Neutrino Mixing Parameters and Unitarity

Tau Lepton Physics 2023

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Establishing Neutrino Mixing Paradigm
Neutrino Oscillations: the Standard Paradigm

Unknown:
\(\delta_{CP}\), mass hierarchy

A Few %:
\(\theta_{12}, \theta_{13}, \theta_{23}\),
\(\Delta m_{21}^2, \Delta m_{32}^2\)
Neutrino Oscillations: the Standard Paradigm

Modified from Song et al., 21
Neutrino Oscillations: the Standard Paradigm

What about new physics?

Modified from Song et al., 21
Unitarity of Neutrino Mixing

Standard Model: 3 neutrinos

\[ \nu_\alpha(x) = \sum_{k=1}^{3} U_{\alpha k} \nu_k(x) \]

Flavor Mass

3-flavor mixing parameterized by:

\[ U_{\text{PMNS}} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} c_{13} & 0 & -s_{13} e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \]

Atmospheric
\( \mu \rightarrow \tau \)
500 km/GeV

Reactor/Interference
\( \mu \leftrightarrow e \)
500 km/GeV

Solar
\( e \rightarrow e \)
15000 km/GeV

Also Parke, Ross-Lonergan 16
Denton, Gehrlein 21
Unitarity of Neutrino Mixing

Standard Model: 3 neutrinos

\[ \nu_\alpha(x) = \sum_{k=1}^{3} U_{\alpha k} \nu_k(x) \]

Flavor Mass

3-flavor mixing parameterized by:

Sterile neutrino searches – unitarity assumed

\[ U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & U_{e4} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} & U_{\mu4} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} & U_{\tau4} \end{pmatrix} = \begin{pmatrix} 1 & 1 & c_{24} & 1 \\ c_{34} & s_{34} & -s_{24} e^{-i\delta_2} & c_{24} \\ -s_{34} & c_{34} & -s_{24} e^{i\delta_2} & c_{24} \end{pmatrix} \begin{pmatrix} c_{14} & 1 & 1 & s_{14} \\ 1 & 1 & c_{14} \end{pmatrix} \]
This Isn’t the Whole Story…

Neutrinos have masses

\[ L \supset c_v \frac{LHLH}{\Lambda} + \text{h.c.} \]

New physics at scale \( \Lambda \) =>
new states =>

\[ (3 \times 3) U_{\alpha k} \rightarrow (n \times n) U_{\alpha k} \]

E.g., non-standard interactions

\[ L = -2\sqrt{2} G_F \sum_{f',P,\alpha,\beta} \epsilon_{\alpha,\beta}^{f',P} (\bar{\nu}_\alpha \gamma^\mu P_L \nu_\beta)(f \gamma_\mu P f') \]

can be mapped onto non-unitarity

How well can we test non-unitarity in neutrino sector?

Blennow et al., 06
This Isn’t the Whole Story…

Neutrinos have 

\[ L \supset c_v \frac{L H i}{\Lambda} \]

New physics at \new states \Rightarrow

\( (3 \times 3) U_{\alpha k} \rightarrow ( \)

How well can we test non-unitarity in neutrino sector?

Andard interactions

\[ \sum_{\alpha,\beta} \epsilon^f P (\bar{\nu}_\alpha \gamma^\mu P L \nu_\beta) (\bar{f} \gamma_\mu P f') \]

\( \text{led onto non-unitarity} \)

Blennow et al., 06

\[ \text{excluded area has } CL > 0.95 \]
What Is Unitarity?

\[ U^\dagger U = U U^\dagger = \mathbb{I} \]

An over-complete set of constraints:

\[ N_\alpha \equiv \sum_{k=1}^{3} |U_{\alpha k}|^2 = 1 \quad \text{for } \alpha = e, \mu, \tau \]

\[ t_{\alpha \beta} \equiv \sum_{k=1}^{3} U_{\alpha k} U_{\beta k}^* = 0 \]

Row / column normalizations

\[ N_k \equiv \sum_{\alpha = e, \mu, \tau} |U_{\alpha k}|^2 = 1 \]

\[ t_{kl} \equiv \sum_{\alpha = e, \mu, \tau} U_{\alpha k} U_{\alpha l}^* = 0 \]

Closures
How Can We Test Normalizations?

Assuming unitarity, infinite precision

All measurements should cross the same point

Ellis, Kelly, SL 2020a
How Well Can We Test Normalizations?

Ellis, Kelly, SL 20a

\( \theta_{13} - \theta_{12} \) plane

Current precisions

Future sensitivities

DUNE solar: Capozzi et al., 18
How Well Can We Test Normalizations?

\[ \theta_{23} - \theta_{13} \text{ plane} \]

Current precisions

Future sensitivities

Ellis, Kelly, SL 20a

Current

95\% CL (dashed)

99\% CL (solid)

Future

DUNE \( P_{\mu\tau} \)

IceCube \( P_{\mu\tau} \)

DUNE/T2HK \( P_{\mu\mu} \)

DUNE/T2HK \( P_{\mu\tau} \)
### How Well Can We Test Normalizations?

Ellis, Kelly, SL 20a

#### \( \theta_{23} - \theta_{13} \) plane

<table>
<thead>
<tr>
<th></th>
<th>Best-fit (current)</th>
<th>3( \sigma ) (current)</th>
<th>3( \sigma ) (future)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N_e )</td>
<td>1.00</td>
<td>0.06</td>
<td>0.03</td>
</tr>
<tr>
<td>( N_\mu )</td>
<td>0.99</td>
<td>0.04</td>
<td>0.03</td>
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<tr>
<td>( N_\tau )</td>
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<td>( N_1 )</td>
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<td>( N_2 )</td>
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<td>0.26</td>
<td>0.09</td>
</tr>
<tr>
<td>( N_3 )</td>
<td>1.05</td>
<td>0.37</td>
<td>0.10</td>
</tr>
</tbody>
</table>

A factor of 2—3 improvement, reaches \( \leq 10\% \) precisions
How Can We Test Closures?

Unitarity triangles

Quark sector

\[ U_{e1} U_{\mu 1}^* + U_{e2} U_{\mu 2}^* + U_{e3} U_{\mu 3}^* = 0 \]

\[ \rho_{e\mu} + i \eta_{e\mu} \equiv -\frac{U_{e1} U_{\mu 1}^*}{U_{e3} U_{\mu 3}^*} \]

Area \propto \text{Jarkslog invariant}
How Well Can We Test Closures?

Ellis, Kelly, SL 20b

Current data
Future data: Disappearance $\nu_\alpha \rightarrow \nu_\alpha$
Appearance $\nu_\alpha \rightarrow \nu_\beta$

Currently no sensitivity to CPV

Future:
- Appearance vs. disappearance
- Only one measurement of $\delta_{CP}$
All Six Unitarity Triangles
Triangles as Tests of Unitarity

Injected non-unitarity

$$U_{e1}U_{\mu1}^* + U_{e2}U_{\mu2}^* + U_{e3}U_{\mu3}^* = 0.01 + 0.04i$$

Tension in triangle

No tension in standard approach

Advocating for experiments to separately analyze appearance and disappearance Data
How Well Can We Test Closures?

A factor of 1—3 improvement but need better tau data!
Conclusions

- In oscillation analysis, unitarity of PMNS matrix is typically assumed
- New physics could show up as unitarity violation
- Standard analysis may not be sensitive to this
- We need more tau data, and different measurements of $\delta_{CP}$