Probing τ flavour change with $\mu \to e$ observables

17th International Workshop on Tau Lepton Physics
Louisville, Kentucky, USA

Marco Ardu
Univ. Valencia & IFIC

Based on *Phys.Rev.D* 105 (2022) 9 with S. Davidson, M. Gorbahn
and
Eur.Phys.J.C 83 (2023) 5, 394 with F. Kirk

VNIVERSITAT DE VALÈNCIA

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Charged Lepton Flavour Violation (cLFV)

- cLFV ≡ contact interaction among the charged leptons that violates flavour
- Neutrino masses and oscillations imply lepton flavour violation
- Accidental symmetries of the SM can be easily violated (cLFV is expected in many models)
- Unambiguous signals of New Physics
- Can probe New Physics scale well above the reach of colliders

Experimental searches

Experimental searches

Process	Current bound on BR	Future Sensitivity
$\mu \to e\gamma$	$< 4.2 \times 10^{-13}$ MEG	10^{-14} MEGII
$\mu \to \bar{e}ee$	$< 1.0 \times 10^{-12}$ SINDRUM	10^{-16} Mu3e
$\mu A \to eA$	$< 7 \times 10^{-13}$ SINDRUMII	$10^{-16} \to 10^{-18}$ COMET, Mu2e

- $\mu \to e$ transitions

Best current sensitivities (and expect a significant improvement)

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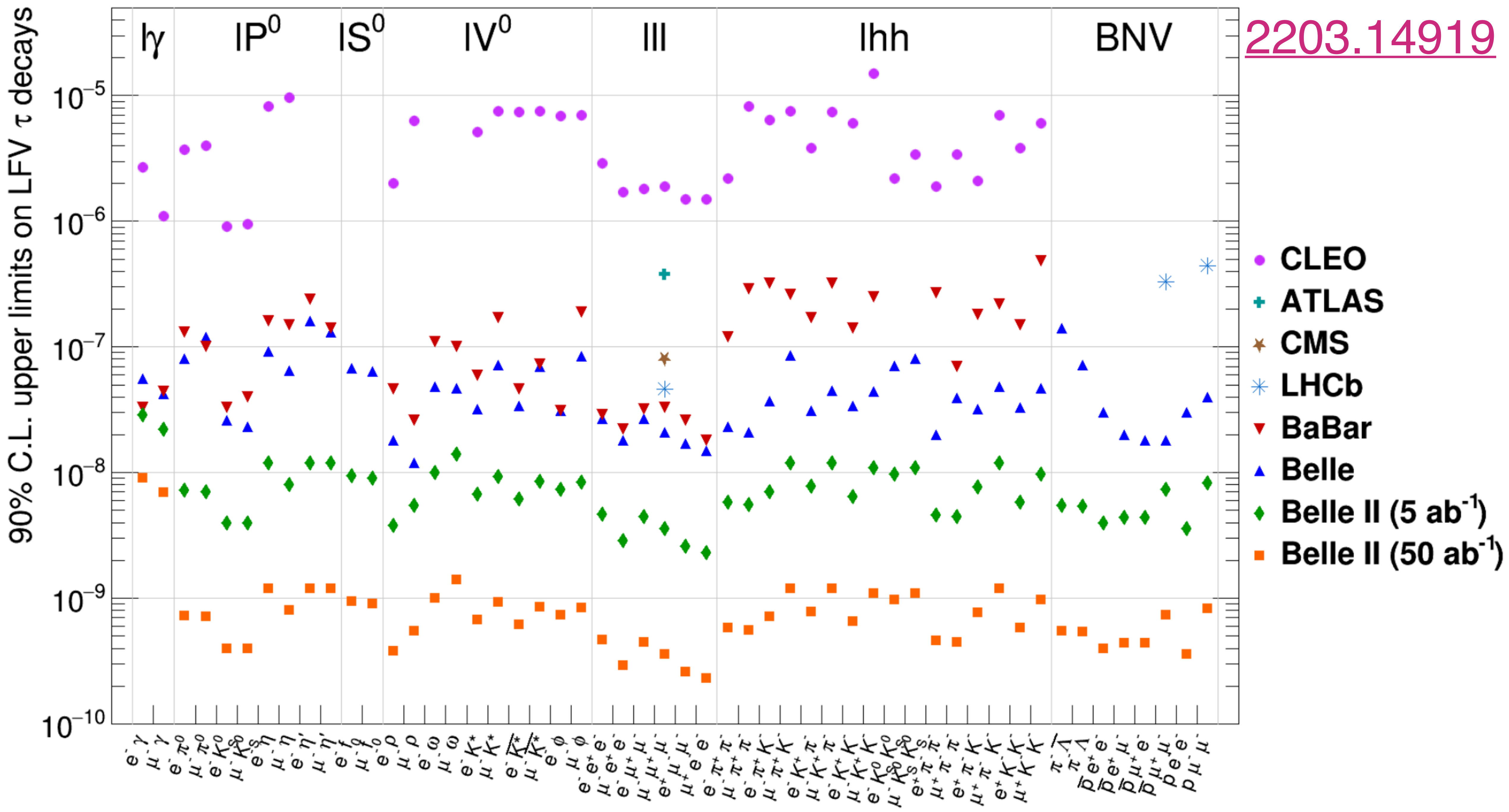

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- $\tau \to l$ decays

Less sensitive but the phase space is larger (multitude of channels → can help distinguishing models)

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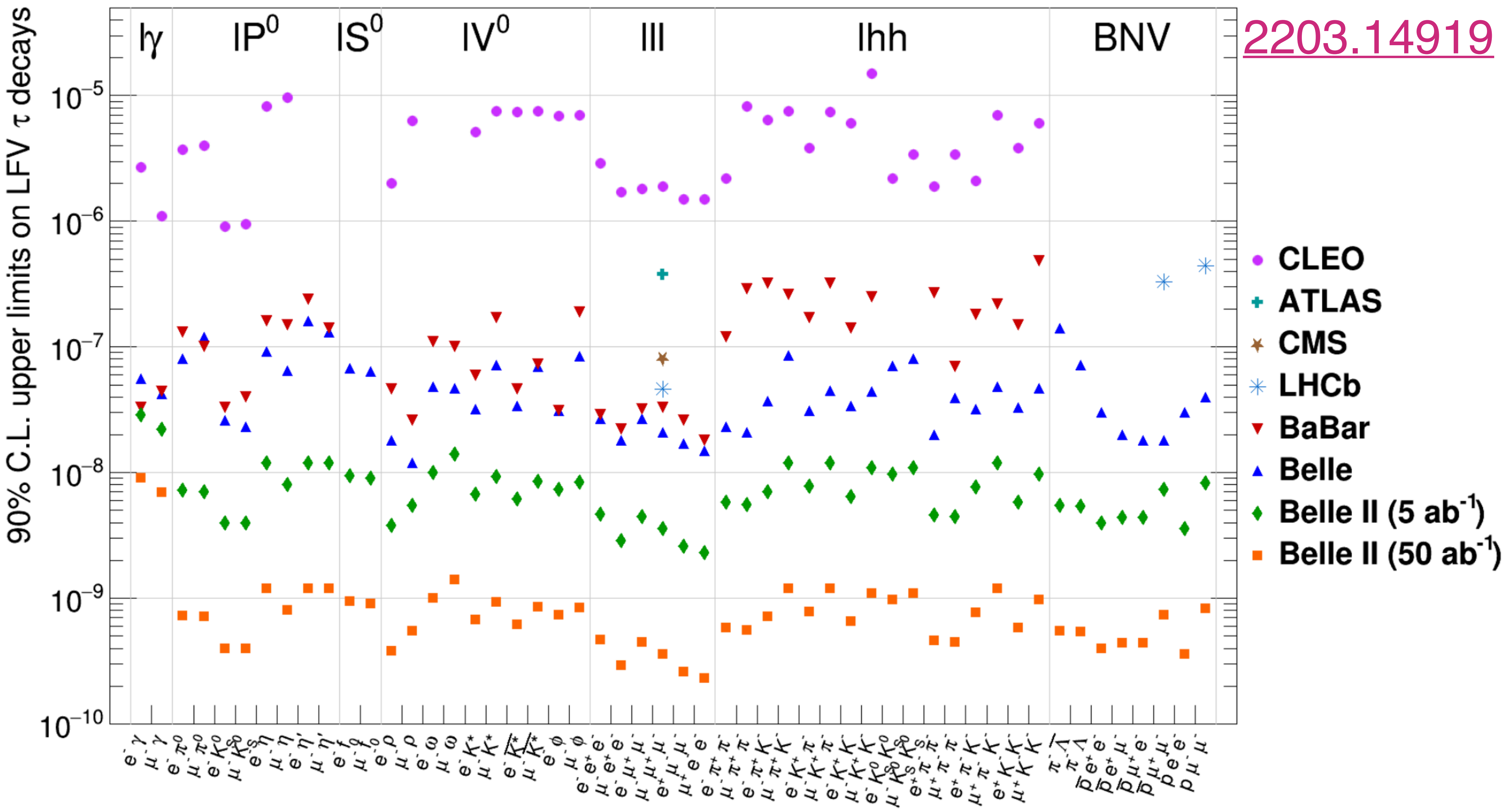
$K^0 \to \mu^{\pm} e^{\mp}$	$< 4.7 \times 10^{-12}$	
$B_d^0 \to \tau^{\pm}\mu^{\mp}$	$< 1.2 \times 10^{-5}$ LHCb	$\sim 10^{-6}$?
...	...	...
$h \to e^{\pm}\mu^{\mp}$	$< 6.1 \times 10^{-5}$ Atlas	2.1×10^{-5}
$h \to e^{\pm}\tau^{\mp}$	$< 2.2 \times 10^{-3}$ CMS	2.4×10^{-4}
$h \to \tau^{\pm}\mu^{\mp}$	$< 1.5 \times 10^{-3}$ CMS	2.3×10^{-4} ILC
$Z \to e^{\pm}\mu^{\mp}$	$< 7.5 \times 10^{-7}$ Atlas	
$Z \to l^{\pm}\tau^{\mp}$	$< 10^{-7}$ Atlas	

- Heavy particles decaying into LFV final states



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$\mu \to e$ vs $\tau \to l$

- Due to the possibility of having very intense muon beams the $\mu \to e$ experimental sensitivities are the frontier in LFV searches

- Taus are heavy and short-lived: LFV τ decays or LFV decays of heavier particles into τ-s are searched for at colliders

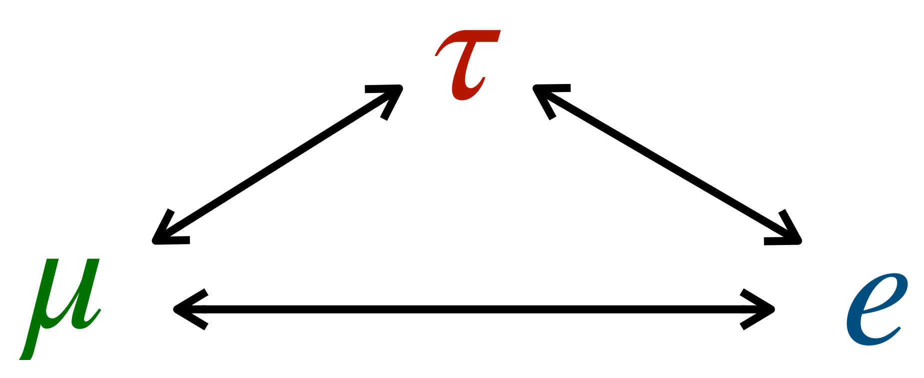
- The difference in sensitivities can be such that future (and sometime current) experimental sensitivities satisfy:

$$Br(\mu \to e) \lesssim Br(\tau \to \mu) Br(\tau \to e)$$

Can we learn something about $\tau \to (e, \mu)$ with $\mu \to e$?

$$\mu \to e = \mu \to \tau^* \times \tau^* \to e$$

When we introduce $\tau \leftrightarrow e$ and $\tau \leftrightarrow \mu$ flavour change, there is no symmetry that forbids $\mu \to e$



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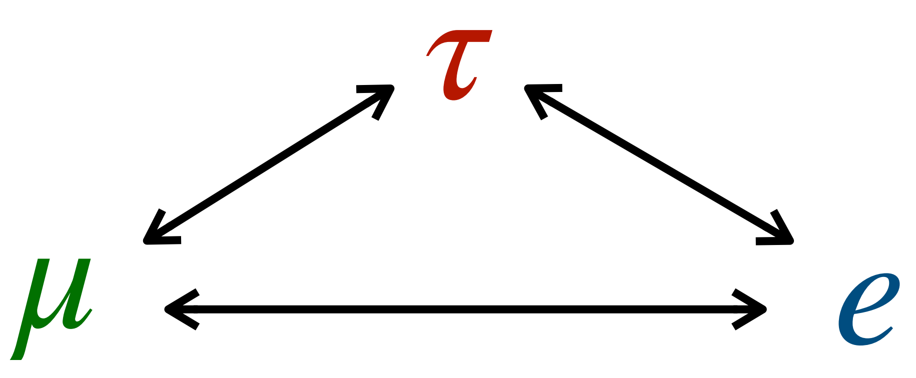
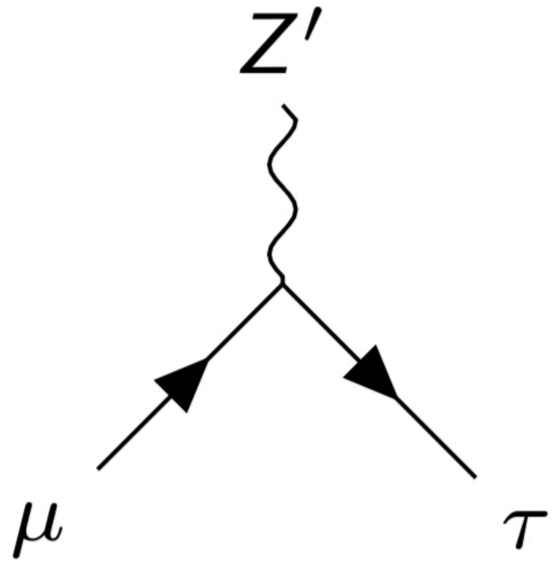
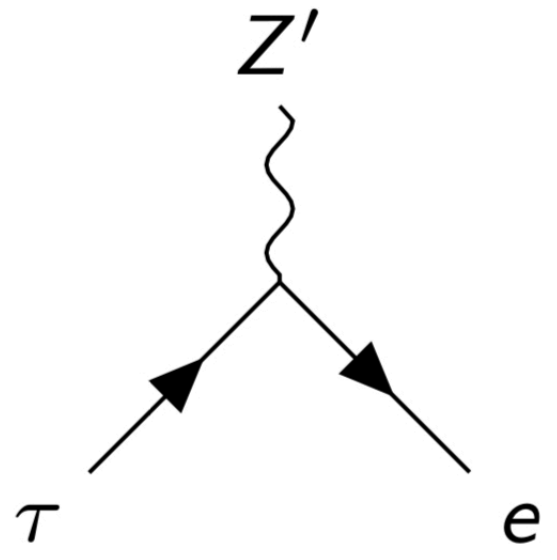


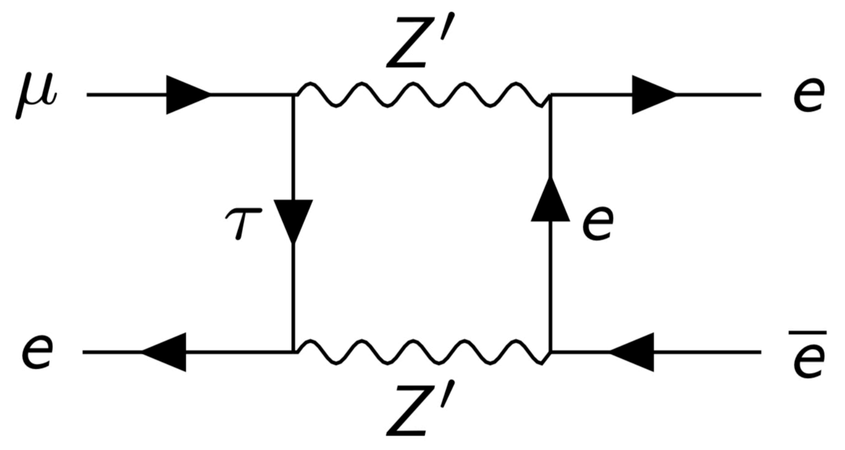
Consider for instance a Z' with tau flavour changing couplings



$\sim g_{\tau\mu}$



$\sim g_{e\tau}$



$\propto \frac{g_{e\tau} g_{\tau\mu}}{16\pi^2 M_{Z'}^2}$

Outline

- $\mu \to \tau^* \times \tau^* \to e$ in a Z' model

- $\mu \to \tau^* \times \tau^* \to e$ in the EFT

- Phenomenology

- Conclusion

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An example: $L_e - L_\mu$ + doublet

MA+F.Kirk, EPJC '23

Extend the Standard Model with an extra $U(1)_{L_e-L_\mu}$ (with a gauge coupling g') and an additional scalar doublet ϕ (with charge -1 under $U(1)_{L_e-L_\mu}$)

$$\ell_1 \sim \ell_e, \quad \ell_2 \sim \ell_\mu + \theta_{23}\ell_\tau, \quad \ell_3 \sim \ell_\tau - \theta_{23}\ell_e$$

$$\mathscr{L}_{\rm Yuk} \supset y_{31}\overline{\ell}_3\phi e_1 + y_{23}\overline{\ell}_2\phi e_3 + {\rm h.c}$$

$$e_1 \sim e_e + \theta_{13}e_\tau, \quad e_2 \sim e_\mu, \quad e_3 \sim e_\tau - \theta_{13}e_e$$

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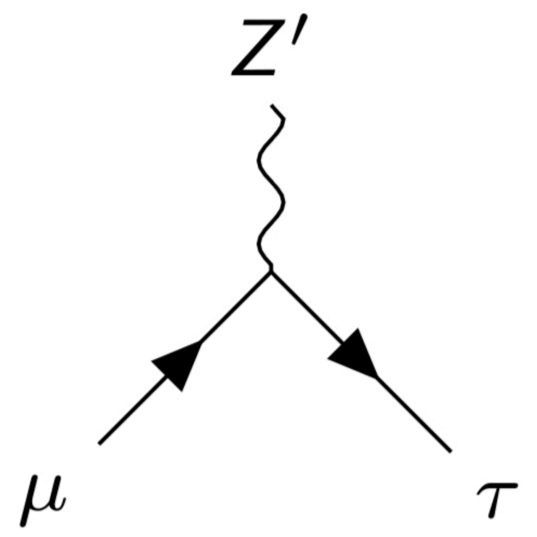
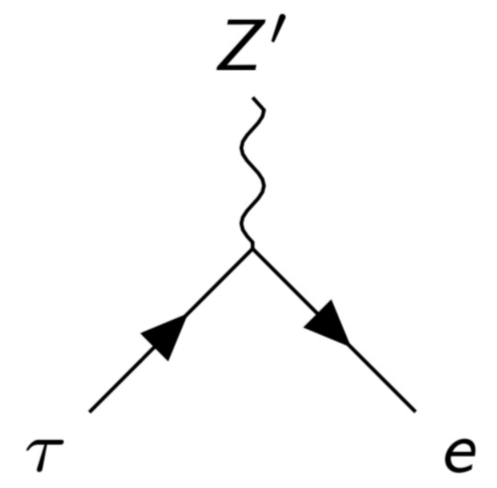
The Z' (and the scalars) acquire LFV couplings $\tau \leftrightarrow e \propto y_{31}$, $\tau \leftrightarrow \mu \propto y_{23}$ at the leading order



$\sim g'\theta_{23} \propto g' y_{23}$



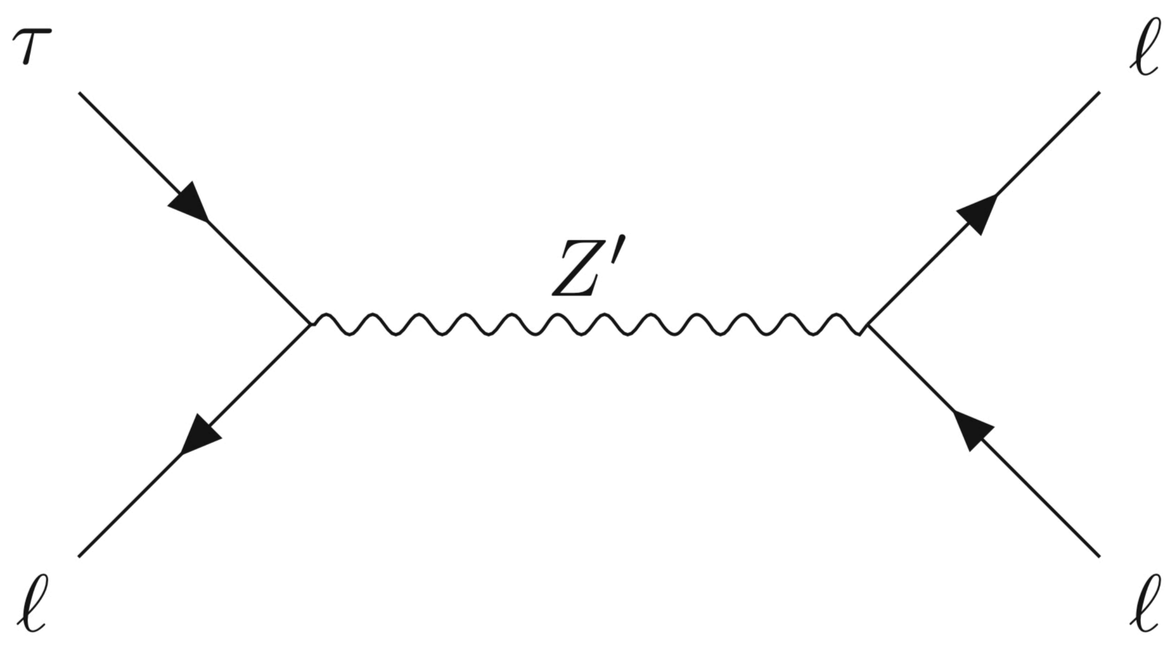
$\sim g'\theta_{13} \propto g' y_{31}$

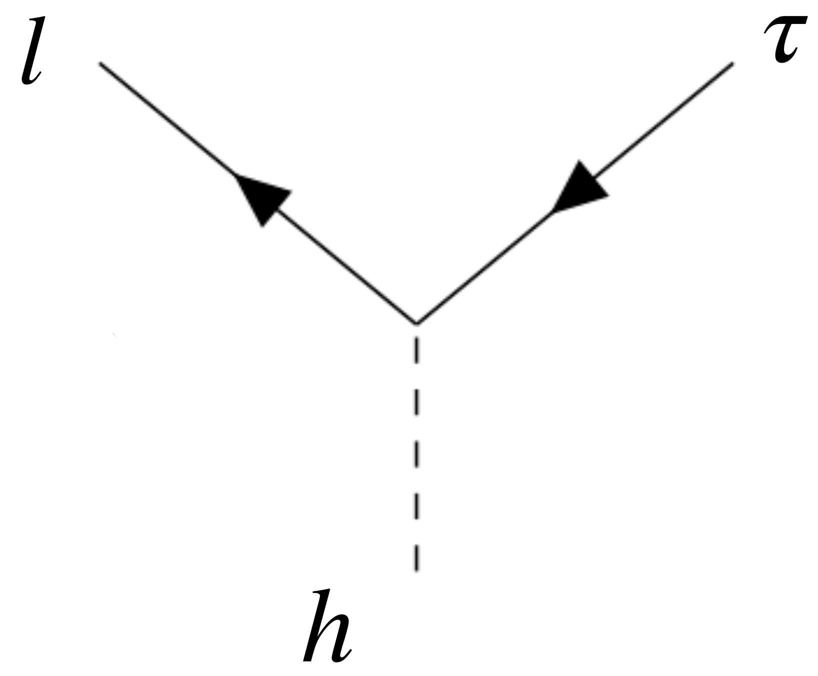
While $\mu \leftrightarrow e$ is always proportional to $y_{23} \times y_{31}$!

$\mu \to e$ *vs* $\tau \to l$ in the model

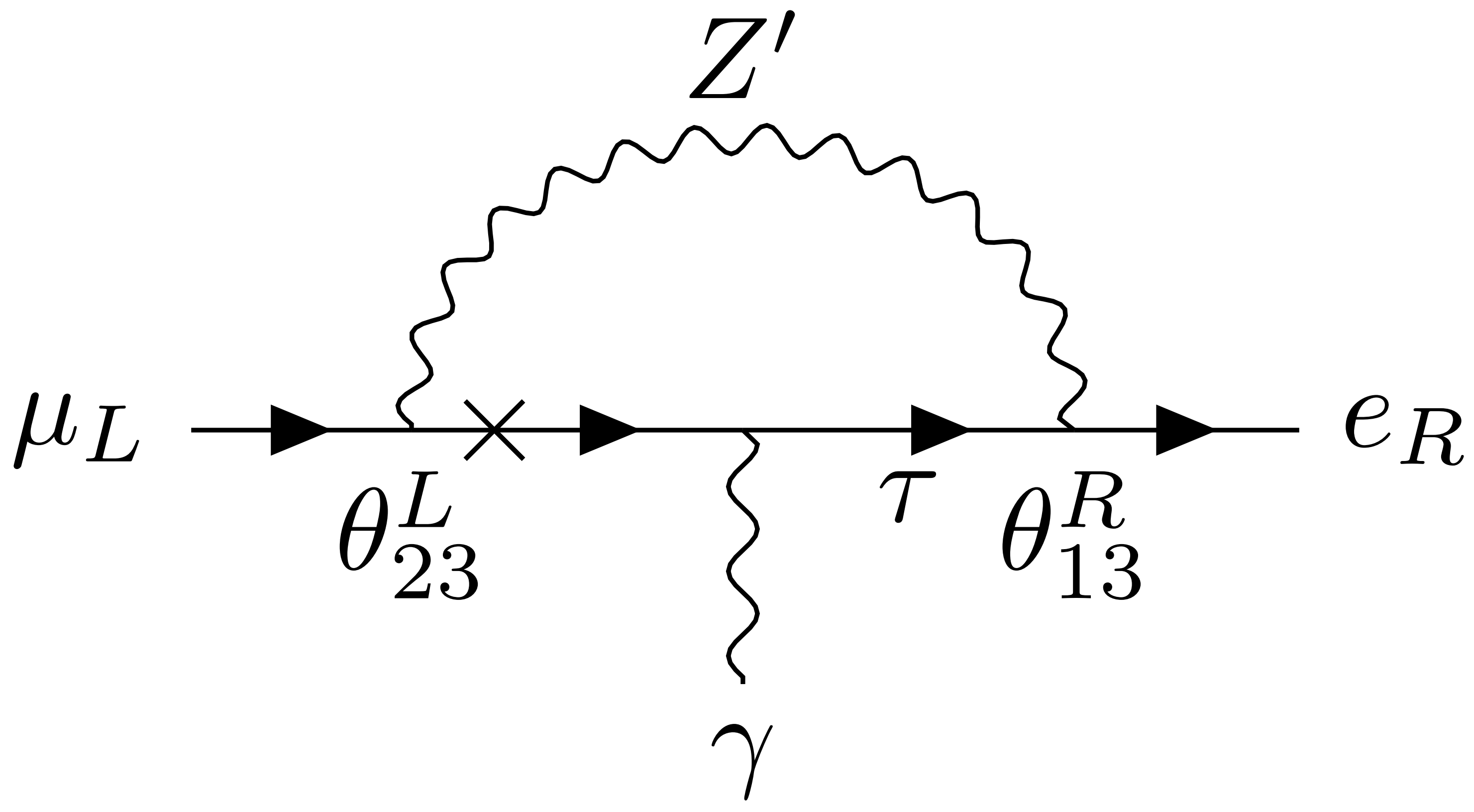
MA+F.Kirk, EPJC '23

- Several $\tau \leftrightarrow l$ processes could potentially constrain the size of the LFV Yukawa couplings y_{23}, y_{31}





- $\mu \leftrightarrow e$ processes are sensitive to the product $y_{23} \times y_{31}$. For instance:



$\mu \to e$ *vs* $\tau \to l$ in the model

MA+F.Kirk, EPJC '23

There are regions of the parameter space where $\tau \leftrightarrow l$ does not constrain the model while $\mu \leftrightarrow e$ does ($g' = 10^{-4}, M_{Z'} = 10$ GeV and M mass scale extra doublet)

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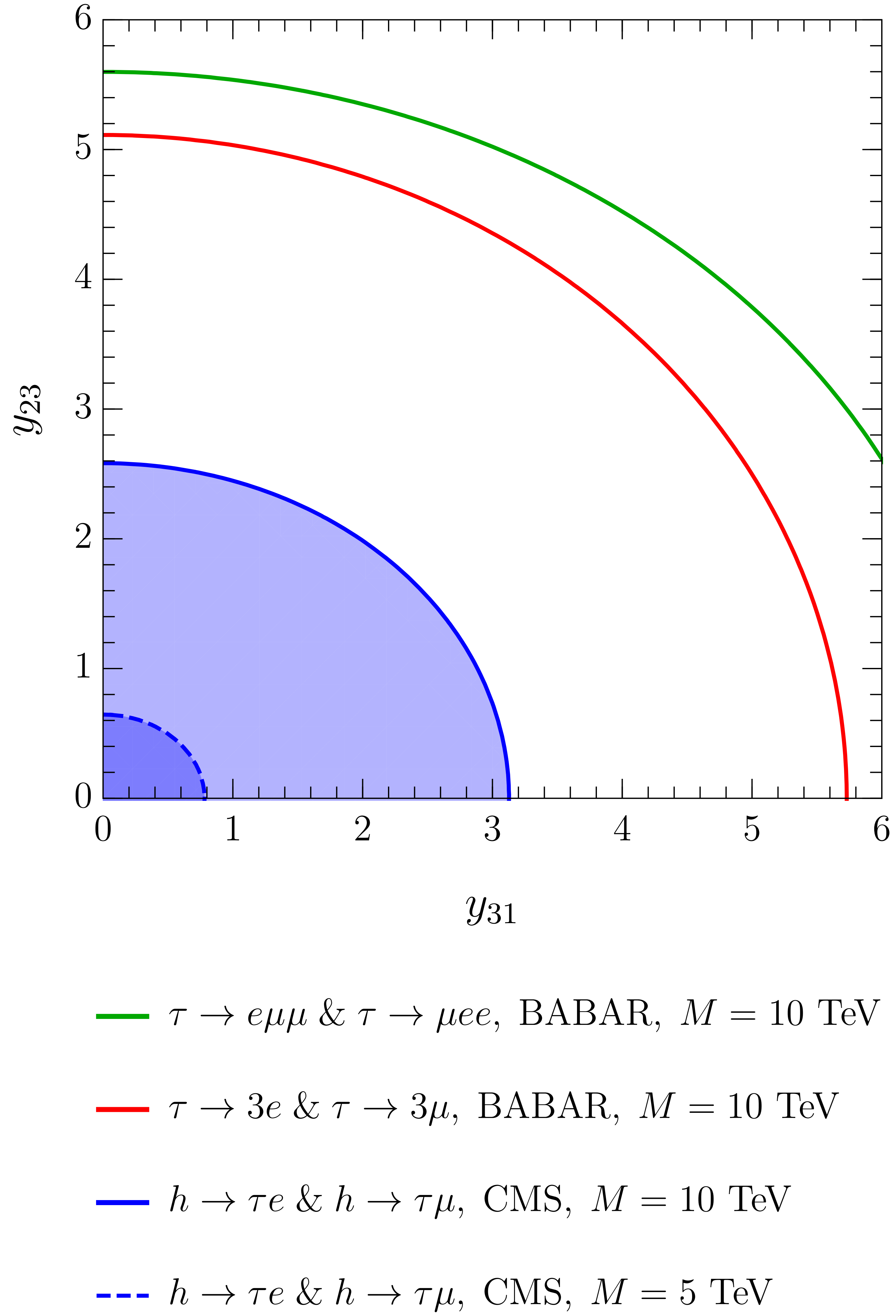

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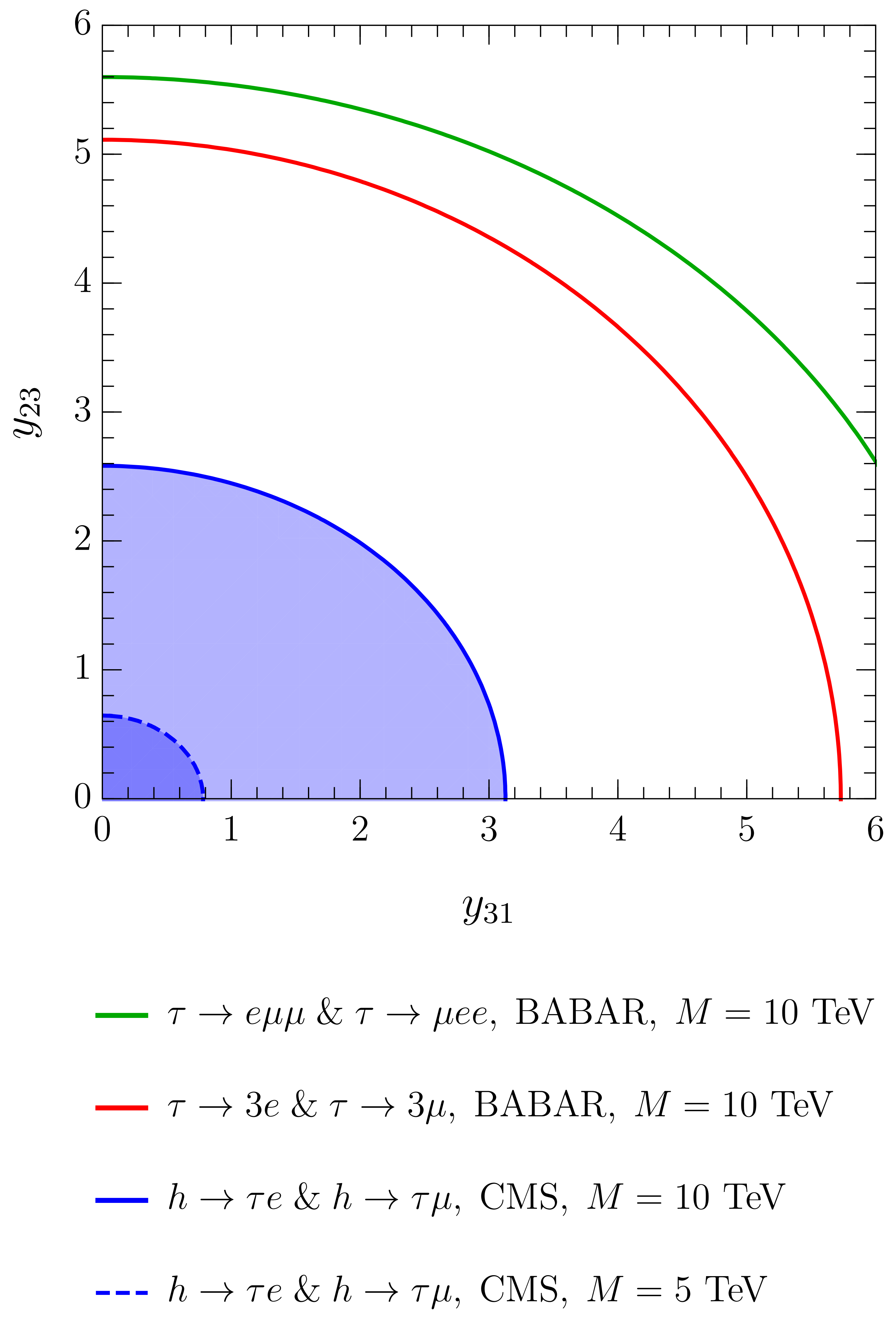

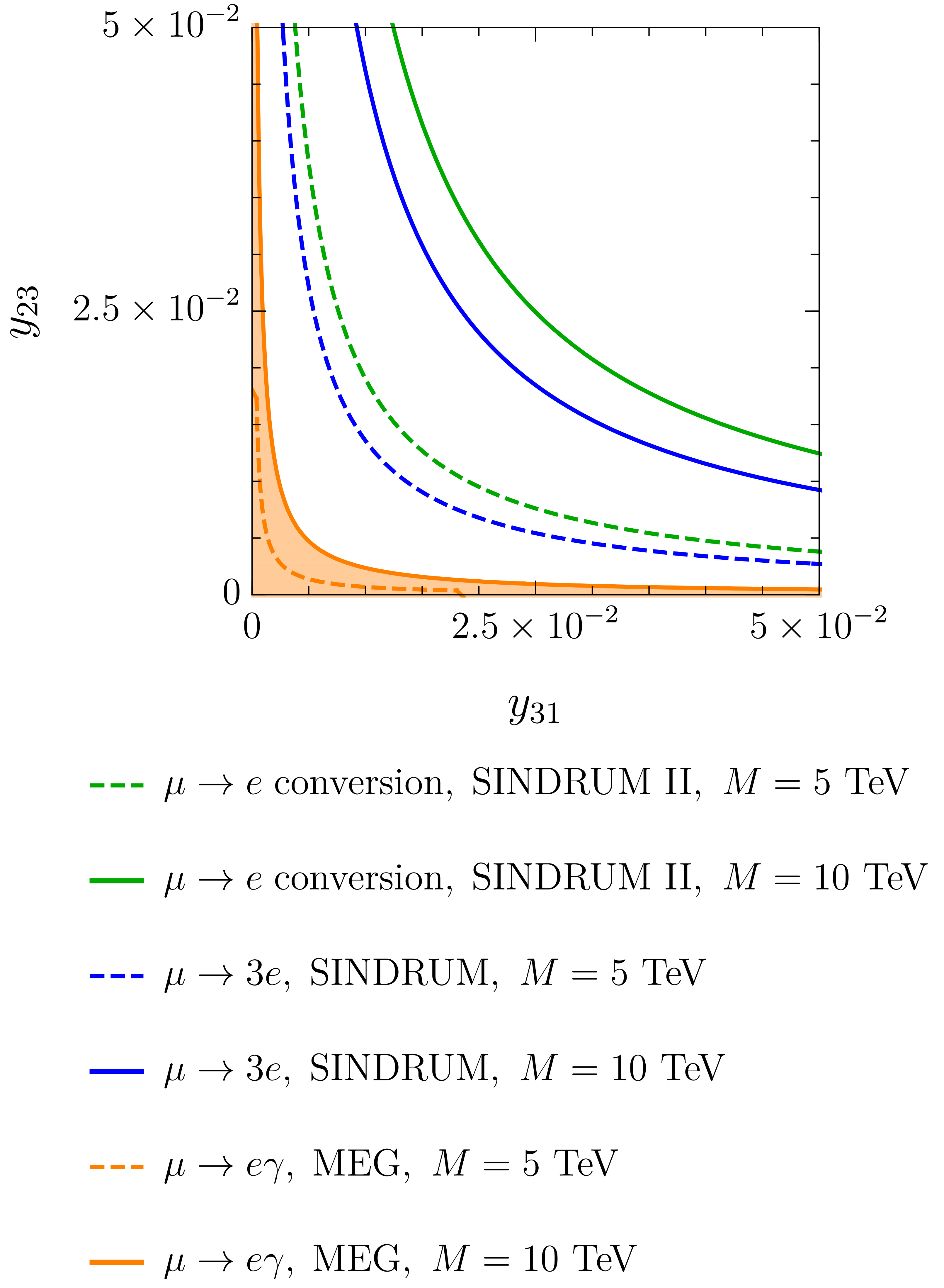

Outline

- $\mu \to \tau^* \times \tau^* \to e$ in a Z' model
- $\mu \to \tau^* \times \tau^* \to e$ in the EFT
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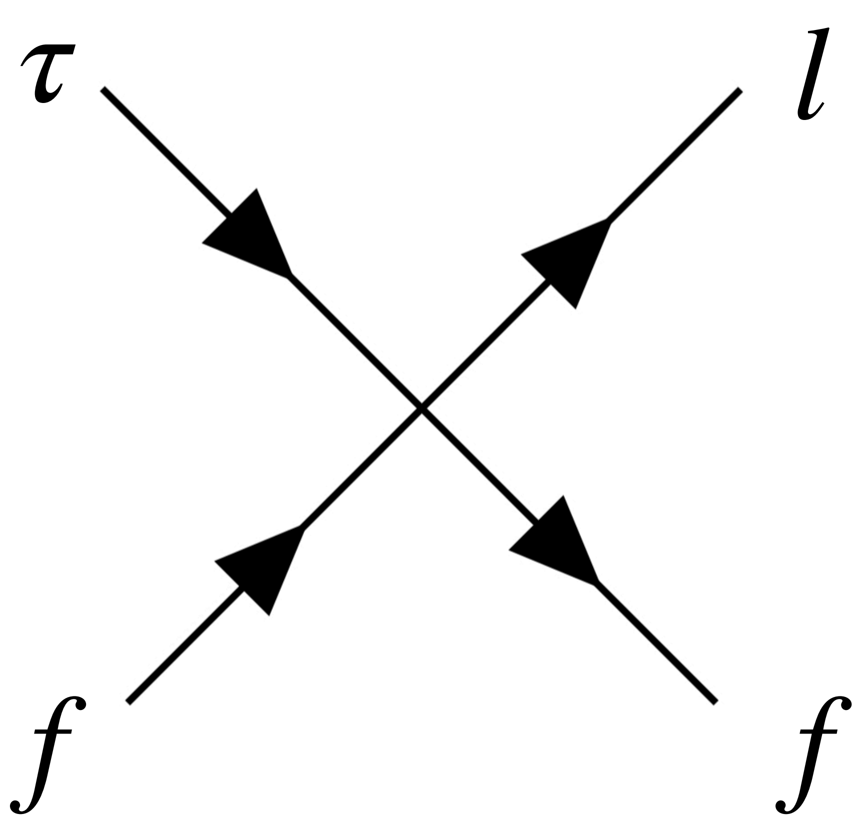
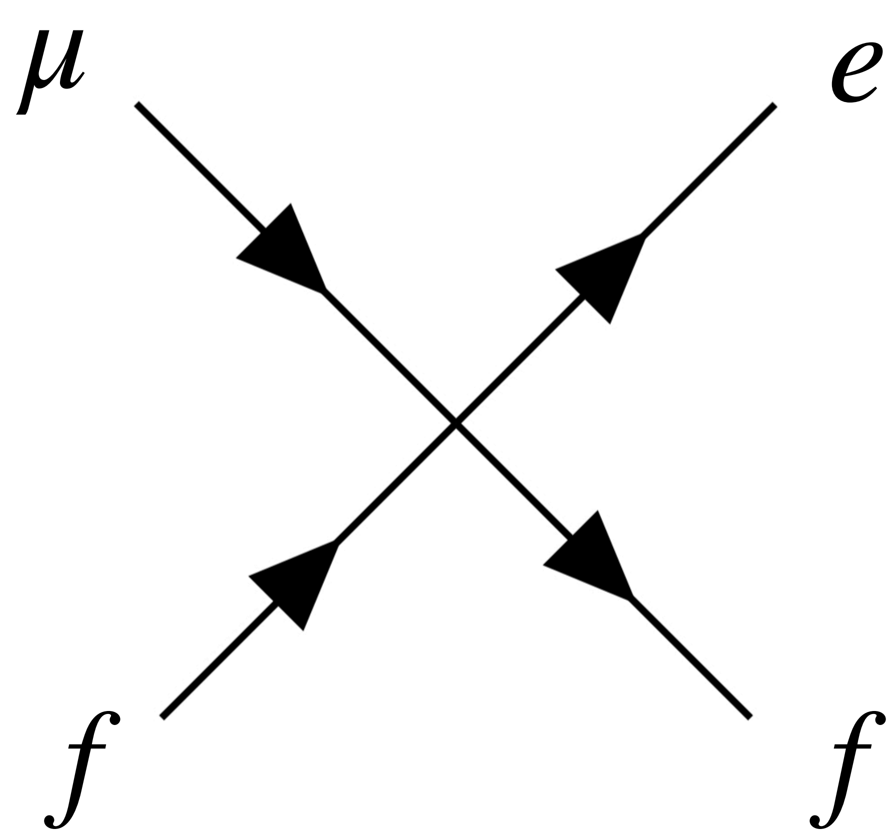
Effective Field Theory for LFV

- Assuming that the New Physics responsible for LFV is heavy, we can describe it with contact interactions



$$\sim \frac{1}{\Lambda^2_{\tau\leftrightarrow l}}$$



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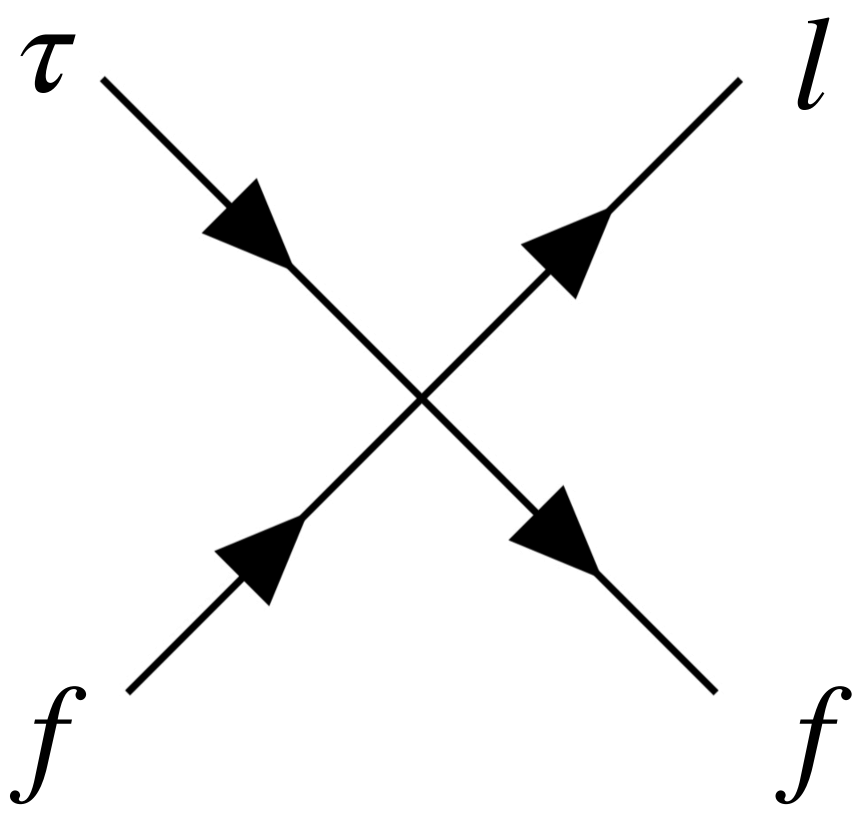

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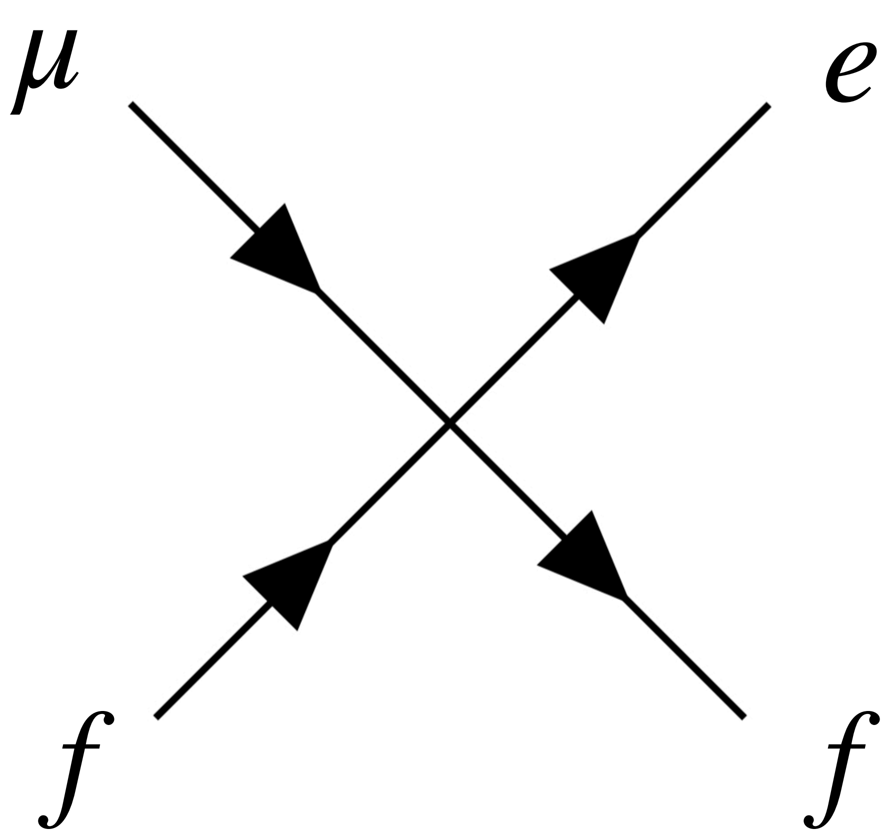


$$\sim \frac{1}{\Lambda^2_{\mu\leftrightarrow e}}$$

- The current (and future) experimental reach in terms of scales is approximately

$$\Lambda_{\tau\leftrightarrow l} \gtrsim 10\ \text{TeV} \xrightarrow{\text{future}} \text{few} \times 10\ \text{TeV}$$

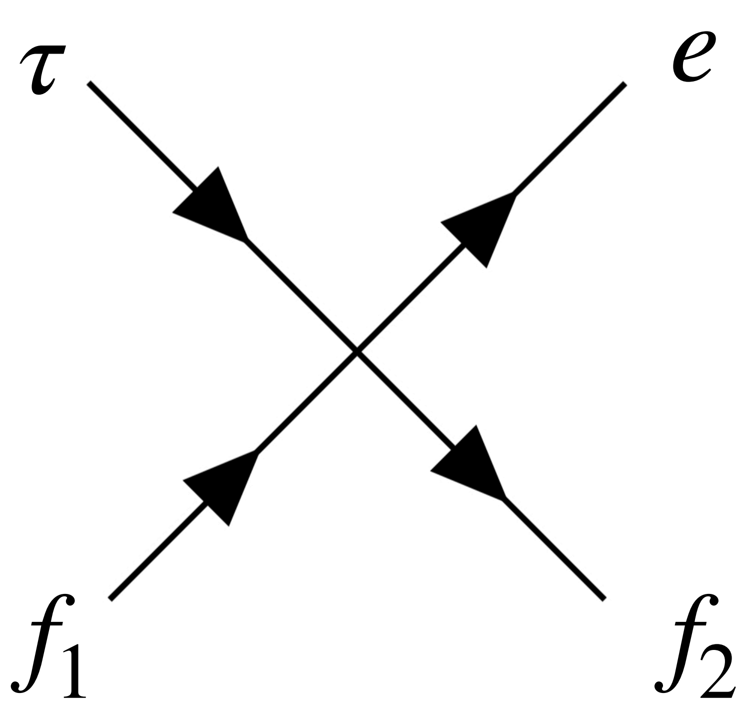
(smaller in some cases)

$$\Lambda_{\mu\leftrightarrow e} \gtrsim \text{few} \times 10^3\ \text{TeV} \xrightarrow{\text{future}} 10^4\ \text{TeV}$$

(smaller in some cases)

$\mu \to e = \mu \to \tau^* \times \tau^* \to e$ in the EFT

- We want to calculate the contribution to $\mu \to e$ arising from $\tau \leftrightarrow l$ operators pairs in the EFT



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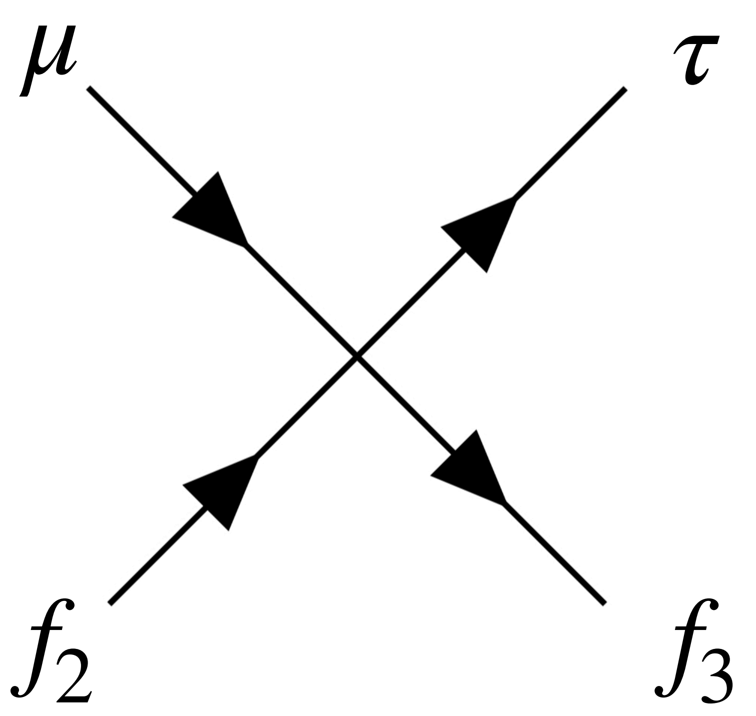

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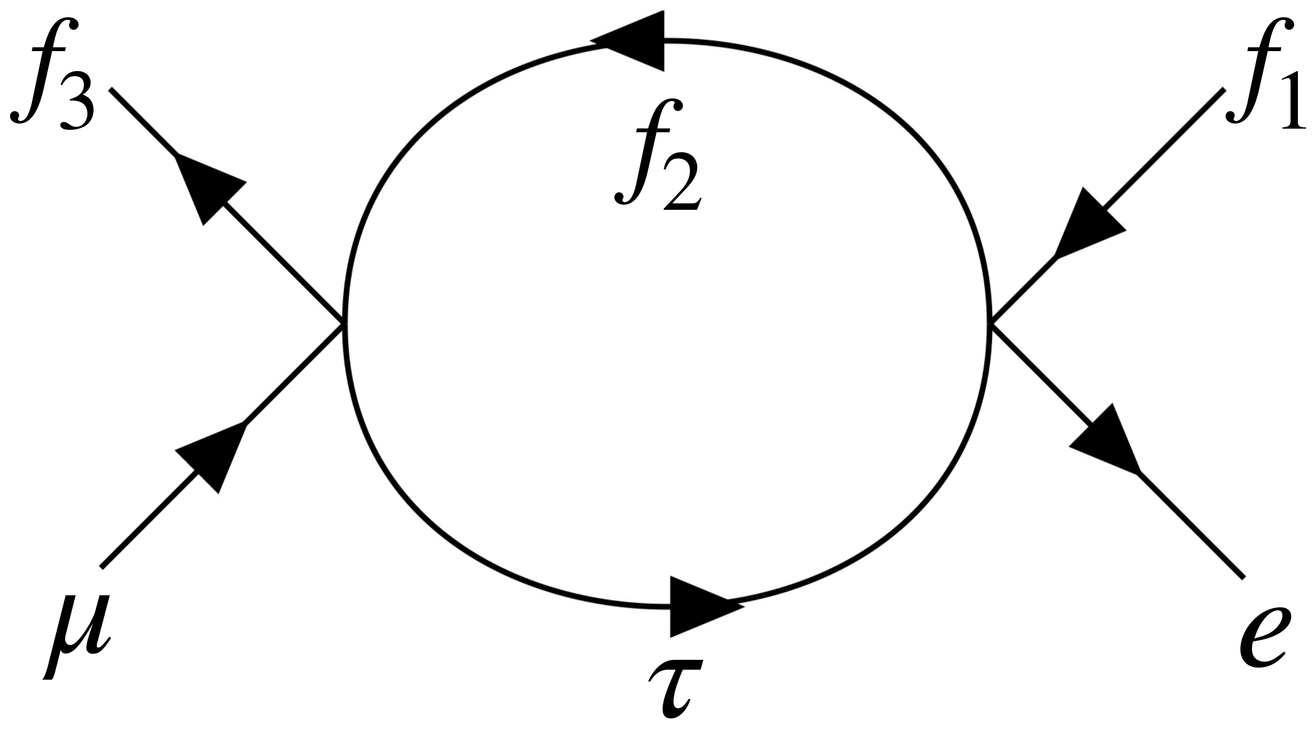

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$$\sim \frac{1}{\Lambda^2_{\tau\leftrightarrow l}} \qquad \sim \frac{1}{\Lambda^2_{\tau\leftrightarrow l}} \qquad \sim \frac{1}{16\pi^2\Lambda^4_{\tau\leftrightarrow l}}$$

- Resulting amplitude at dimension eight, but $\mu \to e$ is (will be) sensitive to some dimension eight operators

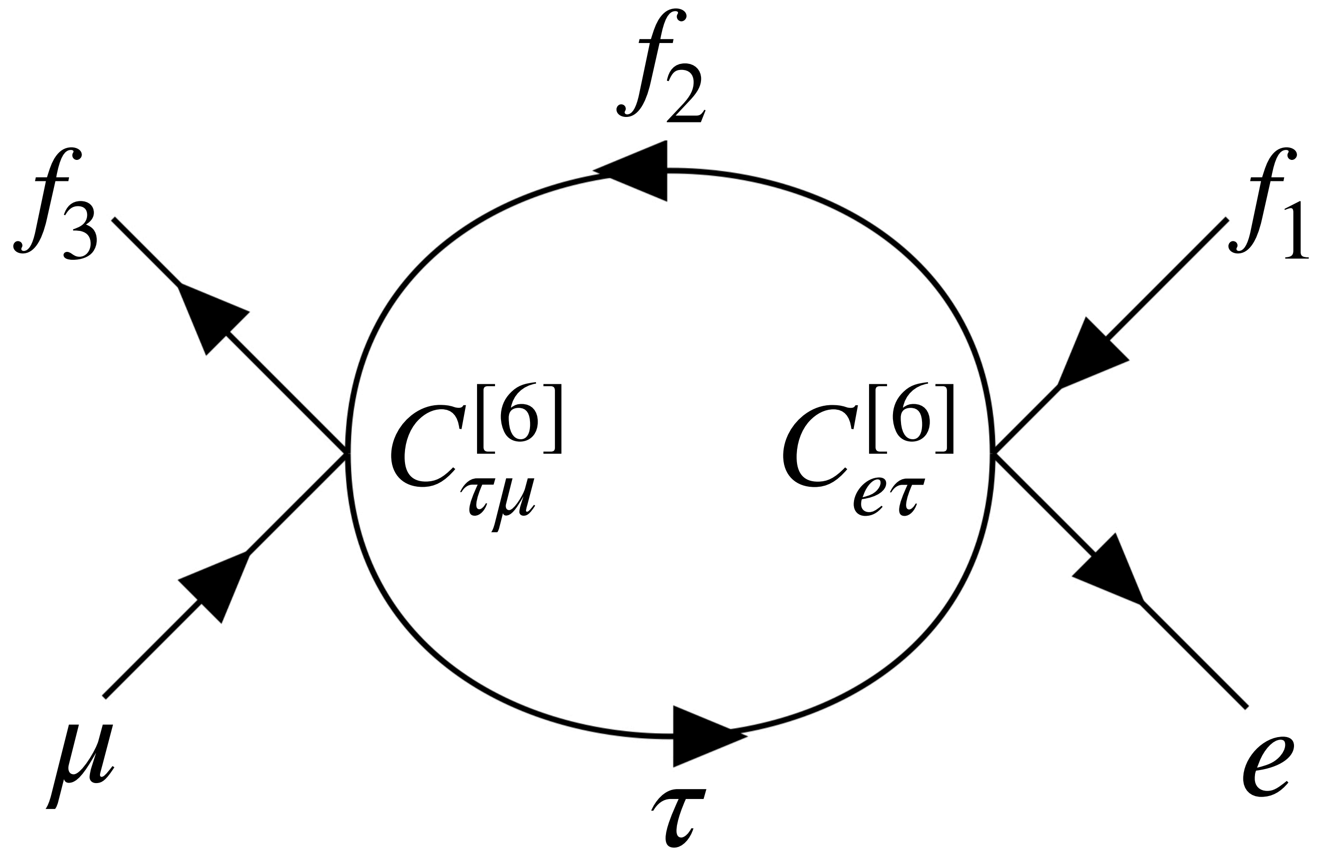
MA+S.Davidson, JHEP '21

- In some cases is possible to probe $\Lambda_{\tau\leftrightarrow l}$ beyond the reach of $\tau \leftrightarrow l$ (Remember that sometimes $Br(\mu \to e) \lesssim Br(\tau \to \mu)Br(\tau \to e)$)

$\mu \to \tau^* \times \tau^* \to e$ EFT calculation

MA+S.Davidson+M.Gorbahn, PRD '22

Divergent diagrams with insertions of $\mu \to \tau \times \tau \to e$ dimension six operators renormalize $\mu \to e$ dimension eight operators



$$C^{[8]}_{e\mu}(E_{\text{exp}}) \sim \frac{C^{[6]}_{\tau\mu} C^{[6]}_{e\tau}}{16\pi^2} \log\left(\frac{\Lambda}{E_{\text{exp}}}\right) \qquad E_{\text{exp}} \equiv \text{exp . scale}$$

- We calculate only the contributions that are (estimated) to be within future experimental sensitivities
- These are included in a subset of dim6xdim6→dim8 one-loop mixing in the Standard Model EFT
- The renormalization group equations for dimension eight are largely unknown and present some technical challenges (Equation of Motion at higher orders, large number of operators/diagrams…)

The EFT calculation

MA+S.Davidson+M.Gorbahn, PRD ‘22

The dimension eight operators that are generated in running are matched onto the low-energy contact interactions when the electroweak symmetry is broken (and the heavy SM particles are integrated out)

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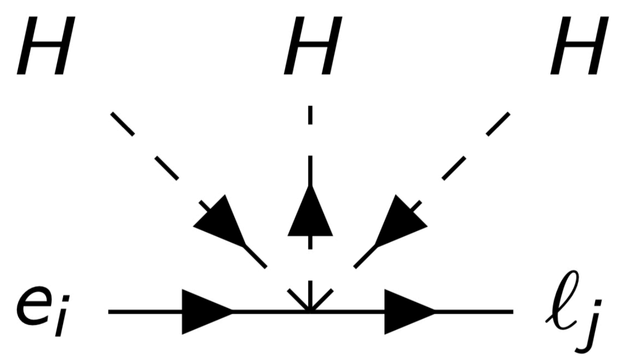
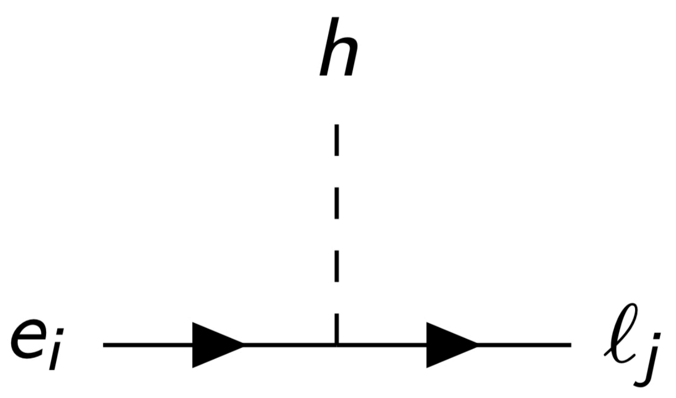
Two dimension six operator can also give additional tree-level matching contribution. For instance, when the Higgs gets a vev $< H > = v$

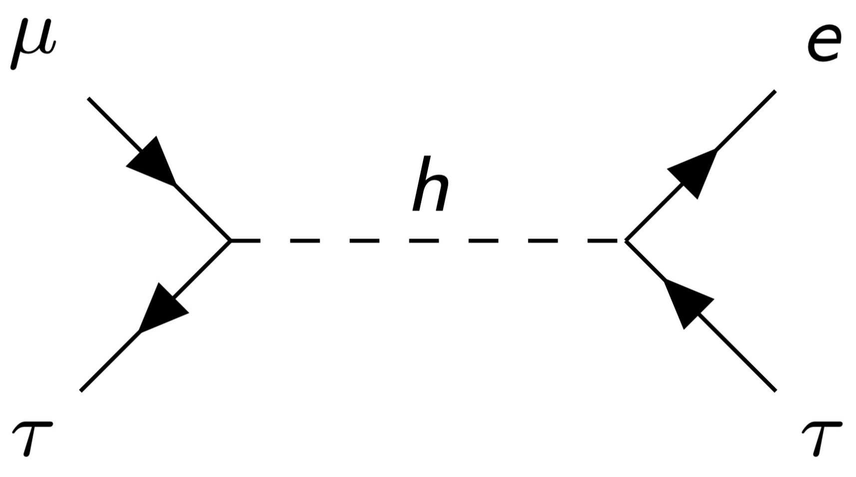


$$= \frac{C_{eH}^{ji}}{\Lambda^2}(\overline{\ell}_j H e_i)(H^\dagger H)$$



$$\propto C_{eH}^{ji}\frac{v^2}{\Lambda^2}$$

Matching contribution when h is removed



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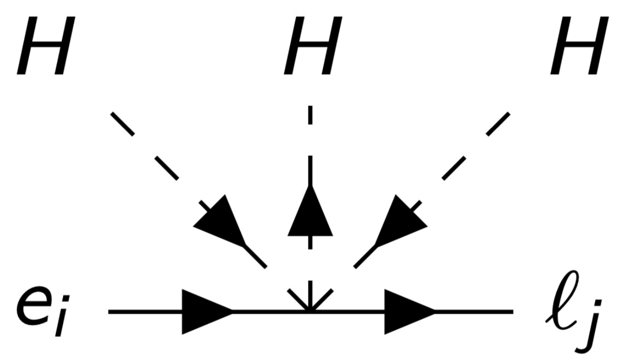

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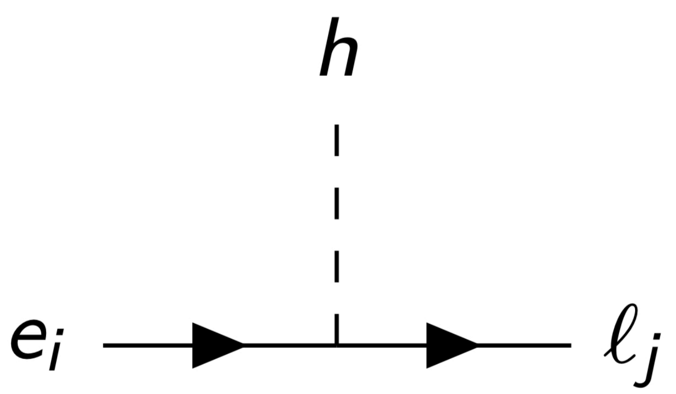

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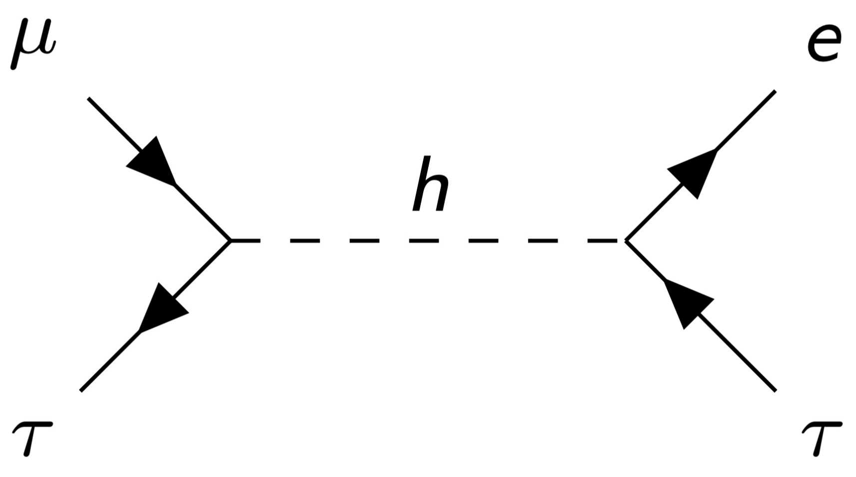

Low energy operators run from the electroweak scale to the scale of the experiment $E_{\rm exp} \longrightarrow$ experiments probe $C^{[6]}_{\tau\mu}C^{[6]}_{e\tau}$

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Hyperbole ($\mu \to e$) vs Ellipses ($\tau \to l$)

In the plane of the $\tau \leftrightarrow e$, $\tau \leftrightarrow \mu$ coefficients $C^{[6]}_{e\tau}$,$C^{[6]}_{\tau\mu}$, direct searches can probe the region outside an ellipse

$$\frac{|C^{[6]}_{e\tau}|^2}{B^2_{\tau\leftrightarrow e}} + \frac{|C^{[6]}_{\tau\mu}|^2}{B^2_{\tau\leftrightarrow\mu}} = 1$$

while $\mu \leftrightarrow e$ is sensitive to region above an hyperbola

$$|C^{[6]}_{e\tau}C^{[6]}_{\tau\mu}| \lesssim B_{\mu\leftrightarrow e}$$

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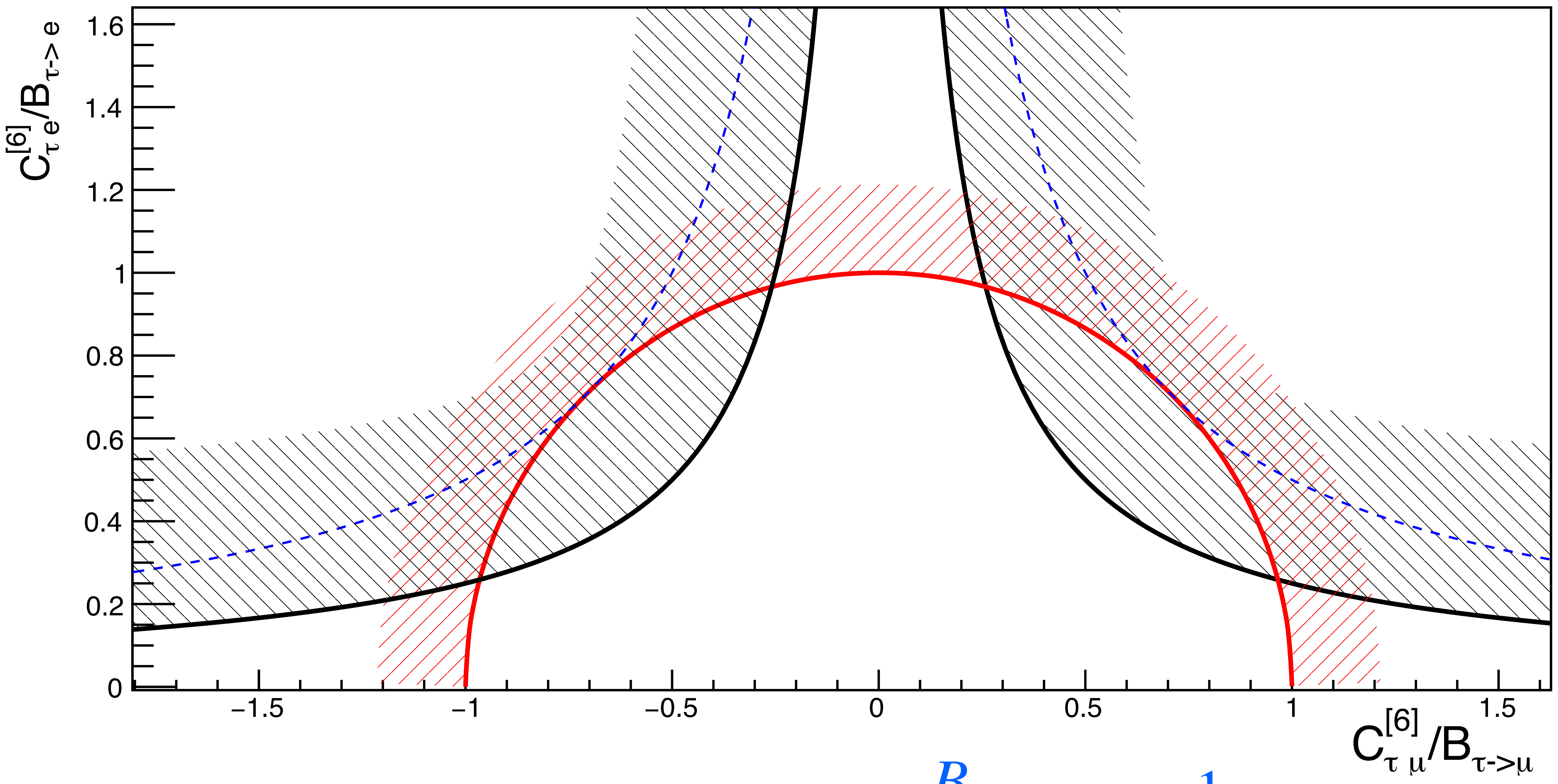
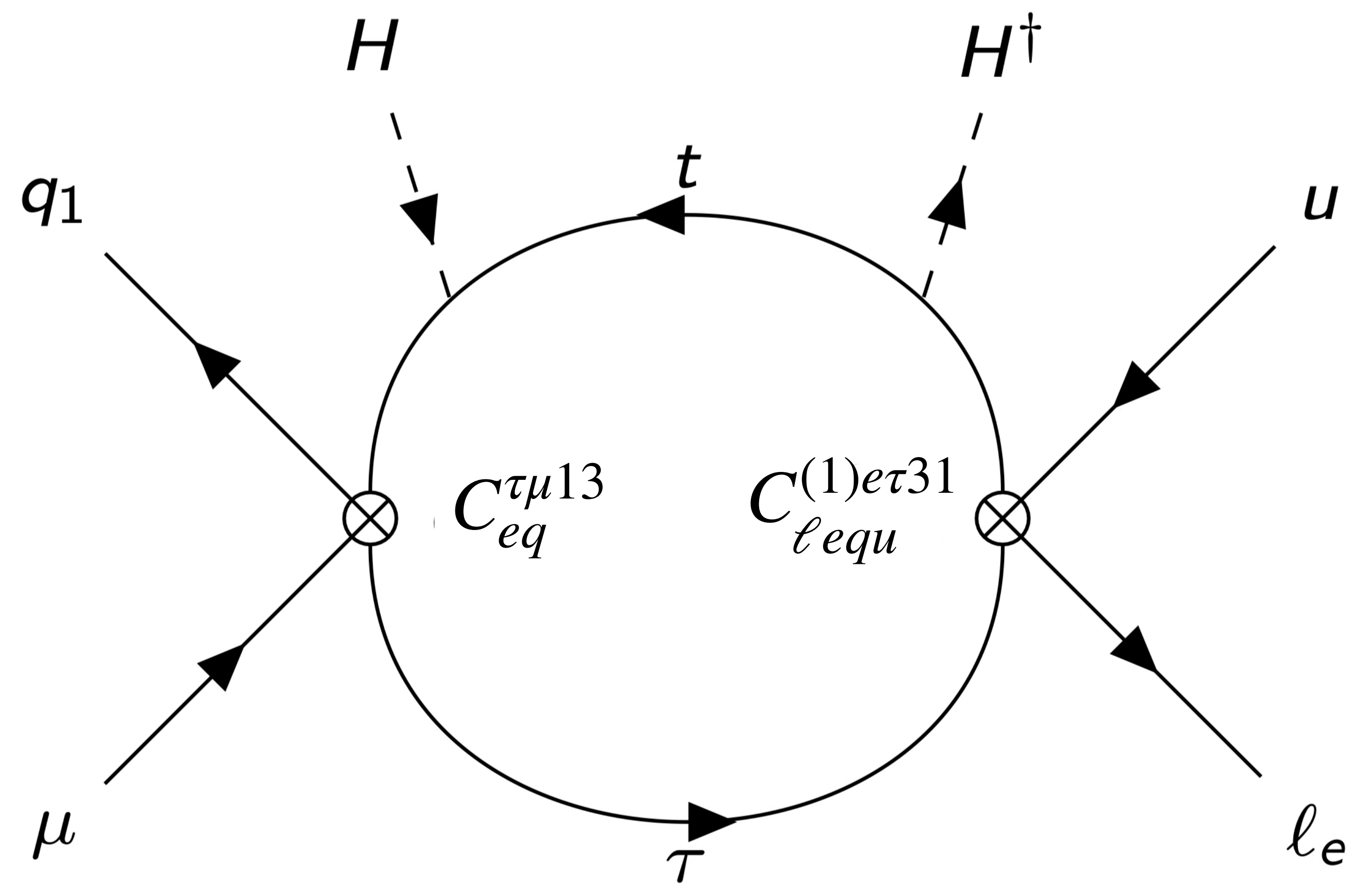


Dashed Hyperbola $\frac{B_{\mu\leftrightarrow e}}{B_{\tau\leftrightarrow e}B_{\tau\leftrightarrow\mu}} = \frac{1}{2}$

Thick Hyperbola $\frac{B_{\mu\leftrightarrow e}}{B_{\tau\leftrightarrow e}B_{\tau\leftrightarrow\mu}} < \frac{1}{2}$

An example

Consider the operators $\mathcal{O}_{eq}^{\tau\mu 13} = 2\sqrt{2}G_F(\bar{\tau}\gamma\mu)(\bar{q}_1\gamma q_3)$ (vector) and $\mathcal{O}_{\ell equ}^{(1)e\tau 31} = 2\sqrt{2}G_F(\bar{\ell}_e P_R \tau)(\bar{q}_3 P_R u)$ (scalar)



The diagram generates the scalar operator $(\bar{e}P_R\mu)(\bar{u}P_R u)$, contributing to $\mu \to e$ conversion in nuclei

$$Br(\mu A \to eA) < 7 \times 10^{-13} \to C_{eq}^{\tau\mu 13} \times C_{\ell equ}^{(1)e\tau 31} \lesssim B_{\mu\leftrightarrow e} = 1.5 \times 10^{-8}$$

B LFV decays are sensitive to the operator pair

$$Br(B_d \to \mu\tau) < 1.4 \times 10^{-5} \to C_{eq}^{\tau\mu 13} \lesssim B_{\tau\leftrightarrow\mu} = 1.5 \times 10^{-3}$$

$$Br(B^+ \to \bar{\tau}\nu) = (1.09 \pm 0.24) \times 10^{-4} \to C_{\ell equ}^{(1)e\tau 31} \lesssim B_{\tau\leftrightarrow e} = 1.8 \times 10^{-3}$$

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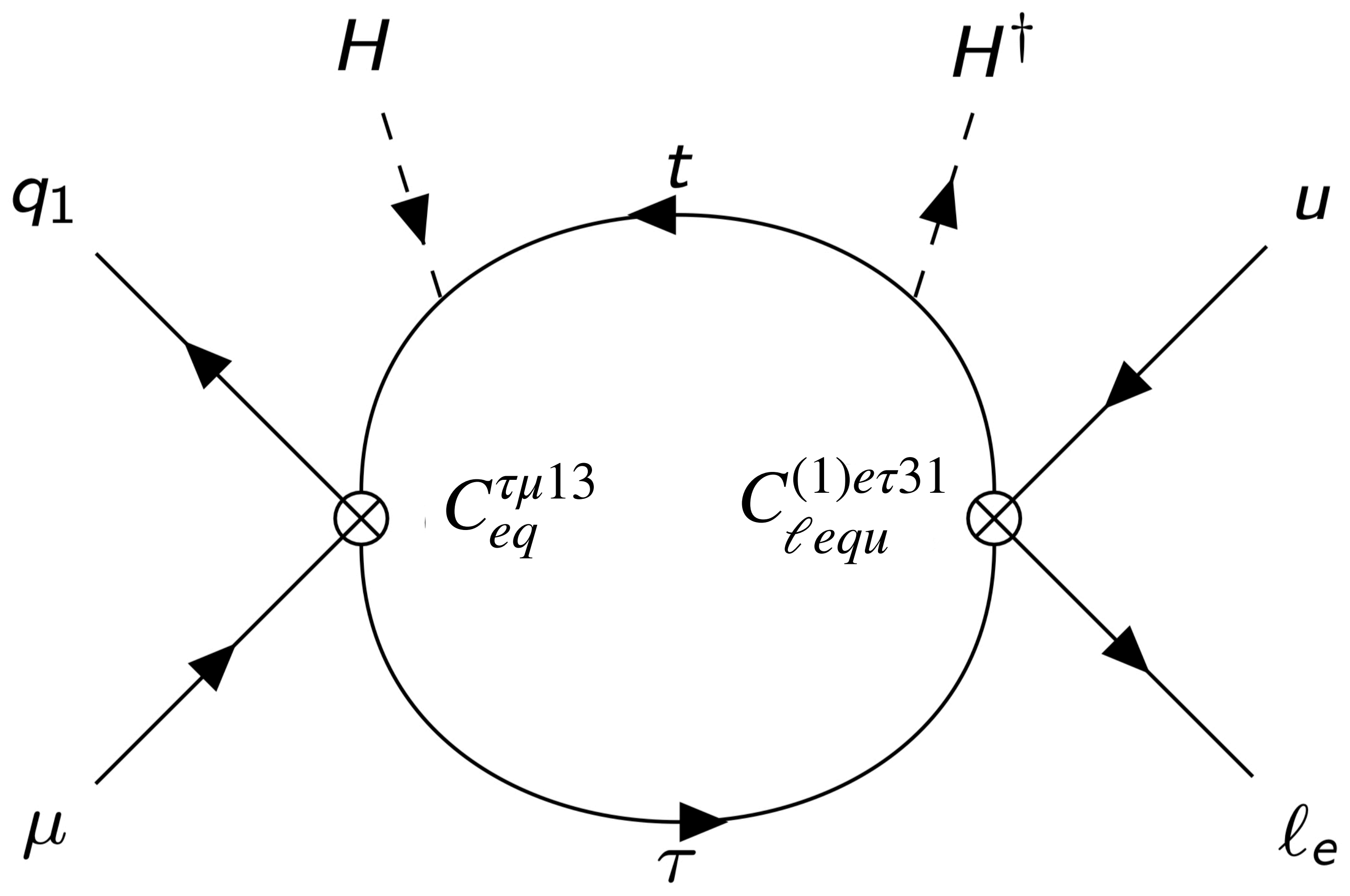


The diagram generates the scalar operator $(\bar{e}P_R\mu)(\bar{u}P_R u)$, contributing to $\mu \to e$ conversion in nuclei

$$Br(\mu A \to eA) < 7 \times 10^{-13} \to C_{eq}^{\tau\mu13} \times C_{\ell equ}^{(1)e\tau31} \lesssim B_{\mu\leftrightarrow e} = 1.5 \times 10^{-8}$$

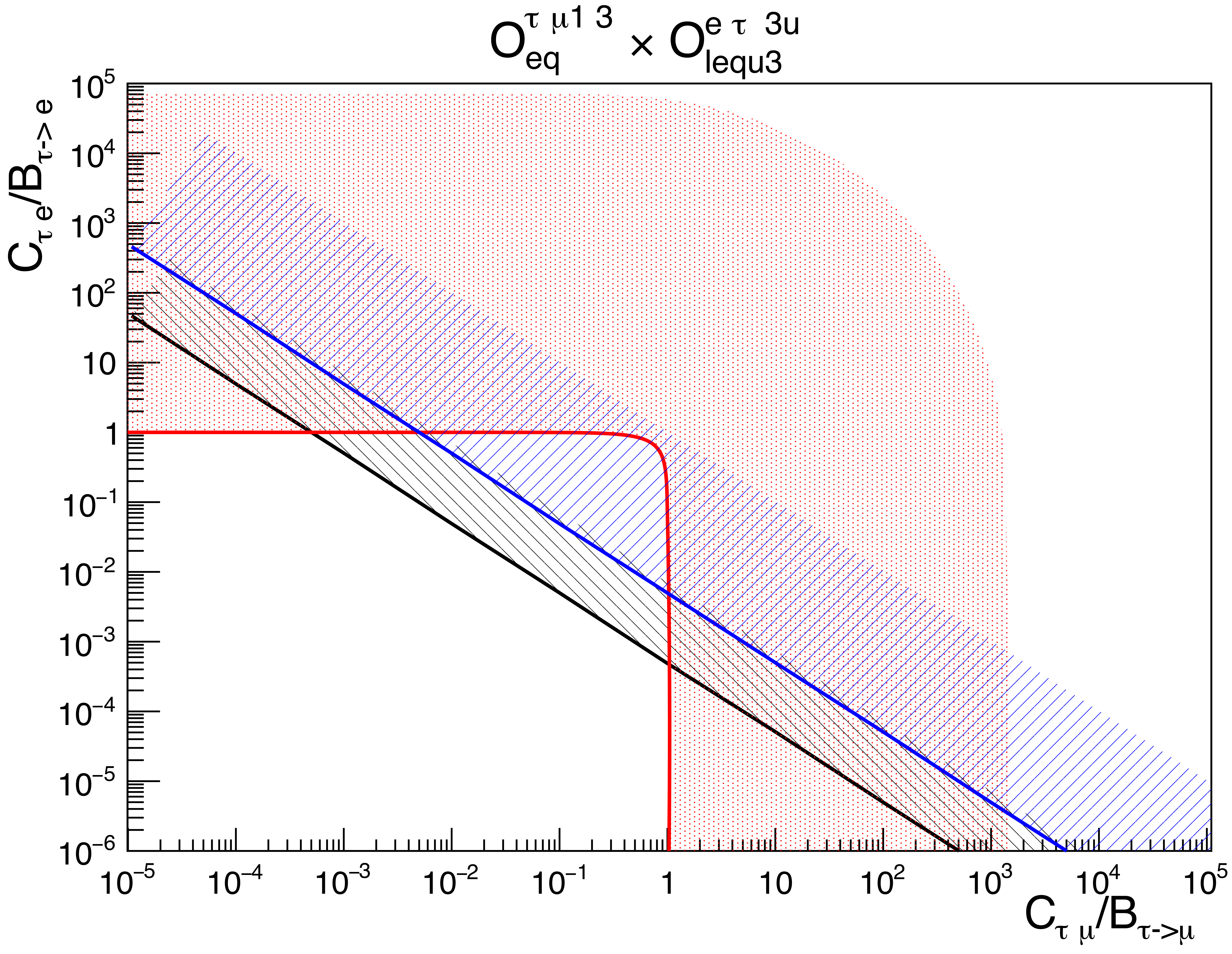
B LFV decays are sensitive to the operator pair

$$Br(B_d \to \mu\tau) < 1.4 \times 10^{-5} \to C_{eq}^{\tau\mu13} \lesssim B_{\tau\leftrightarrow\mu} = 1.5 \times 10^{-3}$$

$$Br(B^+ \to \bar{\tau}\nu) = (1.09 \pm 0.24) \times 10^{-4} \to C_{\ell equ}^{(1)e\tau31} \lesssim B_{\tau\leftrightarrow e} = 1.8 \times 10^{-3}$$

$$\frac{B_{\mu\leftrightarrow e}}{B_{\tau\leftrightarrow e}B_{\tau\leftrightarrow\mu}} \sim 5 \times 10^{-3} \longrightarrow \sim 5 \times 10^{-4}$$ (with future sensitivities)

Example: coefficients space



- Current $\mu A \to eA$
- Future $\mu A \to eA$
- B decays

Relating $\mu \leftrightarrow e,\ \tau \leftrightarrow e,\ \tau \leftrightarrow \mu$

Suppose that we observe a $\tau \leftrightarrow e$ transitions, but $\mu \leftrightarrow e$ is not seen

$$|C_{e\tau}^{[6]}| = B_{\tau\leftrightarrow e} \qquad\qquad |C_{e\tau}^{[6]}C_{\tau\mu}^{[6]}| \lesssim B_{\mu\leftrightarrow e}$$

The results suggest values for $|C_{\tau\mu}^{[6]}|$ that are "unlikely"

$$|C_{\tau\mu}^{[6]}| \lesssim \frac{B_{\mu\leftrightarrow e}}{B_{\tau\leftrightarrow e}}$$

Conclusion

- An impressive experimental improvement for $\mu \to e$ is expected in the near future

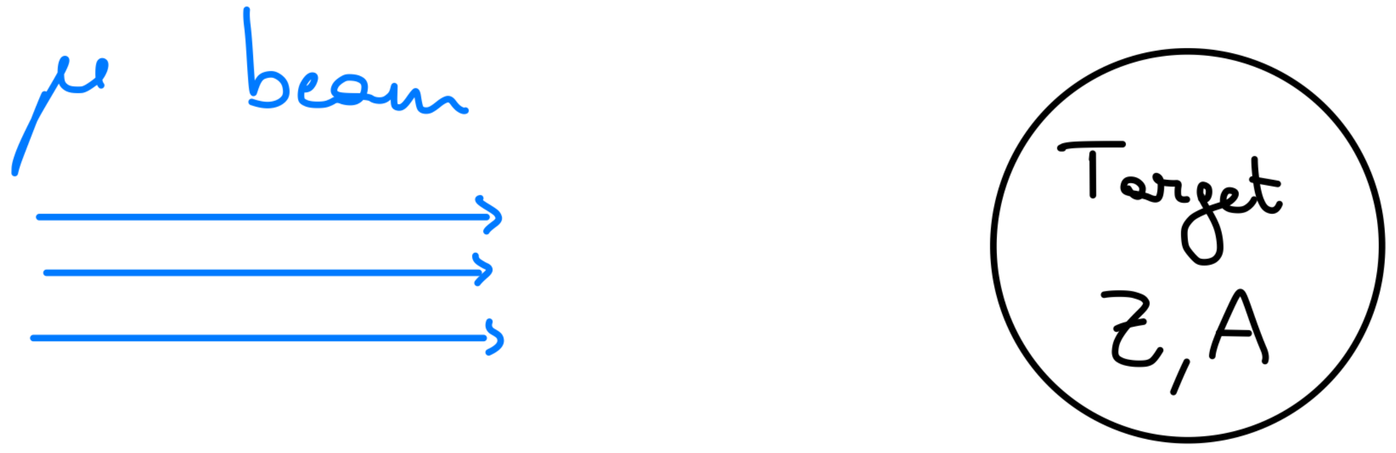
- The sensitivity of $\mu \to e$ to $\mu \to \tau^* \times \tau^* \to e$ contributions can compete with the direct $\tau \to l$ searches and probe out-of-reach regions in the τ flavour changing parameter space

- We explored this in an Z' model and in the Standard Model EFT (which required calculating a subset of RGEs mixing pairs of $\tau \leftrightarrow e, \tau \leftrightarrow \mu$ dimension six operators into $\mu \to e$ dimension eight contact interactions)

- The EFT results can be used to relate the different flavour changing transitions: a detection of $\tau \leftrightarrow e$ $(\tau \leftrightarrow \mu)$ and a null result from $\mu \to e$ can suggest the size of some $\tau \leftrightarrow \mu$ $(\tau \leftrightarrow e)$ couplings

Back-up

$\mu \to e$ conversion in nuclei



Standard calculation in Kuno+Okada hep-ph/9909265

- The muon gets captured by the (Z,A) nucleus and tumbles down to the 1s state

- The SM processes that can happen are:

  A. $\mu + p \to \nu_\mu + n$ (capture)

  B. $\mu \to \nu_\mu + e + \overline{\nu_e}$ (Decay-In-Orbit)

- If there are LFV interactions with nucleons, an electron can be emitted without a neutrino (conversion)

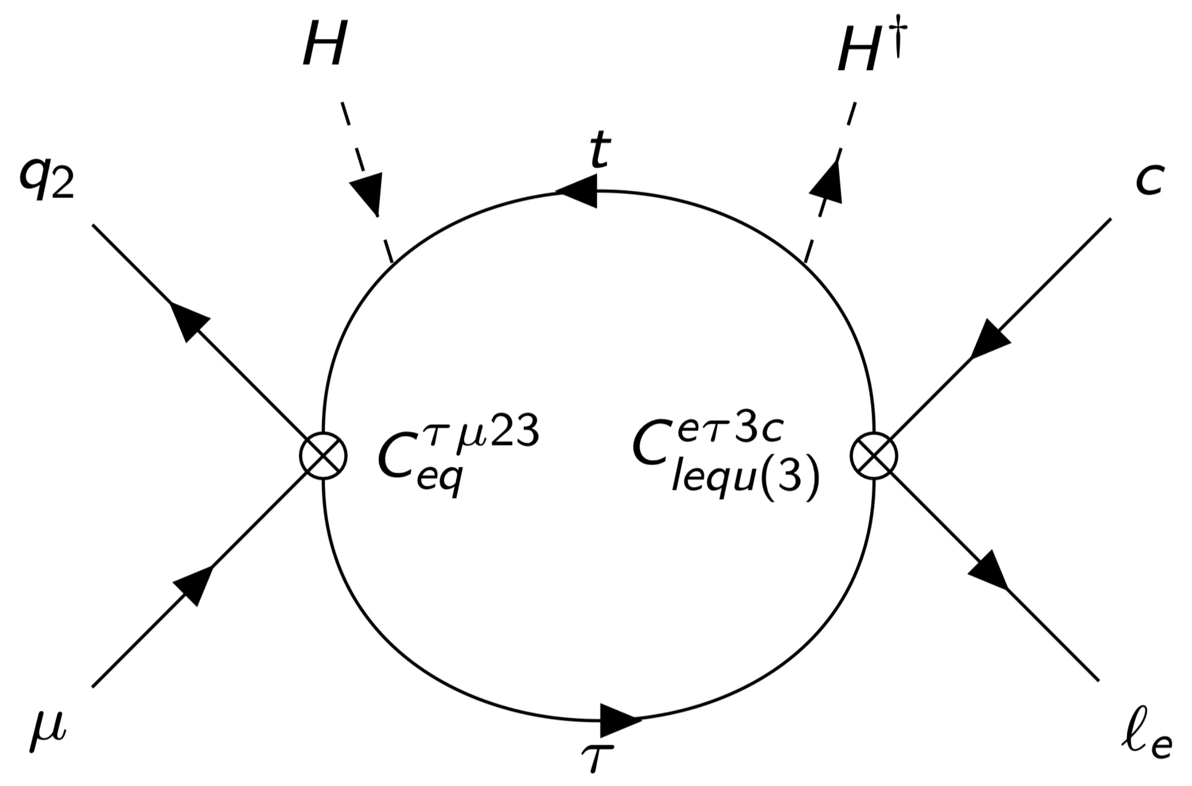
$$\mu + (Z, A) \to e + (Z, A)$$

- Spin-Independent rate is enhanced by $\propto A^2$ because the process is coherent (similar to WIMP scattering)

- The upcoming experiments (COMET, Mu2e) will deliver extremely intense muon beams allowing to probe $Br(\mu A \to eA) \sim 10^{-17}$

I-Relating $\mu \leftrightarrow e,\ \tau \leftrightarrow e,\ \tau \leftrightarrow \mu$

Consider $\mathcal{O}_{eq}^{\tau\mu 23} = 2\sqrt{2}G_F(\bar{\tau}\gamma\mu)(\bar{q}_2\gamma q_3)$ and $\mathcal{O}_{\ell equ}^{(3)e\tau 32} = 2\sqrt{2}G_F(\bar{\ell}_e\sigma\tau)(\bar{q}_3\sigma c)$



The diagram generates a $\mu \to e$ tensor with external charms that mixes with the dipole and contribute to $\mu \to e\gamma$

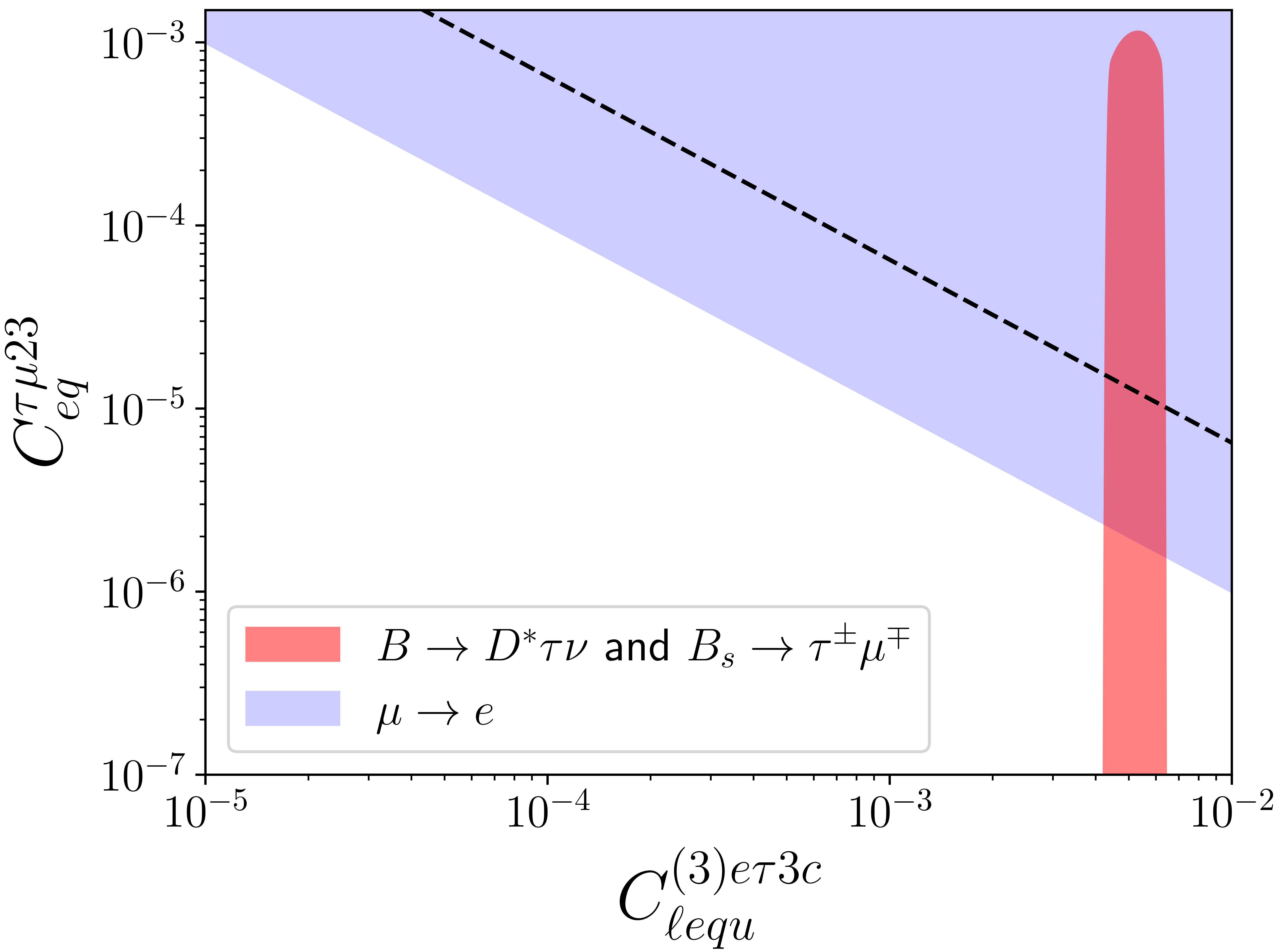
The charged-current anomaly in B decays

$$R_{D^{(*)}} \equiv \frac{Br(B \to D^{(*)}\tau\bar{\nu})}{Br(B \to D^{(*)}l\bar{\nu})}$$

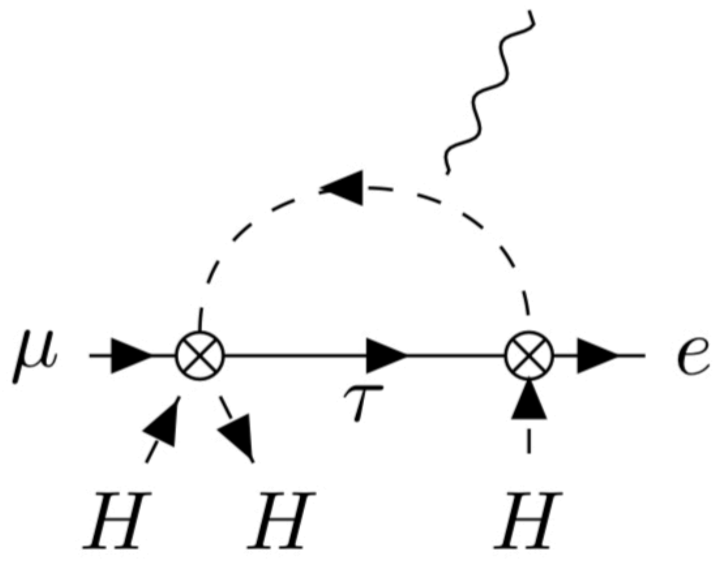
$$R_{D^{(*)}}^{\rm exp} - R_{D^{(*)}}^{\rm SM} \sim +3\sigma$$

can be fitted with a non-zero $\mathcal{O}_{\ell equ}^{(3)e\tau 32} = 2\sqrt{2}G_F(\bar{\ell}_e\sigma\tau)(\bar{q}_3\sigma c)$ (increase the numerator, the neutrino flavour is not identified)

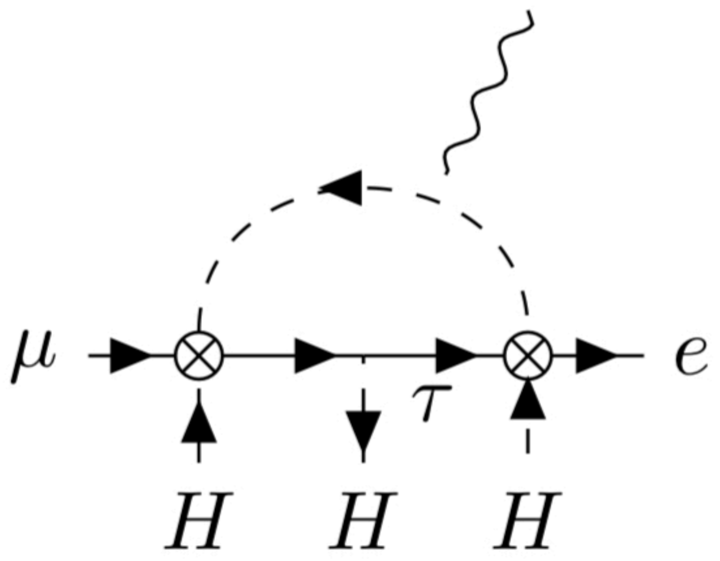
II-Relating $\mu \leftrightarrow e$, $\tau \leftrightarrow e$, $\tau \leftrightarrow \mu$



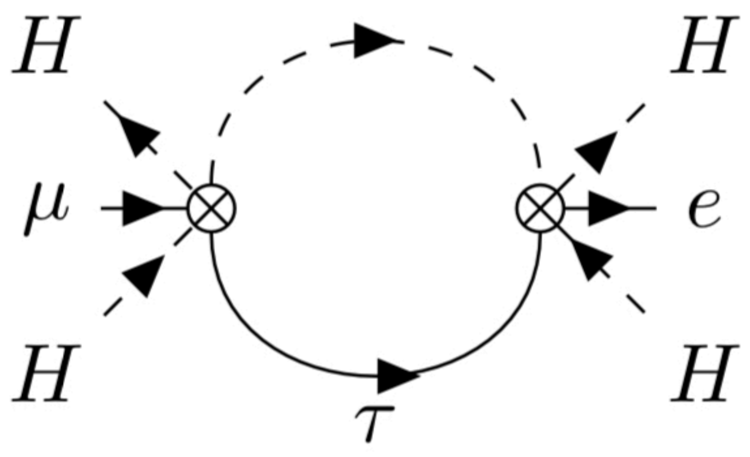
RGEs diagrams



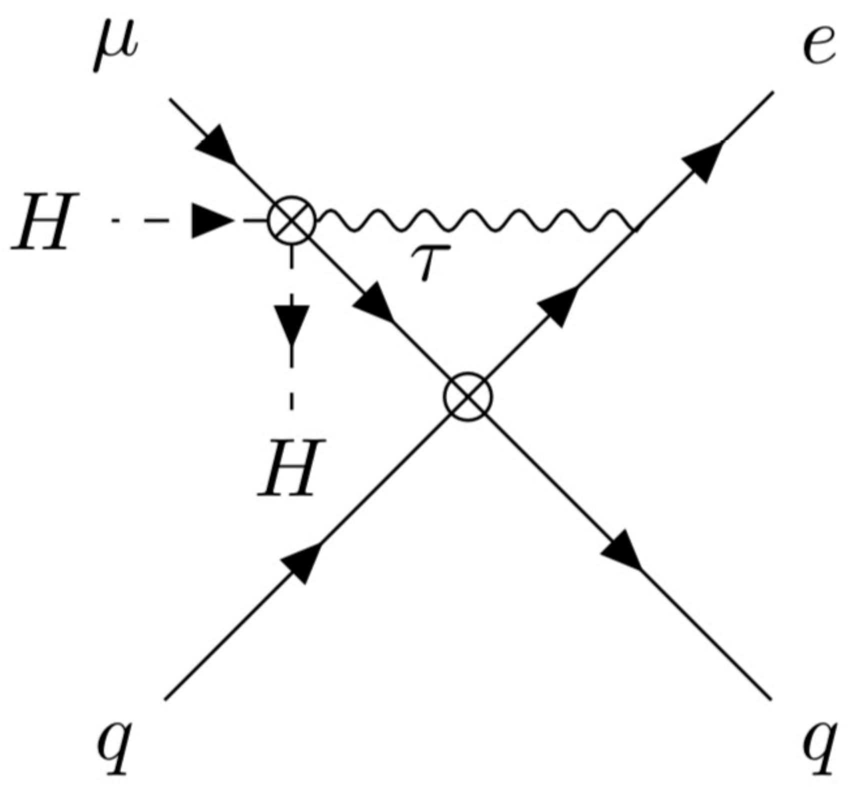
(a) $Y_6 \times P_6 \to D_8$



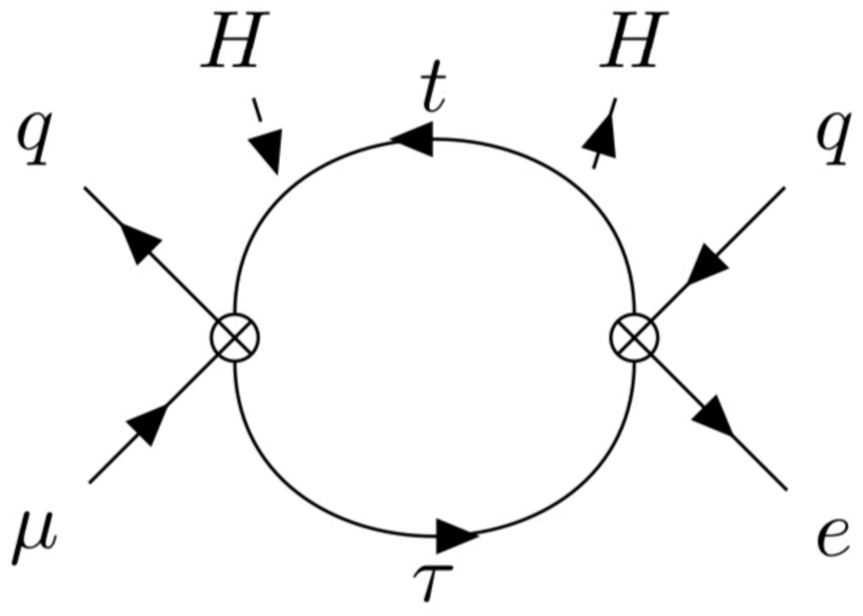
(b) $P_6 \times P_6 \to D_8$



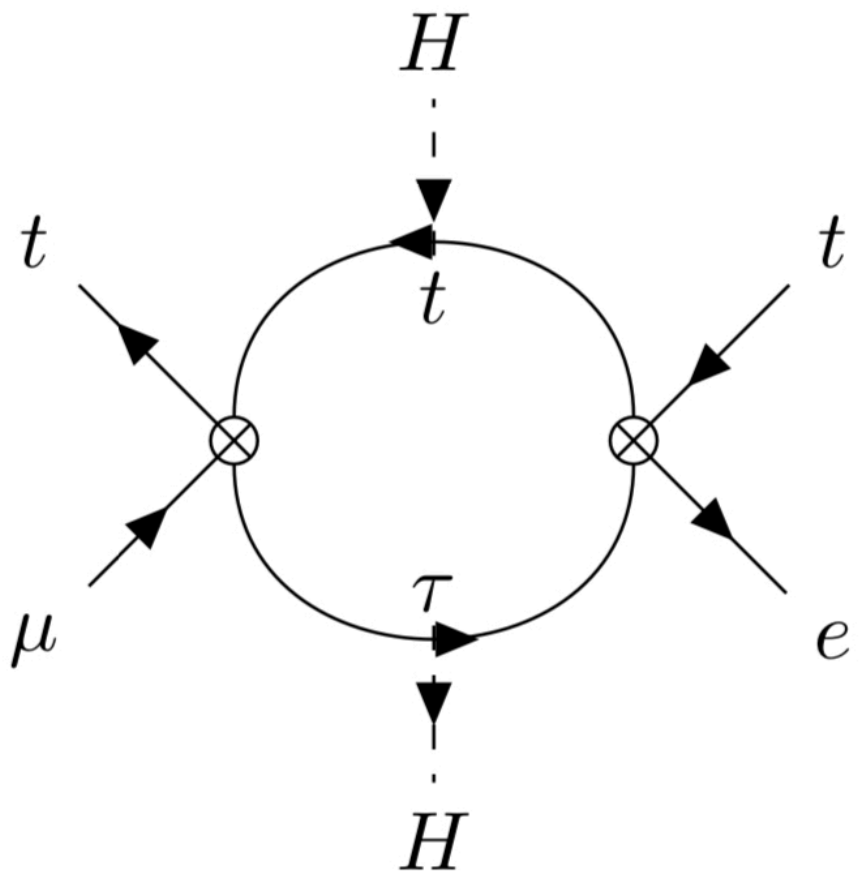
(c) $Y_6 \times Y_6 \to P_8$



(d) $P_6 \times 4f_6 \to 4f_8$

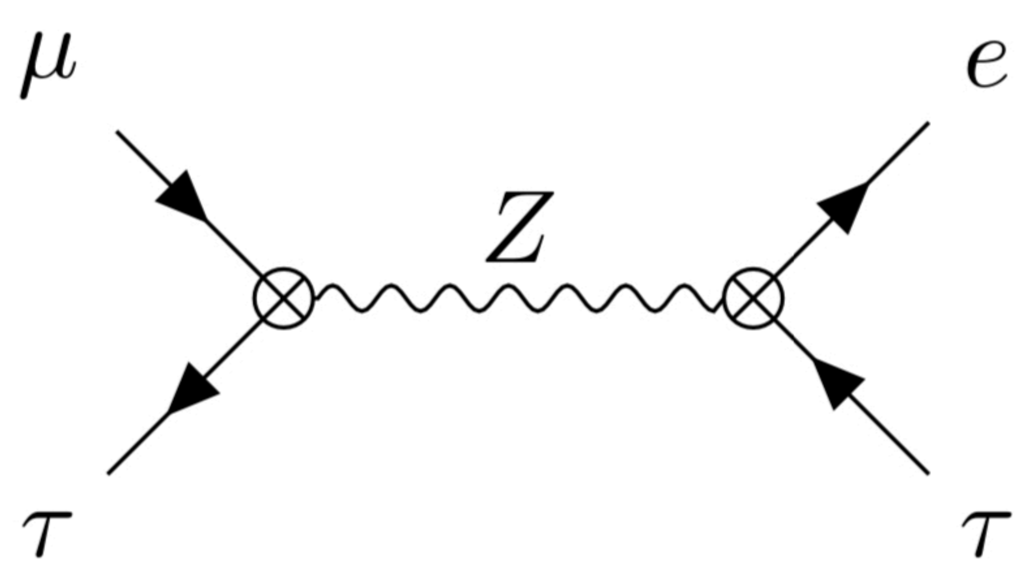


(e) $4f_6 \times 4f_6 \to 4f_8$

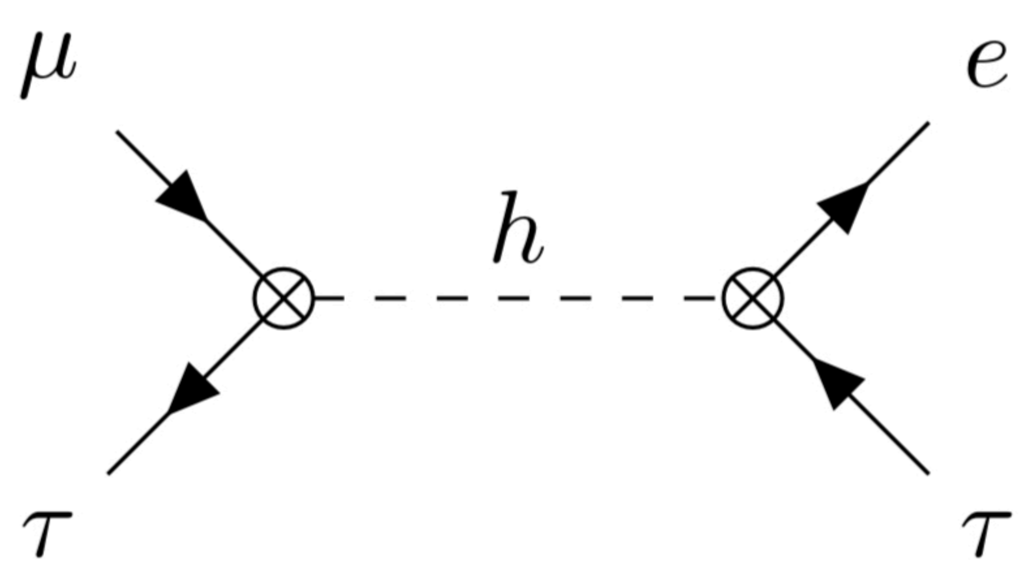


(f) $4f_6 \times 4f_6 \to 4f_8$

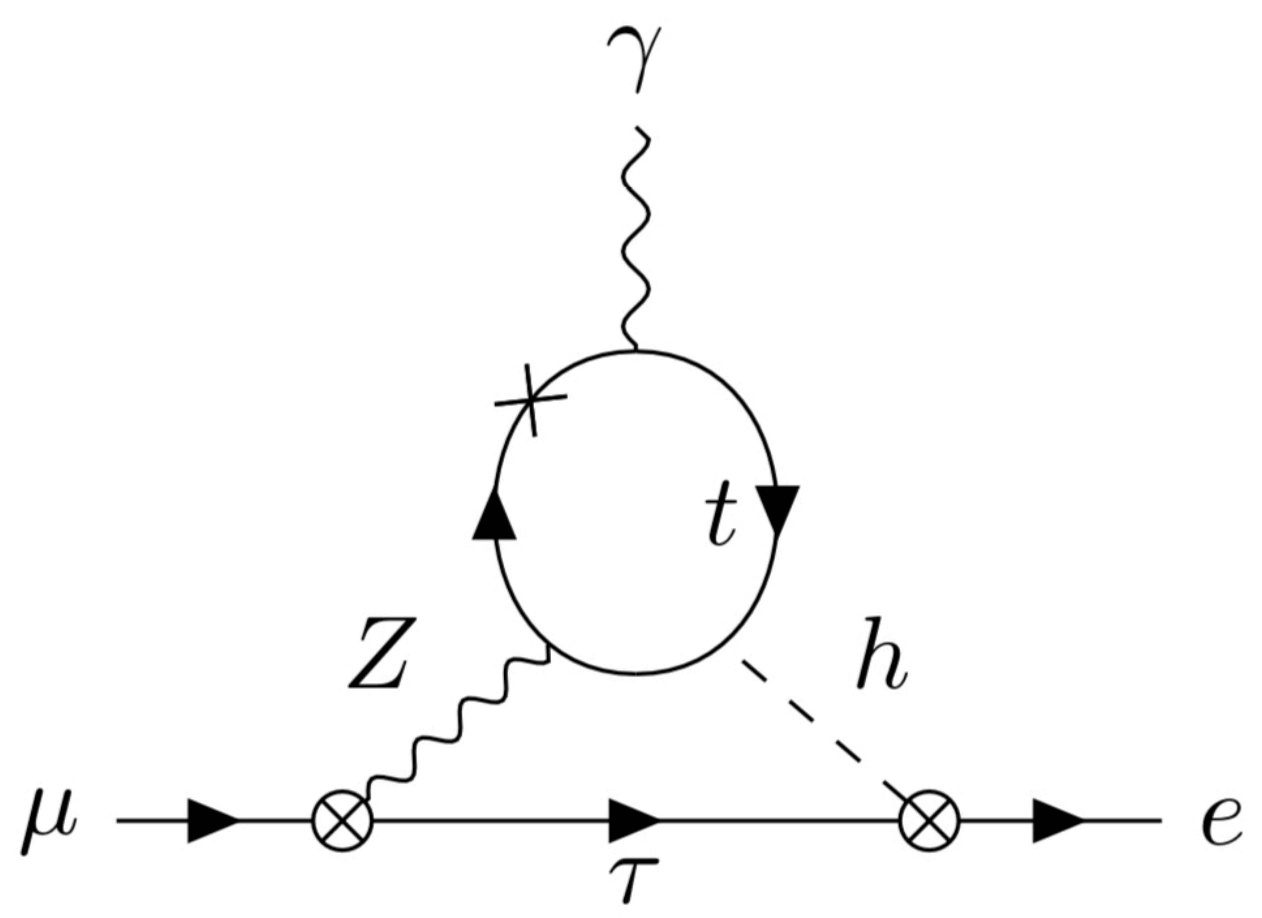
Matching diagrams



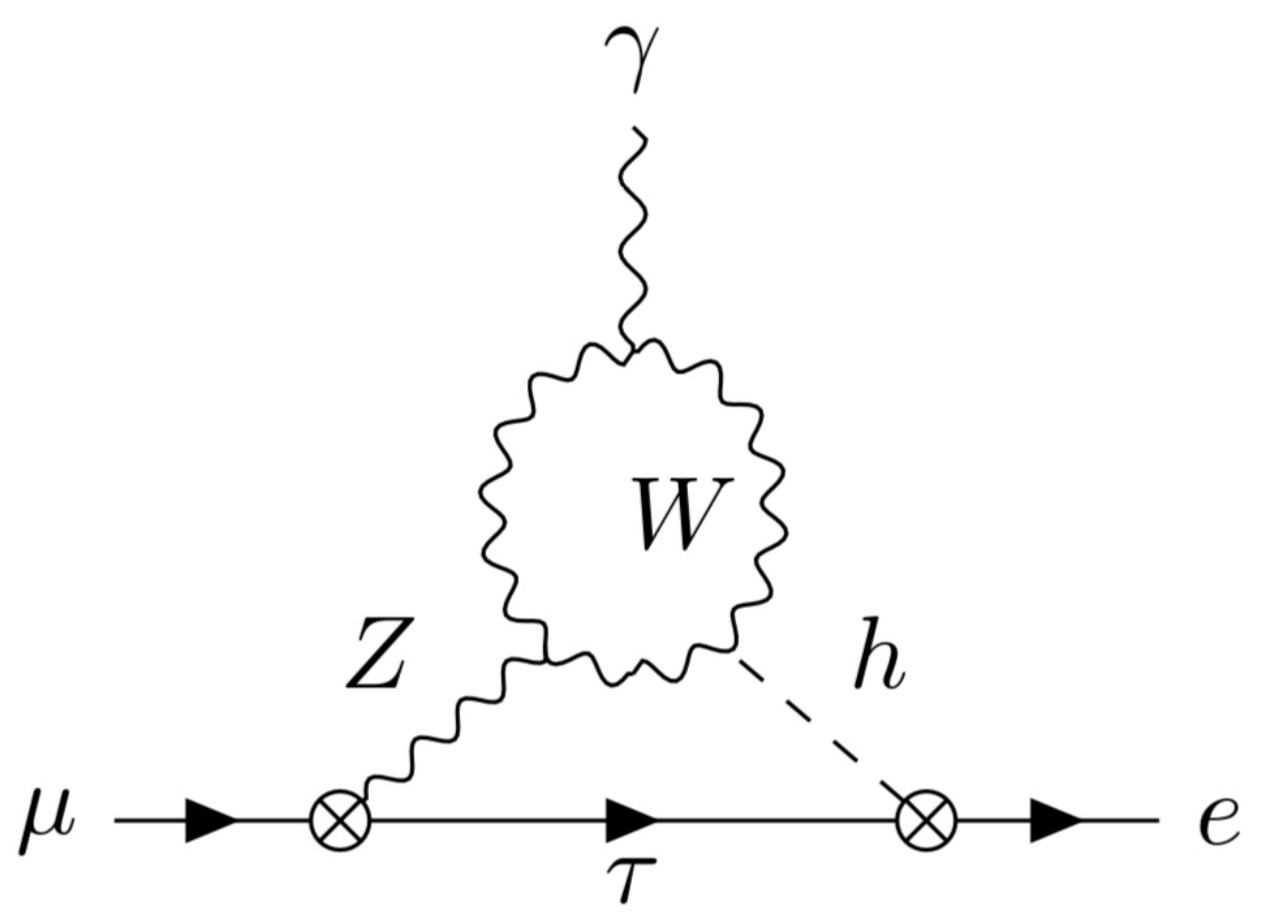
(a)



(b)



(c)



(d)

Equation of motion 1

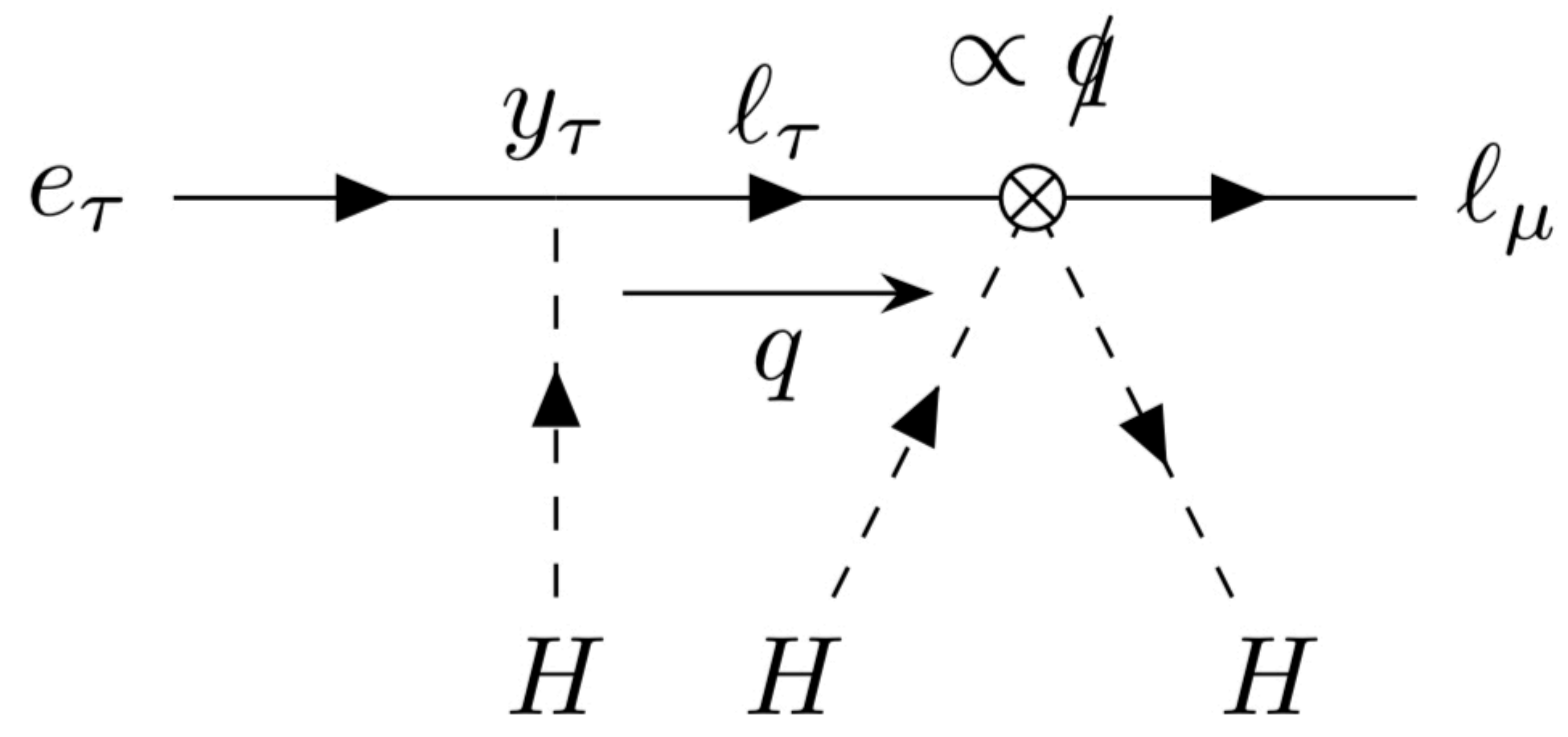
Two operators $\mathcal{O}_1, \mathcal{O}_2$ that differ by an Equation of Motion (EOM) vanishing operator are physically equivalent

$$\mathcal{O}_1 - \mathcal{O}_2 = \mathcal{O}_{EOM} \propto \frac{\delta S}{\delta \phi}$$

because $\mathcal{O}_{EOM}$ has vanishing $S-$matrix elements. For instance, $i(\bar{\ell}_\mu \not{D} \ell_\tau)(H^\dagger H)$ and $y_\tau(\bar{\ell}_\mu H e_\tau)(H^\dagger H)$ are on-shell equivalent

$$\begin{aligned} i(\bar{\ell}_\mu \not{D} \ell_\tau)(H^\dagger H) \to & [i(\bar{\ell}_\mu \not{D} \ell_\tau)(H^\dagger H) - y_\tau(\bar{\ell}_\mu H e_\tau)(H^\dagger H)] \\ & + y_\tau(\bar{\ell}_\mu H e_\tau)(H^\dagger H) \end{aligned}$$

We can understand it diagrammatically:

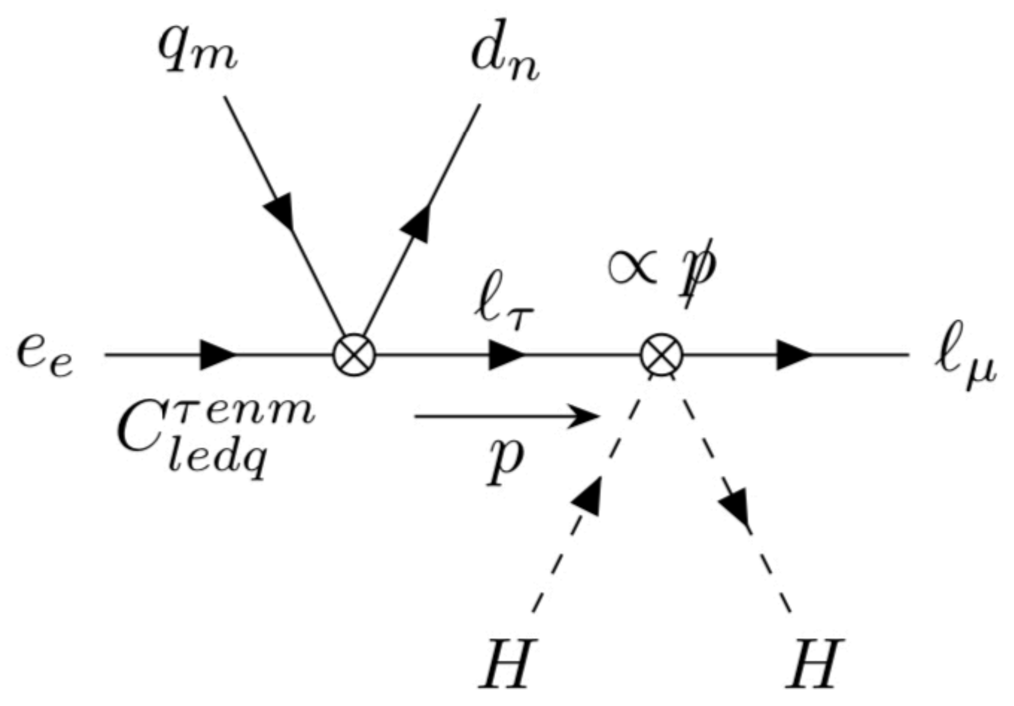


Equation of motion 2

Suppose that the only dimension six present is $\mathcal{O}_{\ell edq}^{e\tau nm} = (\bar{\ell}_\tau e_e)(\bar{d}_n q_m)$. The operators corrects the EOM, such that

$$i(\bar{\ell}_\mu \not{D} \ell_\tau)(H^\dagger H) \to \left[i(\bar{\ell}_\mu \not{D} \ell_\tau)(H^\dagger H) - y_\tau (\bar{\ell}_\mu H e_\tau)(H^\dagger H) + \frac{C_{\ell edq}^{\tau enm}}{\Lambda_{\rm NP}^2} (\bar{\ell}_\mu e_e)(\bar{d}_n q_m)(H^\dagger H) \right]$$
$$+ y_\tau (\bar{\ell}_\mu H e_\tau)(H^\dagger H) - \frac{C_{\ell edq}^{\tau enm}}{\Lambda_{\rm NP}^2} (\bar{\ell}_\mu e_e)(\bar{d}_n q_m)(H^\dagger H)$$

Diagrammatically:



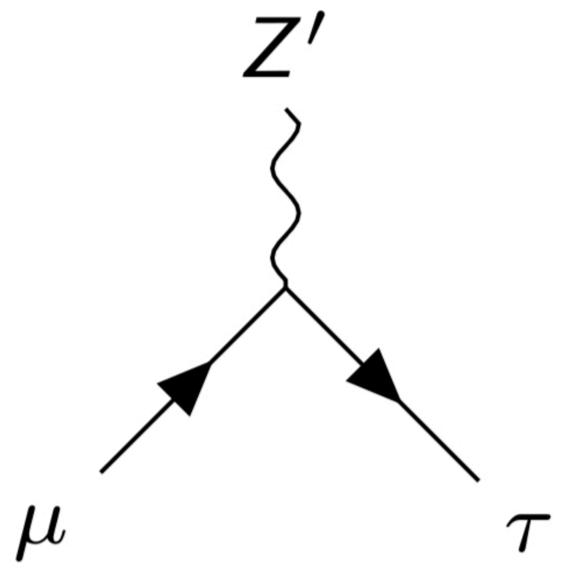
If a redundant operator is generated via loops, in general:

$$\frac{A}{\Lambda_{\rm NP}^2 \epsilon} \left(\mathcal{O}^{[6]} + \frac{\mathcal{O}^{[8]}}{\Lambda_{\rm NP}^2} - \mathcal{O}_{EOM} \right)$$

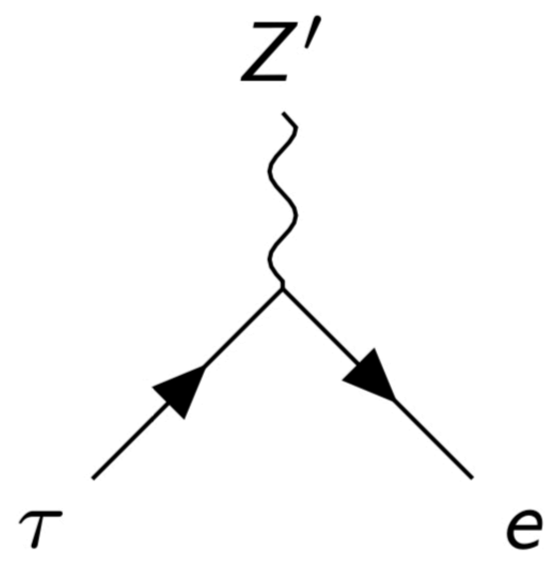
where $\mathcal{O}^{[8]}$ is due to the dimension six correction to the EOM.

Dimension eight may be subleading

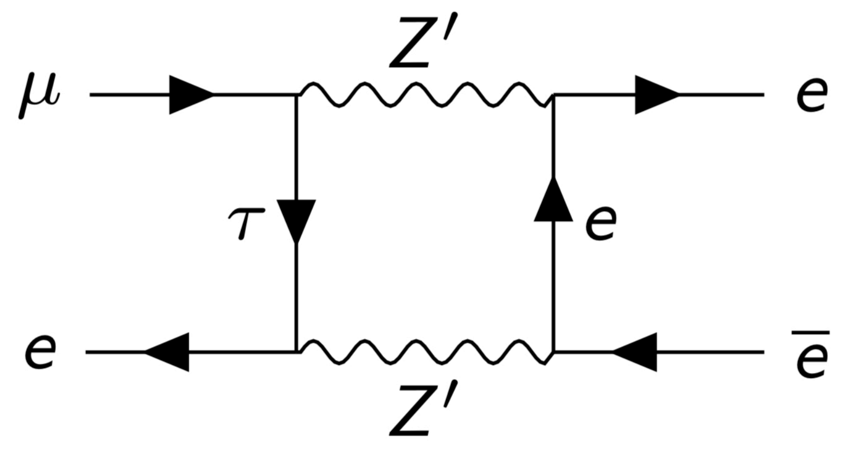
Models may generate $\mu \to e = \mu \to \tau^* \times \tau^* \to e$ amplitudes that match onto dimension six operators (as in a heavy Z' model)



$\sim g_{\tau\mu}$



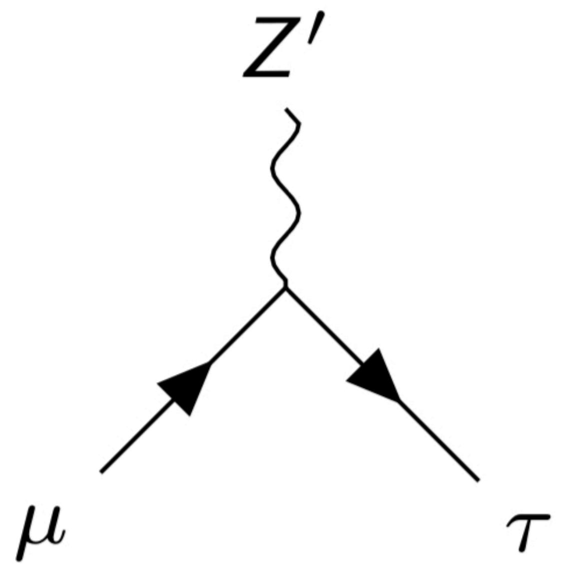
$\sim g_{e\tau}$



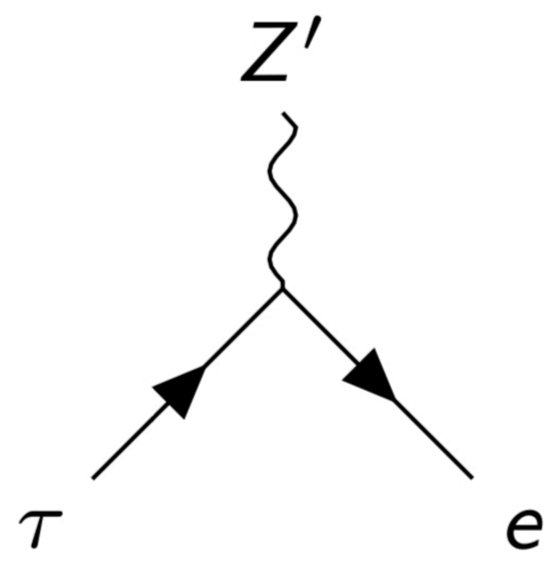
$\propto \frac{g_{e\tau} g_{\tau\mu}}{16\pi^2 M_{Z'}^2}$

Dimension eight may be subleading

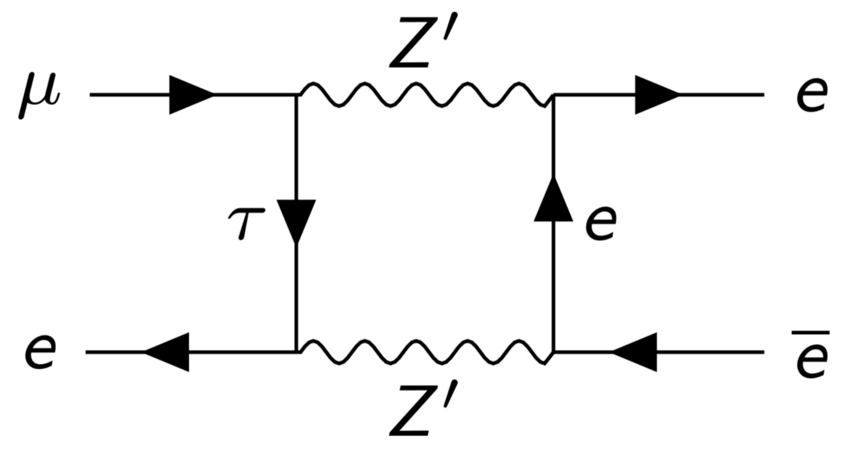
Models may generate $\mu \to e = \mu \to \tau^* \times \tau^* \to e$ amplitudes that match onto dimension six operators (as in a heavy Z' model)



$\sim g_{\tau\mu}$

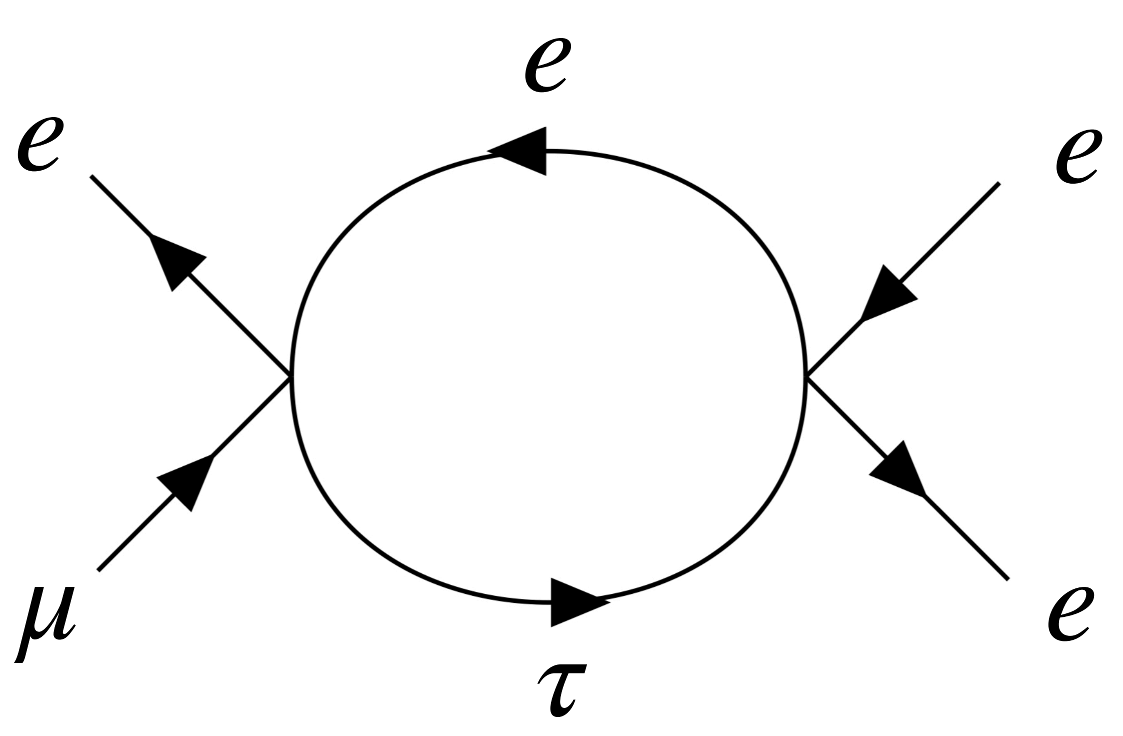


$\sim g_{e\tau}$



$\propto \frac{g_{e\tau} g_{\tau\mu}}{16\pi^2 M_{Z'}^2}$

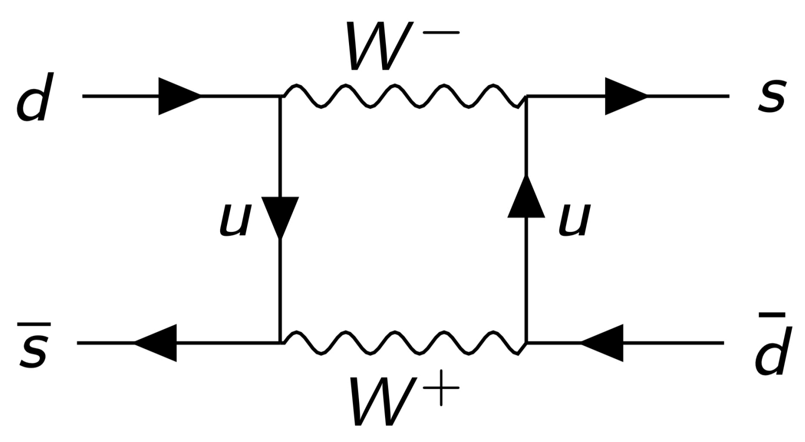
The EFT loop reproduces the (log enhanced) dimension eight contribution of the box



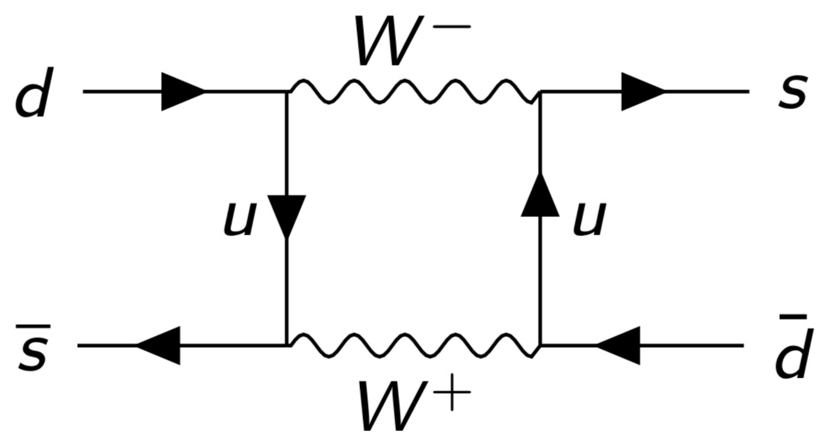
$$\mathcal{A}_{\mathrm{EFT}} \sim \frac{g_{e\tau} g_{\tau\mu}}{16\pi^2 M_{Z'}^2} \frac{m_\tau^2}{M_{Z'}^2} \log\left(\frac{m_\tau^2}{m_{Z'}^2}\right)$$

Dimension eight may be subleading (but sometimes is not)

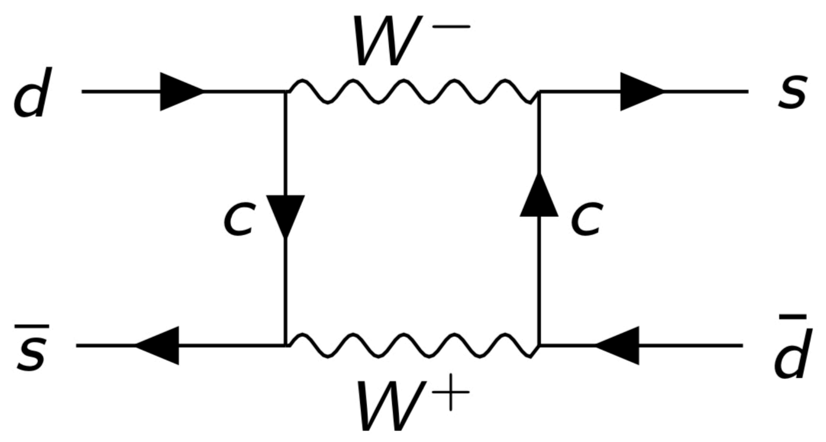
Kaon mixing in the SM is a well known example where the dimension eight amplitude is the leading contribution (GIM)



$\sim \frac{1}{M_W^2}$ but

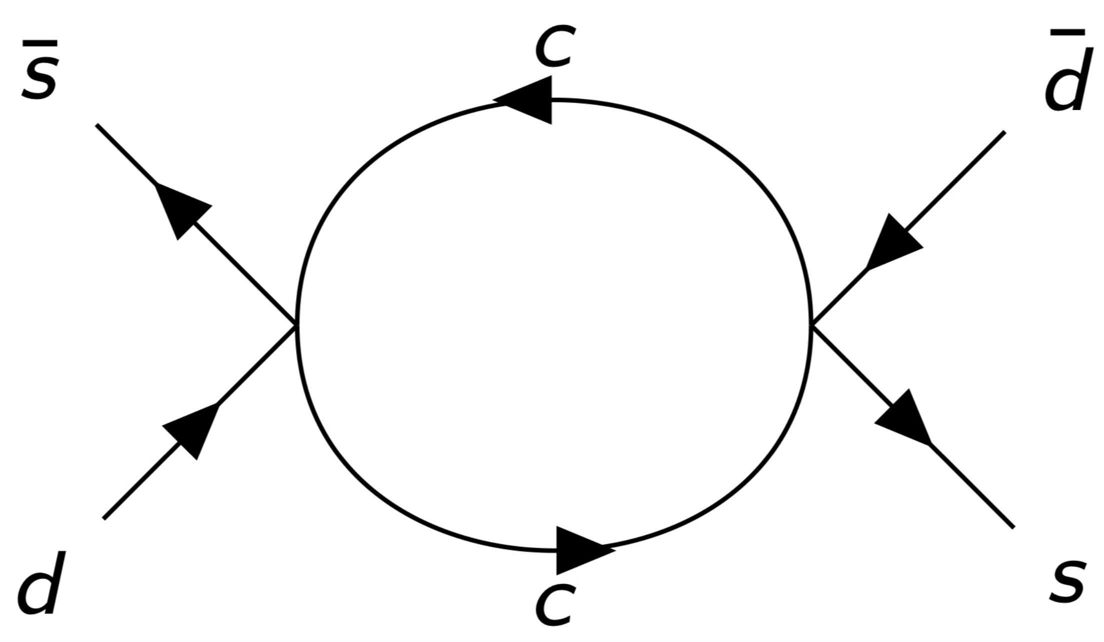


$+$



$\sim G_F^2 m_c^2$ *

The EFT reproduces the log-enhanced theory result



$\sim G_F^2 m_c^2 \log\left(\frac{m_c^2}{M_W^2}\right)$

*The diagram with the top is effectively a dimension six contribution $\propto G_F^2 m_t^2$, but is suppressed by the small CKM mixing $(V_{td}V_{ts}^*)^2$