WINDOW CONTRIBUTIONS TO a_μ & THE LATTICE-DISPERSIVE HVP DISCREPANCY

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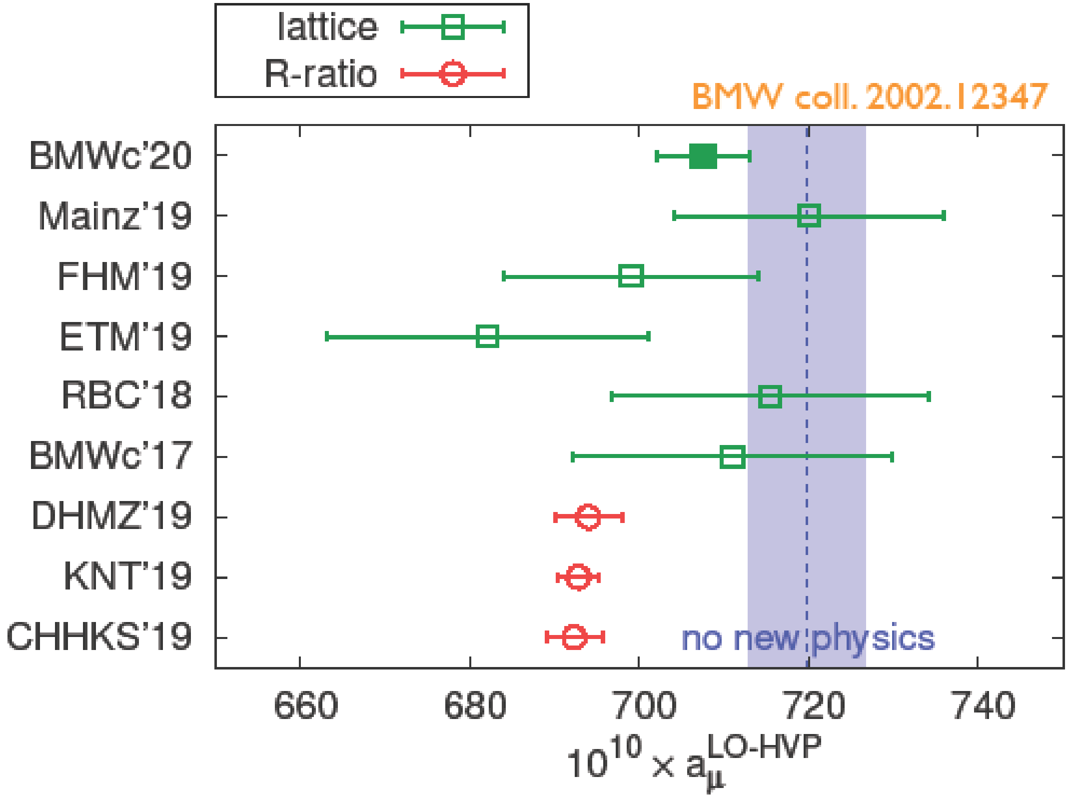
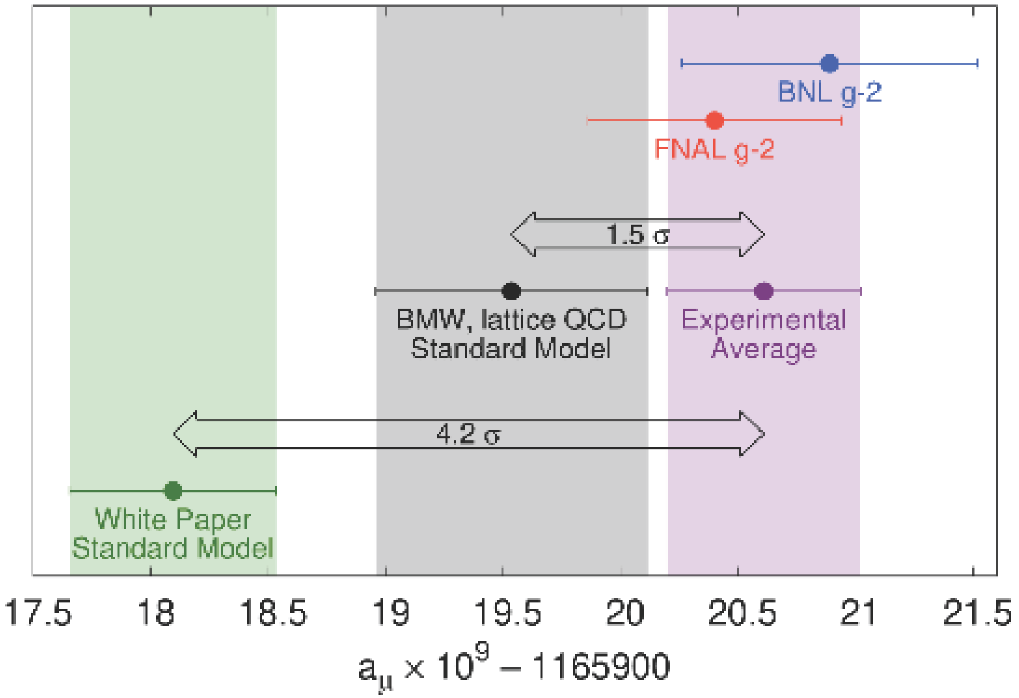
with Genessa Benton, Diogo Boito, Maarten Golterman, Alex Keshavarzi and Santi Peris

DB, MG, KM, SP, PRD105 (2022) 093003 [2203.05070 [hep-ph]]
DB, MG, KM, SP, PRD107 (2023) 034512 [2210.13677 [hep-ph]]
DB, MG, KM, SP, PRD107 (2023) 074001 [2211.11055 [hep-ph]]
GB, DB, MG, AK, KM, SP, PRLxxx (2023) [2306.16808 [hep-ph]]
GB, DB, MG, AK, KM, SP, arXiv: 2311.09523 [hep-ph]

CONTEXT: CURRENT FORMS OF THE LATTICE-DISPERSIVE HVP DISCREPANCY

SM expectations for a_μ with dispersive vs lattice HVP



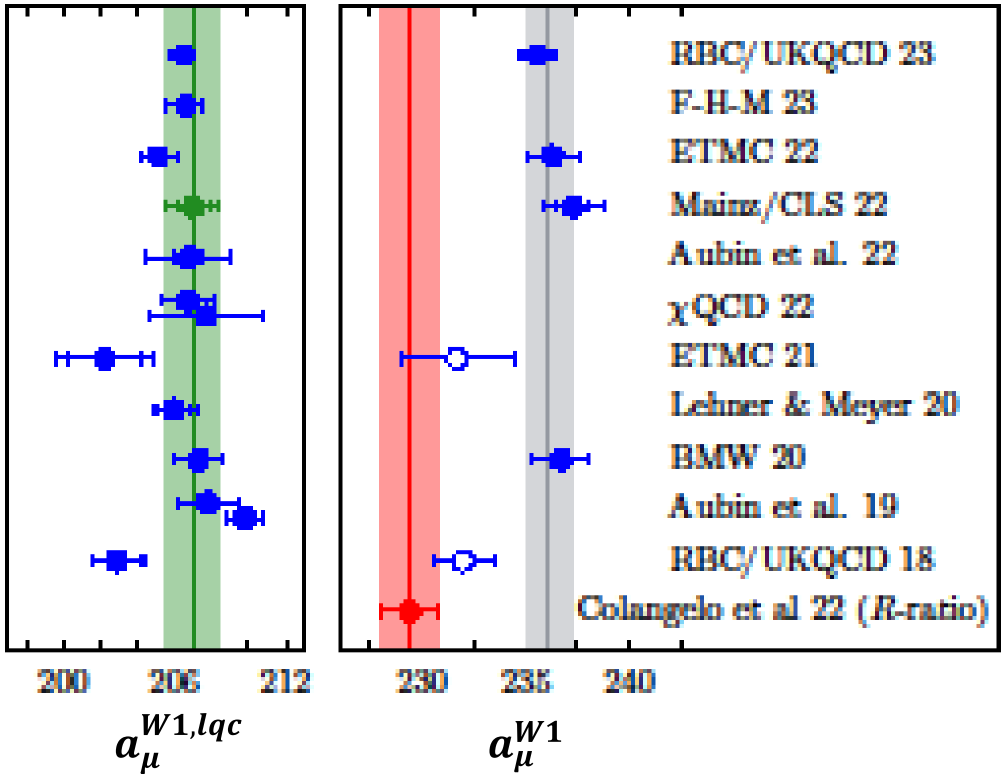


Post-FNAL Run2/3: 4.2 σ→ 5.0 σ

CONTEXT: RBC/UKQCD intermediate window (W1) quantities (x 10^{10})

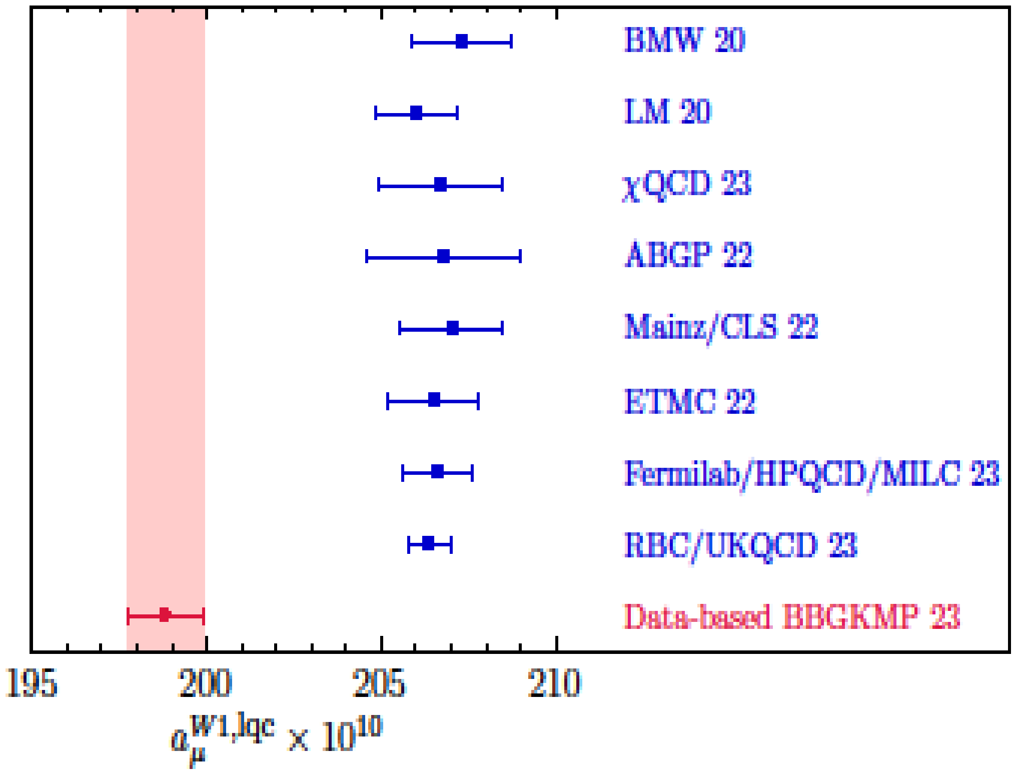
- The full intermediate window a_μ^{W1}

H. Wittig, EW Moriond 2306.04165



- IL, lqc intermediate window $a_\mu^{W1,lqc}$

GB, DB, MG, AK, KM, SP [PRLxxx [2306.16808] and below]



NOTATION, CONVENTIONS, FLAVOR DECOMPOSITIONS

$$\left(q_\mu q_\nu - q^2 g_{\mu\nu}\right)\Pi_{EM}(q^2) = i\int d^4x\, e^{iq\cdot x}\langle O|T\left(J_\mu^{EM}(x)J_\nu^{EM}(0)\right)|O\rangle$$

$$\hat{\Pi}_{EM}(q^2) \equiv \Pi_{EM}(q^2) - \Pi_{EM}(0) = q^2\int_{m_{\pi^0}^2}^{\infty} ds \frac{\rho_{EM}(s)}{s(s-q^2+i\epsilon)} \qquad \rho_{EM}(s) \equiv \frac{1}{\pi} Im\,\Pi_{EM}(s)$$

SU(3)$_F$ decompositions

$$\begin{aligned} J_\mu^{EM} &= V_\mu^3 + \frac{1}{\sqrt{3}}V_\mu^8 \equiv J_\mu^{EM,3} + J_\mu^{EM,8} + \cdots \\ &= \frac{1}{2}\left(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d\right) + \frac{1}{6}\left(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d - 2\bar{s}\gamma_\mu s\right) + \cdots \end{aligned}$$

I=1, G-parity + I=0, G-parity -

$$\begin{aligned} \hat{\Pi}_{EM}(Q^2) &= \hat{\Pi}_{EM}^{33}(Q^2) + \frac{2}{\sqrt{3}}\hat{\Pi}_{EM}^{38}(Q^2) + \frac{1}{3}\hat{\Pi}_{EM}^{88}(Q^2) \\ &= \hat{\Pi}_{EM}^{I=1}(Q^2) + \hat{\Pi}_{EM}^{MI}(Q^2) + \hat{\Pi}_{EM}^{I=0}(Q^2) \end{aligned}$$

+ *similarly for* $\rho_{EM}(s)$

$$a_\mu^X = a_\mu^{X,33} + \frac{2}{\sqrt{3}}a_\mu^{X,38} + \frac{1}{3}a_\mu^{X,88} \equiv a_\mu^{X,I=1} + a_\mu^{X,MI} + a_\mu^{X,I=0}$$

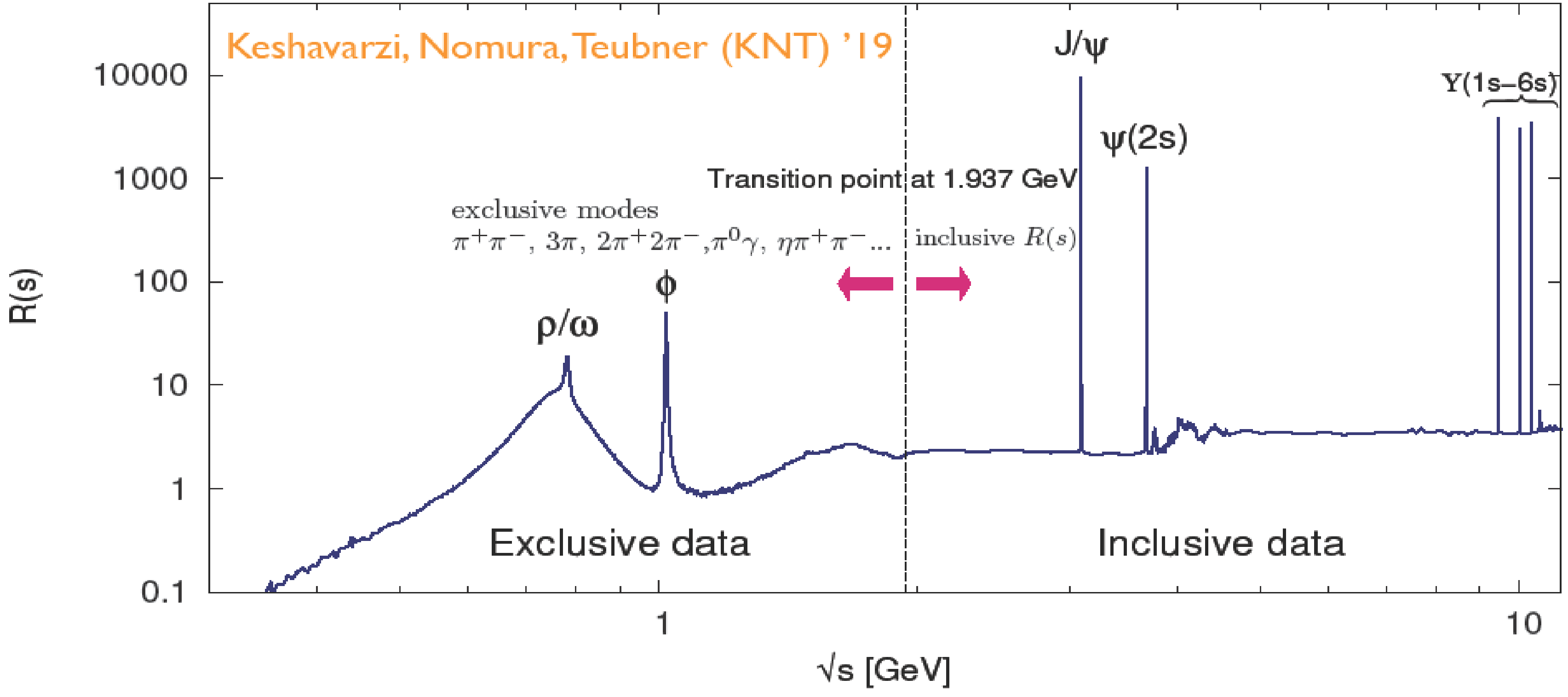
NOTATION and CONVENTIONS: a_μ^{HVP} DISPERSIVE REPRESENTATION

$$a_\mu^{\mathrm{HVP}} = \frac{4\alpha^2 m_\mu^2}{3}\int_{m_\pi^2}^{\infty} ds\, \frac{\hat{K}(s)}{s^2}\rho_{\mathrm{EM}}(s)$$

$$\rho_{\mathrm{EM}}(s) = \frac{1}{12\pi^2}R(s)$$

Compilation of R-ratio data

(see also Davier et al '19, Jegerlehner '16)



NOTATION and CONVENTIONS: a_μ LATTICE REPRESENTATION

$$C(t) = \frac{1}{3}\sum_{i=1}^{3}\int d^3x \langle j_i^{\rm EM}(\vec{x},t) j_i^{\rm EM}(0)\rangle = \frac{1}{2}\int_{m_\pi^2}^{\infty} ds\,\sqrt{s}\,e^{-\sqrt{s}t}\,\rho_{\rm EM}(s) \quad (t>0)$$

Bernecker and Meyer '11

$$\hat{\Pi}(Q^2) = \int_0^\infty dt \left(\frac{4\sin^2(Qt/2)}{Q^2} - t^2\right) C(t)$$

Leading order contribution to $a_\mu^{\rm HVP}$

$$a_\mu^{\rm HVP} = 2\int_0^\infty dt\, w(t) C(t)$$

$$\frac{\hat{K}(s)}{s^2} = \frac{3\sqrt{s}}{4\alpha^2 m_\mu^2}\int_0^\infty dt\, w(t)\, e^{-\sqrt{s}t}$$

Light flavor lattice contributions

- isospin limit (IL) light-quark (u, d) connected (lqc): **I=0 and 1**
- IL strange-quark connected (sconn) and uds disconnected (disc): **I=0 only**
- EM (connected and disconnected): **all of I=0, I=1 and MI**
- strong isospin-breaking (SIB) (connected and disconnected): **to O(m_d-m_u): MI only**

ISOSPIN LIMIT RELATIONS

$$\rho_{EM}^{IL,lqc} = \frac{10}{9}\rho_{EM}^{I=1}$$

$$\hat{\Pi}_{EM}^{IL,lqc} = \frac{10}{9}\hat{\Pi}_{EM}^{I=1}$$

$$\rho_{EM}^{sconn+disc} = \rho_{EM}^{I=0} - \frac{1}{9}\rho_{EM}^{I=1}$$

$$\hat{\Pi}_{EM}^{sconn+disc} = \hat{\Pi}_{EM}^{I=0} - \frac{1}{9}\hat{\Pi}_{EM}^{I=1}$$

$$a_{\mu}^{IL,lqc} = \frac{10}{9}a_{\mu}^{I=1}$$

$$a_{\mu}^{sconn+disc} = a_{\mu}^{I=0} - \frac{1}{9}a_{\mu}^{I=1}$$

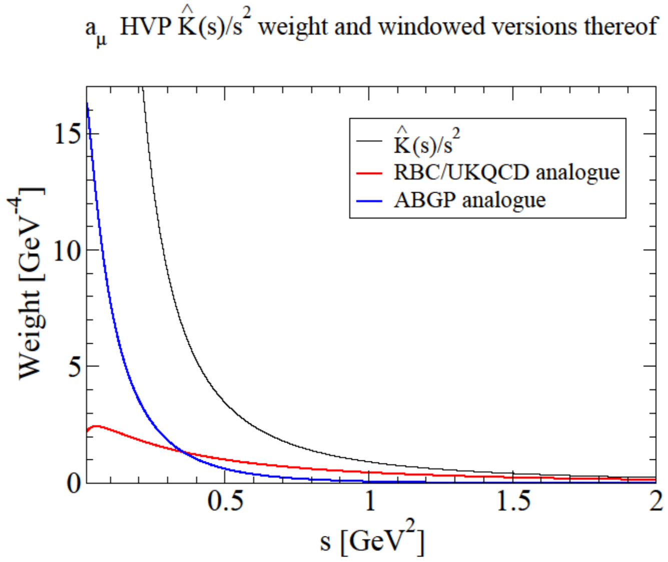
with similar representations for re-weighted (windowed) IL, lqc and sconn+disc integrals

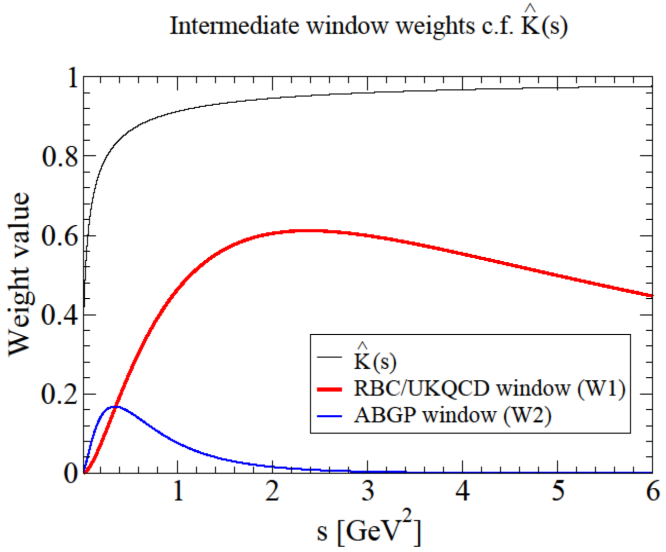
LATTICE-MOTIVATED INTERMEDIATE WINDOW QUANTITIES

- **Reduce lattice errors by cutting out short- and long-t contributions**
- RBC/UKQCD style intermediate-window reweighting:

$$a_\mu^W = 2\int_0^\infty dt\, f_W(t;\, t_0,\, t_1,\, \Delta)\,[w(t)C(t)] \qquad f_W(t;\, t_0,\, t_1,\, \Delta)) = \frac{1}{2}\left[tanh\left(\frac{t-t_0}{\Delta}\right) - tanh\left(\frac{t-t_1}{\Delta}\right)\right]$$

- **RBC/UKQCD (W1): t_0 = 0.4 fm, t_1 = 1.0 fm, Δ=0.15 fm**
- **ABGP (W2): t_0 = 1.5 fm, t_1 = 1.9 fm, Δ=0.15 fm**





"WINDOWS" EMPHASIZING DIFFERENT REGIONS IN s

- **Examples of motivation for focusing on specific regions in s:**
  - BaBar/KLOE/CMD-3 ππ data discrepancies in dominant ρ peak region
  - EM/τ+CVC discrepancies for 4π contributions at higher s

- **Key question: can this be done without blowing up errors on the corresponding lattice quantities? Answer: Yes: exponential-weight sum rules**
  - Appropriately chosen/tuned linear combinations of exponentials in Euclidean t
  - Tuning to focus of desired region of s
  - $\{t_k\}$ chosen to avoid large Euclidean t and control lattice errors

EXPONENTIAL WEIGHT SUM RULES (EWSRs)

$$\text{For } w_n(E) = \sum_{j=1}^{n} x_j E^2 e^{-Et_j}, \quad C(t) = \int_{E_{th}}^{\infty} dE\, E^2 e^{-Et} \rho(E^2), \quad t > 0$$

Hansen, Lupo, Tantalo 2019
Hashimoto, Ishikawa 2020

Exponential-weight sum rules

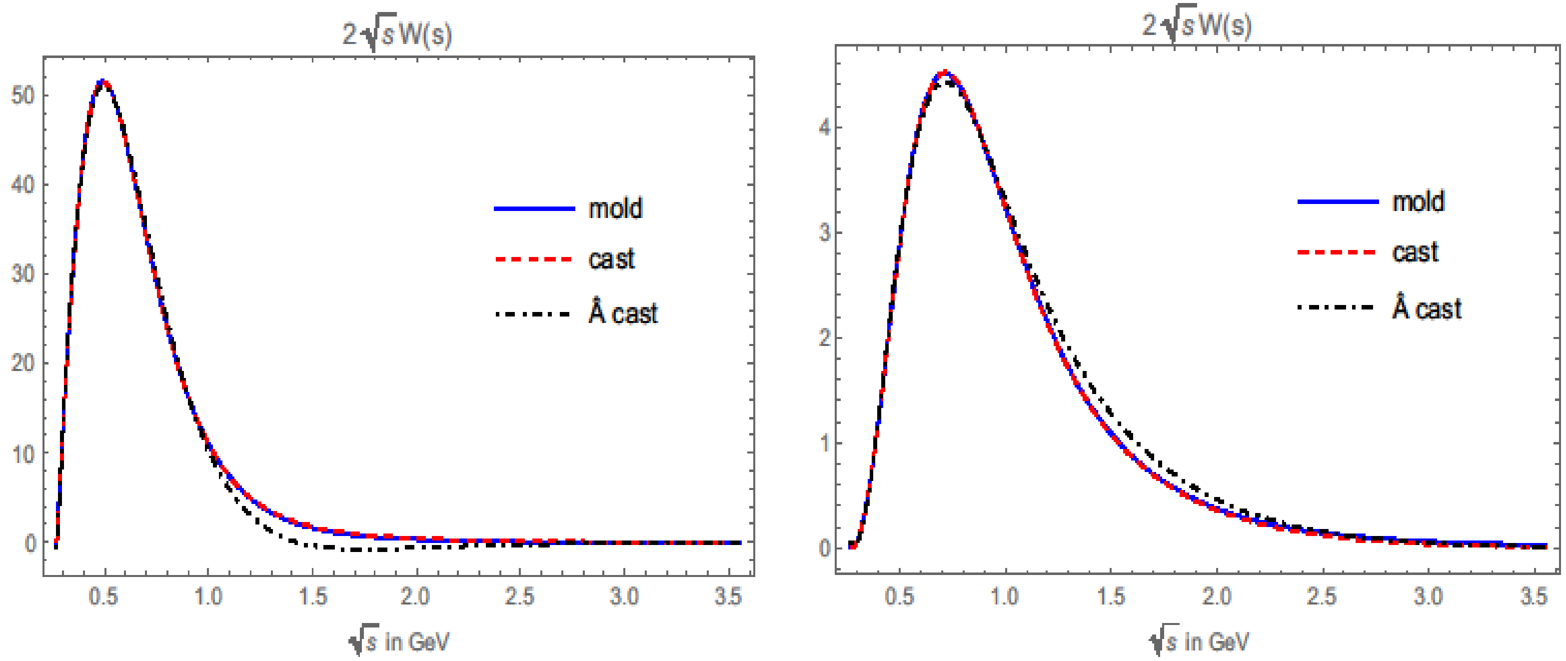
$$\Rightarrow \int_{E_{\rm th}}^{\infty} dE\, w_n(E)\rho(E) = \sum_{j=1}^{n} x_j C(t_j)$$

R(s) data ***lattice***

Basic analysis strategy *[Boito, Golterman, KM, Peris, 2022]*

- Pick a physically interesting w(s)=2E W(E) **(the "mold")**
- Build an approximation, w_n (E; $\{x_j\}$, $\{t_j\}$) **(the "cast")** using a small number of externally chosen $\{t_j\}$ and appropriately adjusted $\{x_j\}$
- Choose $\{t_j\}$ to keep all $C(t_j)$ errors small (avoiding large t by construction)
- Provided the localization-in-s of the "cast" is similar to that of the "mold", throw away the mold and work instead with the exact cast sum rule

“IMPROVED” VS “UNIMPROVED” EWSR CASTS



- “Unimproved” casts from strict HLT implementation; “improved” from reduced fit condition number construction
- “Improved” casts (slightly) less similar to the “molds”
- However, still rather close to those “molds”; hence provide essentially the same localization-in-s as the original “molds” or unimproved “casts”

RESULTS OF THE IMPROVED EWSR "CAST" CONSTRUCTION

R(s) data — rational weight — W' — exponential weight

	R-ratio	rel. error	lattice	rel. error	lattice	rel. error
W_{15}	0.4756(16)	0.3%	0.468(26)	5.6%	0.496(17)	3.4%
W_{25}	0.08912(34)	0.4%	0.0838(33)	3.9%	0.0798(18)	2.3%

Lattice errors: ABGP22 IL, lqc only
avoid comparing central values

improved exponential weights

	lattice	rel. error
$\widehat{W}_{15}$	0.4669(68)	1.5%
$\widehat{W}_{25}$	0.0824(10)	1.2%

DB, Golterman, Maltman, and Peris, '22

- Significant reduction in lattice errors (though still ~3-5 larger than dispersive)
- Further improvement possible (e.g., through modified $\{t_j\}$ set/range choices) but best explored in conjunction with detailed error studies with new lattice data
- Further reduction in lattice errors also expected c.f. ABGP22

PART II: IL, lqc AND sconn+disc RESULTS

- **Data input for the dispersive side of IL, lqc and sconn+disc sum rules**
  - Need s-dependent exclusive-mode input: here, from KNT19 (to E_{CM}=1.937 GeV)
  - G-parity I=0, 1 separation for G-parity eigenstate exclusive modes
  - External input for dominant G-parity-ambiguous modes (I=1 $K\bar{K}$ from BaBar 2018 $\tau\to K\bar{K}\upsilon_\tau$+CVC, BaBar 2007 Dalitz plot I=0/1 separation for $K\bar{K}\pi$, PDG ρ, ω, φ EM decay constants and $\pi^0\gamma$, ηγ widths for $\pi^0\gamma$, ηγ I=0/1/MI separation)
  - Maximally conservative anti-correlated 50±50% split if no external input
  - pQCD(+DVs) for inclusive contributions (E_{CM}>1.937 GeV)
  - Small inclusive I=0, 1 EM IB corrections from lattice
  - Data-based corrections for MI EM+SIB contamination of nominally I=0, 1 sums
    - dominant ρ-ω region 2π, 3π: Hoferichter et al. 2022/23 (dispersive)
    - other nominally I=0, 1 modes: O(1%) additional uncertainty estimate
- **Here: new IL, lqc and sconn+disc results, including windows**

ANATOMY OF THE DATA-BASED DETERMINATION OF $a_\mu^{HVP,lqc}=a_\mu^{lqc,IL}$

- **With KNT19 input:**

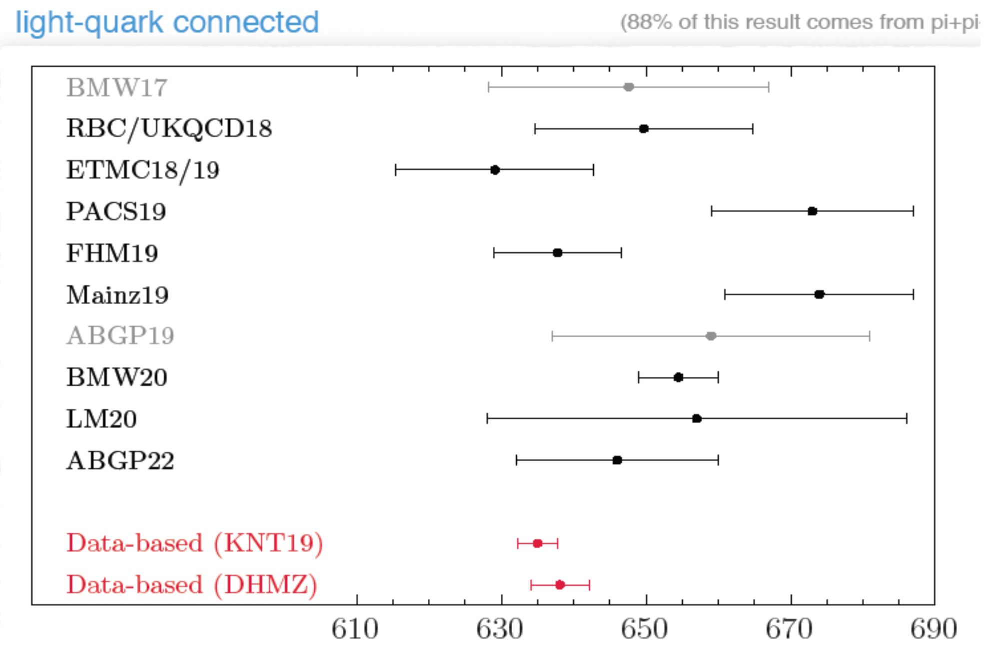
$$a_\mu^{lqc,IL} = \left(\frac{10}{9}\right)\left(543.2(2.1) + 3.2(1.0) + 28.27(2) + 0.26(12)\right) + 0.93(59) - 4.21(47)$$

G-par +	*G-par ambig*	*pQCD*	*DVs*	*EM IB*	*MI IB*
95%	0.56%	5.0%		0.15%	-0.66%

Colangelo, Hoferichter, Kubis, Stoffer 2022 (CHKS22)

- **Full $a_\mu^{lqc,IL}$ x 10^{10} results:**

$a_\mu^{lqc,IL}$ = 635.2(2.7) (KNT)
$a_\mu^{lqc,IL}$ = 638.3(4.1) (DHMZ)



- **Some tension with lattice (BMW)**

RBC/UKQCD INTERMEDIATE WINDOW IL,lqc RESULTS

G=+ mode X	$a_{\mu,X}^{W1}$ x 10^{10}
low-s $\pi^+\pi^-$	0.02(00)
$\pi^+\pi^-$	144.13(49)
$2\pi^+2\pi^-$	9.29(13)
$\pi^+\pi^-2\pi^0$	11.94(48)
$3\pi^+3\pi^-$ (no ω)	0.14(01)
$2\pi^+2\pi^-2\pi^0$ (no η)	0.83(11)
$\pi^+\pi^-4\pi^0$ (no η)	0.13(13)
$\eta\pi^+\pi^-$	0.85(03)
$\eta 2\pi^+2\pi^-$	0.05(01)
$\eta\pi^+\pi^-2\pi^0$	0.07(01)
$\omega(\to\pi^0\gamma)\pi^0$	0.53(01)
$\omega(\to$ npp$)3\pi$	0.10(02)
$\omega\eta\pi^0$	0.15(03)
TOTAL	168.24(72)

$a_\mu^{W1,lqc}$ contributions (in units of 10^{10})

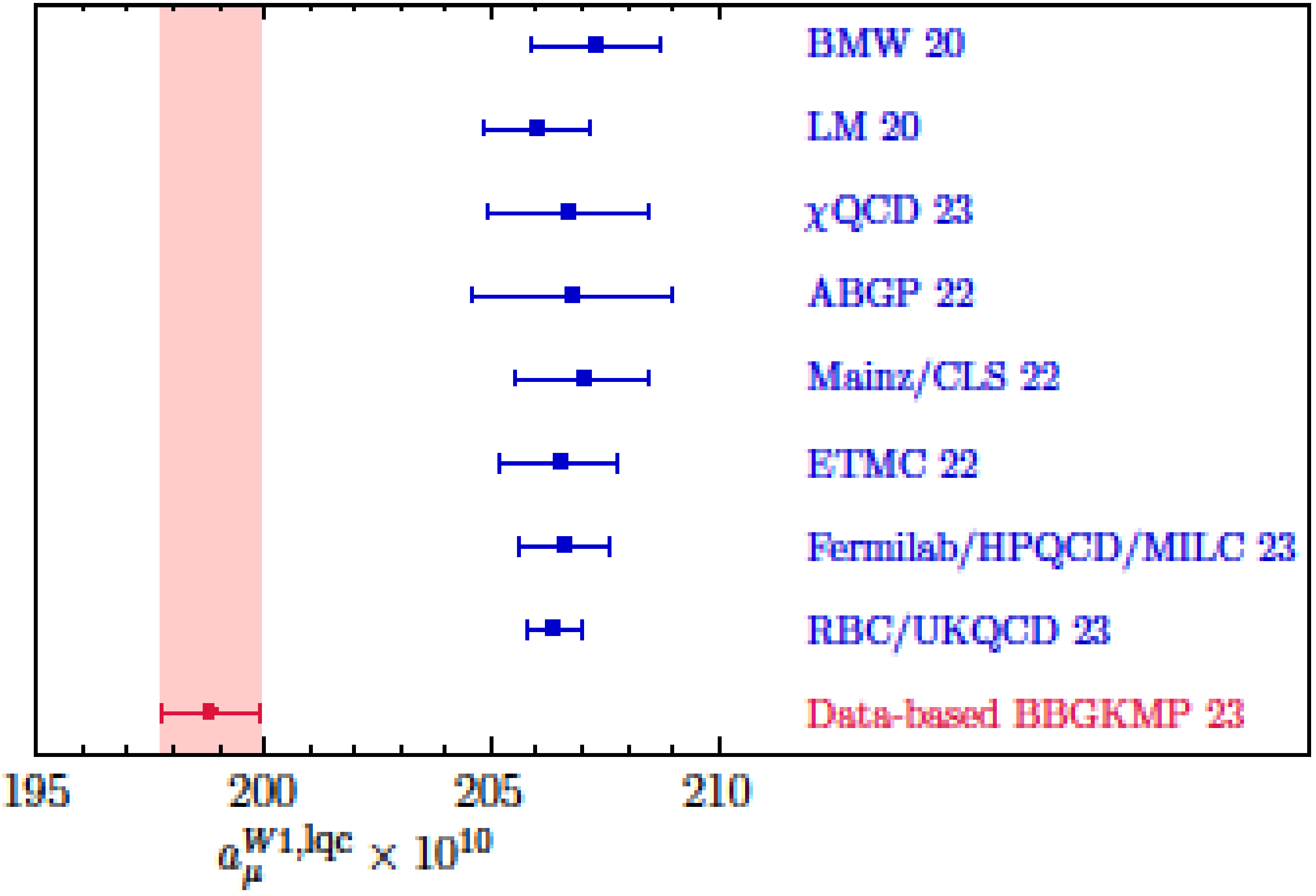
- G=+: 186.93(80)
- $K\bar{K}$: 0.58(7)
- $K\bar{K}\pi$: 0.52(9)
- $K\bar{K}\pi\pi$: 0.60(60)
- $\pi^0\gamma+\eta\gamma$: 0.14(1)
- other mixed G: 0.05(5)
- pQCD$\pm DV$s: 10.90$\pm$0.17
- EM corr'n: -0.04(6)
- 2π ($\pm$other) MI corr'n: -0.96(7)$\pm$0.29

[CHKS22 2π MI corr'n: JHEP 10 (2022) 032]

$$a_\mu^{W1,lqc} = \mathbf{198.9(1.1)}\ x\ 10^{-10}$$

G. Benton, DB, MG, AK, KM, SP [PRLxxx, 2306.16808]

RBC/UKQCD INTERMEDIATE WINDOW DISPERSIVE IL, lqc c.f. LATTICE



- **Very significant tension between dispersive and lattice W1 IL, lqc results**

IL, lqc DISPERSIVE vs LATTICE RESULTS FOR OTHER INTERMEDIATE WINDOWS

- Lattice EM results not available so neglect EM I=0, 1 corrections for now (plausible based on RBC/UKQCD intermediate window result)
- For IL, lqc case, windowed versions of CHKS22 2π MI correction (provided by M. Hoferichter and P. Stoffer: thanks!)
- For sconn+disc case, windowed ρ-ω region 3π MI correction of Hoferichter, Hoid, Kubis, Schuh (JHEP 08 (2023) 208) (HHKS23)
- **Compare IL, lqc dispersive and ABGP22-based lattice results**

ABGP22 INTERMEDIATE WINDOW (W2) RESULTS

- **RBC/UKQCD-style intermediate window, designed to be longer distance, more amenable to possible use of ChPT**

Light-quark connected from KNT19 R(s) data

$$a_\mu^{\mathrm{W2,lqc}} = 93.75(36) \times 10^{-10}$$

lattice results

Aubin, Blum, Golterman, Peris '22

$$a_\mu^{\mathrm{W2,lqc}} = 102.1(2.4) \times 10^{-10}$$

Fermilab/HPQCD/MILC '23

$$a_\mu^{W2,\mathrm{lqc}} = 100.7(3.2) \times 10^{-10}$$

- Once more, dispersive-lattice tension, but reduction in lattice errors needed to sharpen the dispersive-lattice comparison

IMPROVED EWSR WEIGHT ($\widehat{W}_{15}$, $\widehat{W}_{25}$) IL, lqc RESULTS

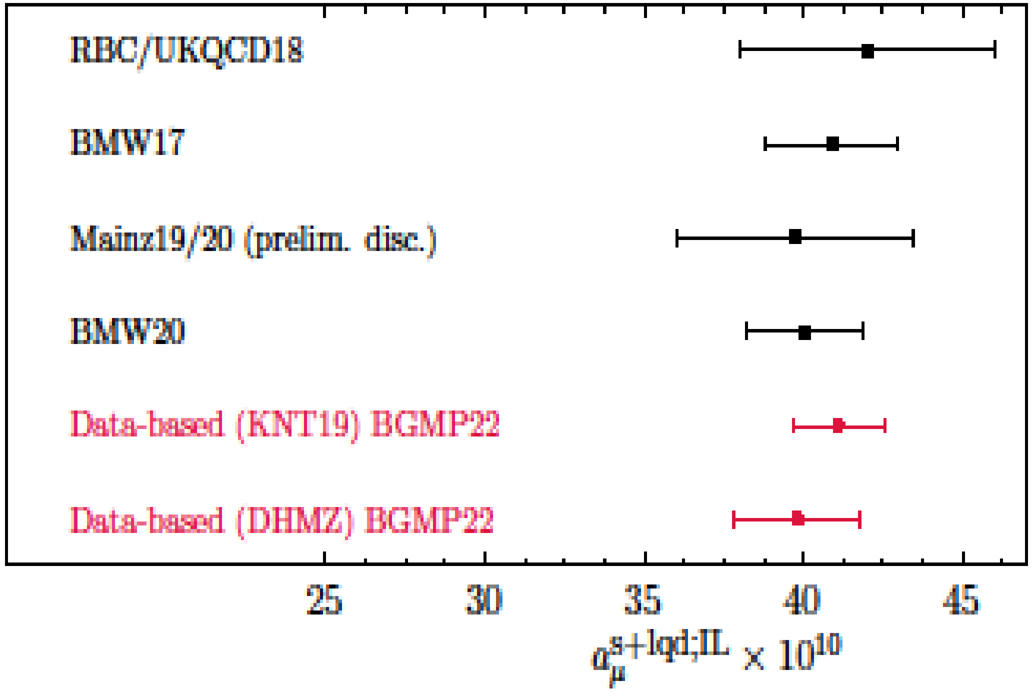
- $I_W^{lqc} \equiv \int_{th}^{\infty} ds\, \mathrm{W(s)}\, \rho_{EM}^{IL,lqc}\mathrm{(s)}$

Dispersive		ABGP Lattice
$I_{\widehat{W}_{15}}^{IL,lqc}$ = 42.80(16) x 10^{-2}	⟺ 5.6 σ	$I_{\widehat{W}_{15}}^{IL,lqc}$ = 46.69(68) x 10^{-2}
$I_{\widehat{W}_{25}}^{IL,lqc}$ = 78.99(45) x 10^{-3}	⟺ 3.1 σ	$I_{\widehat{W}_{25}}^{IL,lqc}$ = 82.4(1.0) x 10^{-3}
Benton, Boito, Golterman, Keshavarzi, Maltman, Peris, 2311.09523		systematic errors on lattice results still to be assessed

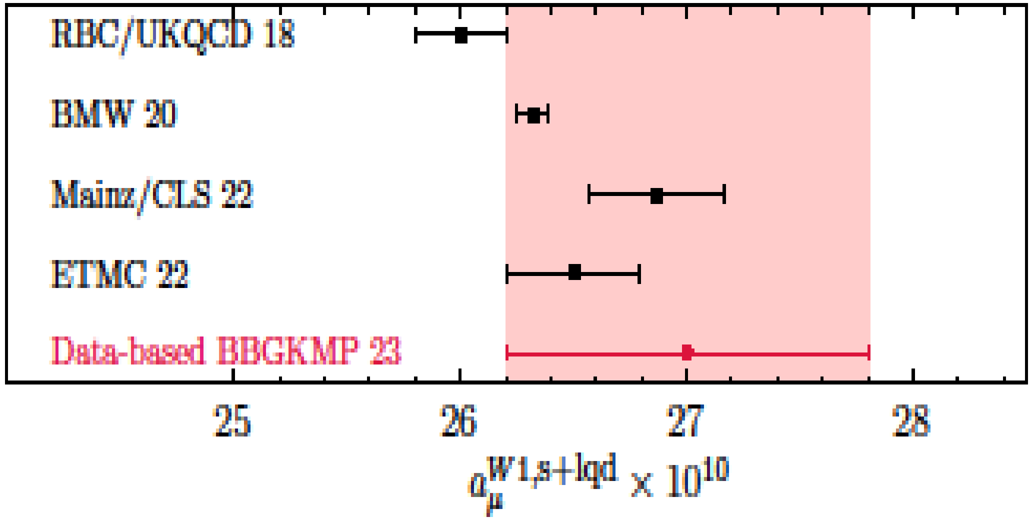
- **Another sign of tension between dispersive and lattice IL, lqc results**

sconn+disc $a_\mu^{sconn+disc} = a_\mu^{s+lqd,IL}$, $a_\mu^{W1,s+lqd}$ results

- Update of PRD105 (2022) 093003 $a_\mu^{s+lqd,IL}$



- RBC/UKQCD window version $a_\mu^{W1,s+lqd}$



POTENTIAL IMPACT OF THE NEW CMD-3 ππ DATA

- **Interim conclusion:** current published cross sections lead to
  - good lattice/data-driven agreement for sconn+disc quantities
  - significant disagreement (data-driven<lattice) for lqc quantities
  - **⇒ lattice vs. data-driven discrepancy essentially all from lqc contribution**

- **However, new CMD-3 study [2302.8834 [hep-ex]] finds ρ peak region ππ cross sections significantly higher than CMD-2, BaBar, KLOE, BESIII, SND**

  - Still undergoing scrutiny, but no obvious problems to date
  - Replacing ππ HVP in CMD-3 region with CMD-3 only reduces SM-expt'l a_μ discrepancy to 0.9 σ [CMD-3 2309.12910 [hep-ex] and many others]
  - **What about the impact on the KNT19-based lqc discrepancies above?**

CAUTION: PRELIMINARY EXPLORATION ONLY

i.e., NOT a new KNT-style 2023 2π combination

A. Keshavarzi Lattice 2023: Impact on $a_\mu^{W1,lqc}$ and a_μ^{HVP}

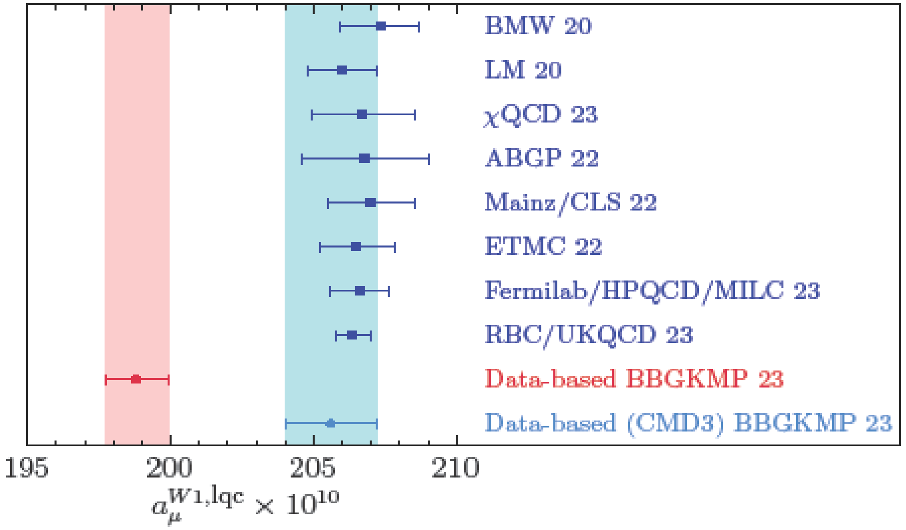
The University of Manchester

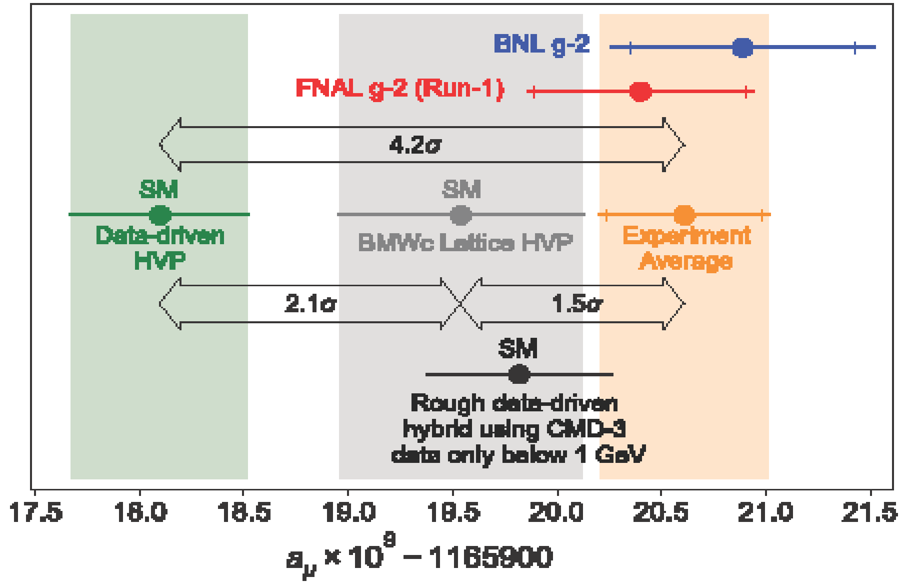
Impact of CMD-3

CMD-3 [F. Ignatov et al, arXiv:2302.08834]

DISCLAIMER: these are <u>NOT</u> new updates or combinations including the CMD-3 data – simply demonstrations of the impact of the CMD-3 data alone.

In collaboration with Genessa Benton, Diogo Boito, Maarten Golterman, Kim Maltman & Santi Peris [arXiv:2306.16808].





IMPORTANT: THIS PLOT IS VERY ROUGH!

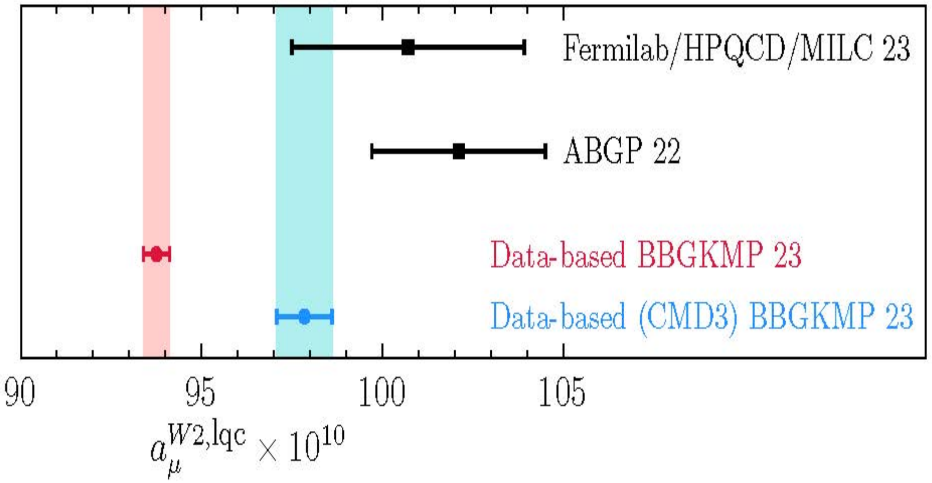
- TI White Paper result has been substituted by CMD-3 only for 0.33 → 1.0 GeV.
- The NLO HVP has not been updated.
- It is purely for demonstration purposes → <u>should not be taken as final!</u>

Until differences are understood, and intense scrutiny of new/old results is complete, no conclusions can be drawn about the validity of SM estimates. A lot of work still to be done…

With CMD-3: 205.6(1.6) x 10^{-10}

Impact on the IL, lqc W2, $\widehat{W}_{15}$ and $\widehat{W}_{25}$ results

- KNT19 $a_\mu^{W2,lqc}$



- KNT19 $I_{\widehat{W}_{15}}^{IL,lqc}$: 0.4280(16)→0.4483(37) c.f. lattice 0.4669(58) (5.6 σ→2.7 σ)
- KNT19 $I_{\widehat{W}2_5}^{IL,lqc}$: 0.0790(5)→0.0815(6) c.f. lattice 0.0824(10) (3.1 σ→0.8 σ)

- **All dispersive-lattice differences strongly reduced with CMD-3 ππ input**

CONCLUSIONS

- With current EM R(s) data, significant data-driven/lattice discrepancies, especially for IL, lqc RBC/UKQCD intermediate window (W1) and improved EWSR ($\hat{W}_{15}$, $\hat{W}_{25}$) weightings

- Currently known lattice/data-driven discrepancies all disappear if CMD-3 ππ cross-section results correct [resolution of CMD-3/BaBar/KLOE discrepancy of high interest]

- (Improved) EWSRs as a potential approach for exploring potential discrepancies in different regions of s (in this talk, focusing on the ρ peak region, but alternatives focusing on other regions of interest as well)

- C(t) results needed to determine a_μ^{HVP} and a_μ^{W1} and/or components thereof also provide results for the lattice side of any related EWSR: further exploration of EWSR weight choices in conjunction with new lattice data thus also of interest

BACKUP SLIDES

HLT IMPLEMENTATION OF THE EWSR "CAST" CONSTRUCTION

Given the mold function, minimize $\int_{E_{\text{th}}}^{\infty} dE \left| w_n(E; \{t_j\}, \{x_j\})/E^2 - 2W(E^2)/E \right|^2$

which has the solution

$$x_i = \sum_{j=1}^{n} A_{ij}^{-1} f_j \qquad \text{with} \qquad A_{ij} = \int_{E_{\text{th}}}^{\infty} dE e^{-(t_i+t_j)E}, \qquad f_i = 2\int_{E_{\text{th}}}^{\infty} dE e^{-t_i E} W(E^2)/E$$

Hansen, Lupo, Tantalo '19

For a chosen set of time values this gives the coefficients x_j

HLT IMPLEMENTATION OF THE EWSR "CAST" CONSTRUCTION

Given the mold function, minimize $\int_{E_{\rm th}}^{\infty} dE \left| w_n(E;\{t_j\},\{x_j\})/E^2 - 2W(E^2)/E \right|^2$

which has the solution

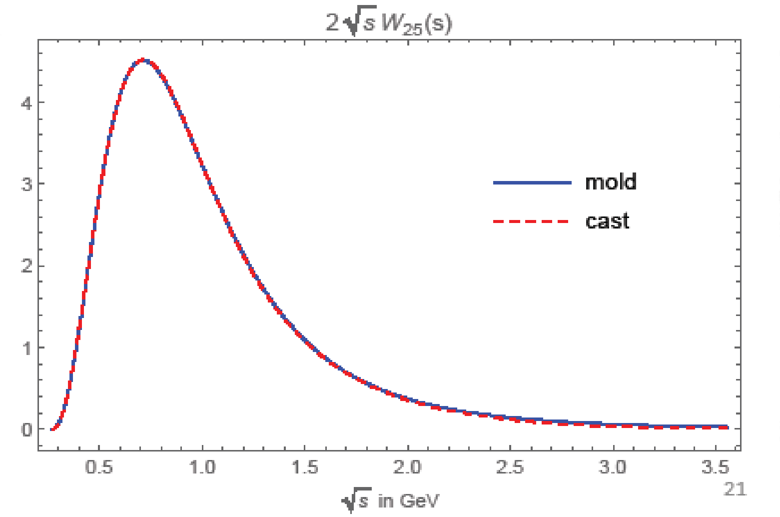
$$x_i = \sum_{j=1}^{n} A_{ij}^{-1} f_j$$ with $$A_{ij} = \int_{E_{\rm th}}^{\infty} dE e^{-(t_i+t_j)E},$$ $$f_i = 2\int_{E_{\rm th}}^{\infty} dE e^{-t_i E} W(E^2)/E$$

Hansen, Lupo, Tantalo '19

For a chosen set of time values this gives the coefficients x_j

$t_j = 3,\ 6,\ 9,\ 12,\ 15\ {\rm GeV}^{-1} \approx 0.6,\ 1.2,\ 1.8,\ 2.4,\ 3\ {\rm fm}$

$W'_{2,5}: x_1 = 34.0249,\quad x_2 = 870.640,\quad x_3 = -5501.14,$
$\quad x_4 = 9933.01,\quad x_5 = -5284.24.$



HLT IMPLEMENTATION OF THE EWSR "CAST" CONSTRUCTION

Given the mold function, minimize $\int_{E_{\rm th}}^{\infty} dE \left| w_n(E;\{t_j\},\{x_j\})/E^2 - 2W(E^2)/E \right|^2$

which has the solution

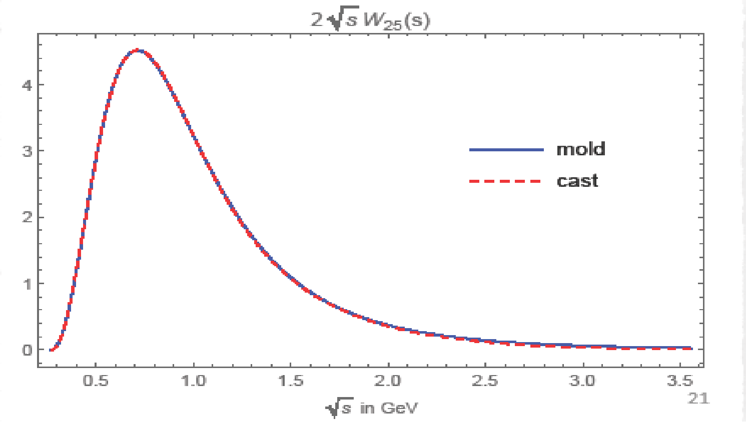
$$x_i = \sum_{j=1}^{n} A_{ij}^{-1} f_j \quad \text{with} \quad A_{ij} = \int_{E_{\rm th}}^{\infty} dE e^{-(t_i+t_j)E}, \quad f_i = 2\int_{E_{\rm th}}^{\infty} dE e^{-t_i E} W(E^2)/E$$

Hansen, Lupo, Tantalo '19

For a chosen set of time values this gives the coefficients x_j

$$t_j = 3,\ 6,\ 9,\ 12,\ 15\ \mathrm{GeV}^{-1} \approx 0.6,\ 1.2,\ 1.8,\ 2.4,\ 3\ \mathrm{fm}$$

$W'_{2,5}: x_1 = 34.0249, \quad x_2 = 870.640, \quad x_3 = -5501.14,$
$\quad x_4 = 9933.01, \quad x_5 = -5284.24.$



R(s) data | rational weight | exponential weight

	R-ratio	rel. error	lattice	rel. error	lattice	rel. error
W_{15}	0.4756(16)	0.3%	0.468(26)	5.6%	0.496(17)	3.4%
W_{25}	0.08912(34)	0.4%	0.0838(33)	3.9%	0.0798(18)	2.3%

DB, Golterman, Maltman, and Peris, '22

Lattice errors: ABGP22 IL, lqc only
avoid comparing central values

AN IMPROVED EWSR "CAST" CONSTRUCTION

- The HLT minimization of $\int_{E_{\rm th}}^{\infty} dE \left| w_n(E;\{t_j\},\{x_j\})/E^2 - 2W(E^2)/E \right|^2$

$$x_i = \sum_{j=1}^{n} A_{ij}^{-1} f_j$$ with $$A_{ij} = \int_{E_{\rm th}}^{\infty} dE e^{-(t_i+t_j)E},$$ $$f_i = 2\int_{E_{\rm th}}^{\infty} dE e^{-t_i E} W(E^2)/E$$

can be modified to remove the small eigenvalues of the matrix *A* via

$$\hat{A}(\lambda) = (1-\lambda)A + \lambda \mathbf{1}_n$$

This removes eigenvalues $< \lambda$ and reduce the range of the values of $\{x_i\}$.

$\lambda = 10^{-9}$

$$\boxed{\hat{W}_{2,5}: x_1 = 44.8916, \quad x_2 = 590.933, \quad x_3 = -3373.53, \quad x_4 = 3716.86, \quad x_5 = 879.149.}$$

significantly reduced values of x_4 and x_5

$$W'_{2,5}: x_1 = 34.0249, \quad x_2 = 870.640, \quad x_3 = -5501.14, \quad x_4 = 9933.01, \quad x_5 = -5284.24.$$

CMD-3 vs THE KNT19 COMBINATION (from A. Keshavarzi, Lattice 2023)

MANCHESTER 1824
The University of Manchester

CMD-3 compared to KNT19

In collaboration with Genessa Benton, Diogo Boito, Maarten Golterman, Kim Maltman & Santi Peris.

To be able to compare CMD-3 with KNT19 data combination:

- Data published as pion form factor, $|F_\pi|^2$.
- Must subtract vacuum polarisation effects using Fedor Ignatov's VP correction update.
- Must include final-state-radiation effects.
- Put data on fine, common binning.

In the full 2π data combination range, the KNT19 analysis found:

$$a_\mu^{\pi^+\pi^-}(0.305 \rightarrow 1.937\ \text{GeV}) = (503.46 \pm 1.91) \times 10^{-10}.$$

Replacing KNT19 2pi data in the region 0.33 $\rightarrow$ 1.20 GeV with CMD-3 data:

$$a_\mu^{\pi^+\pi^-}(0.305 \rightarrow 1.937\ \text{GeV}) = (525.17 \pm 4.18) \times 10^{-10}.$$

Neglecting possible correlations between e.g. CMD-3 and CMD-2, this results in a difference of:

$$\Delta a_\mu^{\pi^+\pi^-} = (21.71 \pm 4.96) \times 10^{-10} \rightarrow 4.4\sigma\ ,$$

This removes the experiment vs. SM Muon g-2 discrepancy.

CMD-3 [F. Ignatov et al, arXiv:2302.08834]