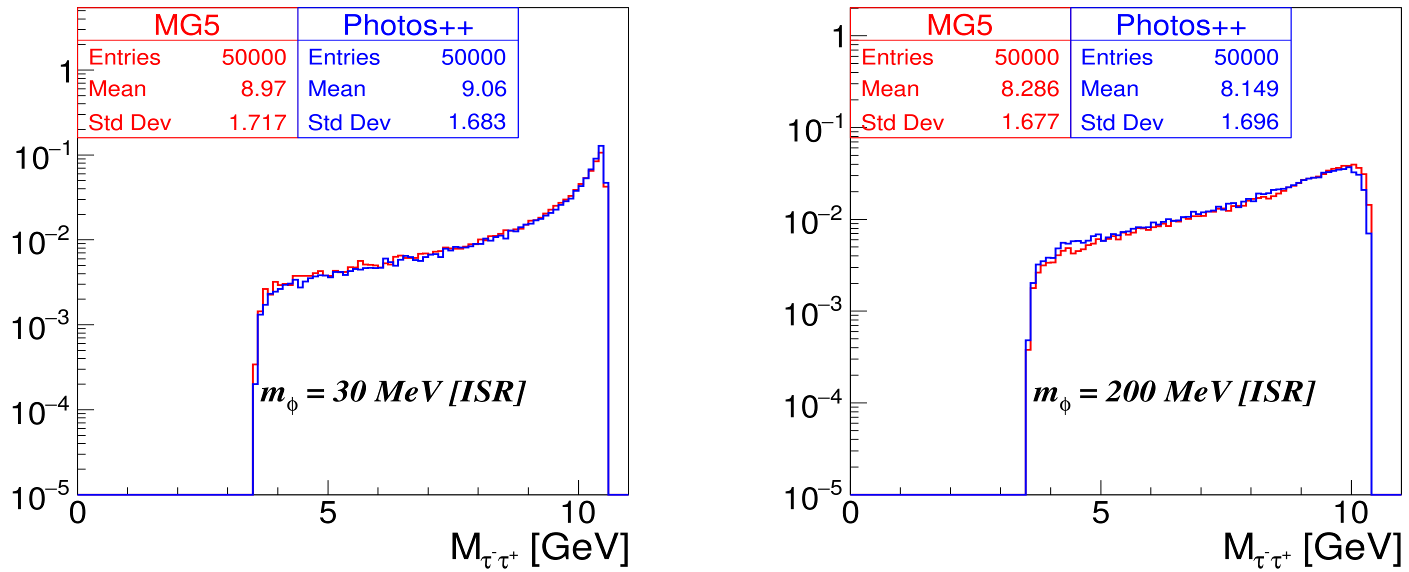


Figure 1: Belle 2 center of mass energies $e^-e^+ \to \tau^-\tau^+\phi_{\rm Dark\ Scalar}(\to e^-e^+)$ Case of dark scalar of 30 and 200 MeV. Simulation of KKMC+Photos is compared with the one based on MadGraph. Emission kernel was inspired from that comparison. At start, QED pair emission kernel was used. Spin correlations of τ-s modified by rotation of τ^- decay products.

Anomalous magnetic and electric dipole moments – details

- **Two examples on technical side:**

(i) `TauSpinner` for weight embedding of anomalous dipole moment into $pp \to \tau^+\tau^- X$ simulation samples.

(ii) Algorithm and results of anomalous moment weights, calculated simultaneously with run of `KKMC` Monte Carlo for $e^+e^- \to l^+l^-(n\gamma)$ at low energies and now also for FCC center of mass energies.

(iii) Results from e-Print: 2307.03526 Sw. Banerjee, A.Yu. Korchin,, E. Richter-Was, Z. Was, *Electron-positron, parton-parton and photon-photon production of τ-lepton pairs: anomalous magnetic and electric dipole moments spin effects*, and from Phys.Rev.D 106 (2022) 11, 113010

Technical side:

To start: M^{SM} and M^{SM+NP} are needed.

- Really OK, for **anomalous magnetic/electric dipole** moments implementation in $e^+e^- \to \tau^+\tau^-(n\gamma)$ process (τ decays included).
- NP distinct interaction scale than (independent from) bremsstrahlung/jet emissions.
- Seem trivial, but one has to keep in mind practical details.
- I will say little about reliability proofs, even though they are essential.
- Important is to preserve precision for SM (interfering-) part of the process!
- Check if factorization properties for NP match with what is in SM. Precision requirements for New Physics implementation are not high. Use of interpolated Improved Born in presence of hard bremsstrahlung required difficult validations.
- For New Physics weights simplified kinematic is used.

Formalism for $\tau^+\tau^-$: phase space $\times$ M.E. squared

- Because narrow τ width (τ propagator works as Dirac δ), cross-section for $f\bar{f} \to \tau^+\tau^- Y; \tau^+ \to X^+\bar{\nu}; \tau^- \to \nu\nu$ reads (norm. const. dropped):

$$d\sigma = \sum_{spin} |\mathcal{M}|^2 d\Omega = \sum_{spin} |\mathcal{M}|^2 d\Omega_{prod}\; d\Omega_{\tau^+}\; d\Omega_{\tau^-}$$

$$\mathcal{M} = \sum_{\lambda_1\lambda_2=1}^{2} \mathcal{M}^{prod}_{\lambda_1\lambda_2}\; \mathcal{M}^{\tau^+}_{\lambda_1} \mathcal{M}^{\tau^-}_{\lambda_2}$$

- Pauli matrices orthogonality $\delta^{\lambda'}_{\lambda}\delta^{\bar{\lambda}'}_{\bar{\lambda}} = \sum_\mu \sigma^\mu_{\lambda\bar{\lambda}}\sigma^{\lambda'\bar{\lambda}'}_\mu$ completes condition for production/decay separation with τ spin states.

- **core formula of spin algorithms, wt is product of density matrices of production and decays**, $0 < wt < 4$, $< wt >= 1$ useful properties.

$$d\sigma = \Big(\sum_{spin} |\mathcal{M}^{prod}|^2\Big)\Big(\sum_{spin} |\mathcal{M}^{\tau^+}|^2\Big)\Big(\sum_{spin} |\mathcal{M}^{\tau^-}|^2\Big) wt\; d\Omega_{prod}\; d\Omega_{\tau^+}\; d\Omega_{\tau^-}$$

Technical side:

Reference frames of host program

- **Host program frames convenient but not essential:** better precision, no need to worry about bremsstrahlung impact etc. Use of internal program variables helps too.
- Profit of well established routines relating τ-lepton rest frame and laboratory frame.
- That looks (and formally is) trivial, but it saves lots of worries how frames are affected in presence of bremsstrahlung photons (jets etc).
- New Physics module may have its own simpler solutions relating to lab frame.
- **On the other hand**, this prevents re-use of events for distinct models

Event record information only

- `TauSpinner` use simplified frames everywhere. It is for pp, not only kinematic is approximated, but on-flight hard process is re-evaluated for every event. Sum over all possible parton level processes is performed.
- Choice of these frames carefully prepared. Final solution was established when CP-sensitive observables were studied: Eur.Phys.J.C 79 (2019) 2, 91
- Re-use of the same event sample, with distinct New Physics models.

Simultaneously with host program run ...

NOTE:

- In KKMC refined solution is used for compatibility with Kleiss-Stirling spinor techniques. See Eur.Phys.J.C 22 (2001) 423 for details.

- The idea was to relate τ leptons quantization frames used in production and decay through consecutive transformation from τ rest frame to lab frame and back to τ rest frame.

- That sound complicated for simple rotation representation, but is safe and independent from number of bremsstrahlung photons.

- The $\tau^{\pm}$ decay products and its $h^i_{\tau^{\pm}}$ vector is transformed to the laboratory frame. Then $h^i_{\tau^{\pm}}$ is transformed back to τ lepton rest-frame, but this time of axes oriented as chosen in Kleiss-Stirling spinor techniques.

- We have explored partly this solution for KKMC anomalous moment event re-weighting.

Now observable for e^+e^- colisions

Simplified kinematic for NP implementation. Cross section:

$$wt_{ME} = \Big(\sum_{spin} |\mathcal{M}^{prod\ SM+NP}|^2\Big) / \Big(\sum_{spin} |\mathcal{M}^{prod\ SM}|^2\Big)$$

Complicated is spin weight

$$wt_{spin} = \Big(\sum_{ij} R_{ij}^{SM+NP} h_+^i h_-^j\Big) / \Big(\sum_{ij} R_{ij}^{SM} h_+^i h_-^j\Big)$$

The R_{ij} depend on kinematic of τ-pair production, $h_\pm^i$ on $\tau^\pm$ decays.

Spin quantization frames orientation need care. It must be the same for production and decay.

We use KKMC routines to transfer $h_\pm^i$ to lab frame and another routines to transfer back to $\tau^\pm$ but oriented as in New Physics calculation.

In this way reference frames are OK and impact of photons on phase space parametrisations is under control.

Solution works for all τ decays!

Now observable for e^+e^- colisions

Main message. Technical side:

- **Use of host program frames is convenient but not essential:** better precision, no need to worry about bremsstrahlung impact etc. Use of internal program variables is helpful too.
- **On the other hand**, this prevents re-use of events for distinct models
- **So far nothing new since last year slides ...**
- **NEW (details next talk):**
  For FCC (`KKMC`): extension of re-weighting algorithm to FCC center of mass energies, electroweak corrections included.
  For LHC (`TauSpinner`): $\gamma\gamma$ parton level processes added, explicit spin correlation matrix R_{ij} prepared for quark initialized processes as well.

but it all helps to develop intuition, and bring fun.

Amplitudes for reweighting

$$\mathcal{M}^{IBA} = \frac{e^2 Q_f Q_i}{s} V_{fi}(s,t)\,\gamma_\mu \otimes \gamma^\mu + \left(\frac{g_Z}{2}\right)^2 \frac{Z_{fi}(s,t)}{d(s)}\,\gamma_\mu [v_i(s,t) - a_i\gamma_5] \otimes \gamma^\mu [v_f(s,t) - a_i\gamma_5], \quad (1)$$

$$v_i(s,t) = T_{3i} - 2Q_i s_W^2 K_i(s,t), \qquad v_f(s,t) = T_{3f} - 2Q_f s_W^2 K_f(s,t), \quad (2)$$

$$V_{fi}(s,t) = \Gamma_{vp}(s) + \left(\frac{g_Z}{e}\right)^2 s_W^4 Z_{fi}(s,t) \frac{s}{d(s)} [K_{fi}(s,t) - K_f(s,t) K_i(s,t)], \quad (3)$$

$$\mathcal{M}^{DM} = \frac{e^2 Q_f Q_i}{s} V_{fi}(s,t)\,\gamma_\mu \otimes [A\gamma^\mu + \frac{(p_+ - p_-)^\mu}{2m}(A - iB\gamma_5)] + \left(\frac{g_Z}{2}\right)^2 \frac{Z_{fi}(s,t)}{d(s)}\,\gamma_\mu [v_i(s,t) - a_i\gamma_5] \otimes [X\gamma^\mu + \frac{(p_+ - p_-)^\mu}{2m}(X - iY\gamma_5)], \quad (4)$$

Complete amplitude $\mathcal{M} = \mathcal{M}^{IBA} + \mathcal{M}^{DM}$ (fermions spinors dropped).

Spin correlation matrix and results

At Z boson peak we get for leading parts:

$$
\begin{aligned}
R_{11}^{(Z)} &= -R_{22}^{(Z)} = -\frac{g_Z^4\, a_\tau^4\, \beta^2\, M_Z^2}{64\, \Gamma_Z^2}\, \sin^2(\theta), \\
R_{12}^{(Z)} &= R_{21}^{(Z)} = -\frac{g_Z^4\, a_\tau^3\, \beta\, M_Z^2}{32\, \Gamma_Z^2}\, \sin^2(\theta)\, \mathrm{Im}(X), \\
R_{13}^{(Z)} &= -R_{31}^{(Z)} = -\frac{g_Z^4\, a_\tau^3\, \beta^2\, M_Z^2}{64\, \Gamma_Z^2}\, \gamma \sin(2\theta)\, \mathrm{Im}(Y), \\
R_{23}^{(Z)} &= R_{32}^{(Z)} = -\frac{g_Z^4\, a_\tau^3\, \beta\, M_Z^2}{64\, \Gamma_Z^2}\, \gamma \sin(2\theta)\, \mathrm{Im}(X), \\
R_{14}^{(Z)} &= R_{41}^{(Z)} = -\frac{g_Z^4\, a_\tau^3\, \beta\, M_Z^2}{64\, \Gamma_Z^2}\, \gamma \sin(2\theta)\, \big[\mathrm{Re}(X) + v_\tau \gamma^{-2}\big], \\
R_{24}^{(Z)} &= -R_{42}^{(Z)} = \frac{g_Z^4\, a_\tau^3\, \beta^2\, M_Z^2}{64\, \Gamma_Z^2}\, \gamma \sin(2\theta)\, \mathrm{Re}(Y), \\
R_{34}^{(Z)} &= R_{43}^{(Z)} = -\frac{g_Z^4\, a_\tau^3\, \beta\, M_Z^2}{32\, \Gamma_Z^2}\, \big\{\big(1+\cos^2(\theta)\big)\, \big[v_\tau + \mathrm{Re}(X)\big] \\
&\qquad +2 v_\tau\, \beta\, \cos(\theta)\big\}, \\
R_{44}^{(Z)} &= R_{33}^{(Z)} = \frac{g_Z^4\, a_\tau^4\, \beta^2\, M_Z^2}{64\, \Gamma_Z^2}\, \big(1+\cos^2(\theta)\big),
\end{aligned} \tag{5}
$$

where $\gamma = M_Z/(2m_\tau) \approx 25.7$ and $\beta \approx 1$.

Spin correlation matrix and results

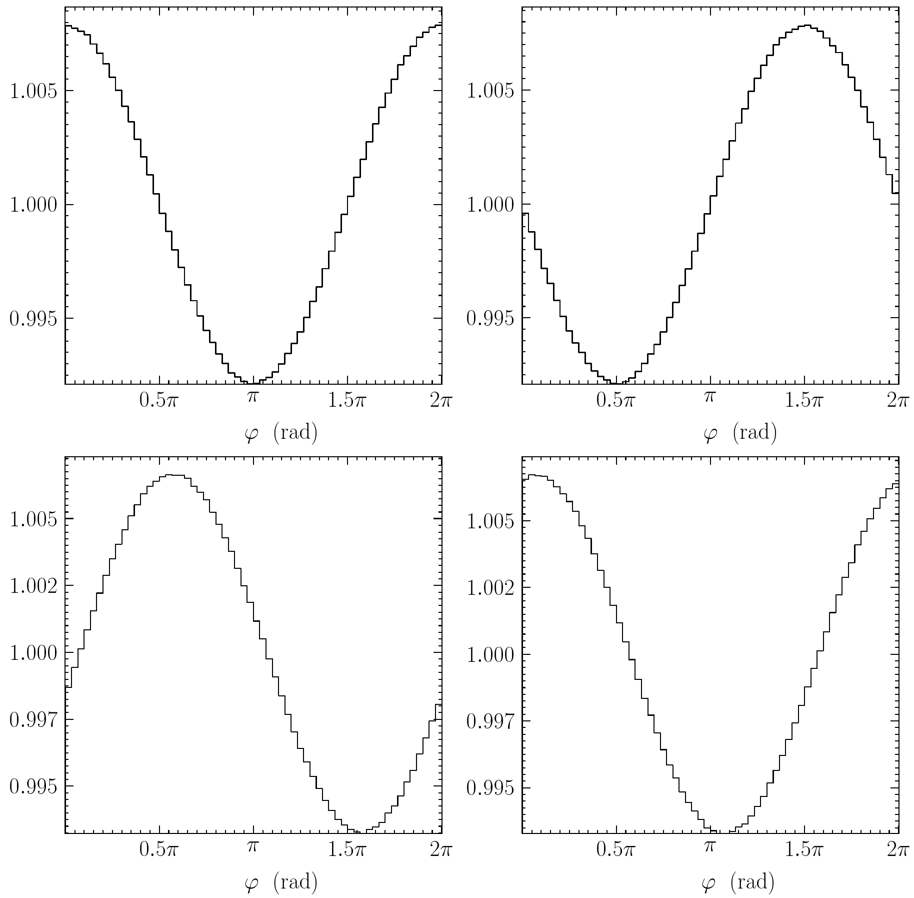


Figure 2: Ratio of number of events with and without weak dipole moments, in function of acoplanarity φ at $\sqrt{s} = M_Z$. The selected events of scattering angles $\cos(\theta) < 0$ are taken. The top left plot for $\mathrm{Re}(X) = 0.0004$, the top right plot for $\mathrm{Re}(Y) = 0.0004$, the bottom left for $\mathrm{Im}(X) = 0.0004$, and the bottom right for $\mathrm{Im}(Y) = 0.0004$ are taken . For the imaginary form-factors, additional constraint $E_{\pi^+} > E_{\bar{\nu}_\tau}$ is taken the τ^+ side. The form-factors $A(M_Z^2) = B(M_Z^2) = 0$ are set. The decays $\tau^- \to \pi^-\pi^0\nu$ and $\tau^+ \to \pi^+\nu$ are taken.

For $\alpha_{QED}(s)$ ambiguities?

- The g-2 measurements revived interest in possible signatures with τ leptons: real and virtual. I have presented some work relevant for that.
- But what about related systematic ambiguities?
- For $\alpha_{QED}(s)$ dispersion relations and measurements of $e^+e^- \to \pi^+\pi^-\pi^0(\pi^+\pi^-)$ at $\sqrt{s} \sim 1GeV^2$ is necessary?.
- Why not with radiative return and Belle 2 data?

Can `KKMC` be adopted for such applications?

1. Compare program predictions for $e^+e^- \to \mu^+\mu^-\gamma n(\gamma)$ at `(C)EEX1` and `(C)EEX2` levels
2. Replace in `KKMC` generation $\mu^\pm$ with $\pi^\pm$, QED initial state bremsstrahlung only.
3. Add to final state π^0 using modified `PHOTOS`
4. Use ratio of $e^+e^- \to \mu^+\mu^-$ and $e^+e^- \to \pi^+\pi^-\pi^0$ matrix elements at correlated phase space points.
5. Reproduce tests with `PHOKARA`, see. https://indico.ph.liv.ac.uk/event/1297/contributions/7323/

For $\alpha_{QED}(s)$ ambiguities?

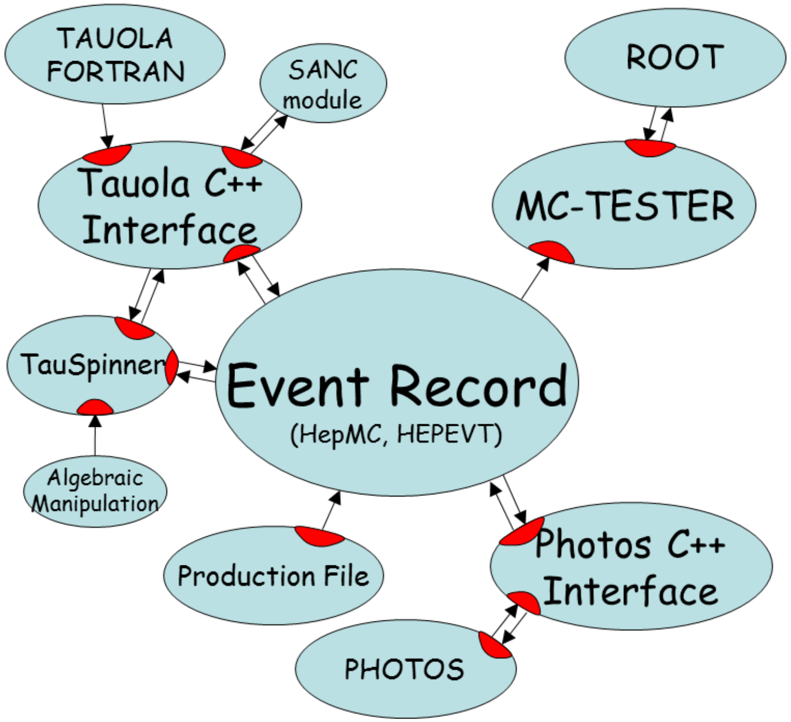
I have not prepared further details (just slides 6 and 7) on:

(iv) Arrangements for τ decays.

(v) Arrangements for software.

Thank you for listening

TauSpinner phase space point from Event Record



● **What is advantageous?** Simultaneous event generation and weight calculation? Re-use the same (stored) events with detector response for many New Physics models?

TauSpinner (re-use) requires:

● Good control of theory on user side.

● Good understanding of tools on user side.

● Challenge: checks (often adjustments) on event record contents. **Beware:** frequent new classes of event record content abuses.

Simultaneous event generation (eg. with `KKMC`**) and weight calculation:**

● easier to control and to assure precision for New Physics.

● Convenient for authors, less so for users in fits, evaluation of detection ambiguities.

TauSpinner phase space point from Event Record

$$\begin{aligned} d\sigma_{Born} & (x_1, x_2, \hat{s}, \cos\theta) = \sum_{q_f, \bar{q}_f} \\ [& f^{q_f}(x_1, ...) f^{\bar{q}_f}(x_2, ...) d\sigma^{q_f \bar{q}_f}_{Born}(\hat{s}, \cos\theta) \\ + & f^{\bar{q}_f}(x_1, ...) f^{q_f}(x_2, ...) d\sigma^{q_f \bar{q}_f}_{Born}(\hat{s}, -\cos\theta)], \end{aligned} \tag{6}$$

where x_1, x_2 denote fractions of incoming protons momenta carried by the corresponding parton, $\hat{s} = x_1\, x_2\, s$ and $f/\bar{f}$ denotes parton (quark-/anti-quark) density functions. We assume that kinematics is reconstructed from four-momenta of the outgoing leptons.

$$x_{1,2} = \frac{1}{2}\Big(\pm \frac{p_z^{ll}}{E} + \sqrt{(\frac{p_z^{ll}}{E})^2 + \frac{m_{ll}^2}{E^2}} \;\Big), \tag{7}$$

where E denotes energy of the proton beam and $p_z^{\ell\ell}$ denotes z-axis momentum of outgoing lepton pair in the laboratory frame and m_{ll} lepton pair virtuality. Note that this formula can be used, as approximation, for the events with hard jets too.

TauSpinner phase space point from Event Record

The $\cos\theta^*$ is then calculated from

$$\cos\theta_1 = \frac{\tau_x^{(1)} b_x^{(1)} + \tau_y^{(1)} b_y^{(1)} + \tau_z^{(1)} b_z^{(1)}}{|\vec{\tau}^{(1)}||\vec{b}^{(1)}|},$$

$$\cos\theta_2 = \frac{\tau_x^{(2)} b_x^{(2)} + \tau_y^{(2)} b_y^{(2)} + \tau_z^{(2)} b_z^{(2)}}{|\vec{\tau}^{(2)}||\vec{b}^{(2)}|}, \tag{8}$$

as follows:

$$\cos\theta^* = \frac{\cos\theta_1 \sin\theta_2 + \cos\theta_2 \sin\theta_1}{\sin\theta_1 + \sin\theta_2} \tag{9}$$

where $\vec{\tau}^{(1)}, \vec{\tau}^{(2)}$ denote 3-vectors of outgoing leptons and $\vec{b}^{(1)}, \vec{b}^{(2)}$ denote 3-vectors of incoming beams' four-momenta.

This polar angle definition, is at present the `TauSpinner` default. For tests we have used variants; *Mustraal* (Berends:1983mi) and *Collins-Soper* (Collins:1977iv) frames, which differ when high p_T jets are present.

I skip presentation of how ϕ orientation angle in (x-y) plane is established.

Spin weight, in TauSpinner ...

The spin weight $wt = wt_{spin}$ is dimensionless, and contains information of all spin effects transmitted from the production to the decay of τ leptons,

$$wt_{spin} = \sum_{i,j=t,x,y,z} R_{i,j} h^i_{\tau^+} h^j_{\tau^-}. \tag{10}$$

The polarimetric vector $h^i_{\tau^\pm}$ is calculated from formula

$$h^i_{\tau^\pm} = \sum_{\lambda,\bar{\lambda}} \sigma^i_{\lambda,\bar{\lambda}} \mathcal{M}^{\tau^\pm}_{\lambda} \mathcal{M}^{\tau^\pm\,\dagger}_{\bar{\lambda}}, \tag{11}$$

where $\sigma^i_{\lambda,\bar{\lambda}}$ stands for Pauli matrices. The $h^i_{\tau^\pm}$ are normalized further to set their time-like component to 1

The spin correlation matrix $R_{i,j}$ is calculated from formula

$$R_{i,j} = \sum_{\lambda_1,\bar{\lambda}_1 \lambda_2,\bar{\lambda}_2} \sigma^i_{\lambda_1,\bar{\lambda}_1} \sigma^j_{\lambda_2,\bar{\lambda}_2} \mathcal{M}^{prod}_{\lambda_1 \lambda_2} \mathcal{M}^{prod\,\dagger}_{\bar{\lambda}_1 \bar{\lambda}_2} \tag{12}$$

Spin weight, in TauSpinner ...

The $R_{i,j}$ normalized, its time-like component set to 1, namely all $R_{i,j} = \frac{R_{i,j}}{R_{t,t}}$.

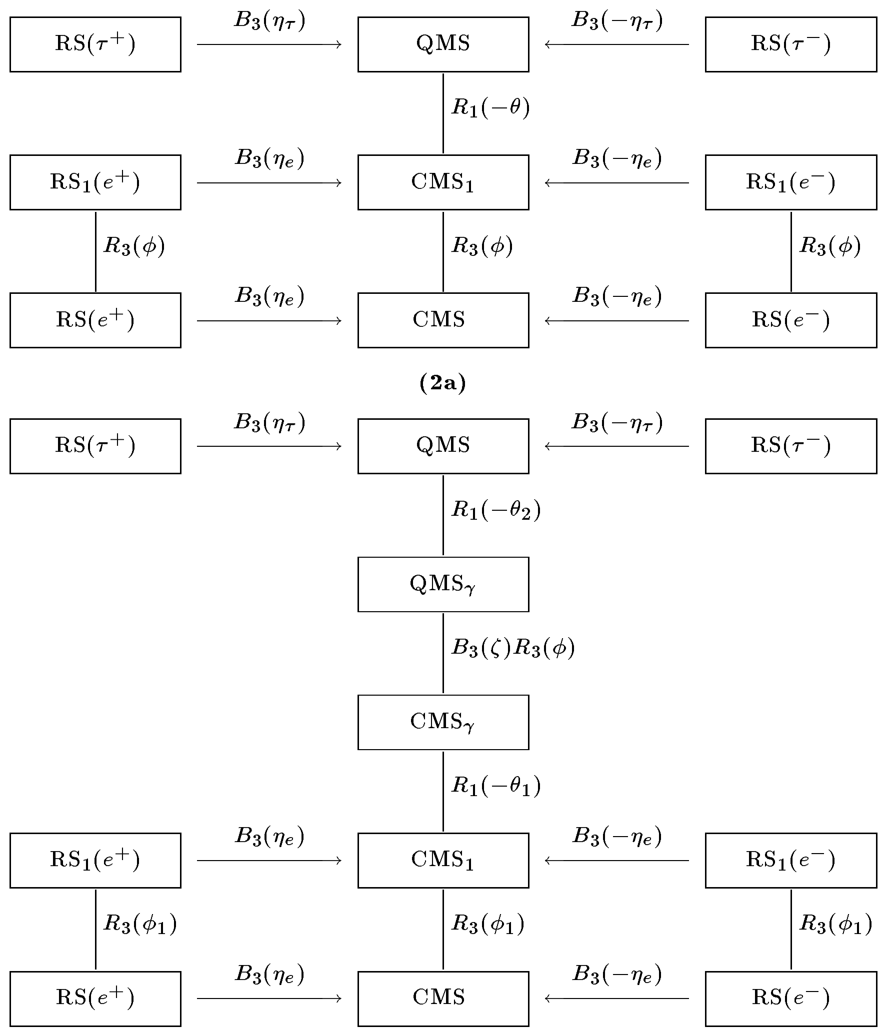
After that average over flavou is taken. No quantume entanglement between incoming parton flavours, but between τ's is taken in:

$$R_{i,j} \to \frac{\sum_{flav.} f(x_1,...)f(x_2,...)\left(\sum_{\lambda_1,\lambda_2} |\mathcal{M}^{prod}_{parton\ level}|^2\right) R_{i,j}}{\sum_{flav.} f(x_1,...)f(x_2,...)\left(\sum_{\lambda_1,\lambda_2} |\mathcal{M}^{prod}_{parton\ level}|^2\right)}. \qquad (13)$$

Spin weight, in TauSpinner ...

Frames, boosts rotations for spin; scheme used in: `KORLAB`, `KORALZ`, `TauSpinner`.

Figure 2



(2b)

NEW: $\gamma\gamma \to \tau^+\tau^-$ parton level process.

We define the factor $D \equiv 1 - \beta^2 \cos^2\theta$. The elements of the matrix $R_{i,j}^{\gamma\gamma}$ and $R_{44}^{\gamma\gamma}$ (for brevity $A \equiv A(0)$ and $B \equiv B(0)$):

$$
\begin{aligned}
R_{11}^{\gamma\gamma} &= \frac{e^4}{8D^2}\big[-\beta^2(\beta^2-4A-2)\cos(4\theta)+4\beta^2(\beta^2-2)\cos(2\theta) \\
&\quad +4A\,(7\beta^2-8)-11\beta^4+22\beta^2-8\big], \\
R_{12}^{\gamma\gamma} &= -R_{21}^{\gamma\gamma} = \frac{e^4 B}{4D^2}\,\beta\,\big(\beta^2\cos(4\theta)+4\cos(2\theta)+15\beta^2-20\big), \\
R_{13}^{\gamma\gamma} &= R_{31}^{\gamma\gamma} = \frac{e^4}{2D^2}\,\gamma\,\beta^2\big[(\beta^2+A\,(\beta^2-2)-1)\cos(2\theta)+A\beta^2-\beta^2+1\big]\,|\sin(2\theta)|, \\
R_{22}^{\gamma\gamma} &= \frac{e^4}{8D^2}\big[-\beta^4\cos(4\theta)+4\beta^2(\beta^2+4A)\cos(2\theta)+16A\,(\beta^2-2) \\
&\quad -11\beta^4+16\beta^2-8\big], \\
R_{23}^{\gamma\gamma} &= -R_{32}^{\gamma\gamma} = \frac{e^4 B}{2D^2}\,\gamma\,\beta\,(\beta^2\cos(2\theta)-3\beta^2+2)\,|\sin(2\theta)|, \\
R_{33}^{\gamma\gamma} &= \frac{e^4}{8D^2}\big[\beta^2(\beta^2-4A-2)\cos(4\theta)-4\beta^4\cos(2\theta)+4A\,(9\beta^2-8) \\
&\quad +11\beta^4+2\beta^2-8\big], \\
R_{44}^{\gamma\gamma} &= \frac{e^4}{8D^2}\big[-\beta^4\cos(4\theta)+4\beta^2(\beta^2-4A-2)\cos(2\theta)-16A\,(\beta^2-2) \\
&\quad -11\beta^4+8\beta^2+8\big].
\end{aligned} \tag{14}
$$

These elements satisfy the condition $R_{i,j}^{\gamma\gamma}(\theta) = R_{i,j}^{\gamma\gamma}(\pi-\theta)$ for $0 \le \theta \le \pi$, as follows from identity of the photons.