Measurement of the Muon Magnetic Anomaly to 0.20 ppm by the Muon $g - 2$ experiment at Fermilab

LORENZO COTROZZI
ON BEHALF OF THE MUON $g - 2$ COLLABORATION
UNIVERSITY OF PISA, INFN PISA

17TH INTERNATIONAL WORKSHOP ON TAU LEPTON PHYSICS
LOUISVILLE, KY – 12/05/2023

FERMILAB-SLIDES-23-397-V
The anomalous magnetic moment

- Particle with spin in a magnetic $B$-field: $\vec{\mu} \equiv g \frac{e}{2m} \vec{S}$

- Torque in $B$-field: $\tau = \vec{\mu} \times \vec{B}$  
  Energy in $B$-field: $U = -\vec{\mu} \cdot \vec{B}$

- Classical mechanics prediction: $g = 1$

- Dirac’s prediction for spin-$1/2$ elementary particles: $g = 2$

- Radiative corrections in Quantum Field Theories: $g \neq 2$

- Kusch and Foley’s measurement, Schwinger’s prediction (1948, electron $g_e - 2$):

  $$\frac{g_e - 2}{2} \equiv a_e \approx 0.00116$$

1st order universal QED term: $\frac{\alpha}{2\pi}$

\[ g^2 \]
Outline

Theory Initiative White Paper 2020

2021 Run-1 results at Fermilab

Run-2/3 upgrades and 2023 results

Puzzles, prospects and outlook

The anomalous magnetic moment of the muon in the Standard Model

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12/05/2023 – TAU2023 – MUON $g-2$
Muon $g - 2$ before 2021

- Previous experiment at BNL (early 2000s): discrepancy between 0.54 ppm experimental uncertainty and 0.55 ppm theoretical uncertainty (1 ppm = part per million)


- HVP in WP2020 based on $e^+ e^-$ hadronic cross section data

- $3.7\sigma$ discrepancy between BNL and WP2020 prediction: $a^B_{\mu} - a^W_{\mu} = 279(76) \times 10^{-11}$
Dispersive method \((e^+ e^-)\) for HVP

- Optical theorem: \(\text{Im} \frac{\gamma}{\gamma_{\text{had}}} \leftrightarrow \left| \frac{\gamma}{\gamma_{\text{had}}} \right|^2\)
  \[
a_H^{\text{HVP-LO}} = \frac{\alpha^2}{3\pi^2} \int_{m_{\pi}^2}^{\infty} ds \frac{K(s)}{s} R(s)
\]

- \(R(s)\) is data-driven hadronic R-Ratio

- 20+ years of \(e^+ e^-\) experiments: CMD-2, SND, KLOE, BaBar, BESIII, CLEO-C included in WP2020

- See talk today by K. Maltman

\[
\pi^+ \pi^- \quad 73\%
\]


Lattice QCD method for HVP

- Ab-initio calculation of HVP from first principles, approximation of discrete space-time
- After WP2020: BMW collaboration published first sub-percent uncertainty
- See talk on Friday by S. Kuberski

Window observables: excellent agreement in the intermediate window

G. Colangelo et al, arxiv:2203.15810 (2022)
Muon $g - 2$ result in 2021 (Run-1, 2018 data)

\[ a_{\mu}^{\text{Exp}} = 116,592,061(41) \times 10^{-11} \text{ (0.35 ppm)} \]

**Run-1 uncertainties**

<table>
<thead>
<tr>
<th></th>
<th>Parts per billion [ppb]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Statistical</td>
<td>434</td>
</tr>
<tr>
<td>Systematic</td>
<td>157</td>
</tr>
<tr>
<td>Total</td>
<td>462</td>
</tr>
</tbody>
</table>

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12/05/2023 – TAU2023 – MUON $g - 2$
• 8 GeV protons collide on target and produce pions

• Pions decay into muons along $\sim 2$ km in Delivery Ring

• Muons are injected in 7 m radius ring
Anomalous precession in B-field

\[ \vec{\omega}_a = \vec{\omega}_s - \vec{\omega}_c \neq 0 \]: spin precesses with respect to momentum

\[ \vec{\omega}_a = -\frac{e}{mc} \left[ a_\mu \vec{B} - \left( a_\mu - \frac{1}{\gamma^2 - 1} \right) \vec{\beta} \times \vec{E} - a_\mu \frac{\gamma}{\gamma + 1} (\vec{\beta} \cdot \vec{B}) \vec{\beta} \right] \]

- \( \vec{E} \) from electrostatic quadrupoles
- Relativistic boost of \( \gamma = 29.3 \)
- 3.1 GeV/c «magic momentum»
- Red term is cancelled

\[ \vec{\omega}_a = -\frac{e}{mc} a_\mu \vec{B} \]
Principle of $\omega_\alpha$ measurement

1. Weak decay violates parity
2. $e^+$ spectrum depends on $\omega_\alpha$ phase
3. Count high-energy $e^+$ over time (muons have lifetime of $\sim 64\mu$s, we measure for $700\mu$s)
Master formula for $a_\mu$

$$a_\mu = \frac{\omega_a}{\omega_p} \times \frac{\mu_p'(T_r) \mu_e(H) m_\mu g_e}{\mu_e(H) \mu_e m_e 2}$$

External factors, known to 25 ppb

Make spin precess slower (E-field, vertical motion)

$$\omega_a \omega_p = \omega_a m \omega_p m$$

corrections for effects that

Measured, blinded ratio

Make phase change within 700μs

Induce transient magnetic fields
**ω_α** analysis in a nutshell

1. Count positrons and subtract pileup

   \[ N_\text{obs} = \frac{1}{2} \left( 1 + A \cdot A_{\text{CBO}}(t) \cos(\omega_{\text{CBO}}t + \phi_\text{CBO}(t)) \right) \cdot N_{\text{CBO}}(t) \cdot N_{\text{up}}(t) \cdot N_{\text{low}}(t) \cdot N_{\text{pileup}}(t) \]

   \[ A_{\text{CBO}}(t) = 1 + A_0 \cos(\omega_{\text{CBO}}t + \phi_0) e^{-t / \tau_{\text{CBO}}} \]

   \[ N_{\text{CBO}}(t) = 1 + A_{\text{CBO}} \cos(\omega_{\text{CBO}}t + \phi_{\text{CBO}}) e^{-t / \tau_{\text{CBO}}} \]

   \[ N_{\text{up}}(t) = 1 + A_{\text{up}} \cos(\omega_{\text{up}}t + \phi_{\text{up}}) e^{-t / \tau_{\text{up}}} \]

   \[ N_{\text{low}}(t) = 1 + A_{\text{low}} \cos(\omega_{\text{low}}t + \phi_{\text{low}}) e^{-t / \tau_{\text{low}}} \]

2. Fit wiggle plots with many parameters to account for beam dynamics effects
ω_p (field) analysis in a nutshell

17 petroleum jelly NMR probes

2D field maps (~8000 points)

Azimuthally-Averaged Variation < 1 ppm

Nuclear Magnetic Resonance (NMR) probes: placed on trolley for special runs, every 2 or 3 days between muon fills

378 fixed NMR probes monitor field during muon storage at 72 azimuthal locations
Run-2/3 Result: FNAL + BNL Combination

2006: 540 ppb

April 7th, 2021   FNAL Run-1
August 10th, 2023  FNAL Run-2/3
(203 ppb)

2023: 190 ppb

- Hardware improvements
- More studies (CBO systematics, pileup reconstruction, ...)

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Run-2/3 hardware improvements

• Fixed 2 out of 32 quadrupole resistors that strongly affected beam dynamics

• Added thermal blanket to ring

• Replaced kicker cables for optimal kick
Coherent Betatron Oscillations (CBO)

CBO systematics: more studies

More statistics → more models tested → reduced systematic

From Run-1 to Run-2/3: fixed bad resistors + Stronger kick → more stable beam dynamics

Run-1
Run-2/3

CBO dominated Run-1 systematics (38 ppb). Now reduced to 21 ppb!
Improvement of statistical uncertainty

On 27 February 2023: proposal Goal of x21 BNL datasets!

Muon g-2 (FNAL)

- Run 1
- Run 2/3
- Combined Run 1 + Run 2/3
- Run 5
- Run 6 ended on 9th July 2023

<table>
<thead>
<tr>
<th>Dataset</th>
<th>Statistical Error [ppb]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Run 1</td>
<td>434</td>
</tr>
<tr>
<td>Run 2/3</td>
<td>201</td>
</tr>
<tr>
<td>Combined Run 1 + Run 2/3</td>
<td>185</td>
</tr>
<tr>
<td>Expected total from Run 1 to Run 6</td>
<td>$\leq 100$</td>
</tr>
</tbody>
</table>

We expect to complete analysis by 2025
Updates since WP2020

Disclaimer from A. Keshavarzi’s Lattice 2023 talk:

IMPORTANT: THIS PLOT IS VERY ROUGH!
- TI White Paper result has been substituted by CMD-3 only for 0.33 → 1.0 GeV.
- The NLO HVP has not been updated.
- It is purely for demonstration purposes → should not be taken as final.
Future experiments: JPARC and MUonE

MUonE (see next talk by R. Pilato):
- Extract leading-order HVP from hadronic running of $\alpha_{\text{QED}}$ in the space-like region
- Measure directly muon-electron elastic scattering differential cross section shape
- Goal of 0.3% uncertainty statistical, similar systematic; comparable with data-driven HVP

J-PARC Muon g-2/EDM (see next talk by T. Mibe and picture of the apparatus above)
Summary and conclusions

❖ Improvements from Run-1 (2021 result) to Run-2/3 (2023 result):
  • **Statistic**: 4.7 times the Run-1 events → **2.2 smaller**
  • **Systematic**: many hardware and software improvements, like new reconstruction, more studies on beam dynamics → **2.2 times smaller**

❖ New experimental average has **unprecedented precision of 190 ppb**

❖ **Puzzles** in the muon $g - 2$ theory: a firm comparison between theory and experiment cannot be established (see [https://muon-gm2-theory.illinois.edu/](https://muon-gm2-theory.illinois.edu/))

❖ Talk on Friday by B. Kiburg: «Wishlist of G-2 results for Tau2025»
THANK YOU FOR YOUR ATTENTION!

ANY QUESTIONS?

LORENZO COTROZZI – LORENZO.COTROZZI@PHD.UNIPI.IT

UNIVERSITY OF PISA, INFN PISA

July 2023 collaboration meeting @ Liverpool, UK
BACKUP SLIDES
Extracting $a_\mu$

FNAL Projected Errors:
140 ppb (total) = 
100 (stat) $\oplus$ 100 (syst)

$$a_\mu = \frac{\omega_a}{\tilde{\omega}_p} \frac{\mu_p}{\mu_e} \frac{m_\mu}{m_e} \frac{g_e}{2}$$

$2017$ CODATA

-0.001 519 270 380(5) [3 ppb] Hydrogen Maser

206.768 2826(46) [22 ppb] Muonium Hyperfine

-2.002 319 304 361 82(52) [0.26 ppt] Electron $g$-2/QED
Muon g-2 detectors

24 calorimeters
- Each made of 6x9 PbF$_2$ crystals read out by large-area SiPMs
- 1296 channels individually calibrated by state-of-the-art laser system

2 straw trackers
- Each consisting of 8 modules
- Gas filled straws

More auxiliary detectors for dedicated runs, muon beam profile, ...
$$\Lambda(t) = 1 - k_{LM} \cdot J(t)$$

$$J(t) = \frac{\int_{t_0}^{t} L(t')e^{t'/\gamma\tau} dt'}{\int_{t_0}^{t_{end}} L(t)e^{t'/\gamma\tau} dt}$$
28-Parameter fit and FFT of residuals

\[ N(t) = N_0 \cdot e^{-\frac{t}{\gamma \tau}} \cdot [1 + A \cdot A_{BD}(t) \cdot \cos(\omega_a t + \phi_{BD}(t) - \phi_0)] \]
\[ \cdot N_{CBO}(t) \cdot N_{VW}(t) \cdot N_y(t) \cdot N_{2CBO}(t) \cdot \Lambda(t) \cdot N_{CHOVW}(t) \]

\[ N_{CBO}(t) = 1 + A_{CBO} \cdot \cos(\omega_{CBO}(t) \cdot t - \phi_{CBO}) \cdot D_{CBO}(t) \]

\[ D_{CBO}(t) = e^{-\frac{t}{\tau_{CBO} + C_{CBO}}} \]

\[ \Lambda(t) = 1 - k_{LM} \cdot J(t) \]

\[ J(t) = \frac{\int_{t_0}^{t} L(t')e^{t'/\gamma \tau} dt'}{\int_{t_0}^{t_{\text{end}}} L(t)e^{t'/\gamma \tau} dt'} \]

- No beam dynamic terms
- Full fit function

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Two-level blinding

• Hardware: secret clock frequency, unknown to collaboration

• Software: avoid biases among analyzers from 7 different groups
Software unblinding for Run-2/3

Common blinding among the 6 analyses
### Detailed systematic effects on $\omega_\alpha$

<table>
<thead>
<tr>
<th>Source</th>
<th>E989 goal [ppb]</th>
<th>Run-1 [ppb]</th>
<th>Run-2/3 [ppb]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gain</td>
<td>20</td>
<td>8</td>
<td>5</td>
</tr>
<tr>
<td>Pileup</td>
<td>40</td>
<td>35</td>
<td>7</td>
</tr>
<tr>
<td>CBO</td>
<td>30</td>
<td>38</td>
<td>21</td>
</tr>
<tr>
<td>Total (including all contributions)</td>
<td>70</td>
<td>157</td>
<td>70</td>
</tr>
</tbody>
</table>
Reconstruction improvements

Seed-and-propagation algorithms that take into account time and energy resolution of detectors.

Reduced pileup in un-physical region (decay $e^+$ with more energy than muons) by $\sim 2$.

Pileup dominated Run-1 systematics (35 ppb). Now reduced to 7 ppb!
Energy methods to build wiggle plot

- **T-Method:** count high-energy (>1.7 GeV) positrons over time
- **A-Method:** positron rate weighted by $A(E)$ i.e. the asymmetry for each energy bin
  - More positrons (>1.0 GeV)
  - Maximize statistical power

A-Method was the standard in Run-1
# Beam dynamics uncertainties

<table>
<thead>
<tr>
<th>Systematics</th>
<th>Description</th>
<th>Run-1 [ppb]</th>
<th>Run-2/3 [ppb]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_e )</td>
<td>Electric field effect on ( \omega_a ), minimized by «magic momentum» of 3.1 Gev/c. Improved analysis of momentum spread vs bunch time, and complimentary tracker information to reduce uncertainty</td>
<td>53</td>
<td>25</td>
</tr>
<tr>
<td>( C_p )</td>
<td>Caused by vertical betatron oscillations</td>
<td>13</td>
<td>10</td>
</tr>
<tr>
<td>( C_{pa} )</td>
<td>Correlation between muon decay position and ensemble-averaged initial phase. Mostly reduced because we fixed the damaged ESQ resistors that enhanced it in Run-1</td>
<td>75</td>
<td>15</td>
</tr>
<tr>
<td>( C_{dd} )</td>
<td>Higher-momentum muons have longer boosted lifetime</td>
<td>-</td>
<td>17</td>
</tr>
<tr>
<td>( C_{ml} )</td>
<td>Muons that are scattered away from the ring change the momentum distribution. Also reduced by fixing resistors</td>
<td>5</td>
<td>3</td>
</tr>
</tbody>
</table>

\( C_{dd} \) was not evaluated in Run-1 but it was known to be significantly smaller than other evaluated beam dynamics systematics.
# Field uncertainties

<table>
<thead>
<tr>
<th>Systematics</th>
<th>Description</th>
<th>Run-1 [ppb]</th>
<th>Run-2/3 [ppb]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_q$</td>
<td>Vibrations caused by Quadrupole pulsing. Better mapping in azimuth in Run-2/3</td>
<td>92</td>
<td>20</td>
</tr>
<tr>
<td>$B_k$</td>
<td>Kicker-induced Eddy currents. Improved magnetometer measurements thanks to more stable setup that greatly reduced vibrations</td>
<td>37</td>
<td>13</td>
</tr>
<tr>
<td>Trolley calibration, tracking, muon weighting</td>
<td>Improvements in analysis and more trolley measurements. Stabilized temperature in the hall to reduce diurnal variations in the field</td>
<td>56</td>
<td>46</td>
</tr>
</tbody>
</table>
Field shimming: 2015-2016 campaign

Before

72 steel poles to increase homogeneity
864 wedges for dipole/quadrupole fields
144 edge shims for quadrupole/sextupole fields
48 iron top hats + 8000 surface iron foils to achieve desired uniformity

After

Dipole Field

\( g - 2 \)