Neutrino Mixing Parameters and Unitarity

Tau Lepton Physics 2023

Shirley Li (UC Irvine)

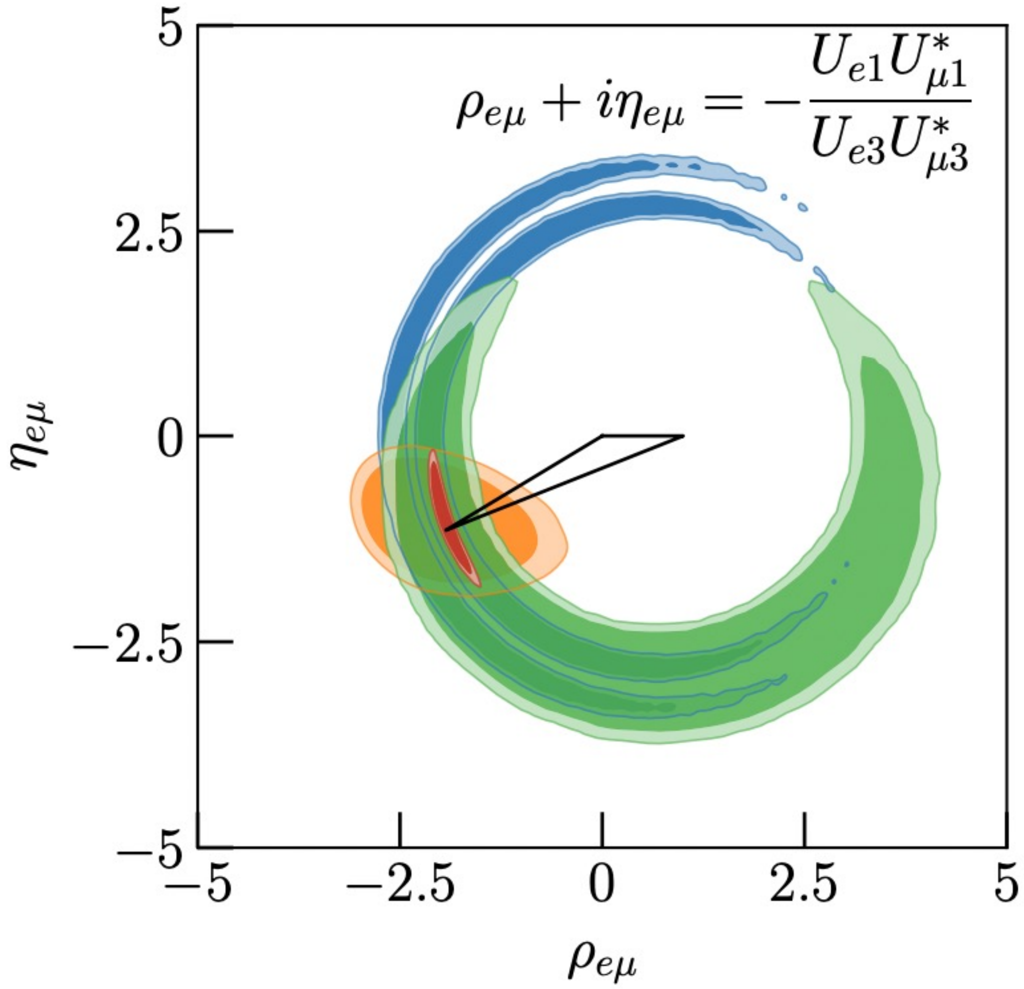


Figure credit: P. Denton

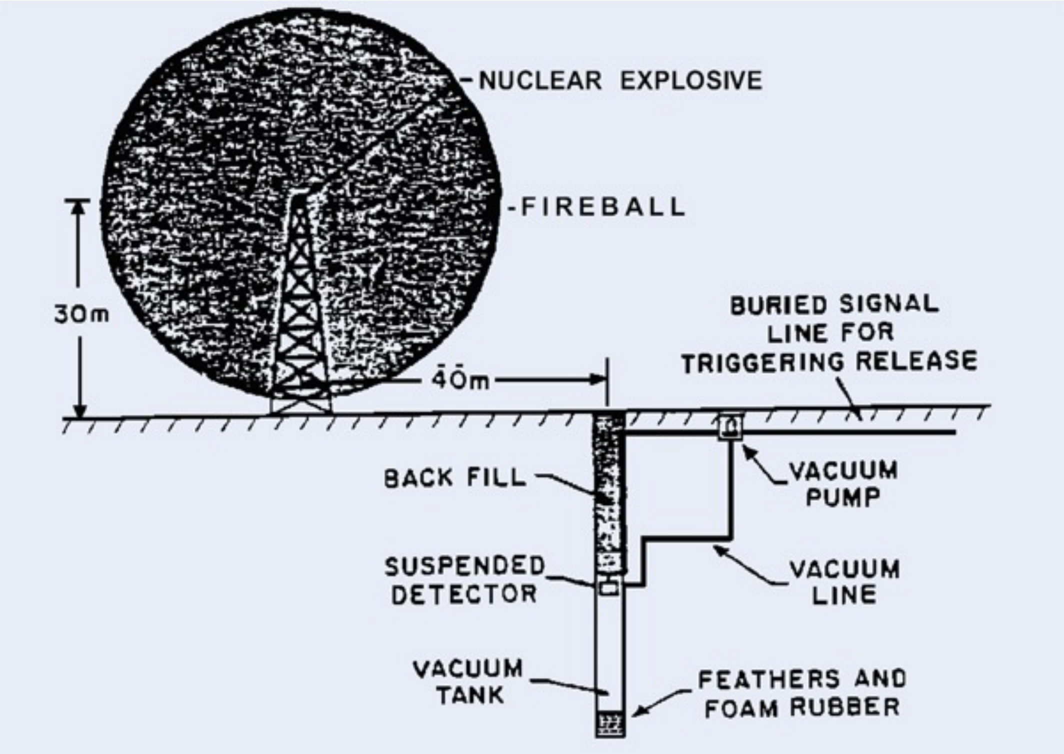
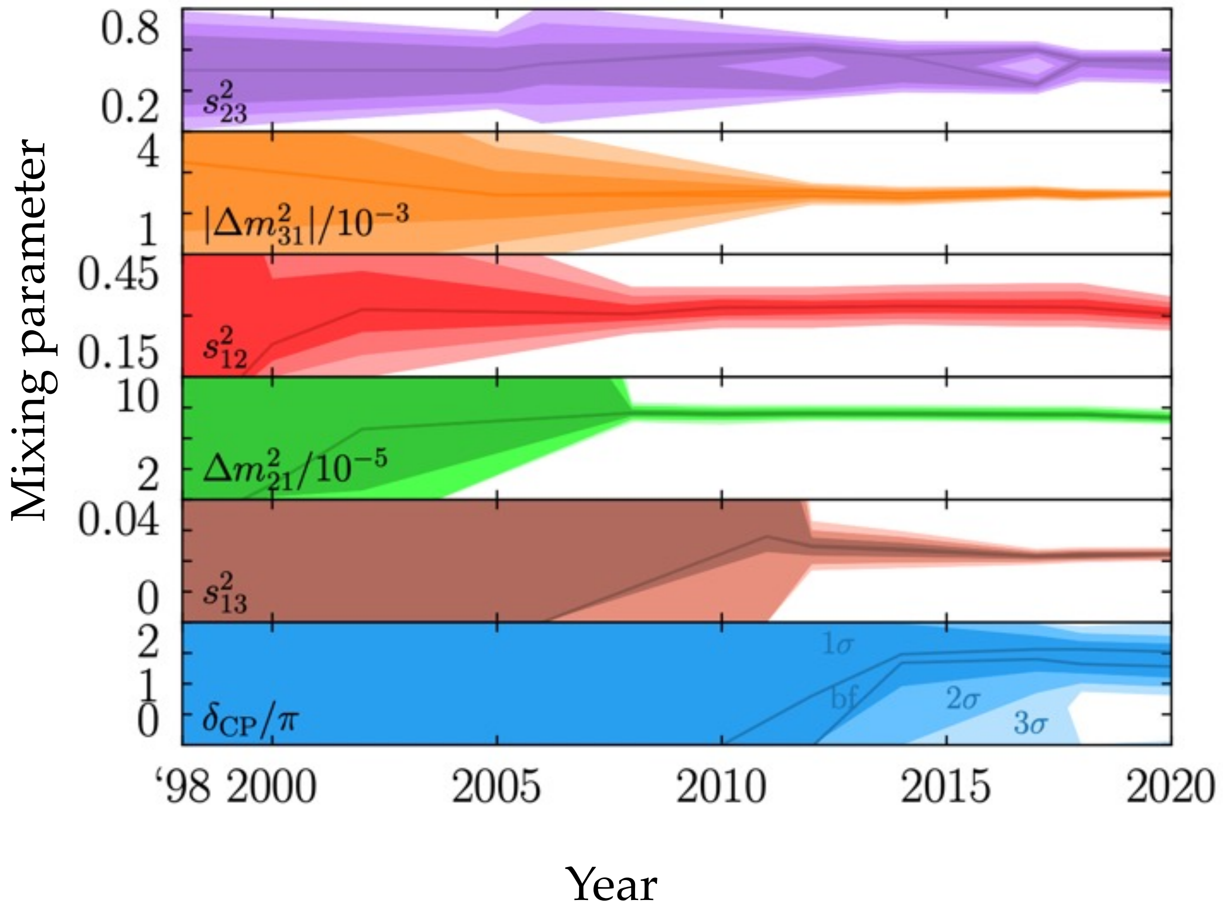


Figure credit: F. Reines



Establishing Neutrino Mixing Paradigm

Neutrino Oscillations: the Standard Paradigm

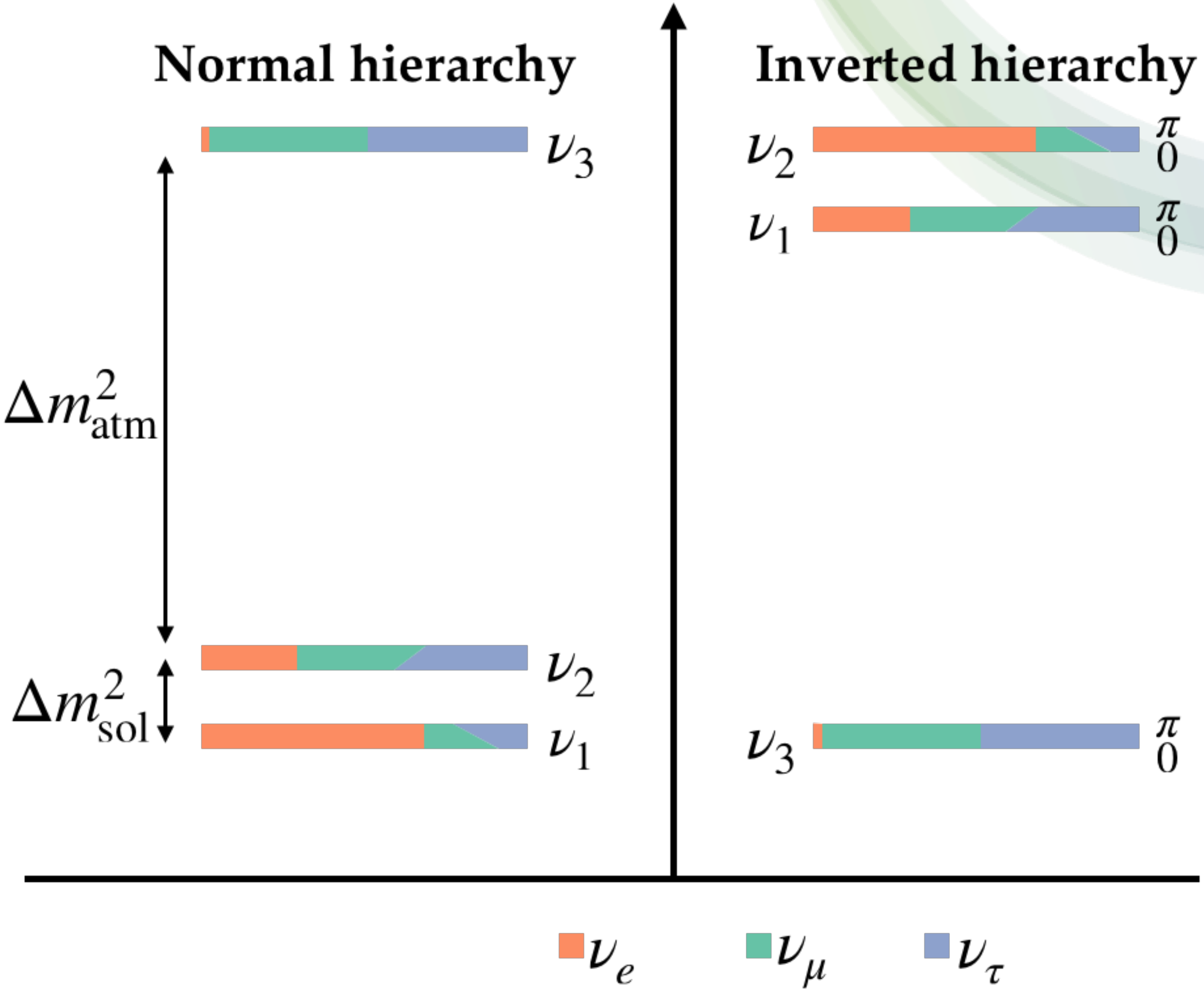
Unknown:

δ_{CP}, mass hierarchy

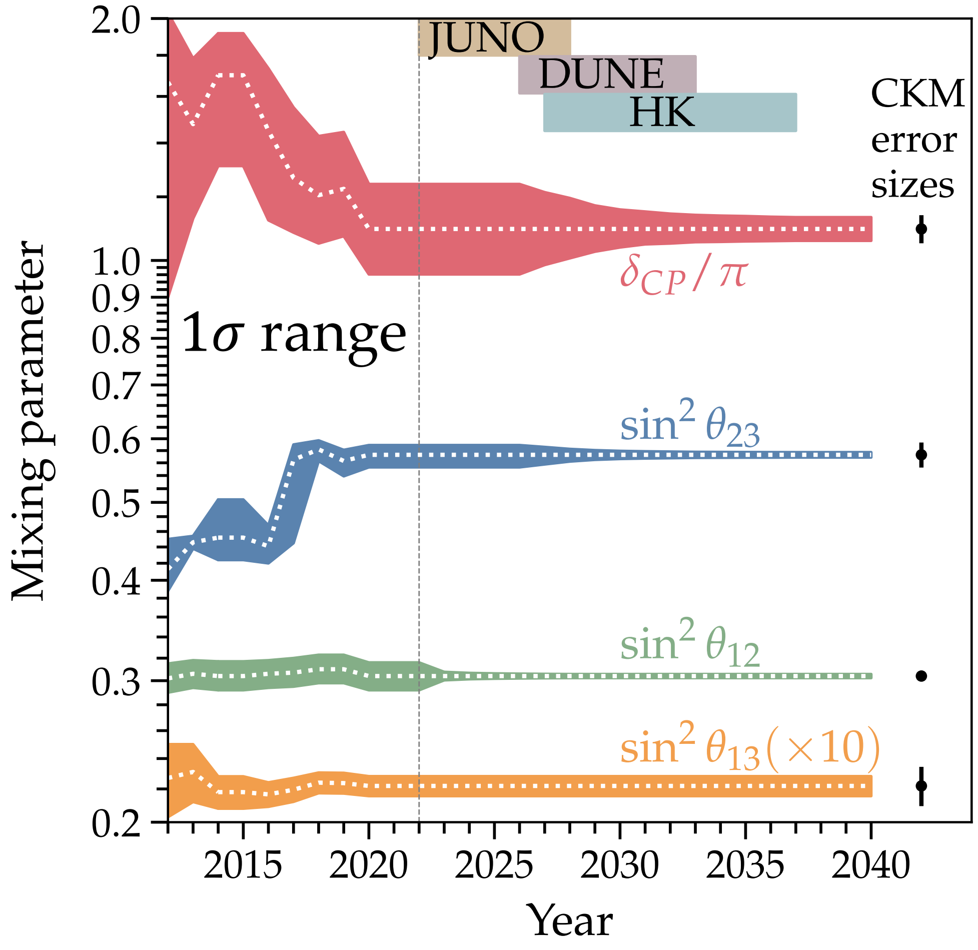
A Few %:

$\theta_{12}, \theta_{13}, \theta_{23}$,

$\Delta m^2_{21}, \Delta m^2_{32}$

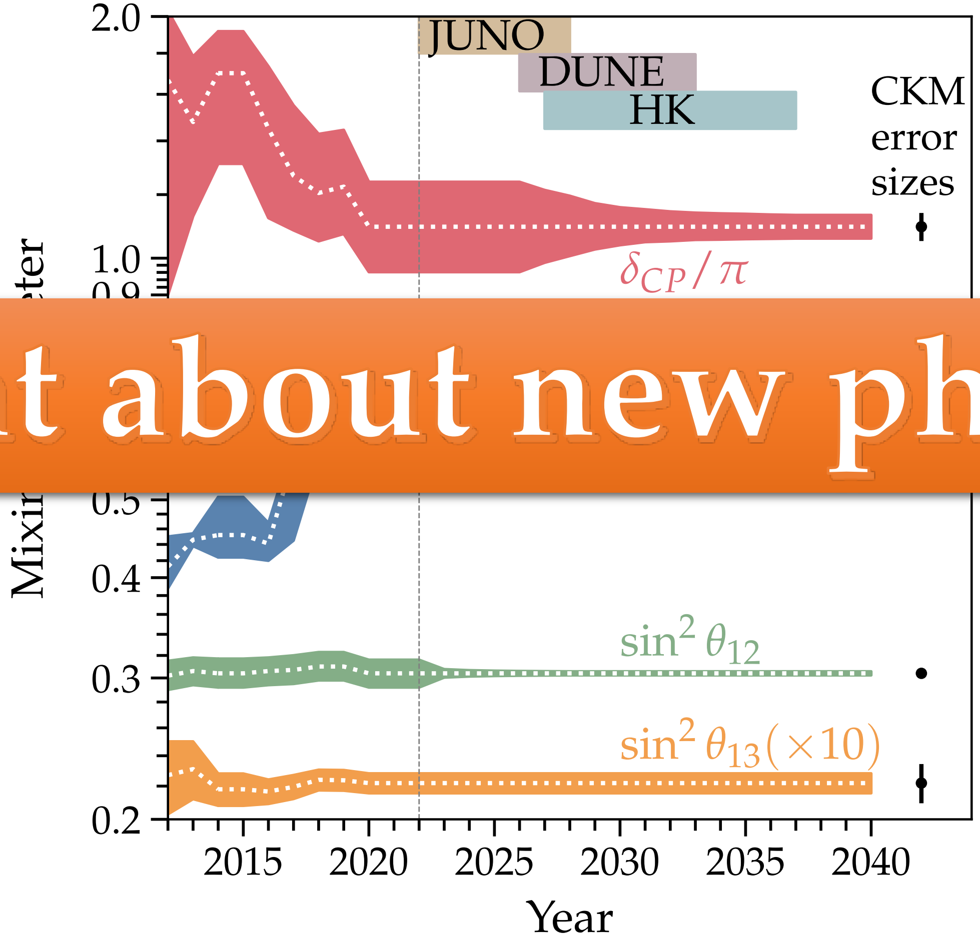


Neutrino Oscillations: the Standard Paradigm



Modified from Song *et al.*, 21

Neutrino Oscillations: the Standard Paradigm



What about new physics?

Modified from Song *et al.*, 21

Unitarity of Neutrino Mixing

Standard Model: 3 neutrinos

$$\nu_\alpha(x) = \sum_{k=1}^{3} U_{\alpha k}\nu_k(x)$$

Flavor Mass

3-flavor mixing parameterized by:

$$U_{\text{PMNS}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Atmospheric	Reactor/Interference	Solar
$\mu \to \tau$ 500 km/GeV	$\mu \leftrightarrow e$ 500 km/GeV	$e \to e$ 15000 km/GeV

Also Parke, Ross-Lonergan 16
Denton, Gehrlein 21

Unitarity of Neutrino Mixing

Standard Model: 3 neutrinos

$$\nu_\alpha(x) = \sum_{k=1}^{3} U_{\alpha k}\nu_k(x)$$

Flavor Mass

3-flavor mixing parameterized by:

Sterile neutrino searches – unitarity assumed

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & U_{e4} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} & U_{\mu 4} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} & U_{\tau 4} \\ & \dots & & \end{pmatrix} = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & c_{34} & s_{34} \\ & & -s_{34} & c_{34} \end{pmatrix} \begin{pmatrix} 1 & & & \\ & c_{24} & & s_{24}e^{-i\delta 2} \\ & & 1 & \\ & -s_{24}e^{i\delta 2} & & c_{24} \end{pmatrix} \begin{pmatrix} c_{14} & & & s_{14} \\ & 1 & & \\ & & 1 & \\ -s_{14} & & & c_{14} \end{pmatrix} U_{\text{PMNS}}$$

This Isn't the Whole Story…

Neutrinos have masses

$$L \supset c_\nu \frac{LHLH}{\Lambda} + \text{h.c.}$$

New physics at scale Λ =>
new states =>

$$(3\times3)\ U_{\alpha k} \rightarrow (n\times n)\ \mathcal{U}_{\alpha k}$$

E.g., non-standard interactions

$$L = -2\sqrt{2}G_F \sum_{f,P,\alpha,\beta} \epsilon_{\alpha,\beta}^{f,P} (\bar{\nu}_\alpha \gamma^\mu P_L \nu_\beta)(\bar{f}\gamma_\mu P f')$$

can be mapped onto non-unitarity

Blennow *et al.*, 06

How well can we test non-unitarity in neutrino sector?

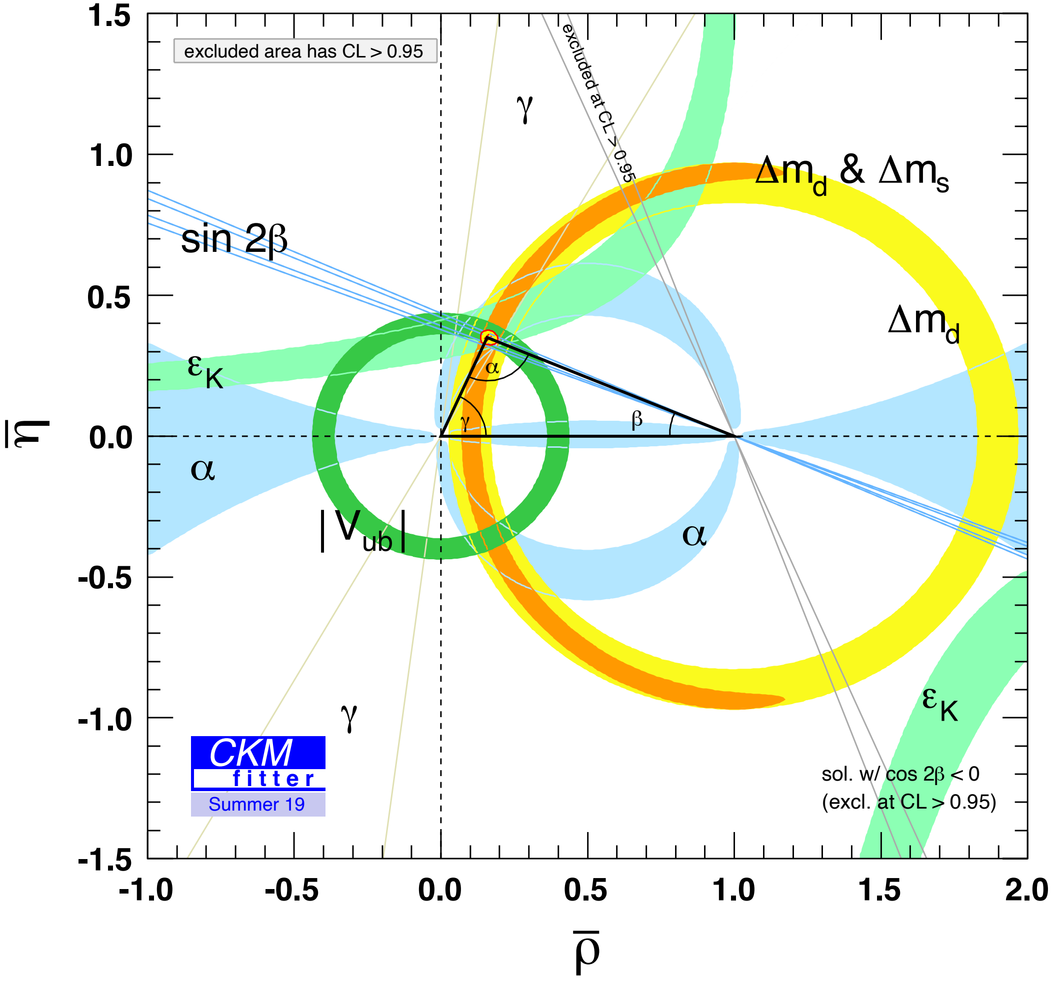
This Isn't the Whole Story…

Neutrinos have

$L \supset c_\nu \frac{LHI}{\Lambda}$

New physics at
new ν States =>

$(3\times3)\ U_{\alpha k} \rightarrow ($



andard interactions

$\epsilon^{f,P}_{\alpha,\beta}\,(\bar{\nu}_\alpha \gamma^\mu P_L \nu_\beta)(\bar{f}\gamma_\mu P f')$

α,β

ed onto non-unitarity

Blennow *et al.*, 06

How well can we test non-unitarity in neutrino sector?

What Is Unitarity?

$$U^\dagger U = UU^\dagger = \mathbb{I}$$

An over-complete set of constraints:

$$N_\alpha \equiv \sum_{k=1}^{3} |U_{\alpha k}|^2 = 1$$

$$t_{\alpha\beta} \equiv \sum_{k=1}^{3} U_{\alpha k} U_{\beta k}^* = 0$$

$$N_k \equiv \sum_{\alpha=e,\mu,\tau} |U_{\alpha k}|^2 = 1$$

$$t_{kl} \equiv \sum_{\alpha=e,\mu,\tau} U_{\alpha k} U_{\alpha l}^* = 0$$

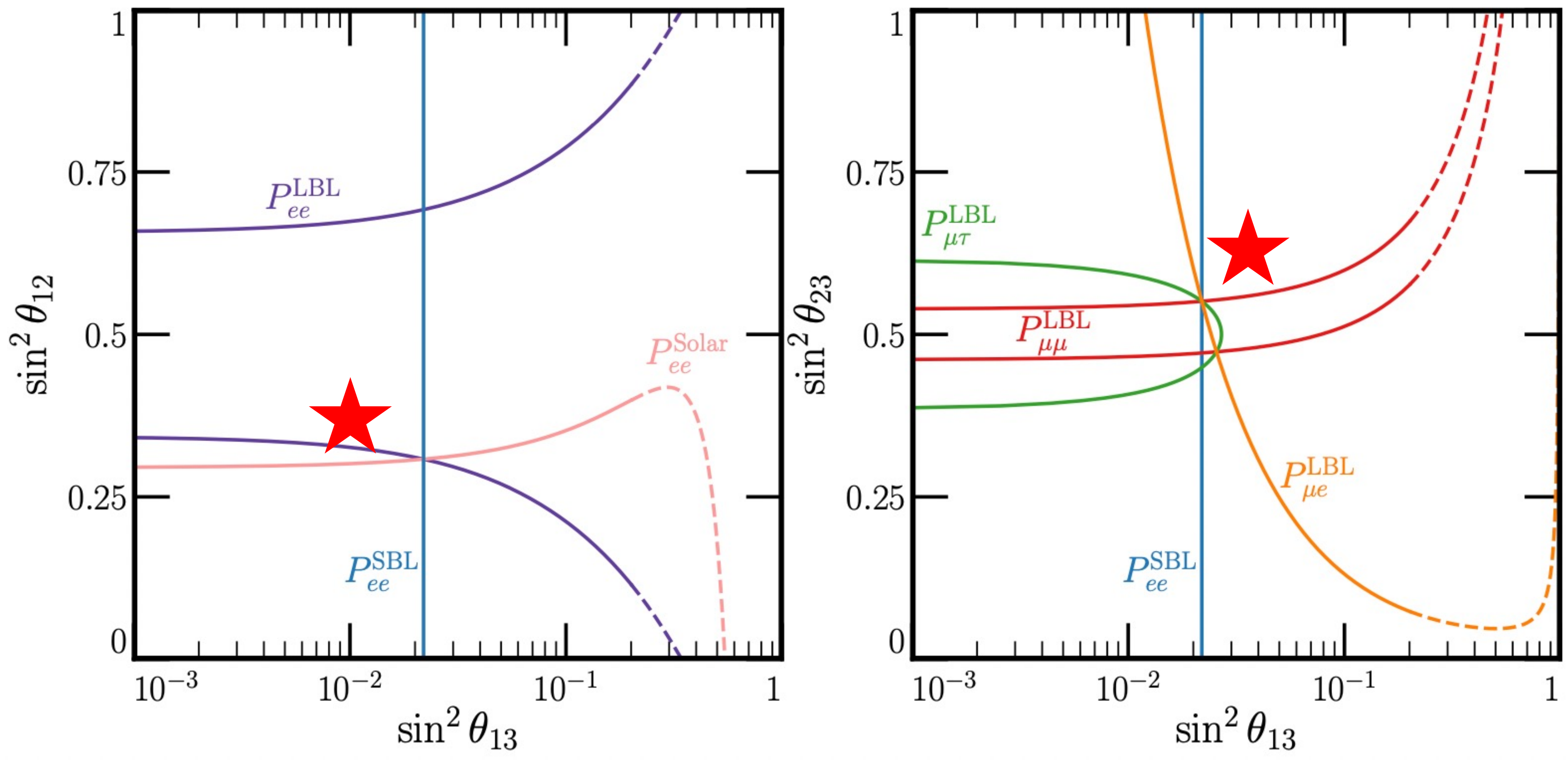
Row/column normalizations

closures

How Can We Test Normalizations?

Assuming unitarity, infinite precision

Ellis, Kelly, SL 2020a



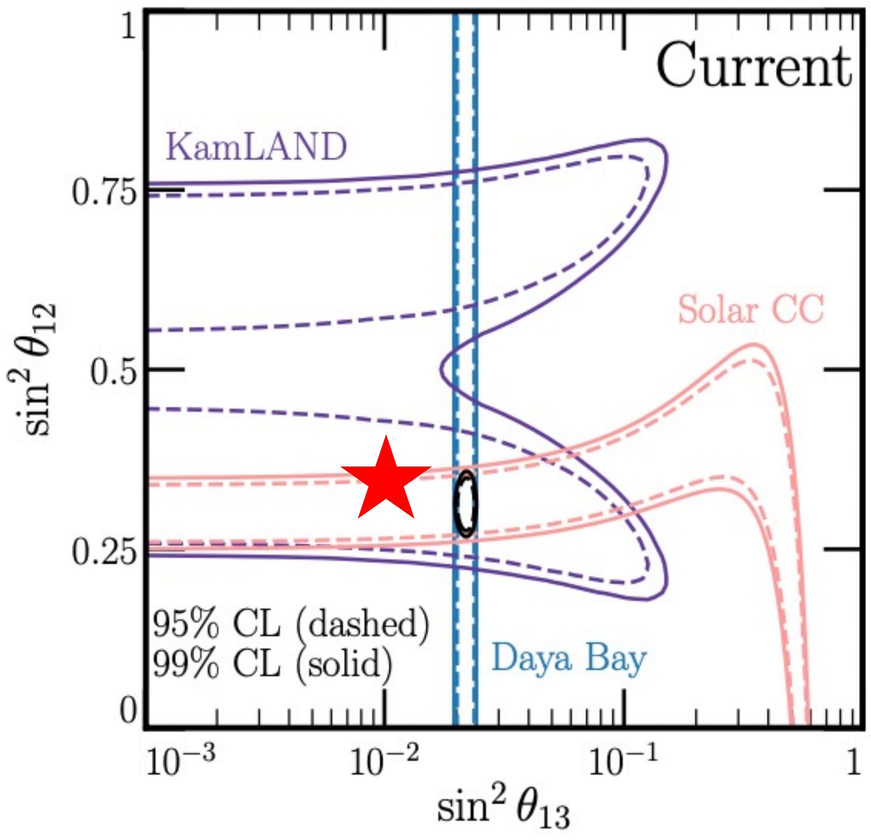
All measurements should cross the same point

How Well Can We Test Normalizations?

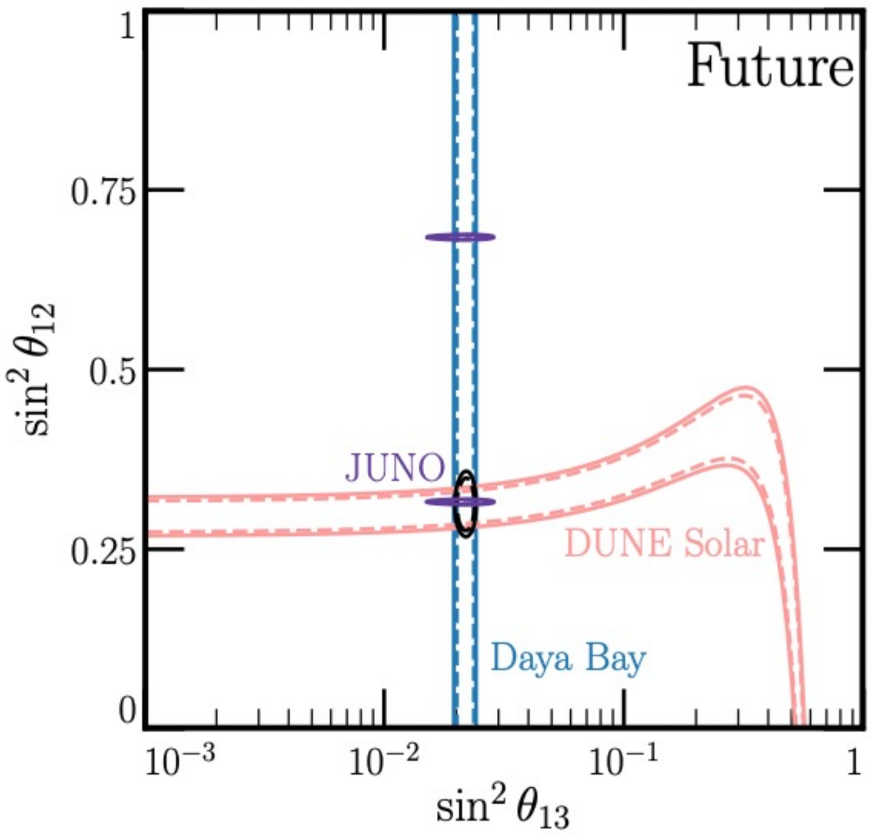
Ellis, Kelly, SL 20a

$\theta_{13} - \theta_{12}$ plane

Current precisions



Future sensitivities



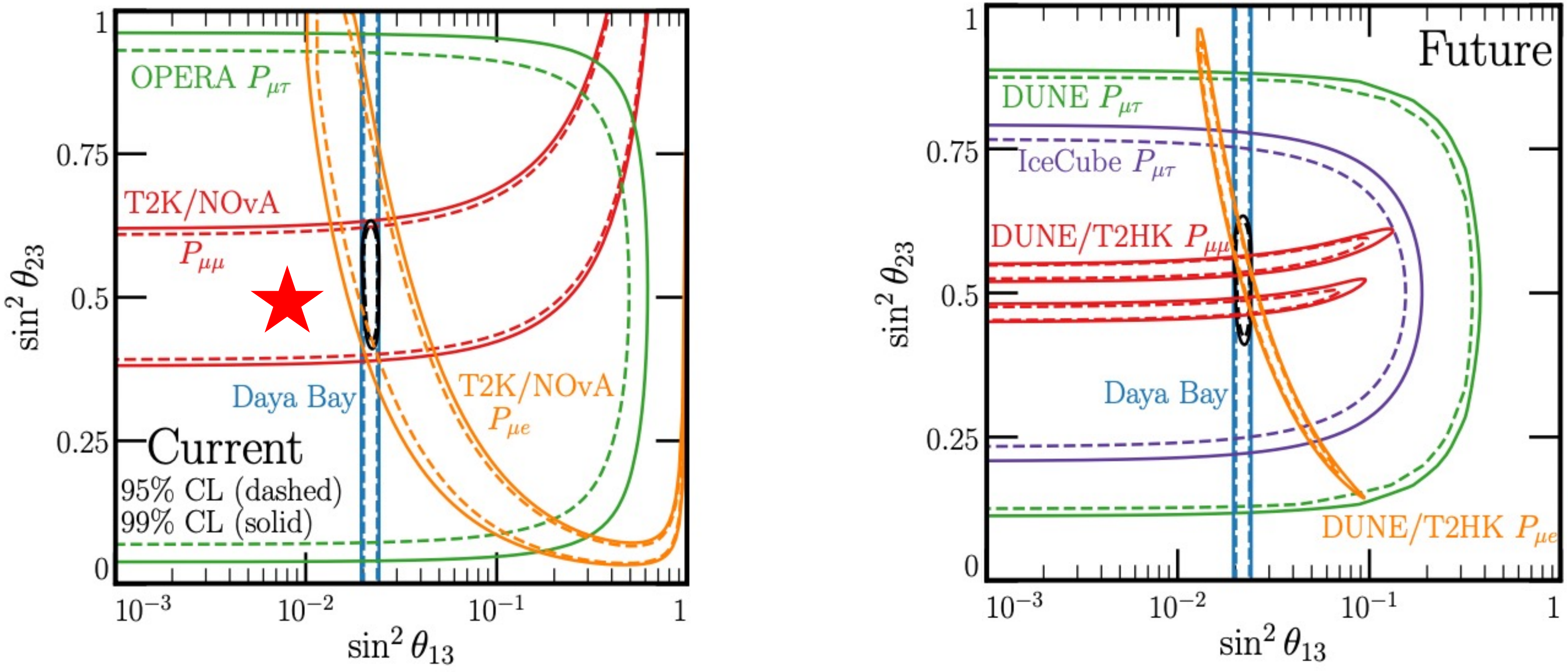
DUNE solar: Capozzi *et al.*, 18

How Well Can We Test Normalizations?

Ellis, Kelly, SL 20a

$\theta_{23} - \theta_{13}$ plane

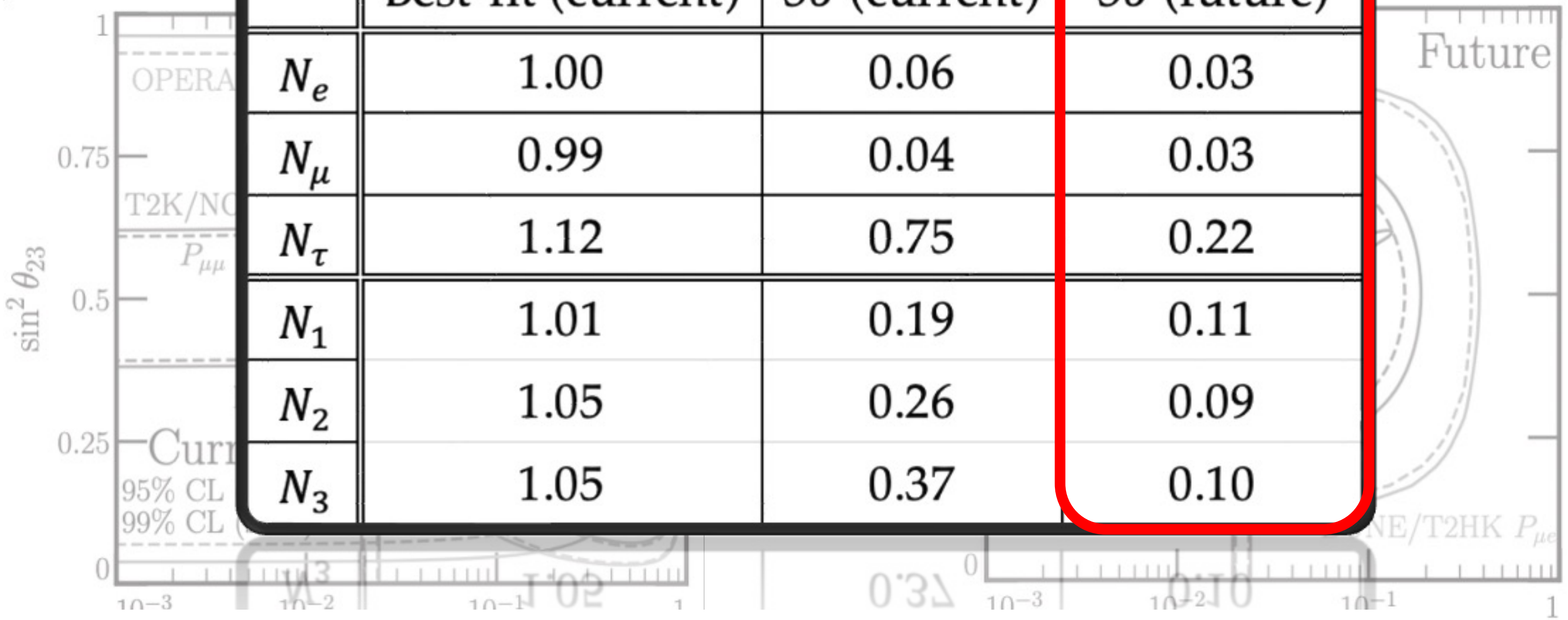
Current precisions | Future sensitivities



How Well Can We Test Normalizations?

Ellis, Kelly, SL 20a

$\theta_{23} - \theta_{13}$ plane



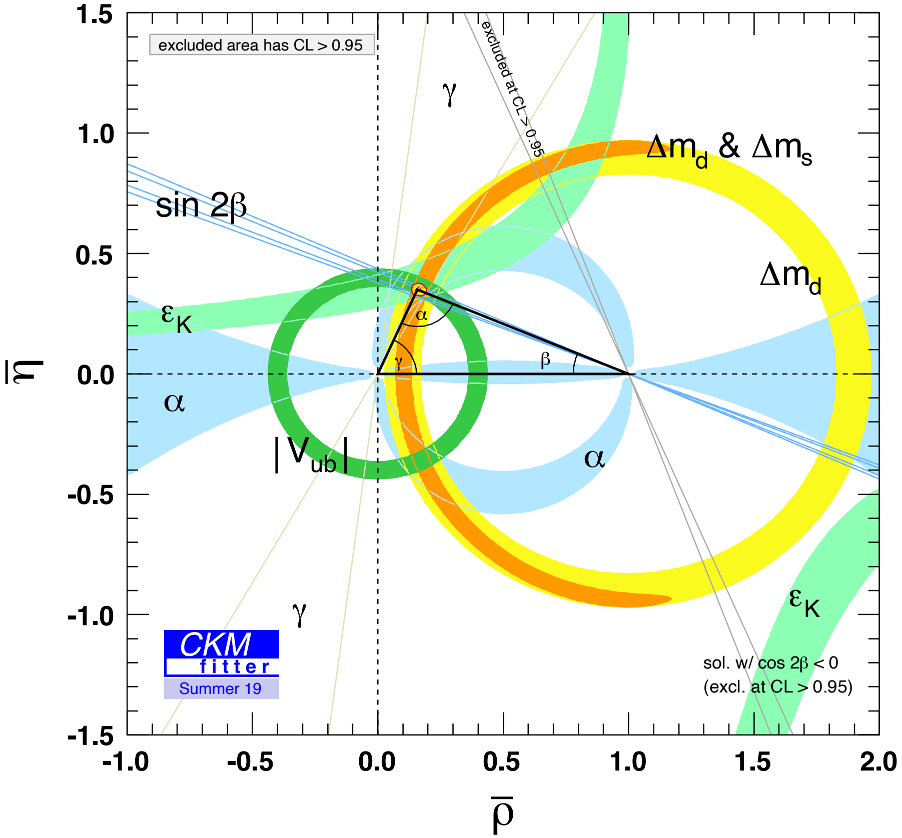
	Best-fit (current)	3σ (current)	3σ (future)
N_e	1.00	0.06	0.03
N_μ	0.99	0.04	0.03
N_τ	1.12	0.75	0.22
N_1	1.01	0.19	0.11
N_2	1.05	0.26	0.09
N_3	1.05	0.37	0.10

A factor of 2—3 improvement, reaches ≤ 10% precisions

How Can We Test Closures?

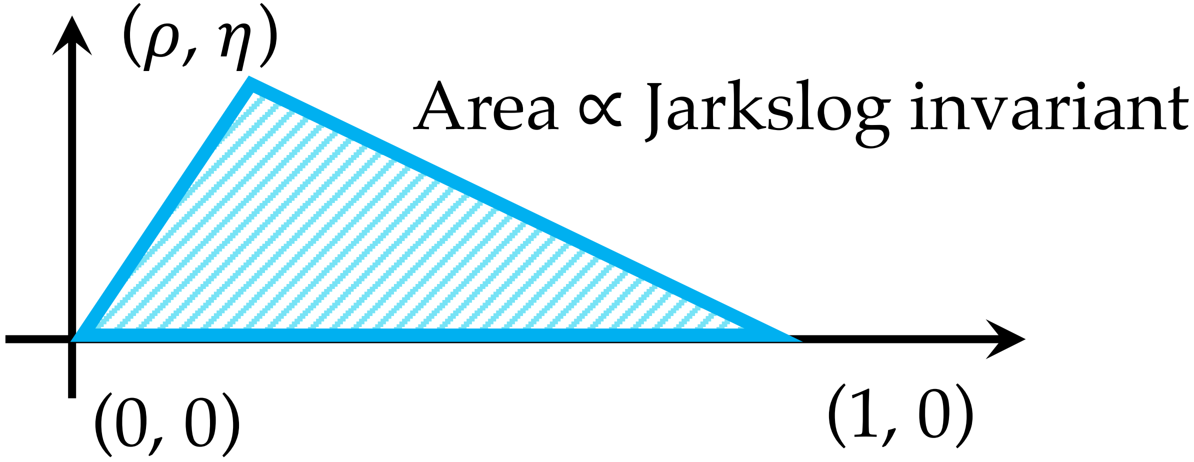
Unitarity triangles

Quark sector



$$U_{e1}U_{\mu 1}^* + U_{e2}U_{\mu 2}^* + U_{e3}U_{\mu 3}^* = 0$$

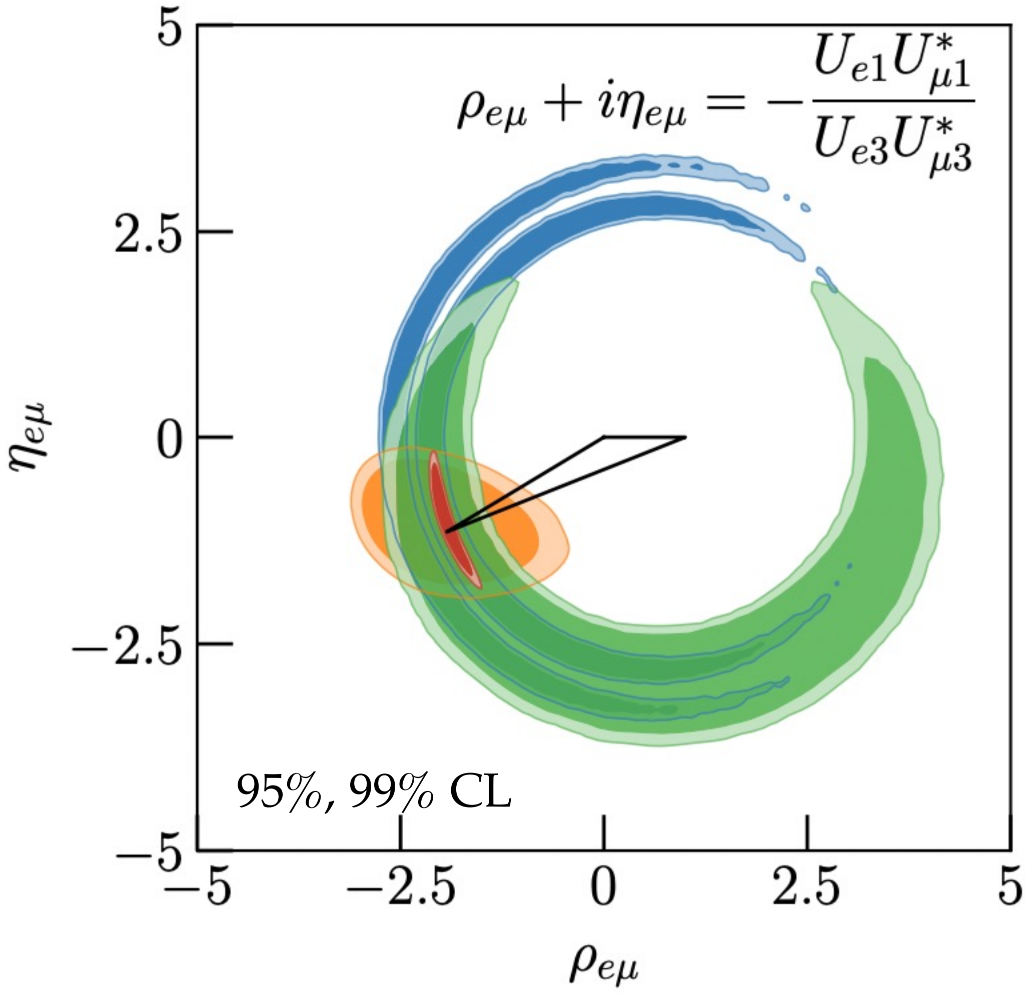
$$\rho_{e\mu} + i\,\eta_{e\mu} \equiv -\frac{U_{e1}U_{\mu 1}^*}{U_{e3}U_{\mu 3}^*}$$



How Well Can We Test Closures?

Ellis, Kelly, SL 20b

Current data
Future data:
- Disappearance
  $\nu_\alpha \to \nu_\alpha$
- Appearance
  $\nu_\alpha \to \nu_\beta$

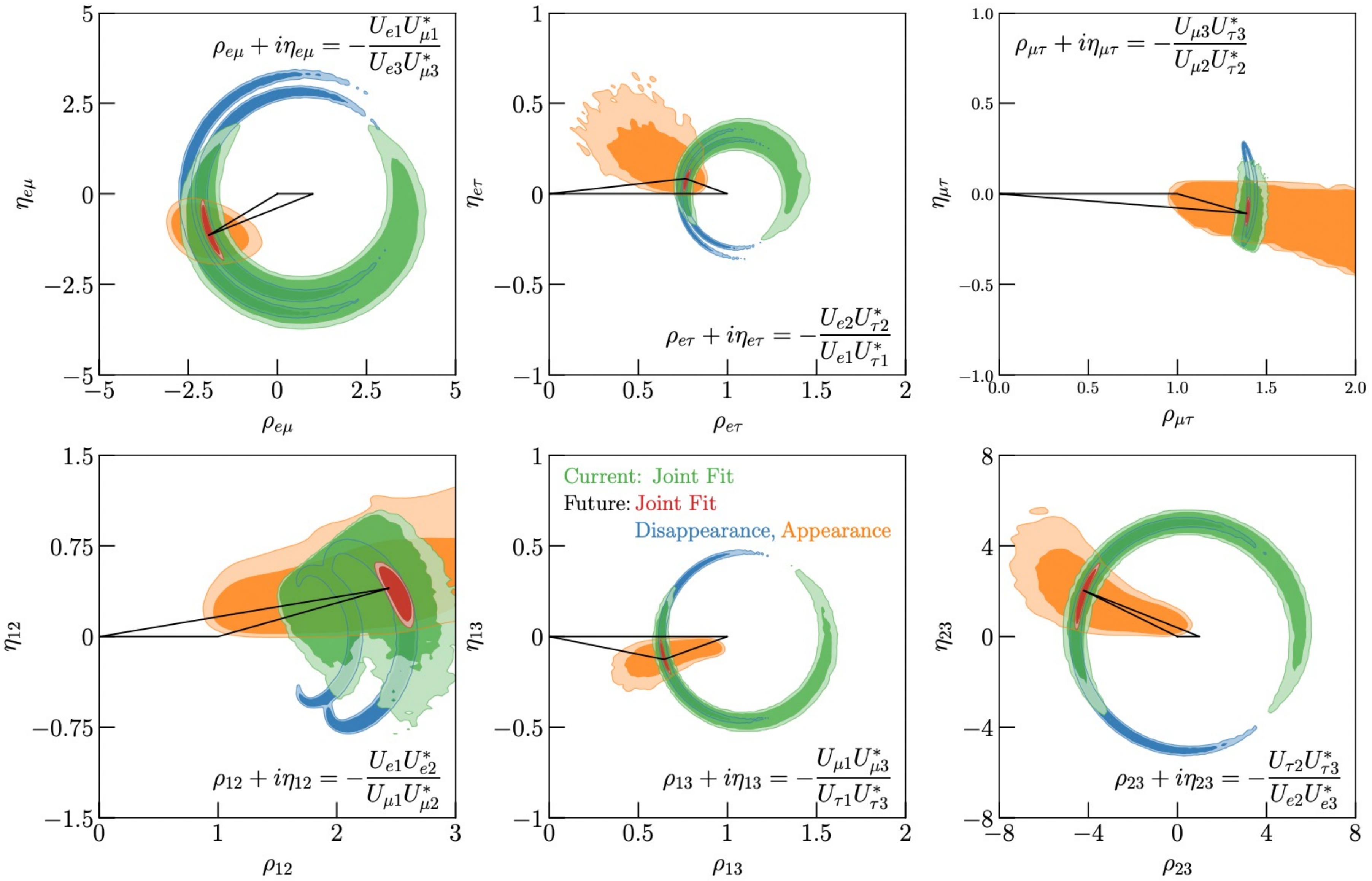


Currently no sensitivity to CPV

Future:
- ➢ Appearance vs. disappearance
- ➢ Only one measurement of δ_{CP}

All Six Unitarity Triangles

Ellis, Kelly, SL 20b



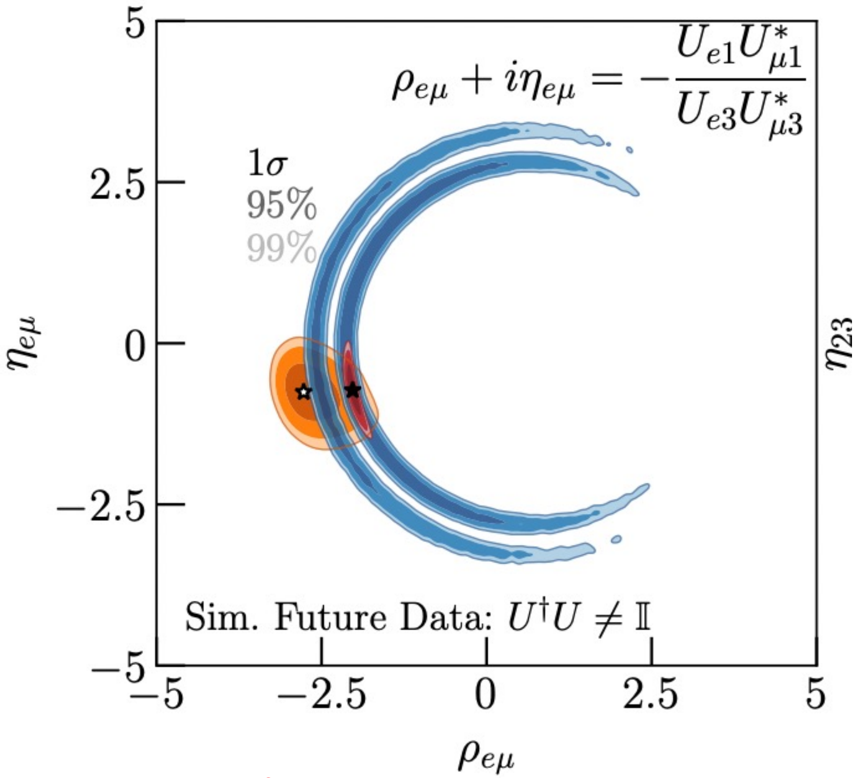
Triangles as Tests of Unitarity

Ellis, Kelly, SL 20b

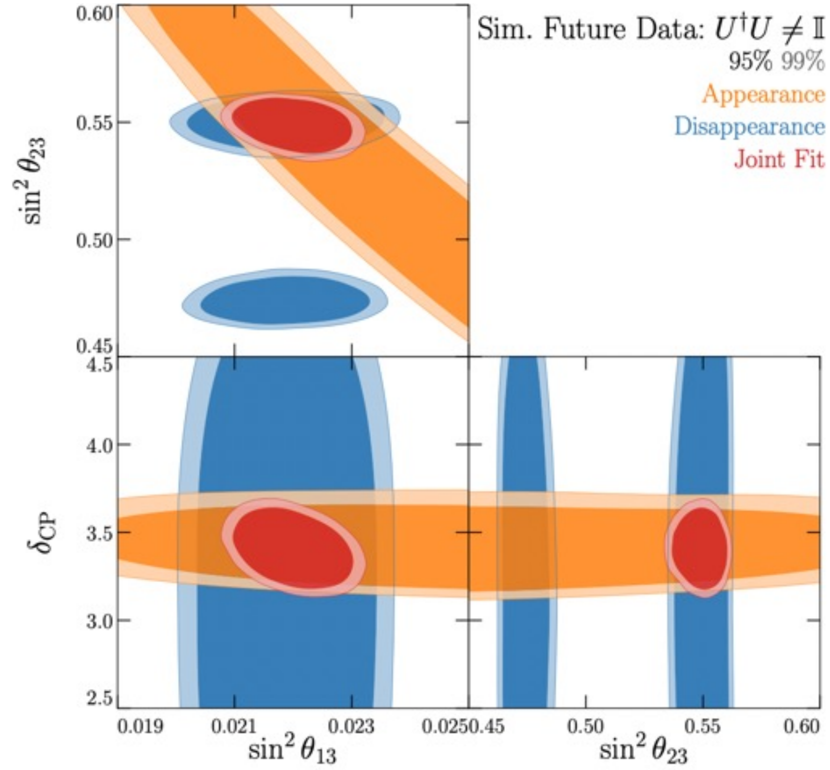
Injected non-unitarity

$$U_{e1}U^*_{\mu1} + U_{e2}U^*_{\mu2} + U_{e3}U^*_{\mu3} = 0.01 + 0.04i$$

Tension in triangle



No tension in standard approach



Advocating for experiments to separately analyze appearance and disappearance Data

How Well Can We Test Closures?

Ellis, Kelly, SL 20a

	Current 3σ Upper Limit	Future 3σ Upper Limit
$\lvert t_{e\mu} \rvert$	3.2×10^{-2}	2.5×10^{-2}
$\lvert t_{e\tau} \rvert$	1.3×10^{-1}	No Improvement
$\lvert t_{\mu\tau} \rvert$	1.6×10^{-2}	No Improvement
$\lvert t_{12} \rvert$	2.5×10^{-1}	1.0×10^{-1}
$\lvert t_{13} \rvert$	3.2×10^{-1}	1.2×10^{-1}
$\lvert t_{23} \rvert$	3.3×10^{-1}	1.1×10^{-1}

A factor of 1—3 improvement but need better tau data!

Conclusions

- In oscillation analysis, unitarity of PMNS matrix is typically assumed
- New physics could show up as unitarity violation
- Standard analysis may not be sensitive to this
- We need more tau data, and different measurements of δ_{CP}